NASA Technical Memorandum 85717

NASA-TM-85717 19840012437

DEVELOPMENT OF A VOR/DME MODEL FOR AN ADVANCED CONCEPTS SIMULATOR

GEORGE G. STEINMETZ
ROLAND L. BOWLES

MARCH 1984

National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

SUMMARY/ABSTRACT

The report presents a definition of a VOR/DME, airborne and ground systems simulation model. This description was drafted in response to a need in the creation of an advanced concepts simulation in which flight station design for the 1980 era can be postulated and examined. The simulation model described herein provides a reasonable representation of VOR/DME station in the continental United States including area coverage by type and noise errors. The detail in which the model has been cast provides the interested researcher with a moderate fidelity level simulator tool for conducting research and evaluation of navigator algorithms. Assumptions made within the development are listed and place certain responsibilities (data bases, communication with other simulation modules, uniform round earth, etc.) upon the researcher.

INTRODUCTION

In order to better facilitate future flight station design in light of rapidly advancing technology, a joint effort involving two National Aeronautics and Space Administration centers is underway in conjunction with a major aircraft company. In this effort, a preliminary design of an advanced concepts simulator was formatted and is being constructed at each participating organization. While individual research focuses will be maintained at each organization, a high degree of commonality has been a goal in the design so that maximum cooperation and less costly exchanges can more effectively occur during the research phases.

The software and hardware development efforts for such a simulation facility are massive and thus the burden has been shared among the three organizations. The aircraft and ground systems software has been broken down into modules in order to maintain flexibility and growth. Specific module requirements and definition have been assigned to organizations. The requirements and definition of the VOR/DME systems module is the subject of this document. The requirements and definitions, including some limiting assumption, which are listed, are aimed at a moderate level of fidelity.

LIST OF VARIABLES AND UNITS

<u>Input</u>

PVORi, i=1,2	Logical,	A/C power status, VOR
PDMEi, i=1,2	Logical,	A/C power status, DME
Lat (A/C)	(deg)	A/C Latitude
Lon (A/C)	(deg)	A/C Longitude

N84-20505#

ALT (A/C)	(ft)	A/C altitude mean sea level
Vg (A/C)	(kts)	A/C ground speed
VORFi, i=1,2	Mhz	Selected frequency

<u>DATA BASE all for VORFi, i = 1 or 2</u>

VLATi	(deg)	station latitude
FLONi	(deg)	station longitude
ALTi	(ft)	station altitude, mean sea level
Identi	(Text)	call letters, morse code
Typei	0=T, 1=L, 2=H	station classification
MVARi	(deg)	magnetic variation
flag1i	Logical	station on/off status, VOR
flag2i	Logical	station on/off status, DME

<u>Internal</u>

TBi	(deg)	True bearing (station to A/C)
GRi	(nm)	ground range (great circle)
SRi	(nm)	slant range
$C\Theta i$	(deg)	cone of confusion angle
n_v	(deg)	VOR noise term (total)
N_r	(deg)	VOR noise term (receiver)
N_t	(deg)	VOR noise term (transmitter)
N_c	(deg)	VOR noise term (course roughness)
σ_r	(deg)	VOR standard deviation (receiver)
σ_t	(deg)	VOR standard deviation (transmitter)
t	(sec)	time
τ	(sec)	time correlation parameters

σ_C	(deg)	VOR standard deviation (course roughness)
w_C	rad/sec/kts	VOR frequency (course roughness)
w_V	dimensionless	white noise source
N_D	(nm)	DME noise term (total)
N_B	(nm)	DME noise term (transmitter and receiver)
σ_D	(nm)	DME standard deviation (transmitter and receiver)
N_{DC}	(nm)	DME noise term (course roughness)
W_{DC}	(rad/sec/kts)	DME frequency (course roughness)
σ_{DC}	(nm)	DME standard deviation (course roughness)
Output		
VORFi	(Mhz) (To data base)	selected frequency
BOi	(deg)	mag. bearing (station to A/C)
DMEOi	(deg)	slant range
VFLOi	logical	validity status, VOR
DFLOi	logical	validity status, DME
R_o	ft	Mean Earth Radius (20,887,749.4 ft)

VOR/DME System Modeling Description

This description describes a simulation model for station and receiver portions of a VOR/DME system.

The advanced concepts simulator contains the provision for simultaneously tuning two VOR/DME stations. The information received from these ground stations is used by other modules within the simulation to manage navigation functions. The VOR and DME portions are contained in the same module but can be failed individually. This approach is consistent with the assumption that the VOR and DME occur in pairs only. The VOR/DME models are activated by the selection of VHF frequencies on the integrated comm/nav management panel. This automatically serves as the selection process for the DME channel. The frequency ranges will generally be between 108 to 118 MHz. The output of the station will be a bearing and slant range from the station location of the inquiring aircraft. The update rate is normally 1/15 of a second. The station bearing will contain the magnetic variation associated with the station location. Distances for the DME portion will be slant range in n miles (6076.1 ft/nm).

The model assumes the existence of a data base containing relevant parameters. Given selected frequency, the data base will provide station latitude, longitude, and altitudes, as well as type, identification and an on/off failure flag. Station and aircraft locations are combined to determine bearing and slant range. The range, altitude and station type are compared to ICAO standards for a valid region check, including the cone of confusion. No consideration of ground features (such as mountains, hills, buildings, etc., that could occult the signals) is included in the modeling process.

Noise and mean bias errors are added to the signal bearing and slant range. Parameters governing the magnitudes will be suggested but should be alterable by any experimenter. The bias errors (both VOR and DME) will attempt to represent both station and receiver terms plus a colored course roughness term.

There are two types of failures that can be introduced in this model; a power failure can disable either the VOR or DME portion of the model and is assumed to be passed on to the appropriate other modules, a station failure of either the VOR or DME process imbedded in the data base will cause a nonvalid signal in a similar manner as an outside-proper-region coverage failure. The latter failure will be passed on in the validity variable.

Summary of Assumptions and Conditions

- VOR and DME stations occur together
- Valid regions of coverage given by United States standard (station type) and no transient effect concerning in/out of coverage are modeled
- Noise and mean bias are gaussian
- Magnetic variation included
- Provisions included for a/c power and/or station failure
- Module suuported from appropriate data base.
- Ground features and/or higher order error terms are not included.

