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CONSTRUCTION OF A MODEL OF THE VENUS SURFACE AND ITS USE IN PROCESSING RADAR OBSERVATIONS

V.A. Borodin, V.A. Stepan'yants and V.A. Shishov

Translation of "Postroyeniye Modeli Poverkhnosti Venery i Yeye Ispol'zovaniye pri Obrabotke Radiolokatsionnykh Nablyudeniy," Academy of Sciences USSR, Institute of Applied Mathematics imeni M.V. Keldysh, Moscow, Preprint No. 20, 1983, pp. 1-19.

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16. Abstract

An algorithm is described for constructing the model of the Venus surface as an expansion in spherical functions. The relief expansion coefficients were obtained up to the coefficient S_{99}. The surface picture representation is given according to this expansion. The surface model constructed was used for processing radar observations. The paper contains data showing that the use of the surface model allows improved agreement between the design and measured values of radar ranges.

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ANNOTATION

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Key words and phrases: Venus, surface relief, spherical functions, radar observations.

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CONSTRUCTION OF A MODEL OF THE VENUS SURFACE AND ITS USE IN PROCESSING RADAR OBSERVATIONS

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Introduction

Radar ranging of Venus, Mars and Mercury has been successfully conducted since the beginning of the 1960s. Due to improvement in equipment, the accuracy of radar measurements is continually increasing. The errors of the most accurate measurements of the delay time of a signal reflected from the surface of the planet is not over 2-3 microseconds, which corresponds to a few hundred meters in distance. Radar measurements presently are widely used for the development of highly precise theories of motion of the solar system planets. They permit determination of the distance from the nearest section of the surface of the planet to a ground measuring point. To determine the location of the center of mass of a planet relative to the observation point, it is extremely important to take account of its topography. Thus, irregularities of the surface of Venus reach 5-10 km in altitude, which significantly affects the accuracy of determination of the distance between the centers of mass of the earth and Venus. /4*

In the work of Ye.V. Pit'yeva [4], based on a hypsometric chart of the surface of Venus for approximation of the relief of the planet, an altitude interpolation grid has been constructed. Data on the relief were successfully used for reduction of the averaged values of the radar observations.

It is advisable to present information on the surface relief of the planet in a form which is convenient for computer processing of radar observations. The volume of this information is limited by the place assigned to it in the computer memory. It should also be con-

*Numbers in the margin indicate pagination in the foreign text.

sidered that a radar signal is reflected from a quite extensive region of the planet (approximately 100 km in diameter). Therefore, it is sufficient to know the average relief characteristics in processing radar measurements.

With this situation taken into account, it is most convenient to present the description of the model of the planet surface in the form of an expansion in spherical functions. Such an expansion was obtained for Mars in [6]. The task of approximating the surface relief of Venus in a similar manner is carried out in this work.

The most accurate data on the surface relief of Venus was obtained with the aid of the American Pioneer-Venus-1 artificial Venus satellite, which was injected into a highly eccentric orbit with the following characteristics: /5

Rotation period	24 hours
Semimajor axis of orbit	39,500 km
Eccentricity	0.84
Distance to pericenter	150 km
Distance to apocenter	66,600 km
Pericenter longitude	17°
Inclination of orbit	104°.

The pericenter longitude and inclination of the orbit are reduced to the planetographic coordinate system, a description of which is in Section 1.

The equipment installed aboard the satellite permitted determination of the distance from the spacecraft to the planet surface. As a result of processing altimeter measurements, a color topographic map of the surface of Venus was obtained [5].

The following conclusions were drawn from analysis of the experiment conducted:

the distances of all points of the surface of Venus at which measurements were made from the center of mass of Venus is in the range from 6049 km to 6062 km;

approximately 5% of the territory investigated is elevated more than 2 km above the average radius sphere (6051.5±0.1 km);

approximately 60% of the territory investigated is within ±0.5 km of the mean level.

Accuracy of presentation of the relief was 100-150 km horizontally (linear resolution) and 200 m in altitude. The distances from the center of mass of Venus to different points on the surface of the planet can be recovered from the map to within the color gradations. This accuracy is 0.5 km in the 6049.5-6056 km distance range and 1 km in the 6056-6062 km range.

