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ON THE GRID GENERATION METHODS BY HARMONIC MAPPING ON PLANE AND CURVED SURFACES

S. S. Sritharan

and

Philip W. Smith

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NASA Langley Research Center, Hampton, Virginia 23665

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AUTH: A/SRITHARAN, S. S.; B/SMITH, P. W. PAA: B/(Old Dominion Univ.)

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ABS: Harmonic grid generation methods for multiply connected plane regions and regions on curved surfaces are discussed. In particular, using a general formulation on an analytic Riemannian manifold, it is proved that these mappings are globally one-to-one and onto.

ENTER:

ON THE GRID GENERATION METHODS BY HARMONIC MAPPING ON PLANE AND CURVED SURFACES

S. S. Sritharan
Institute for Computer Applications in Science and Engineering

Philip W. Smith
Old Dominion University

Abstract

Harmonic grid generation methods for multiply connected plane regions and regions on curved surfaces are discussed. In particular, using a general formulation on an analytic Riemannian manifold, it is proved that these mappings are globally one-to-one and onto.

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INTRODUCTION

One of the most important tasks of computational physics is the generation of boundary conforming grids in the computational domain. This problem can be framed mathematically as the construction of certain coordinate charts on a given Riemannian manifold. In this paper the method of harmonic mappings for two-dimensional manifolds is considered. The method of two-dimensional harmonic mappings has been used, for example, to generate grids in multiply connected plane regions [1] and to generate coordinate surfaces for three-dimensional domains using the Gauss equations [2]. We will see that the harmonic mapping method can also be used to generate grids on a portion of a given analytic curved surface. We will describe and analyze these methods by unifying them to a general mapping problem on a simply connected Riemannian manifold.

2. HARMONIC MAPPING METHOD FOR AN ANALYTIC RIEMANNIAN MANIFOLD

The simplest grid generation problem of this kind can be formulated as follows. Let Ω be a simply connected region on a two-dimensional manifold M^2 with compact closure (see Figure 1). Suppose (θ,ϕ) is a given (natural) coordinate system on this manifold. The problem is to find a boundary conforming coordinate system (ξ,η). This is accomplished by first mapping $\partial\Omega_0$ in a continuous 1-1 manner to the boundary of the rectangle Ω_1 in the (ξ,η) plane. To map the interior of Ω_0 to the interior of Ω_1, we use the following conditions:

$$\Delta\xi = 0, \qquad \Delta\eta = 0 \tag{2.1}$$

where Δ is the Laplace-Beltrami operator.

We will see later that more complicated grid generation problems can be treated in this manner. Let us investigate more closely this technique. We set $(\bar{x}^1,\bar{x}^2) = (\theta,\psi)$ and $(x^1,x^2) = (\xi,\eta)$ for ease of notation. Then $\bar{g}_{\alpha\beta}$ and $g_{\alpha\beta}$ are respectively the metric tensors of these coordinate systems. Then from the elementary theory of Riemannian geometry we have the following explicit expression for the Laplacian (Beltrami) of any scalar function ϕ on M^2

$$\Delta\phi = g^{\alpha\beta}\left[\frac{\partial^2\phi}{\partial x^\alpha\,\partial x^\beta} - \Gamma^\lambda_{\alpha\beta}\frac{\partial\phi}{\partial x^\lambda}\right] \tag{2.2}$$

where $\alpha,\beta,\lambda = 1,2$. In this context we have used the Christoffel symbol of the second kind in the x^α coordinates. It is well known that this symbol obeys the transformation laws,

$$\Gamma^\lambda_{\alpha\beta} = \frac{\partial x^\lambda}{\partial\bar{x}^\gamma}\frac{\partial^2\bar{x}^\gamma}{\partial x^\alpha\,\partial x^\beta} + \bar{\Gamma}^\mu_{\gamma\delta}\frac{\partial\bar{x}^\gamma}{\partial x^\alpha}\frac{\partial\bar{x}^\delta}{\partial x^\beta}\frac{\partial x^\lambda}{\partial\bar{x}^\mu}, \tag{2.3}$$

where, of course, $\bar{\Gamma}^\mu_{\gamma\delta}$ is the Christoffel symbol of the second kind for the coordinate system $\bar{x}^\alpha$.