Figure 1 is a basic block diagram of the module.

Preliminary Design Layout

A preliminary version of the ordering of events in the model is proposed in table 1. This process could be repeated for the second frequency or both frequencies handled simultaneously. The data base is assumed to exist and contains as a minimum the information as supplied in the check case setups of this paper.

Logic Details

The names assigned to variables in this document should be changed by the coders or systems integrators to properly correspond to the global variables structures. Hence, the names used herein are to be considered temporary for purposes of illustrations. The output logic variables will be validity flags, VFLOi, i = 1,2 and DMEFOi, i = 1,2. The logic for both VOR and DME is split into two phases: (1) an A/C power status, and (2) station status or coverage. These are separated in that a data base call can be avoided if A/C power to both the VOR and DME is in a failed state. Station status and regional coverage are treated deeper within the modules and must involve a data base call.

Let VFLOi = status of valid tests for VOR including coverage, cone of confusion, or station failure i = 1,2; and

DFLOi = status of valid tests for DME including coverage, cone of confusion or station failure i = 1,2.

Then IF(VFLOi or PVORi) then IDENT = no signal (Blank) and BOi - o + n_V,

IF(DFLOi or PDMEi) then DMEOi = o + n_D

where PVORi = power status of VORi, determined via input logic i = 1,2, and PDMEi = power status of DMEi, determined via input logic, i = 1,2. Note it is assumed that power status logic to VOR/DME is routed elsewhere as required for caution and warning.

Mathematical Equations

For each VORFi i = 1,2.

$$\text{GRi (ground range)} = 60\text{x}\ \text{COS}^{-1}\Big[\text{SIN(VLATi(station))SIN(Lat(A/C))} + \text{COS(VLATi)COS(Lat(A/C)COS(Lon(A/C)-VLONi)}\Big]$$

$$\text{TBi (true bearing)} = \text{COS}^{-1}\left[\frac{\text{SIN(Lat(A/C))COS(GRi/60)SIN(VLATi)}}{\text{SIN (GRi/60)COS(VLATi)}}\right]$$

IF SIN (VLONi - Lon) < 0 then TBi = 360 - TBi. (If LONi and Lon(A/C) are given in negative values for west such as United States then this logic must be reversed)

$$\text{SR}_i\ \text{(slant range)} = \text{Abs}\left[(\text{ALT(A/C)-ALTi(station)})^2 + 4\ \text{SIN}^2\ \frac{\text{GRi}}{2\text{Ro}}\ (\text{Ro+ALTi(station)})(\text{Ro+ALT(A/C)})\right]^{1/2}$$

$$\text{BOi (Bearing output)} = \text{TBi} + n_V + \text{MVARi}$$

The error term n_v includes receiver, transmitter and a course roughness component such that $n_v = N_r + N_t + N_c$

N_r is a random constant, selected from a normal distribution, mean = zero and $\sigma_r = 0.2$ (deg) $N_r = N\left[0,\sigma_r\right]$.

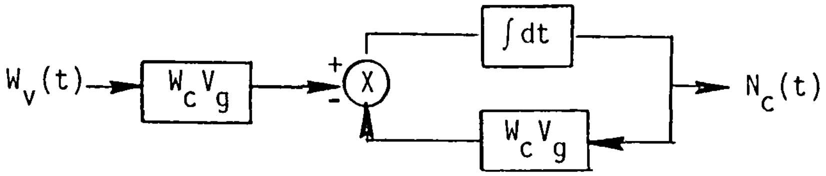
N_t is likewise a random constant term such that $N_t = N\left[0,\sigma_t\right]$ where $\sigma_t = 0.15$ (deg).

The course roughness component is treated as colored noise with correlation time inversely proportional to A/C ground speed (V_g)

$$E\left[N_c(t)N_c(t+\tau)\right] = \sigma_c^2 e^{-w_c V_g|\tau|}$$

let $\sigma_c = 0.2$ (deg) and $w_c = 4 \times 10^{-3}$ rad/sec/kts.

Note that the course roughness error is generated from Gaussian white noise $W_v(t)$ as shown below.



and the autocorrelation function of the driving white noise is

$$E\left[W_v(t)W_v(t+\tau)\right] = \left(2\sigma_c^2 \; w_c V_g\right) \delta(\tau)$$

where τ = time correlation parameter. Correspondingly

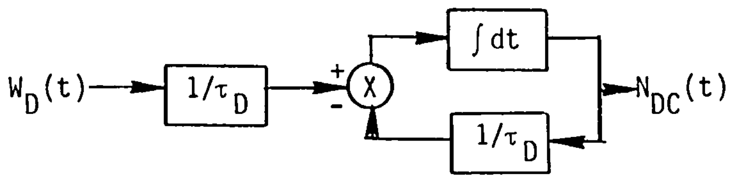
$$DMEOi = SRi + N_D$$

where

$$N_D = N_B + N_{DC}$$

$$N_B = N\left[0, \sigma_D\right] \text{for } \sigma_D = 0.5 \text{ nm or } N_B = (.03SRi)$$

whichever is greater, and N_{DC} = is similar to N_c of the VOR process with equivalent definitions for W_D, and σ_{DC}. Specifically



where the DME range error is modeled as colored noise, i.e.,

$$E\left[N_{DC}(t)N_{DC}(t+\tau)\right] = \sigma_{DC}^2\, e^{\frac{|\tau|}{\tau_D}}$$

with $\tau_D = 400$ sec. and $\sigma_{DC} = 0.1$ nm.

As shown in the above diagram, the DME correlated error is generated from white noise with the autocorrelation given by $E\left[W_D(t)W_D(t+\tau)\right] = 2\sigma_{DC}\tau_D\,\delta(\tau)$. For implementation guide see Appendix B.

Coverage Limits

The following is an excerpt from reference 1 concerning the VOR/DME coverage volumes. These volumes shall be used to define valid regions for this model. The station type or class designator will be obtained from the data base.

Standard Service Volumes (SSV).- Ground stations are classified according to their intended use. These stations are available for use within their service volume. Outside the service volume, reliable service may not be available. For standard use, the airspace boundaries are called standard service volumes. They are defined, in the table below, for the three station classes.