1. Coordinate System

The motions of Venus and other planets of the solar system are described in the geoequatorial coordinate system of the 1950.0 epoch, $(XYZ)_{50.0}$. /6

For tie in of surface details of Venus, the aphroditographic coordinate system $X_B Y_B Z_B$ is used (see Fig. 1). In this system, the Z_B axis is directed to the north pole of Venus. The X_B axis is rigidly tied to the surface of Venus, and it is in the zero meridian plane. The Y_B axis supplements the coordinate system to the right.

Auxiliary planetoequatorial coordinate system $X_B^e Y_B^e Z_B^e$ also is introduced. The Z_B^e axis of this system coincides with the Z_B axis of the $X_B Y_B Z_B$ coordinate system. The X_B^e axis is directed to the point of intersection of the equators of the earth and Venus. The Y_B^e axis supplements the system to the right.

The transition matrix from the geoequatorial $(XYZ)_{50.0}$ system to the aphroditographic $X_BY_BZ_B$ system

$$P=P_2 \cdot P_1$$

is expressed by the product of transition matrix P_2 from the $X_B^eY_B^eZ_B^e$ coordinate system to the $X_BY_BZ_B$ coordinate system and transition matrix P_1 from the $(XYZ)_{50.0}$ coordinate system to the $X_B^eY_B^eZ_B^e$ coordinate system.

The elements of matrix P_1 are determined by the equations [3]

$$\begin{aligned}
p_{11} &= \cos\Omega_B \\
p_{12} &= \sin\Omega_B \\
p_{13} &= 0 \\
p_{21} &= -\sin\Omega_B \cos i_B \\
p_{22} &= \cos\Omega_B \cos i_B \\
p_{23} &= \sin i_B \\
p_{31} &= \sin\Omega_B \sin i_B \\
p_{32} &= -\cos\Omega_B \sin i_B \\
p_{33} &= \cos i_B ,
\end{aligned}$$

where $\Omega_B=\alpha_0+90°$ is the angular distance from the spring equinox of the earth (γ) to the ascending node of the equator of Venus on the equator of the earth; $i_B=90°-\delta_0$ is the relative inclination of the equatorial planes of Venus and the earth; $\alpha_0=272.8°$, $\delta_0=67.2°$ are the coordinates of the North Pole of Venus [7]. /7

The matrix

$$P_2 = \begin{pmatrix} \sin\omega & \cos\omega & 0 \\ -\cos\omega & \sin\omega & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

gives the rotation from the $X_B^eY_B^eZ_B^e$ coordinate system to the $X_BY_BZ_B$ coordinate system by the angle

$$\omega=213.63° - 1.4814205° \cdot d$$

between the point of intersection of the equators of the earth and Venus and the zero meridian of Venus [7]. In this equation, d is the number of Julian ephemeris days which have elapsed from the start of the 1950.0 epoch to the current moment.

2. Representation of Planet Relief in the Form of Expansion in Spherical Functions

The shape of the surface of the planet can be represented in the form of an expansion in spherical functions in the following manner [6]

$$R_i = R_0\left\{1+\sum_{n=1}^{M'}\sum_{m=0}^{n}\begin{bmatrix}C_{nm}\\ S_{nm}\end{bmatrix}^T \bar{P}_n^m(\sin\varphi_i)\begin{bmatrix}\cos m\lambda_i\\ \sin m\lambda_i\end{bmatrix}\right\}, \quad (1)$$

where R_i is the distance from the center of mass of the planet to points of the surface with angular coordinates ϕ_i, λ_i, R_0 is the mean radius of the planet, C_{nm}, S_{nm} are coefficients of expansion, M' is the maximum power of the Legendre polynomials used in the expansion: /8

$$\bar{P}_n^m(x) = N_n^m P_n^m(x), \quad (2)$$

$$N_n^m = \left[(2-\delta_{m0})(2n+1)\frac{(n-m)!}{(n+m)!}\right]^{1/2}, \quad (3)$$

$$P_n^m(x) = \frac{(1-x^2)^{m/2}}{2^n\cdot n!}\,\frac{d^{n+m}(x^2-1)^n}{dx^{n+m}}; \quad (4)$$

δ_{mo} is the Kroneker symbol.