2.2. Formulation on a Given Manifold

We note here that if the manifold $M^2 \equiv E^2$ (Euclidean space) then $\bar{x}^\alpha$ can be chosen as the Cartesian coordinate system yielding $\bar{\Gamma}^\alpha_{\beta\gamma} \equiv 0$ [5].

This is not possible for a general manifold. However, since any two-dimensional Riemannian manifold is conformally flat [5], it is always possible to choose (locally) a coordinate system $\bar{x}^2$ known as the isothermal coordinate system which satisfies

$$\bar{g}_{\alpha\beta} = \lambda^2(\bar{x}^1, \bar{x}^2)\, \delta_{\alpha\beta}. \tag{2.4}$$

In this case, one can verify that

$$\bar{\Gamma}^\lambda_{\alpha\beta}\, \bar{g}^{\alpha\beta} = 0 \qquad \text{yielding}$$

$$\Delta\phi = g^{\alpha\beta} \frac{\partial^2 \phi}{\partial x^\alpha\, \partial x^\beta} - \frac{\partial x^\lambda}{\partial \bar{x}^\gamma} \frac{\partial \phi}{\partial x^\lambda}\, g^{\alpha\beta} \frac{\partial^2 \bar{x}^\gamma}{\partial x^\alpha\, \partial x^\beta}. \tag{2.5}$$

If we choose $\phi = x^\delta$ (one of the coordinate functions) then considerable simplification takes place and we have

$$\Delta x^\delta = - \frac{\partial x^\delta}{\partial \bar{x}^\gamma}\, g^{\alpha\beta} \frac{\partial^2 \bar{x}^\gamma}{\partial x^\alpha\, \partial x^\beta}.$$

By imposing the condition $\Delta x^\delta = 0$, we in effect impose conditions on the resulting metric tensor. Yielding the equation

$$g^{\alpha\beta} \frac{\partial^2 \bar{x}^\lambda}{\partial x^\alpha\, \partial x^\beta} = 0, \qquad \lambda = 1,2$$

as the generating equations to be solved in the (x^1, x^2) plane. For example, we consider a portion of a spherical surface. In this case we can obtain an isothermal natural coordinate system $\bar{x}^\lambda$ using the stereographic projector

$$\bar{x}^1 + i\bar{x}^2 = \tan(\psi/2)\ \exp(i\theta)$$

where (θ,ψ) are the spherical coordinates. Imposing $\Delta x^{\delta} = 0$, $\delta=1,2$ yields as before

$$g^{\alpha\beta} \frac{\partial^2 \bar{x}^{\lambda}}{\partial x^{\alpha}\, \partial x^{\beta}} = 0, \qquad \lambda = 1,2$$

with Dirichlet boundary conditions. A practical grid system for a conical wing shape [3] is given in Figure 2. Similar computations could be performed for other simple surfaces where some isothermal coordinates are explicitly known. For certain simple surfaces, isothermal coordinates are given in [4,5].

2.3. Surface Construction Using Harmonic Mappings

We now consider the problem of generating a coordinate surface in E^3 connecting two one-dimensional boundaries (lying in two boundary surfaces). Again, the idea is to place implicitly restrictions on the metric tensor (see Figure 3). Let the unknown manifold M^2 be immersed in E^3 with Cartesian coordinate system x^i, $i = 1,2,3$. The tangent vectors are

$$B^i_{\alpha} := \frac{\partial x^i}{\partial \xi^{\alpha}}, \qquad i = 1,2,3, \quad \alpha = 1,2$$

where (ξ^1,ξ^2) are some coordinate system for M^2. The Gauss [7] equations are

$$B^i_{\alpha\|\beta} = \Omega_{\alpha\beta}\ N^i$$

where "$\|$" denotes covariant differentiation, $\Omega_{\alpha\beta}$ is the second fundamental form and N^i is the normal to the manifold. Multiplying by $g^{\alpha\beta}$ yields