SSV CLASS DESIGNATOR	ALTITUDE AND RANGE BOUNDARIES
T (Terminal)	From 1000 feet (305 m) AGL up to and including 12,000 feet (3,658 m) AGL at radial distances out to 25 n. mi. (46 km). See Figures 4 and 5.
L (Low Altitude)	From 1000 feet (305 m) AGL up to and including 18,000 feet (5,486 m) AGL at radial distances out to 40 n. mi. (74 km). See Figures 3 and 6.
H (High Altitude)	From 1000 feet (305 m) AGL up to and including 14,500 feet (4,420 m) AGL at radial distances out to 40 n. mi. (74 km). From 14,500 feet (4,420 m) AGL up to and including 60,000 feet (18,288 m) at radial distances out to 100 n. mi. (185 km). From 18,000 feet (5,486 m) AGL up to and including 45,000 feet (13,716 m) at radial distances out to 130 n. mi. (241 km). See Figures 2 and 6.

These SSV's are graphically shown in Figures 2 through 6.

Within 25 n. mi. (46 km), the bottom of the T service is defined by the curve in Figure 5. Within 40 n. mi. (74 km), bottoms of the L and H service volumes are defined by the curve in Figure 6. (Note metric measurements are given for convenience and are approximations.) The distance parameter to be compared against the defined boundaries is:

$$qi = [R_0 + ALTi(station) + ALT(A/C)]\ SIN\left[\frac{GRi}{2Ro}\right] \text{(see Fig. 7)}$$

If qi is outside the defined boundary, then VFLOi and DFLOi = false.

The following excerpt from reference 1 defines the cone of confusion to be included within this module.

Vertical Angle Coverage Limitations.- Within the operational service volume of each station, azimuth signal information permitting satisfactory performance of airborne components is normally provided from the radio horizon up to an elevation angle of approximately 60° for VOR components and approximately 40° for TACAN components. At higher elevation angles, the azimuth signal information may not be usable. Distance information provided by DME will permit satisfactory performance of airborne components from the radio horizon up to an elevation angle of 60°. Thus given $C\theta = 60°$ then VFLOi and DFLOi = false, where $COS(C\theta i) = qi/SRi$. In addition, a ground station failure can be inserted in the data base through the FLAG1i and FLAG2i variables such that

If FLAG1i = false then VFLOi = false.

If FLAG2i = false then DFLOi = false.

Test Conditions

The following cases are designed to exercise the VOR/DME module by specifying the input conditions and examining the expected outputs. Data base values are also supplied for appropriate conditions. Results are contained in Appendix A.

Case #	Objective of test
1. a,b	Power failure logic, bearing quadrants and noise sources
2.	Remaining quadrants of bearing and various ranges
3. a-c	Valid coverage regions for station types
4. a,b	Station failure

In some of the above cases, noise sources are eliminated in order to unmask possible sources of error. In other cases, the noise sources themselves provide a proper indication and should be examined for the bounds. Tables 2 and 3 contain the necessary data base and aircraft information for checks.

Concluding Remarks

The simulation model described herein provides a reasonable representation of VOR/DME stations in the continental United States. The detail in which the model has been cast provides the interested researcher with a moderate fidelity level simulator tool for conducting research and evaluation of navigator algorithms. Assumptions made within the development are consistent with other portions of the Advanced Concepts Simulation and place certain responsibilities (data bases, communication with other modules, etc.) upon the researcher.

REFERENCES

1. U. S. National Aviation Standard for the VOR/DME/TACAN SYSTEMS. September 2, 1982, Dept. of Transportation Federal Aviation Administration, 9848.1.

TABLE 1.- PRELIMINARY DESIGN FLOW

Proposed step by step process:

(a) Check master power/proceed/exit

(b) Compare frequency to previous/proceed/skip (optional)

(c) Issue call to data base subroutine

(d) Return from data base with lat, long, alt, type, and indent, flag, MVAR

(e) Check flag/proceed/issue non-valid

(f) Calculate bearing and range

(g) Determine validity; given range, type, ALT/Exit

(h) Add bias and noise (function of type)

(i) Add MVAR (station location, data base)

(j) Output, valid discrete, ident code, slant range, bearing

TABLE 2.- PSUEDO DATA BASE

Flat Rock VOR/DME

IDENT = FAK, MVAR = 6.5W

113.3 (ch80) frequency

LAT = 37° 31 min 30 sec (37.525°)

LON = 77° 49 min 30 sec (77.825°)

ALT = 400 ft

TYPE = H, FLAG1, FLAG2

CAPE CHARLES VOR/DME

IDENT = CCV, MVAR = 8°W

112.2 (ch50) frequency

LAT = 37°, 21 min, 0.0 sec (37.350°)

LON = 76° 0.0 min, 0.0 sec (76.000°)

ALT = 20 ft

TYPE = H, FLAG1, FLAG2

NORFOLK VOR/DME

IDENT = ORE, MVAR = 7.5°W

116.9 (ch116) frequency

LAT = 36°, 53 min, 40 sec (36.894°)

LON = 76°, 12 min, 0.0 sec (76.200°)

ALT = 27 ft

TYPE = T, FLAG1, FLAG2

FRANKLIN VOR/DME

IDENT = FKN, MVAR = 6.75°W

110.6 (ch43) frequency

LAT = 36°, 42 min, 50 sec (36.714°)

LON = 77°, 0.0 min, 30 sec (77.008°)

ALT = 37 ft

TYPE = L, FLAG1, FLAG2

TABLE 3.- AIRCRAFT POSITIONS FOR TEST CASES

<u>A/C position #1 a,b</u>
LAT = 36°, 30s (36.05°)
LON = 76°, 30s (76.05°)
ALT = (a) 30,000 ft; (b) 15,000 ft

<u>A/C position #2 a,b</u>
LAT = 37°, 50s (37.0833)
LON = 77° (77.0°)
ALT = (a) 15,000 ft; (b) 10,000 ft

<u>A/C position #3 a,b</u>
LAT = 37° (37.0°)
LON = 76° (76.0°)
ALT = (a) 500 ft; (b) 10,000 ft

<u>A/C position #4</u>
LAT = 37° 31 min, 30 sec (37.525)
LON = 77° 48 min, 0 sec (77.800)
ALT = 30,000 ft