Coeficients of expansion C_{nm}, S_{nm} should be selected so as to ensure sufficiently accurate representation of the distances from the center of mass of the planet to its surface. These values were obtained by processing the altimeter data of the Venus satellite, and it is used as the measurement material in determination of coefficients C_{nm}, S_{nm}. In this case, the number of measurements is considerably greater than the number of coefficients of expansion. Thus, we arrive at the statistical problem of determination of the parameters of the excess composition of the observations, which is reduced to determination of the minimum of the functional [2]

$$\Phi(\bar{Q}) = \sum_{i=1}^{N} p_i\left[R_i^{meas} - R_i(\bar{Q})\right]^2, \quad (5)$$

were $\bar{Q}(q_1, q_2, \ldots, q_m)$ is the vector of the refined coefficients of expansion, R_i^{meas} are the measured distances from the center of mass of Venus to its surface, $q_1=C_{10}$, $q_2=C_{11}$, $q_3=S_{11}$, $q_4=C_{20}$, $q_5=C_{21}$, . . .,

$q_M = S_{M'M'}$, $M = M'(M'+2)$ is the number of parameters refined, N is the number of observations, $p_i = I/\sigma_i^2$ is the weight of the i-th observation, β_i is the a priori observation accuracy, $R_i(\overline{Q})$ are calculated by Eq. (1)-(4).

A necessary condition of the minimum of functional (5) is satis- /9
faction of the following relationships:

$$\sum_{i=1}^{N} p_i \left(R_i^{\text{изм}} - R_i(\overline{Q})\right) \frac{\partial R_i}{\partial q_k} = 0 \qquad (k = I, 2, \ldots, M) \tag{6}$$

On consideration that function $R_i(\overline{Q})$ is linear, system of Eq. (6) can be written in matrix form

$$W \cdot \overline{Q} = \overline{B}, \tag{7}$$

where

$$W_{kl} = R_0^2 \cdot \sum_{i=1}^{N} p_i \Pi_k^i \Pi_l^i, \quad B_l = R_0 \cdot \sum_{i=1}^{N} p_i \left(R_i^{\text{изм}} - R_0\right) \Pi_l^i.$$

$$\Pi_1^i = \overline{P}_1^0(\sin\varphi_i)\cdot\cos(0\cdot\lambda_i), \quad \Pi_2^i = \overline{P}_1^1(\sin\varphi_i)\cdot\cos(I\cdot\lambda_i), \quad \Pi_3^i = \overline{P}_1^1(\sin\varphi_i)\sin(I\cdot\lambda_i), \tag{8}$$

$$\Pi_4^i = \overline{P}_2^0(\sin\varphi_i)\cdot\cos(0\cdot\lambda_i), \Pi_5^i = \overline{P}_2^1(\sin\varphi_i)\cos(I\cdot\lambda_i), \ldots, \Pi_M^i = \overline{P}_{M'}^{M'}(\sin\varphi_i)\sin(M'\cdot\lambda_i).$$

The solution of system of Eq. (7)

$$\overline{Q} = W^{-1} \cdot \overline{B}$$

is the vector of the desired coefficients of expansion.

The values of the joined Legendre functions are calculated by means of the recurrent relationships [1]

$$(2n+I)P_n^m(x) = (n-m+I)P_{n+1}^m(x) + (n+m)P_{n-1}^m(x), \tag{9}$$

$$P_n^{m+2}(x) + 2(m+I)\frac{x}{\sqrt{1-x^2}}P_n^{m+1}(x) + (n-m)(n+m+I)P_n^m(x) = 0. \tag{10}$$

In this case, the explicit expressions of joined functions $\overline{P}_0^0(x) = I$, $\overline{P}_1^0(x) = \sqrt{3}\cdot x$, $\overline{P}_1^1(x) = \sqrt{6}\sqrt{I-x^2}$, $\overline{P}_2^1(x) = \sqrt{I5}\cdot x\sqrt{I-x^2}$ are obtained from Eq. (2)-(4). The values of $\overline{P}_\ell^k(x)$ are calculated as a result of successive use of Eq. (9) and (10).

3. Numerical Results

The algorithm for determination of the coefficients of expansion /10
were in the form of a program in FORTRAN language. The calculations were carried out in a BESM-6 computer. 1944 measurements were processed.

In processing, component sums (8) which correspond to the next measurement are determined successively. Accumulation of matrix elements and vector B occurs as a result. The memory area required for solution of the problem does not exceed the memory assigned for loading the observation material, matrix W (dimension M·M), vector $\bar{B}$ (dimension M), the working cells, program and results.