$$g^{\alpha\beta} \frac{\partial^2 x^i}{\partial\xi^\alpha \partial\xi^\beta} - g^{\alpha\beta} \Gamma^\lambda_{\alpha\beta} B^i_\lambda + \Gamma^i_{hk} B^h_\alpha B^k_\beta g^{\alpha\beta} = (k_1 + k_2)N^i,$$

where $\Gamma^\lambda_{\alpha\beta}$ and Γ^i_{hk} are respectively the Christoffel symbols of the second kind of the ξ^α coordinate system on the manifold and the x^i coordinate system on E^3. Also $(k_1 + k_2)$ is twice the mean curvature. If x^i are Cartesian coordinates then $\Gamma^i_{hk} = 0$ and recalling our earlier development, we obtain

$$g^{\alpha\beta} \frac{\partial^2 x^i}{\partial\xi^\alpha \partial\xi^\beta} + B^i_\lambda \Delta\xi^\lambda = (k_1 + k_2)N^i.$$

If we impose $\Delta\xi^\lambda = 0$ for $\lambda = 1,2$, we get

$$g^{\alpha\beta} \frac{\partial^2 x^i}{\partial\xi^\alpha \partial\xi^\beta} = (k_1 + k_2)N^i$$

which could be used as generating equations for M^2. This technique has been exploited by Warsi [2], [6].

3. MULTIPLY CONNECTED DOMAINS AND THE CONSTRUCTION OF AN ABSTRACT RIEMANN SURFACE

We will first describe briefly the construction of an abstract Riemann surface for an algebraic function as a compact and orientable surface that could be realized in the Euclidean space E^3. Consider the function

$$w^2 = \prod_{i=1}^{2N} (z - r_i).$$

Then w will be single-valued in the two-sheeted cut plane as shown in Figure 4. Projecting each of these planes stereographically to their Riemann spheres and joining the spheres along the cuts after appropriate rotations, we get a closed surface that is topologically equivalent to a surface of genus $N-1$. We have thus obtained a surface on which w is single-valued, in a form which could be realized in E^3.

If we are now interested in generating grids by harmonic mapping on a mulitply connected domain Ω with appropriate cuts as shown in Figure 5, we see that this domain will have a corresponding image on a suitable abstract Riemann surface S. We will now introduce the Riemannian metric in this surface to convert it to a metric Riemann surface (conversely, an analytic Riemannian manifold with isothermal coordinates will become an abstract Riemann surface). Thus, we see that the problem of generating grids on a multiply connected plane domain and that on a portion of an analytic curved surface are equivalent. Let $\hat{S}$ be the universal covering surface (simply connected) of our Riemann surface S. On this covering manifold the domain of interest Ω will have many (disconnected) images and these images can be transformed to each other by what is called the covering transformation. We will thus work with the fundamental domain on this manifold that is left invariant by the group of covering transformations. Since any simply connected Riemann surface can be conformally mapped to a plane [4], the covering surface $\hat{S}$ will have a one-to-one conformal transformation with a domain G on the plane (see Figure 6).

Let P be a point on the Riemann surface S and $\hat{P}$ be one of its images on the universal cover $\hat{S}$. The projection mapping π is defined by

$$\pi(\hat{P}) = P,$$

(note π^{-1} is not single-valued). Let f map $\hat{S}$ to G, then f is univalent and

$$\sigma = \bar{x}^1 + i\,\bar{x}^2 = f(\hat{P})$$

or

$$P = \left(\pi \circ f^{-1}\right)(\sigma) = \mathscr{F}(\sigma).$$

Thus, $\sigma = \mathscr{F}^{-1}(P)$ is the uniformizer for the Riemann surface S. This procedure has provided us with a global family of isothermal coordinates $(\bar{x}^1,\bar{x}^2)$ on the manifold S. We have to note that $\mathscr{F}^{-1}$ is not single-valued if S is not simply connected and the potential σ may have stationary points (for a surface of genus g, a potential with m poles will have $2m + 2g - 2$ stationary points). However, it is to be expected that a suitable σ can be found with no singularities in the domain of interest.