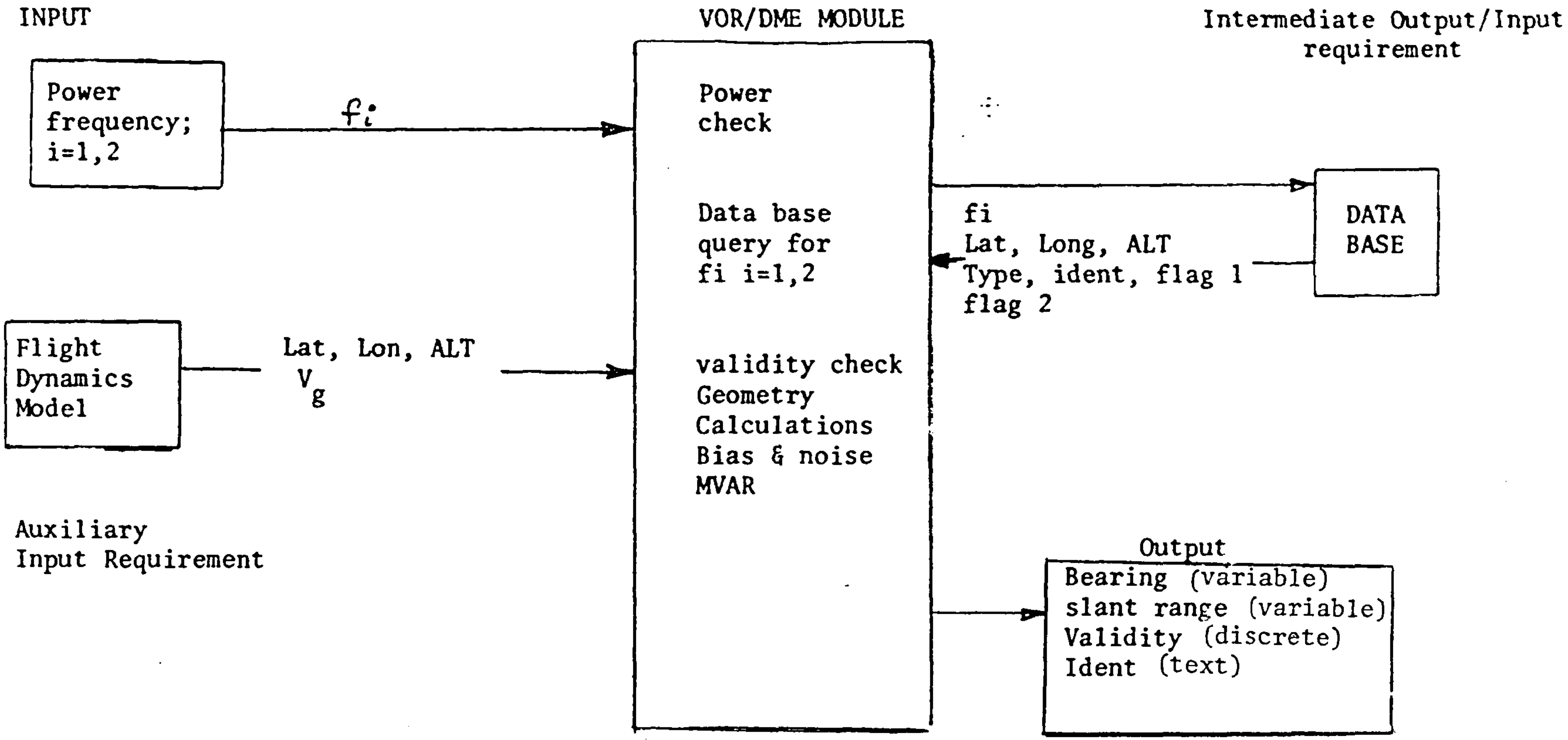


Figure 1; VOR/DME SIMULATION MODEL

FIGURE 2. STANDARD HIGH ALTITUDE SERVICE VOLUME

(refer to FIGURE 6 for altitudes below 1000 feet (305 m)

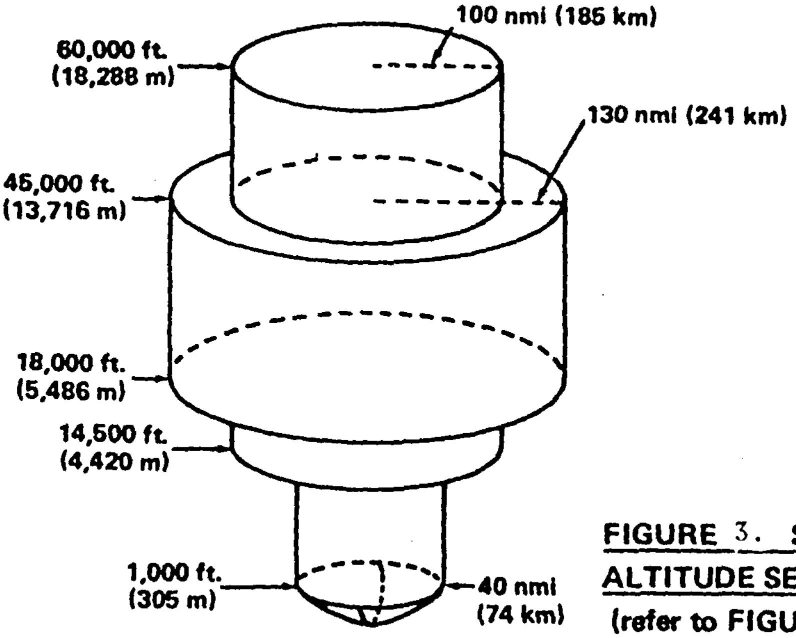


FIGURE 3. STANDARD LOW ALTITUDE SERVICE VOLUME

(refer to FIGURE 6 for altitudes below 1000 feet (305 m)

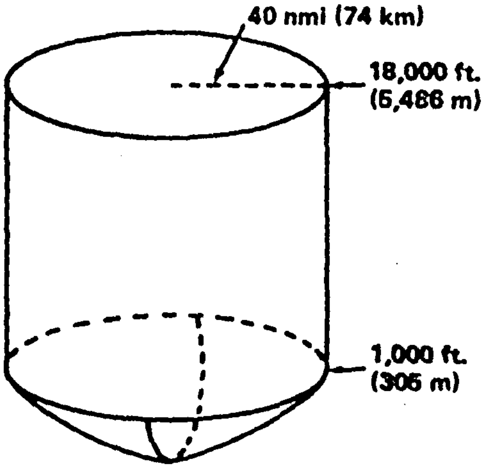
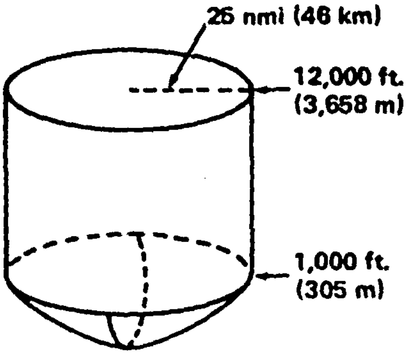


FIGURE 4. STANDARD TERMINAL SERVICE VOLUME

(refer to FIGURE 5 for altitudes below 1000 feet (305 m)



NOTE: All elevations shown are with respect to the station's site elevation (AGL). Metric Measurements are given for convenience and are approximations.