The program developed can be used for representation of the surface of any planet in the form of expansion in spherical functions. For this, it is sufficient to assign the required mean radius of the planet R_0 and incorporate the corresponding observations.

Table 1 presents the results of determination of the coefficients of expansion in spherical functions of the surface relief of Venus up to harmonic S_{99} inclusive. The two right hand columns contain accuracy characteristics of determination of coefficients $-\beta(C_{nm})$ and $\beta(S_{nm})$. The surface relief of Venus which corresponds to this expansion is represented graphically in Fig. 3. The contour lines are reckoned from a sphere of radius 6048 km, and they are 0.4 km apart.

Table 2 contains the parameters which characterize the matching of the constructed model with the measurements:

$\bar{\beta}$ is the root mean difference of the calculated (obtained by means of the Table 1 coefficients) and observed distances between the center of mass of Venus and points on its surface;

TABLE 1.

n	m	$C_{nm}\cdot 10^6$	$S_{nm}\cdot 10^6$	$\sigma(C_{nm})10^6$	$\sigma(S_{nm})10^6$
I	0	23.I		4.4	
I	I	-26.8	I3.2	2.I	2.I
2	0	-32.7		4.5	
2	I	I6.2	-II.5	2.2	2.2
2	2	-26.3	- 5.8	2.5	2.5
3	0	7I.0		5.5	
3	I	64.3	- 9.6	2.6	2.6
3	2	2.2	39.7	2.4	2.4
3	3	- 3.6	-I4.7	2.2	2.2
4	0	30.4		4.2	
4	I	5.I	I7.I	2.8	2.8
4	2	32.9	II.4	2.7	2.7
4	3	- 7.0	- 3.5	2.2	2.2
4	4	0.4	7.5	I.4	I.4
5	0	I5.8		5.2	
5	I	36.2	3I.6	3.2	3.2
5	2	I.8	-30.8	2.7	2.7
5	3	8.I	25.2	2.2	2.2
5	4	6.8	- 5.5	2.3	2.3
5	5	- 0.5	- 0.5	0.8	0.8
6	0	-I2.4		3.4	
6	I	26.I	-2I.9	3.I	3.I
6	2	5.9	I5.5	2.8	2.8
6	3	26.0	0.0	2.2	2.2
6	4	- 7.0	5.9	2.3	2.3
6	5	4.I	6.4	2.2	2.2
6	6	- 2.4	- 2.4	0.8	0.8
7	0	I5.2		4.I	
7	I	32.2	-I9.4	3.2	3.2
7	2	I9.I	2.3	3.0	3.0
7	3	8.I	-I5.5	2.2	2.2
7	4	I.2	0.0	2.3	2.3
7	5	0.9	- I.2	2.2	2.2
7	6	- 2.2	- 5.4	I.6	I.6
7	7	- I.4	- I.5	0.5	0.5
8	0	-I7.5		2.8	
8	I	- 2.5	- 2.9	3.I	3.I
8	2	3I.8	-I3.0	3.2	3.2
8	3	I7.6	5.4	2.2	2.2
8	4	- 8.8	6.3	2.2	2.2
8	5	- I.0	- 3.7	2.2	2.2
8	6	- 0.2	-II.0	2.0	2.0
8	7	3.3	0.3	I.2	I.2
8	8	- 0.8	- 0.4	0.2	0.2
9	0	-I6.3		3.3	
9	I	9.7	7.8	2.9	2.9
9	2	-25.8	-20.2	3.2	3.2
9	3	I5.7	I2.2	2.3	2.3
9	4	- 7.2	5.4	2.3	2.3
9	5	6.8	7.8	2.2	2.2
9	6	- I.7	- 6.3	2.2	2.2
9	7	- 3.5	3.4	I.7	I.7
9	8	2.8	I.4	0.6	0.6
9	9	0.0	0.0	0.0	0.0

Δ is the maximum deviation of the calculated value from the observed value.

TABLE 2.

		б, км	Δ, км
a	сферическая модель	I.I6	8.00
b	модель с использованием коэффициентов до S_{99}	0.65	3.3I

Key: a. Spherical model
b. Model using coefficients up to S_{99}

Similar parameters for a spherical model of the surface of Venus are presented for comparison.