We will now consider the problem of producing a grid on Ω as above using harmonic mapping techniques. This method will provide non-isothermal coordinates in general. However, we will use the fact that the domain of interest Ω on the manifold could be covered by a global isothermal coordinate system to develop the rest of the theory. Thus, the harmonic mapping problem on a curved surface or a multiply connected plane domain with multivalued coordinates (x^1,x^2) has been reduced to a problem of finding single-valued coordinates $\left(x^1(\bar{x}^1,\bar{x}^2),\ x^2(\bar{x}^1,\bar{x}^2)\right)$ on a simply connected domain in the uniformizing plane G.

We will now express the Laplace-Beltrami operator in these uniformizing variables,

$$\Delta = \frac{1}{\lambda^2}\left(\frac{\partial^2}{\partial(\bar{x}^1)^2} + \frac{\partial^2}{\partial(\bar{x}^2)^2}\right), \qquad \lambda = \lambda(\bar{x}^1,\bar{x}^2).$$

Thus, we have $\Delta x^\alpha = 0$, $\alpha = 1,2$, in the image Ω_0 on the plane region G.

Let us now show that this grid generation procedure does map the interior of Ω_0 to the interior of Ω_1 in a one-to-one manner. The problem has been reduced to the uniformizing plane and therefore the following theorem applies [8], we include the proof for completeness.

THEOREM 1: The coordinate functions (x^1,x^2) obtained by harmonic mapping techniques map Ω_0 into the rectangle Ω_1 have non-vanishing gradients in Ω_0.

Proof: Suppose $\nabla x^1 = 0$ at $\sigma \varepsilon \Omega_0$, then

$$W(\sigma) = x^1 + i\, x^{1*},$$

where x^{1*} is an harmonic conjugate of x^1, must satisfy

$$\frac{\partial W}{\partial \sigma} = 0 \qquad \text{at } \sigma_0,$$

which implies that $Z(\sigma) = W(\sigma) - W(\sigma_0)$ should have a zero of order greater than or equal two. If this is true, then the argument of $Z(\sigma)$ around $\partial\Omega_0$ is at least 4π which means that $x^1(\sigma) - x^1(\sigma_0)$ should vanish at least four times on $\partial\Omega_0$. But we chose Ω_1 to be a rectangle and therefore this function has exactly two zeroes on $\partial\Omega_0$. Therefore, we conclude that ∇x^1 and ∇x^2 are never zero in $\overset{\circ}{\Omega}_0$. We remark that if Ω_1 had been a more complicated region, then this proof would be no longer valid.

THEOREM 2: Let T be the harmonic mapping of Ω_0 into Ω_1. That is,

$$T : \partial\Omega_0 \to \partial\Omega_1 \qquad \text{homeomorphically}$$

and

$$\Delta T = 0 \qquad \text{on } \overset{\circ}{\Omega}_0 .$$

Then if T is C^1 on $\Omega_0 \setminus \{P_i\}_{i=1}^N$ where $\{P_i\}_{i=1}^N \subset \partial\Omega_0$, then the Jacobian of T never vanishes in $\overset{\circ}{\Omega}_0$.

Proof: Let

$$P \in \partial\Omega_0 \setminus \{P_i\}_{i=1}^N .$$

Then $\partial\Omega_0$ is smooth at P and we may unambiguously define an orthogonal coordinate system $\bar{s}(P), \bar{n}(P)$ where $\bar{s}(P)$ is the tangent vector on $\partial\Omega_0$ obtained by traversing $\partial\Omega_0$ at constant velocity and $\bar{n}(P)$ points into Ω_0. This yields an orientation of $\partial\Omega_0$. Now let $\{R_n\}_{n=1}^\infty \subset \overset{\circ}{\Omega}$ with $R_n \to P$. Then computing the Jacobian of T at R_n in the coordinates $\bar{s}(P), \bar{n}(P)$, we have for $C \neq 0$,

$$C.\left(x_s^1\, x_n^2 - x_s^2\, x_n^1\right)(R_n) = J(T)(R_n).$$

Since T is C^1, these values converge to the value at P, namely

$$C.\left(x_s^1\, x_n^2 - x_s^2\, x_n^1\right)(P).$$

But since P is a boundary point and Ω_1 is a rectangle, then one of the two terms vanishes and the remaining term is nonnegative if both curves have been oriented counter-clockwise. Thus, we conclude that the Jacobian of this mapping is well-defined at the boundary and nonnegative.