FIGURE 5. DEFINITION OF THE LOWER EDGE OF THE STANDARD T (TERMINAL) SERVICE VOLUME

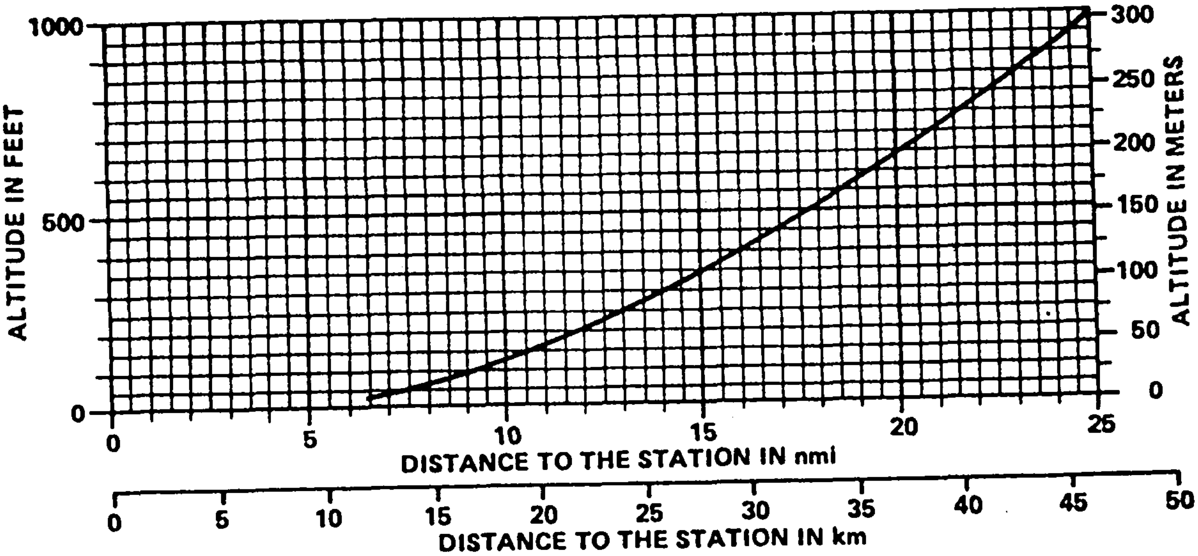
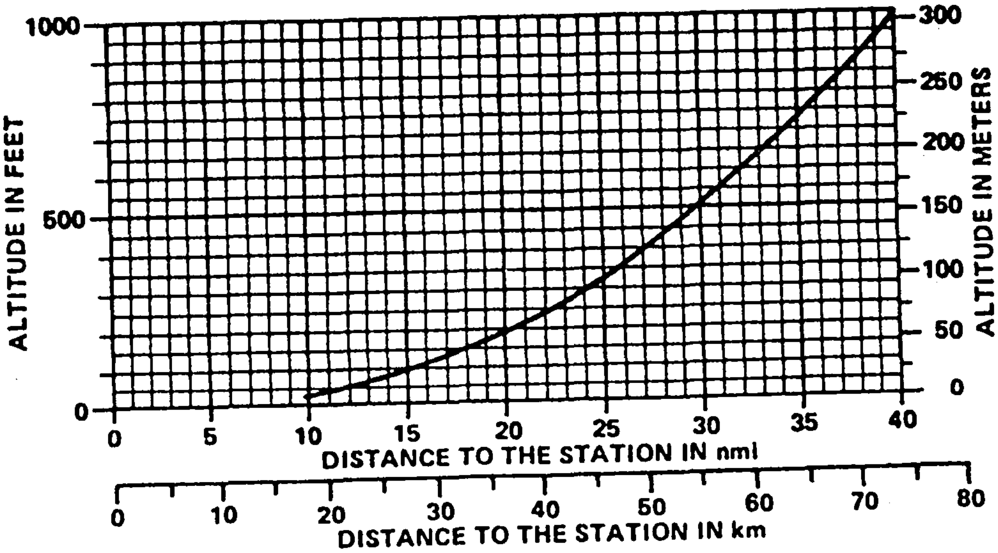