4. Accounting for Asphericity of Surface of Venus in Processing Radar Observations

In construction of a theory of motion of the planet with the use of radar measurements, the delay times of the signal reflected from the surface of Venus must be calculated. The distance from the center of mass of the planet to point C (Fig. 2) on its surface which reflects the radio signal (subradar point) must be known for this. In order to determine this distance, the planetographic coordinates of the subradar point at the time of reflection of the radio signal should be calculated and then, by using available data on the topography of the planet, the excess of the specified point of its surface above a sphere of a given radius should be determined. /13

Let $\bar{e}_1$ and $\bar{e}_2$ be unit vectors which characterize the direction of the incident and reflected radio beams from the surface of the planet,

$$\bar{e}_1=\bar{r}_1/r_1, \qquad \bar{e}_2=\bar{r}_2/r_2,$$

where $\bar{r}_1$ are the venerocentric geoequatorial coordinates of the transmitter at signal emission time t_1, $\bar{r}_2$ are the venerocentric geoequatorial coordinates of the receiver at time of reception of the signal on the earth t_2.

The location of the subradar point is determined by the intersection of surface of the planet with a straight line from the center of mass of the planet in the direction

$$\bar{e}_n = \frac{\bar{e}_1 \cdot \bar{e}_2}{|\bar{e}_1 \cdot \bar{e}_2|}. \quad (11)$$

In this case, angle ACE is considered to be approximately equal to angle BCE.

In the aphroditographic coordinate system, unit vector of the direction to the subradar point $\bar{e}_n'$ is calculated from the relationship

$$\bar{e}_n' = P \cdot \bar{e}_n,$$

where $\bar{e}_n$ is the unit vector of the direction to the subradar point in the geoequatorial system calculated by Eq. (11) and P is the transition /14 matrix from the $(XYZ)_{50.0}$ coordinate system to the $X_B Y_B Z_B$ coordinate system (see Section 1) at signal reflection time t.

Aphroditographic latitude ϕ and longitude λ of the subradar point are tied to the vector

$$\bar{e}_{n'}\{e'_{xn}, e'_{yn}, e'_{zn}\}$$

by the equations

$$\varphi = \arcsin e'_{zn}, \quad \lambda = \mathrm{arctg}\,(e'_{yn}/e'_{xn}).$$

Accounting for asphericity of the relief of Venus, on which the model of the surface of the planet constructed above is based, appreciably decreases the discrepancies of the measured and calculated radar distances.

These discrepancies are presented in Fig. 4 by longitudes of the subradar points on the equator, without allowance for the relief. The excess of the points of the surface above a sphere of radius 6051.5 km is plotted on the same figure with a solid line. The discrepancies noticeably duplicate the relief of the planet. The root mean error is 1.24 km in this case.

The discrepancies are represented with relief taken into account in Fig. 5. In this case, the root mean error is 0.99 km. Thus, allowance for the relief of Venus in processing the radar measurements provides a significant improvement in matching the calculated and measured radar distances.

It should be taken into account that the effect of the relief is reduced by the circumstance that the basic portion of the radar measurements is made in the area of inferior conjunctions of Venus with the earth (with maximum power of the earth-Venus-earth radio line). At inferior conjunctions, Venus turns one side to the earth which approximately corresponds to longitude 320°. In connection with this, the major portion of the measurements are concentrated near this longitude, where the surface of Venus is quite level.

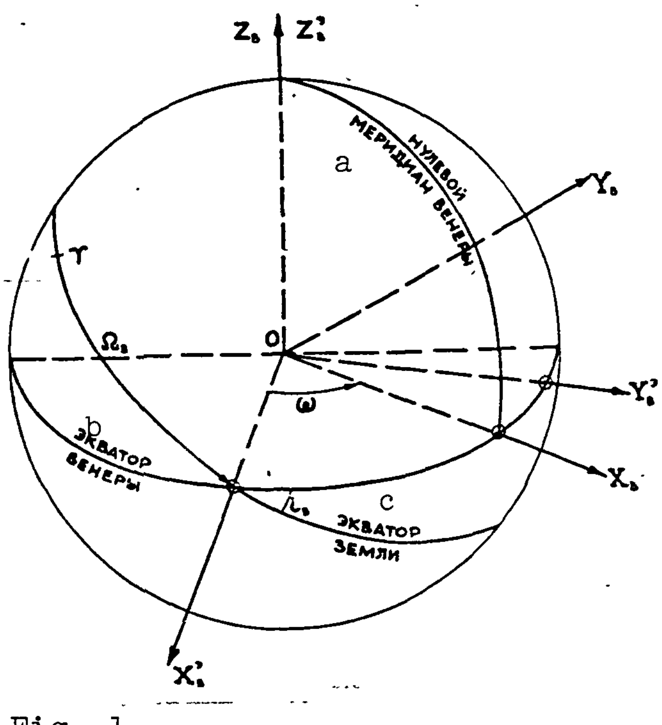

Fig. 1.