Now consider

$$\chi(\sigma) = \frac{(\partial x^2)/(\partial \bar{x}^1) + i\left[(\partial x^{2*})/(\partial \bar{x}^1)\right]}{(\partial x^1)/(\partial \bar{x}^2) + i\left((\partial x^{1*})/(\partial \bar{x}^2)\right)} ,$$

where x^{2*} is the conjugate harmonic function to x^2. Thus, making use of the Cauchy-Riemann equations we have

$$\mathrm{Re}\left(\chi(\sigma)\right) = \frac{\partial(x^1,x^2)}{\partial(\bar{x}^1,\bar{x}^2)} \Big/ |\nabla x^1|^2,$$

which is harmonic in Ω_0 and hence, is positive in $\mathring{\Omega}_0$ by the maximum principle. Here we have used the fact that $\nabla x^1 \neq 0$ and the nonnegativity of the Jacobian in the boundary.

Thus, the harmonic mapping method produces for multiply connected plane domains as well as portions of analytic Riemann manifolds, a locally 1-1 mapping function. We will now prove that these mappings are globally 1-1.

Although our present focus is on mappings in R^2, the next two lemmas are presented in R^N for future extensions to mapping problems in higher dimensions.

LEMMA 1: *Let* Ω_0 *and* Ω_1 *be compact subsets of* $\mathbb{R}^N$ *with* $\overline{\mathring{\Omega}}_0 = \Omega_0$ *and* Ω_1 *convex. Let* T *be a continuous mapping from* Ω_0 *into* $\mathbb{R}^N$ *which is differentiable in* $\mathring{\Omega}$ *and satisfies*

1. $T(\partial\Omega_0) = \partial\Omega_1$

2. $|T'(\underline{x})| \neq 0$ *for all* $\underline{x} \in \mathring{\Omega}_1$

then T *maps* Ω_0 *onto* Ω_1.

Proof: The first thing to notice is that T is an open map on $\mathring{\Omega}_0$ and hence, $T(\mathring{\Omega}_0)$ is open. We claim that $T(\Omega_0) \subset \Omega_1$. If not by compactness, there is an $\underline{x}^* \in \mathring{\Omega}_0$ satisfying

$$0 < \mathrm{dist}\big(T(x^*), \Omega_1\big) = \max \mathrm{dist}\big(T(\underline{x}), \Omega_1\big),$$

But T is open on $\mathring{\Omega}_0$ and Ω_1 is convex, which implies that $y \to \mathrm{dist}(y, \Omega_1)$ has no local maxima. This contradicts the assumption that $T(\Omega_0) \not\subset \Omega_1$. Thus, we have $T(\Omega_0) \subset \Omega_1$ and even $T(\mathring{\Omega}_0) \subset \mathring{\Omega}_1$. If we fail to have equality, there is a $y \in \Omega_1 \backslash T(\mathring{\Omega}_0)$. But for any such y, we must have

$$\inf_{\underline{x} \in \Omega_0} \|T(\underline{x}) - y\| = \inf_{\underline{x} \in \partial\Omega_0} \|T(\underline{x}) - y\|,$$

since T is open on $\mathring{\Omega}_1$. However, one can always produce a $y \in \mathring{\Omega}_1 \backslash T(\mathring{\Omega}_o)$ arbitrarily close to $T(\mathring{\Omega}_0)$ and relatively far from $\partial\Omega_1 = T(\partial\Omega_0)$ to violate this condition. This completes the proof of this lemma.