FIGURE 6. DEFINITION OF THE LOWER EDGE OF THE STANDARD H (HIGH) AND L (LOW) SERVICE VOLUMES



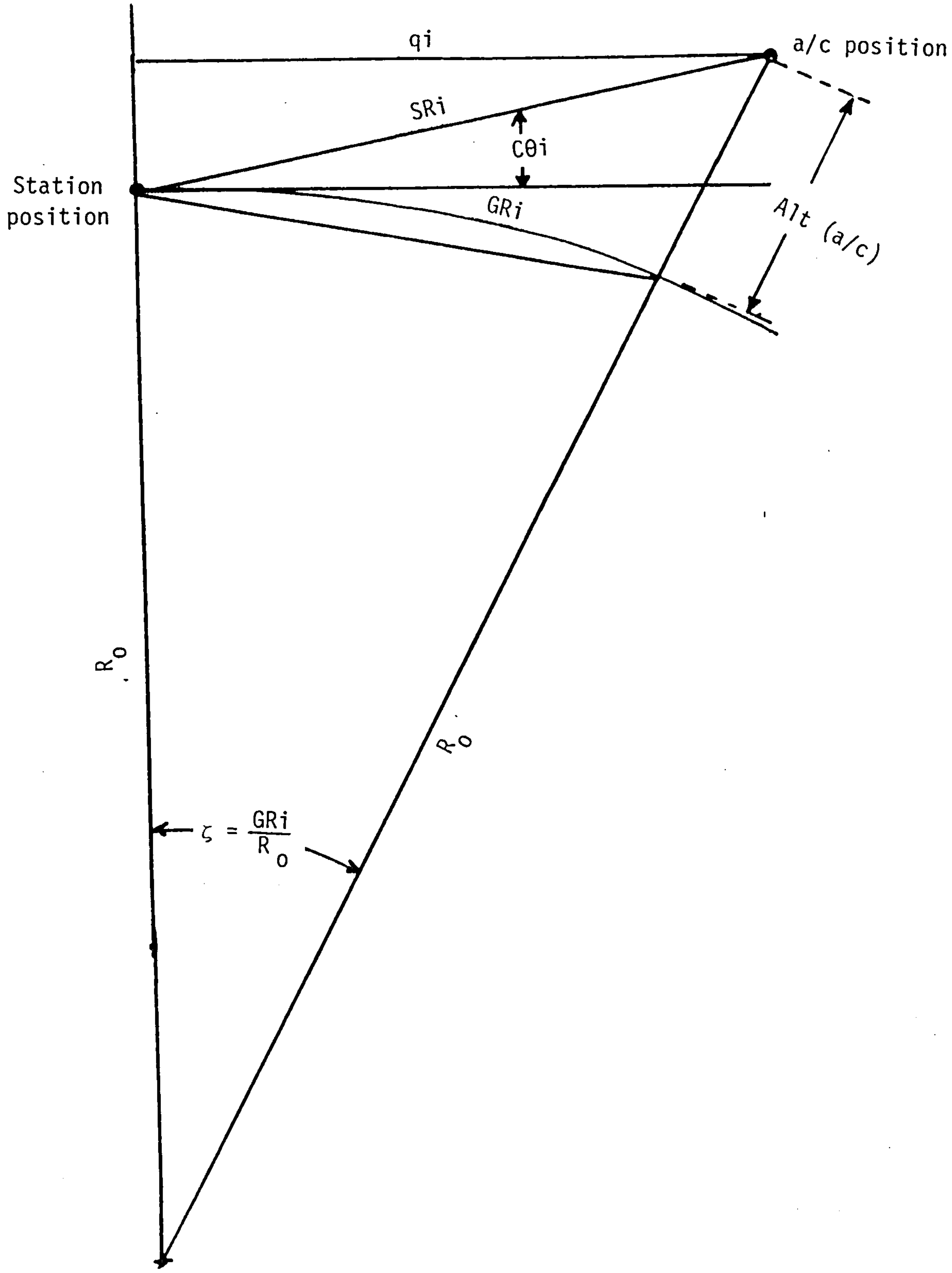


Figure 7.- Geometric relationships. (Not to scale)

APPENDIX A
A/C Power Failure to VOR #1
Case, 1a (noise sources on)

Input

PVOR1 = false

PVOR2 = true

PDME1 = true

PDME2 = true

VORF1 = 113.3

VORF2 = 112.2

A/C position = 1a

Vg = 250

Output

Ident1 = blank

Ident2 = CCV

VFLO1 = false

VFLO2 = true

DFLO1 = true

DFLO2 = true

BO1 = 0 $+n_V$

BO2 = + n_V + 213.3241

DMEO1 = + N_D + 88.0158

DMEO2 = + N_D + 56.6075

Note

n_V and N_D terms should be examined to insure proper bounds and colored noise relationship.

APPENDIX A Continued
A/C Power Failure to VOR #2
Case, 1b (noise sources on)

Input

PVOR1 = true

PVOR2 = false

PDME1 = true

PDME2 = true

VORF1 = 113.3

VORF2 = 112.2

A/C position = 1b

Vg = 250

Output

Ident1 = FAK

Ident2 = blank

VFLO1 = true

VFLO2 = false

DFLO1 = true

DFLO2 = true

BO1 = + n_V + 140.1943

BO2 = 0 + n_V

DMEO1 = + N_D + 87.5665

DMEO2 = + N_D + 56.4257

Note

n_V and N_D terms should be examined to insure proper bounds and colored noise relationship.

APPENDIX A Continued
A/C Power Failure to DME #1
Case, 1c (noise sources on)

Input

PVOR1 = true

PVOR2 = true

PDME1 = false

VORF1 = 112.2

VORF2 = 110.6

A/C position = 1b

Vg = 250

Output

Ident1 = CCV

Ident2 = FKN

VFLO1 = true

VFLO2 = true

DFLO1 = false

DFLO2 = true

BO1 = $+ n_v + 213.3241$

BO2 = $+ n_v + 124.3151$

DMEO1 = $0 + N_D$

DMEO2 = $+ N_D + 27.7270$

Note

n_v and N_D terms should be examined to insure proper bounds and colored noise relationship.

A/C Power failure to DME #2
Case, 1d (noise sources on)

Input

PVOR1 = true

PVOR2 = true

PDME1 = true

PDME2 = false

VORF1 = 116.9

VORF2 = 112.2

A/C position = 3b

Vg = 150

Output

Ident1 = ORD

Ident2 = CCV

VFLO1 = true

VFLO2 = true

DFLO1 = true

DFLO2 = false

BO1 = + n_v + 63.8901

BO2 = + n_v + 187.7436

DMEO1 = + N_D + 11.6823

DMEO2 = 0 + N_D

Note

Recheck noise terms for bounds and correctness of colored terms

APPENDIX A Continued
Case 2, Station bearing checks
(noise sources off)

Input

PVOR1 = true

PVOR2 = true

PDME1 = true

PDME2 = true

VORF1 = 113.3

VORF2 = 112.2

A/C position = 2a

Vg = 200

Output

Ident1 = FAK

Ident2 = CCV

VFLO1 = true

VFLO2 = true

DFLO1 = true

DFLO2 = true

BO1 = 70.9864

BO2 = 309.6803

DMEO1 = 43.1044

DMEO2 = 55.7343

APPENDIX A Continued
Case 3a, Valid geometry check
(noise sources on)