Key: a. Zero meridian of Venus
b. Equator of Venus
c. Equator of earth

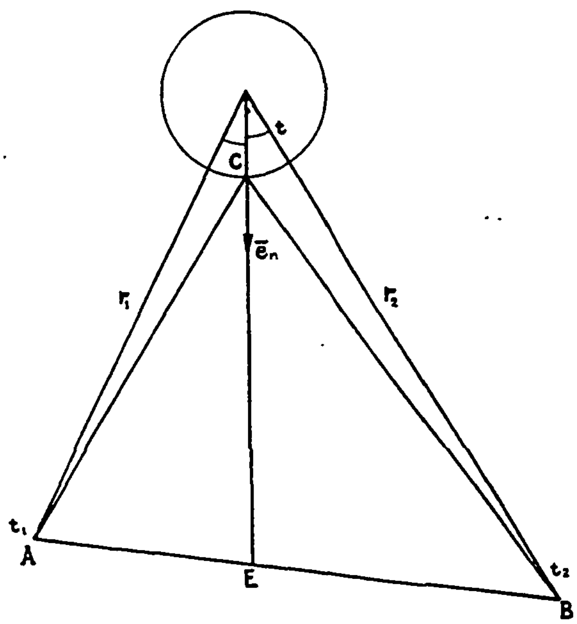

Fig. 2.

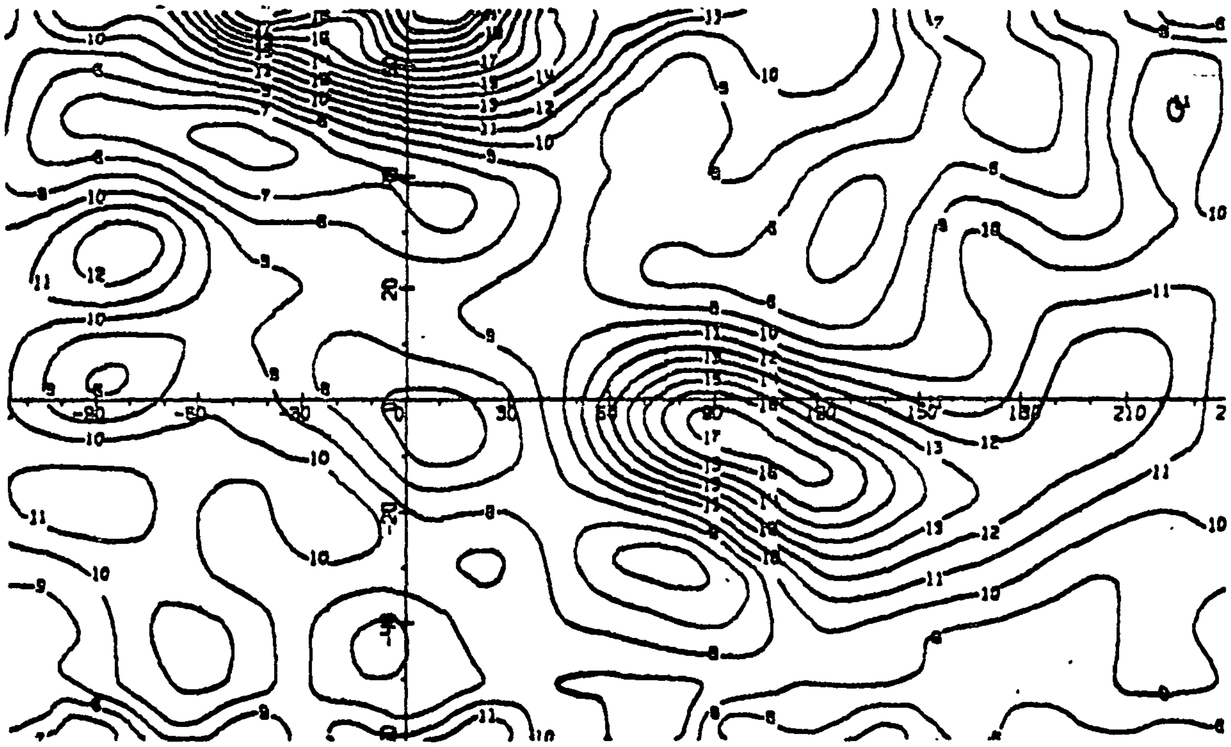

Fig. 3.