LEMMA 2: Let Ω_0 and Ω_1 be compact homotopic subsets of $\mathbb{R}^N$ with Ω_1 convex. Let R_α, $0 \leq \alpha \leq 1$ be the homotopy with

$$R_\alpha(\Omega_1) = \Omega_\alpha$$

$$\overline{\overset{\circ}{\Omega}}_\alpha = \Omega_\alpha$$

$$R_1(\underline{x}) = \underline{x} \qquad \text{for } \underline{x} \in \Omega_1$$

$$|R'_\alpha(\underline{x})| \neq 0 \qquad \text{for } x \in \overset{\circ}{\Omega}_1.$$

Let T_α, $0 \leq \alpha \leq 1$ be the mapping from $\Omega_\alpha \to \mathbb{R}^N$ satisfying,

$$T_1(\underline{x}) = \underline{x} \qquad \text{for } \underline{x} \in \Omega_1$$

$$|T'_\alpha(x)| \neq 0 \qquad \text{for } \underline{x} \in \overset{\circ}{\Omega}_1$$

$$T_\alpha(\partial\Omega_\alpha) = \partial\Omega_1$$

$T_\alpha \circ R_\alpha$ is a homotopy on Ω_1. Then T_α is a one-to-one and onto transformation from Ω_α into Ω_0. In particular, T_0 maps Ω_0 one-to-one and onto Ω_1.

Proof: Recall that for a differentiable map $T : \Omega \to \mathbb{R}^N$ (for Ω compact subset of $\mathbb{R}^N$) we can define

(a) $$d = \text{degree}\,(T,y,\Omega) = \sum_{\substack{T(x) = y \\ \underline{x} \in \overset{\circ}{\Omega}}} \text{sign}|T'(\underline{x})|$$

provided $y \notin T(\partial\Omega)$ and for each $\underline{x} \in \overset{\circ}{\Omega}$ above, $|T'(\underline{x})| \neq 0$. Furthermore, this integer d is invariant under homotopy (homotopy invariance theorem) provided no solutions are introduced on the boundary [9]. Now to prove the lemma, set

$$S_\alpha = T_\alpha \circ R_\alpha .$$

Thus S_α is a homotopy on Ω_1 and for each $\underline{x} \in \overset{\circ}{\Omega}_1$,

$$|S_\alpha'(\underline{x})| \neq 0$$

and hence, $|S_\alpha'(\underline{x})|$ is always positive or always negative. But $|S_1'(\underline{x})| = 1$ and hence, $|S_\alpha'(\underline{x})| > 0$ for all $\underline{x} \in \overset{\circ}{\Omega}_1$. We know from Lemma 1 that T_α maps Ω_α onto Ω_1 and $\overset{\circ}{\Omega}_\alpha$ onto $\overset{\circ}{\Omega}_1$. Thus, S_α maps $\partial\Omega_1$ onto $\partial\Omega_1$ and $\overset{\circ}{\Omega}_1$ onto $\overset{\circ}{\Omega}_1$. Let $y \in \overset{\circ}{\Omega}_1$ and note

$$\text{degree}(S_1, y, \Omega_1) = 1$$

since S_1 is the identity. Both S_α is a homotopy which introduces no solutions on the boundary and hence, by degree theory

$$\text{degree}(S_\alpha, y, \Omega_1) = 1,$$

since $|S_\alpha'(\underline{x})| > 0$ for all $\underline{x} \in \overset{\circ}{\Omega}_1$. Thus, (a) actually counts the number of

$\underline{x}$'s, so that

$$S_\alpha(\underline{x}) = y.$$

Since this number is one we conclude that, S_α maps $\overset{\circ}{\Omega}_1$, one-to-one and onto $\overset{\circ}{\Omega}_1$, and hence T_α maps Ω_α one-to-one and onto Ω_1 and finally Ω_0 one-to-one onto Ω_1 as was to be shown.

CONCLUSION

Grid generation methods using harmonic mappings have been analyzed in a unified framework and a rigorous justification for these methods is given. We have proved that for these mapping methods, mesh intersection, or overspill cannot occur. Our conclusions are limited to two-dimensional manifolds, although substantial portion of the theory developed is applicable for higher dimensions. An extension of this theory should include the generalization to higher dimensions and to the method of nonharmonic mappings.