Input

PVOR1 = true

PVOR2 = true

PDME1 = true

PDME2 = true

VORF1 = 113.3

VORF2 = 112.2

A/C position = 4

Vg = 350

Output

Ident1 = Blank

Ident2 = CCV

VFLO1 = false

VFLO2 = true

DFLO1 = false

DFLO2 = true

BO1 = $0 + n_v$

BO2 = $+ n_v + 285.5277$

DMEO1 = $0 + N_D$

DMEO2 = $+ N_D + 86.5889$

Notes

1) A/C inside cone of confusion for VOR/DME #1
2) check noise terms

Case 3b, Valid geometry check
(noise sources on)

Input

PVOR1 = true

PVOR2 = true

PDME1 = true

PDME2 = true

VORF1 = 110.6

VORF2 = 113.3

A/C position = 3a

Vg = 120

Output

Ident1 = Blank

Ident2 = Blank

VFLO1 = false

VFLO2 = false

DFLO1 = false

DFLO2 = false

BO1 = $0 + n_V$

BO2 = $0 + n_V$

DMEO1 = $0 + N_D$

DMEO2 = $0 + N_D$

APPENDIX A Continued
Case 3c, Valid geometry check
(noise sources on)

Input

PVOR1 = true

PVOR2 = true

PDME1 = true

PDME2 = true

VORF1 = 112.2

VORF2 = 116.9

A/C position = 3a

Vg = 120

Output

Ident1 = CCV

Ident2 = ORF

VFLO1 = true

VFLO2 = true

DFLO1 = true

DFLO2 = true

BO1 = + n_v + 187.7436

BO2 = + n_v + 63.8901

DMEO1 = + N_D + 21.0594

DMEO2 = + N_D + 11.5082

Note

check noise terms

APPENDIX A Continued
Case 4a, Station failure check
(noise source on)

Input

PVOR1 = true

PVOR2 = true

PDME1 = true

PDME2 = true

VORF1 = 113.3

VORF2 = 112.2

A/C position = 2a

Vg = 175

Output

Ident1 = Blank

Ident2 = CCV

VFLO1 = false

VFLO2 = true

DFLO1 = true

DFLO2 = true

BO1 = 0

BO2 = $+ n_V + 309.6803$

DMEO1 = $+ N_D + 43.6810$

DMEO2 = $+ N_D + 55.7343$

Note

Data Base Flag11 = false

Case 4b, Station failure check
(noise sources off)

Input

PVOR1 = true

PVOR2 = true

PDME1 = true

PDME2 = true

VORF1 = 113.3

VORF2 = 112.2

A/C position = 2a

Vg = 175

Output

Ident1 = FAK

Ident2 = CCV

VFLO1 = true

VFLO2 = true

DFLO1 = true

DFLO2 = false

BO1 = 70.9864

BO2 = 309.6803

DMEO1 = 43.8248

DMEO2 = 0

Note

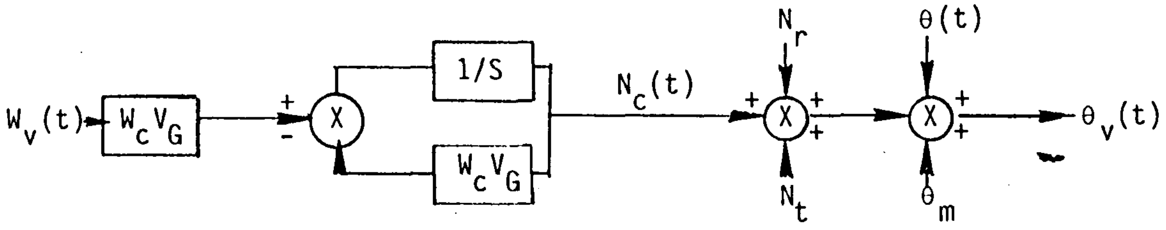
Data Base Flag22 = false

APPENDIX B

Gauss-Marko Process for VOR/DME Error Models to Support ACS Project

VOR:

The VOR error model as synthesized previously is:



Where:

$W_v(t)$ = Gaussian White Noise with autocorrelation given as

$$R_{W_v}(\tau) \triangleq E\left[W_v(t)\ W_v(t+\tau)\right] = \frac{2\sigma_c^2}{W_cV_G}\,\delta(\tau) \qquad \text{(B1)}$$

and δ_c^2 is course roughness variance (specified) and $\int_0^\infty \sigma(\tau)d\tau = 1.$

W_c = course roughness frequency (specified)

V_G = aircraft ground speed

$N_c(t)$ = course roughness error

N_r = a random constant defining receiver bias (a constant for each a/c but could change from run-to-run)

i.e., $N_r = N(0, \sigma_r)$ with σ_r specified.

N_t = a random constant for each VOR station defining transmitter biases

i.e., $N_t = N(0, \sigma_t)$ with σ_t specified.

$\theta(t)$ = correct magnetic bearing from the VOR station to the aircraft.

θ_m = magnetic variation

$\theta_v(t)$ = indicated VOR bearing

The course roughness error, $N_c(t)$, is treated as colored noise with correlation times inversely proportional to A/C ground speed V_G. The autocorrelation of N_c is given by

$$R_{N_c}(\tau) \triangleq E[N_c(t)N_c(t+\tau)] = \sigma_c^2 \, e^{-W_c V_g |\tau|} \tag{B2}$$

The power spectral density $\Phi_{N_c}(W)$ and $R_{N_c}(\tau)$ are related by a Fourier transform pair, therefore,

$$\Phi_{N_c}(W) = \frac{2}{\pi}\int_0^\infty R_{N_c}(\tau)\cos w\tau \, d\tau \tag{B3}$$

$$R_{N_c}(\tau) = \int_0^\infty \Phi_{N_c}(w)\cos w\tau \, dw. \tag{B4}$$

With $\tau = 0$ equation (B4) reduces to

$$R_{N_c}(0) = \int_0^\infty \Phi_{N_c}(w)\, dw = \sigma_c^2$$

Evaluating equation (B3) using equation (B2) gives:

$$\Phi_{N_c}(w) = \frac{2}{\pi}\int_0^\infty \sigma_c^2 \, e^{-W_c V_G |\tau|} d\tau$$

Therefore,

$$\Phi_{N_c}(w) = \frac{2}{\pi}\,\frac{W_c V_G}{W^2 + W_c^2 V_G^{\,2}}\,\sigma_c^2 \tag{B5}$$

Thus N_c can be simulated by computing the following response:

$$n(t) \longrightarrow \boxed{\sqrt{\frac{2W_cV_G}{\pi}}\quad\frac{\sigma_c}{S + W_cV_G}} \longrightarrow N_c(t) \tag{B6}$$

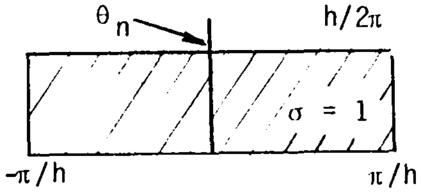
where $n(t)$ is Gaussian white noise with $E(n) = 0$; $VAR(n) = 1$.