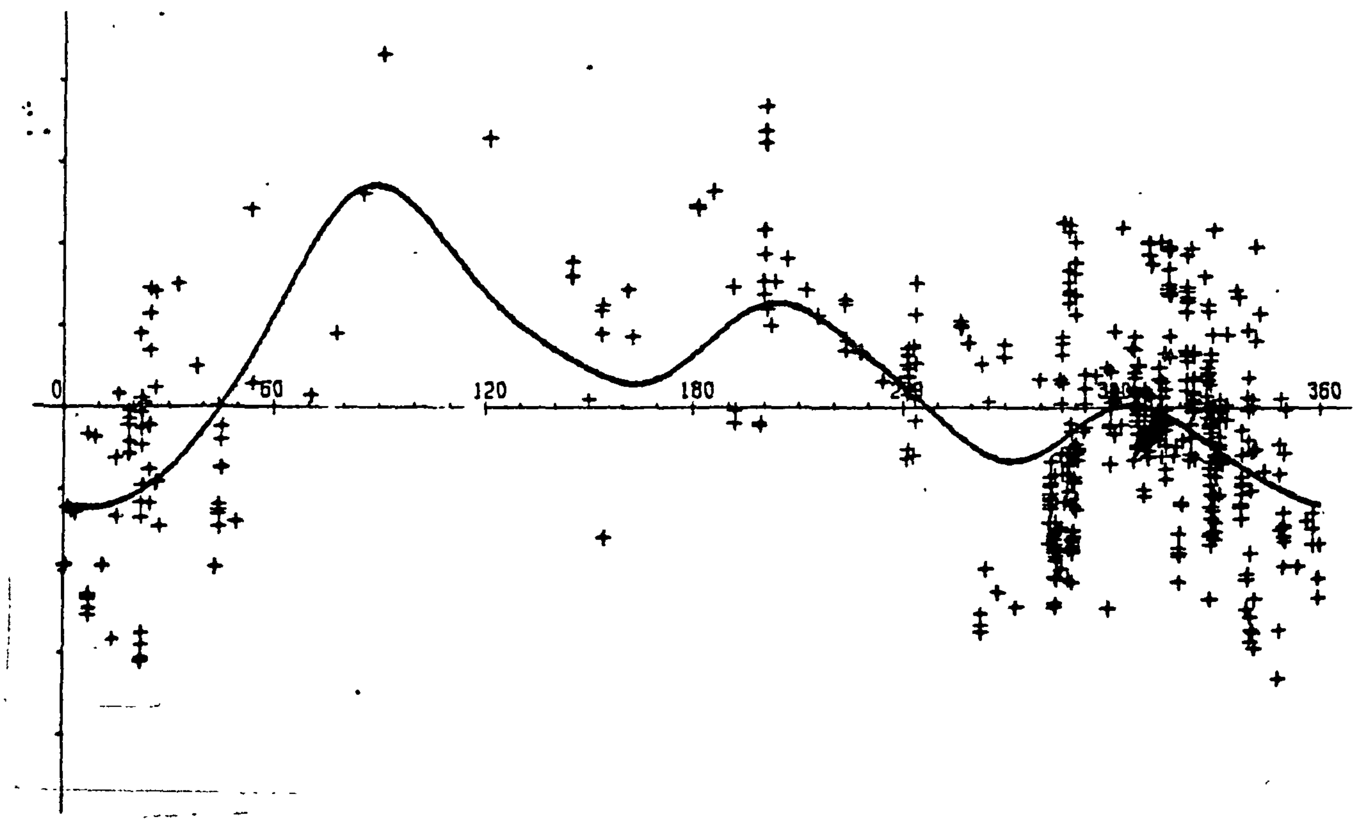

Fig. 4.