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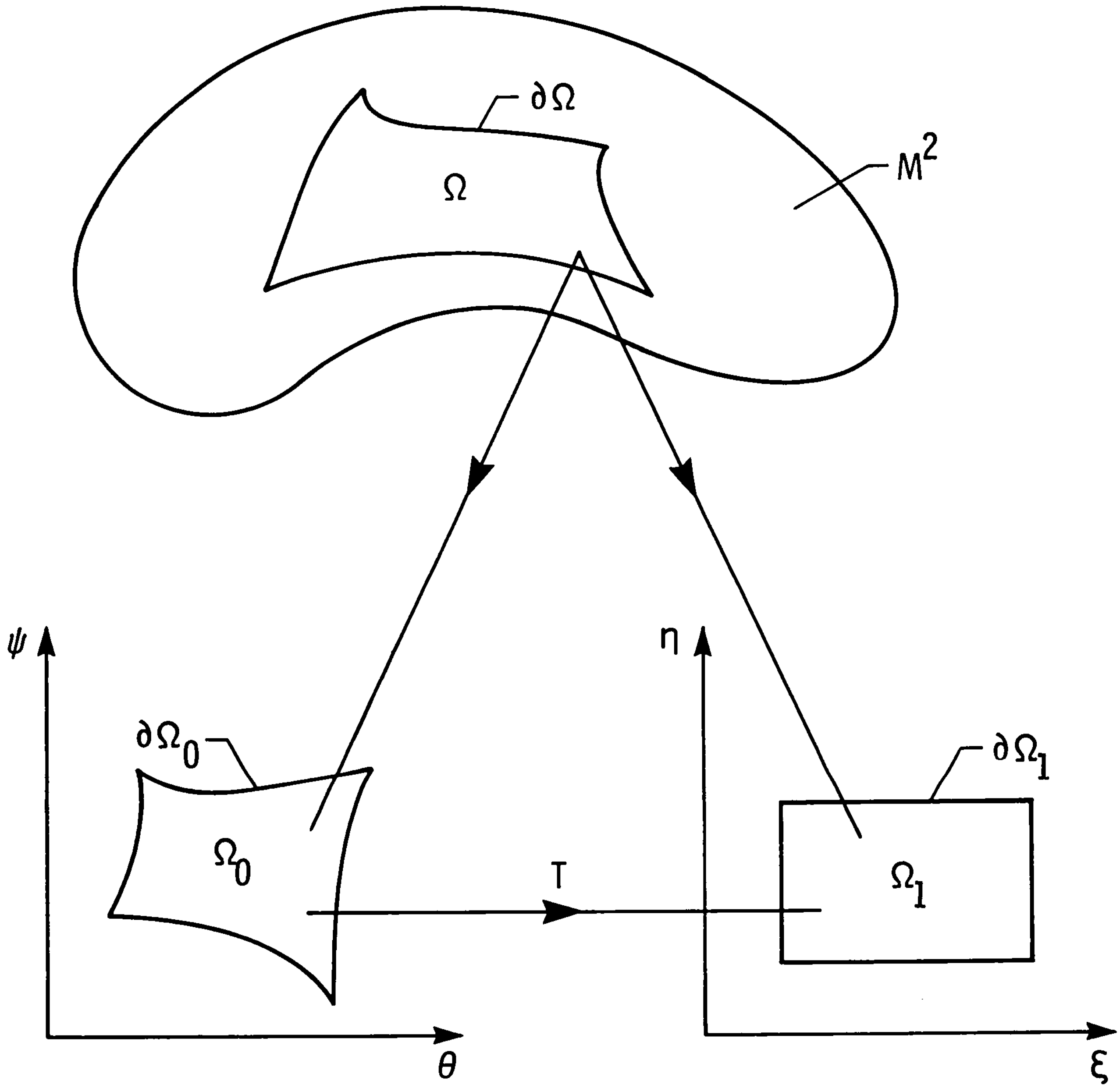