Digital Redesign of Continuous Process

A low pass digital filter that will solve eq. (B6) is given by:

$$N_c(K) = \alpha N_c(k - 1) + (1 - \alpha)\ K\ n(k - 1) \qquad (B7)$$

where $\alpha = e^{-W_c V_G h}$ (h = computation interval) and K is a constant gain to be determined which will insure the proper statistics for $N_c(k)$; and $n(k)$ is a Gaussian white noise sequence with power spectral density



i.e., $E(n(k)) = 0$
$VAR(n(k)) = 1$

Therefore,

$$\Phi_n\ (W) = h/2\pi \quad \pi/h < W < \pi/h$$

The Z-transform of (B7) is given as

$$\frac{N_c(Z)}{N(Z)} = \frac{(1 - \alpha)K}{Z - \alpha} = H(Z) \qquad (B8)$$

Therefore, the power spectral density of the discrete sequence $N_c(k)$ is

$$\Phi_{N_c}(W) = |H(Z)|^2\ \Phi_n(W) \qquad (B9)$$

where by definition, $Z = e^{i\ wh}$. The variance of N_c is (using eqs. (B8) and (B9)

$$\bar{\sigma}_c^2 \int_{\pi/h}^{\pi/h} \Phi_{N_c}(W)dw = \frac{h}{2\pi} \int_{\pi/h}^{\pi/h} \frac{(1 - \alpha)^2 K^2 dw}{1 + \alpha^2 - 2\ \alpha\ \cos\ wh}$$

Integrating we obtain

$$\bar{\sigma}_c^2 = \frac{1 - \sigma}{1 + \sigma} K^2$$

We now determine K such that

$$\bar{\sigma}_c^2 = \sigma_c^2 \quad \text{(the desired variance based on definition of error model)}$$

Therefore,

$$K = \sqrt{\frac{1 + \sigma}{1 - \sigma}}\ \sigma_c$$

Substituting the above K into eq. (B7)

$$N_c(k + 1) = \alpha N_c(k) + \sqrt{1 - \alpha^2}\ \sigma_c\ n\ (k)$$

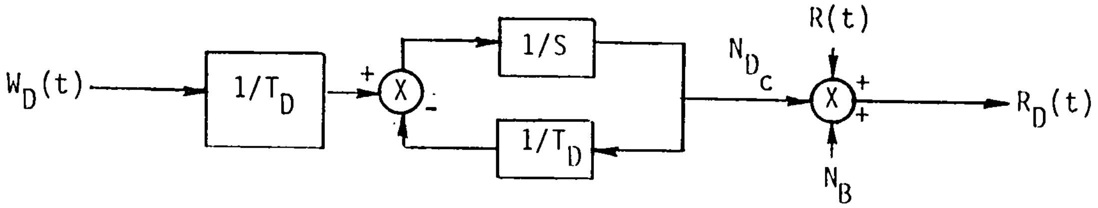
The digital implementation of this difference equation simulates the proper random process for the course roughness error. Finally, the indicated VOR bearing computation is given by

$$\theta_v(k + 1) = N_c(k + 1) + \theta(k + 1) + N_r + N_t + \theta_m$$

where all quantities are defined above.

DME ERROR MODEL

The DME Error Model is synthesized as:



Using the same techniques as used in VOR development; the digital redesign of the above provides the slant range computation as indicated by the DME. The resulting equations used to simulate this process are

$$R_D(k + 1) = N_{D_c}\ (k + 1) + R(k + 1) + N_B$$

where:

$$N_{D_c}(k+1) = \beta N_{D_c}(k) + \sqrt{1-\beta^2}\,\sigma_{D_c}\,n(k)$$

$$\beta = e^{-h/\tau_D};\ N_\beta = N(0,\sigma_D)$$

and

$$E(n(k)) = 0;\quad VAR(n(k)) = 1$$

$$\Phi_n(W) = \frac{h}{2\pi},\ -\pi/h < w < \pi/h \qquad \left\{\begin{matrix}\text{Gaussian}\\ +\\ \text{White}\end{matrix}\right.$$

The constants σ_{D_c}, σ_D, and τ_D are specified as before.

1. Report No.
NASA TM-85717

2. Government Accession No.

3. Recipient's Catalog No.

4. Title and Subtitle
DEVELOPMENT OF A VOR/DME MODEL FOR AN ADVANCED CONCEPTS SIMULATOR

5. Report Date
March 1984

6. Performing Organization Code
505-45-33-06

7. Author(s)
George G. Steinmetz
Roland L. Bowles

8. Performing Organization Report No.

9. Performing Organization Name and Address
NASA Langley Research Center
Hampton, VA 23665

10. Work Unit No.

11. Contract or Grant No.

12. Sponsoring Agency Name and Address
National Aeronautics and Space Administration
Washington, DC 20546

13. Type of Report and Period Covered
Technical Memorandum

14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

The report presents a definition of a VOR/DME, airborne and ground systems simulation model. This description was drafted in response to a need in the creation of an advanced concepts simulation in which flight station design for the 1980 era can be postulated and examined. The simulation model described herein provides a reasonable representation of VOR/DME station in the continental United States including area coverage by type and noise errors. The detail in which the model has been cast provides the interested researcher with a moderate fidelity level simulator tool for conducting research and evaluation of navigator algorithms. Assumptions made within the development are listed and place certain responsibilities (data bases, communication with other simulation modules, uniform round earth, etc.) upon the researcher.

17. Key Words (Suggested by Author(s))
simulation
Math Model
VOR/DME

18. Distribution Statement
Unclassified - Unlimited
Subject Category 04

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	32	A03

N-305

For sale by the National Technical Information Service, Springfield, Virginia 22161