Figure 1

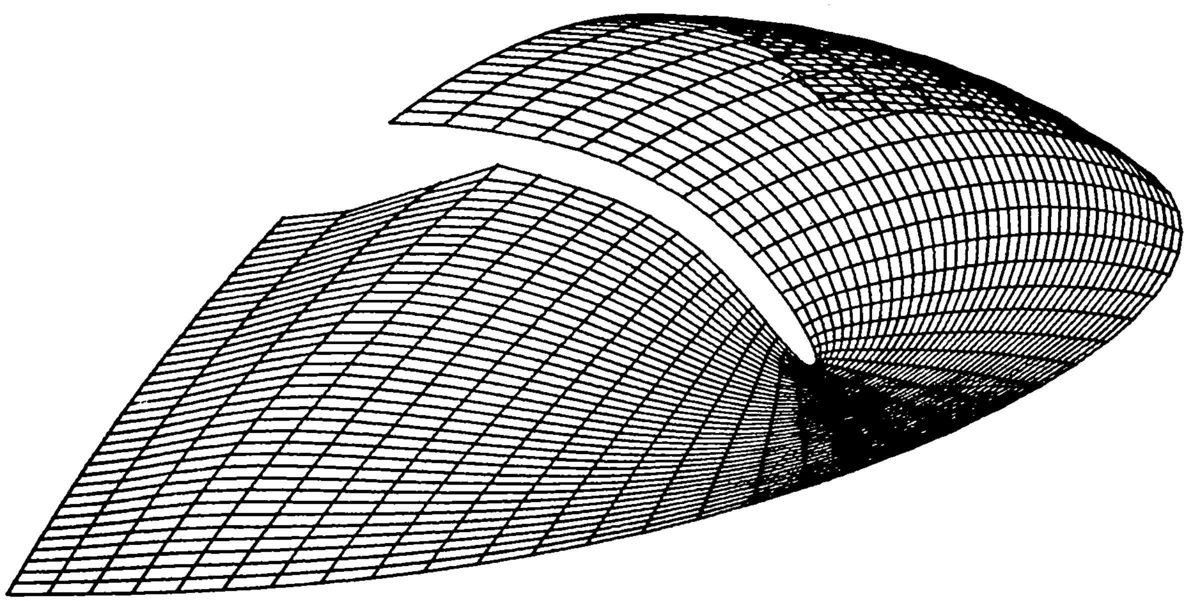

Figure 2

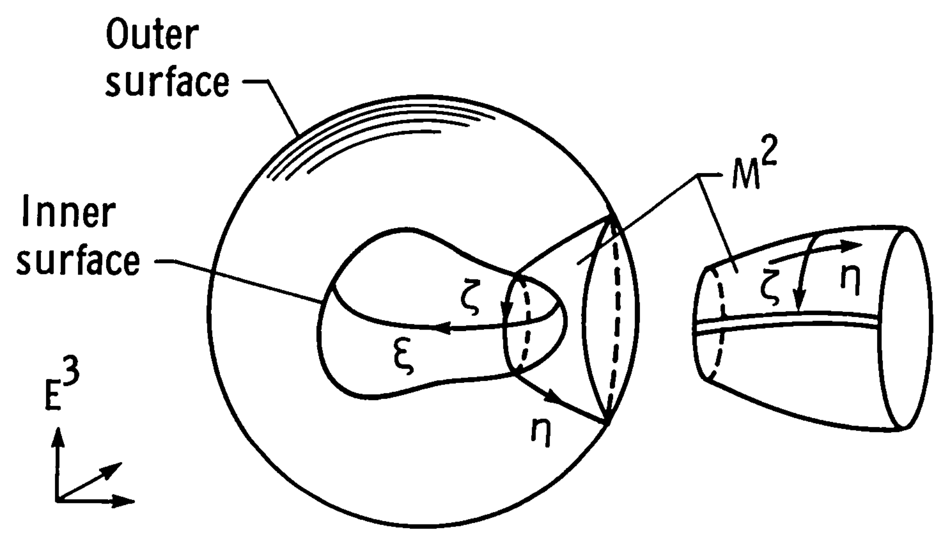

Figure 3

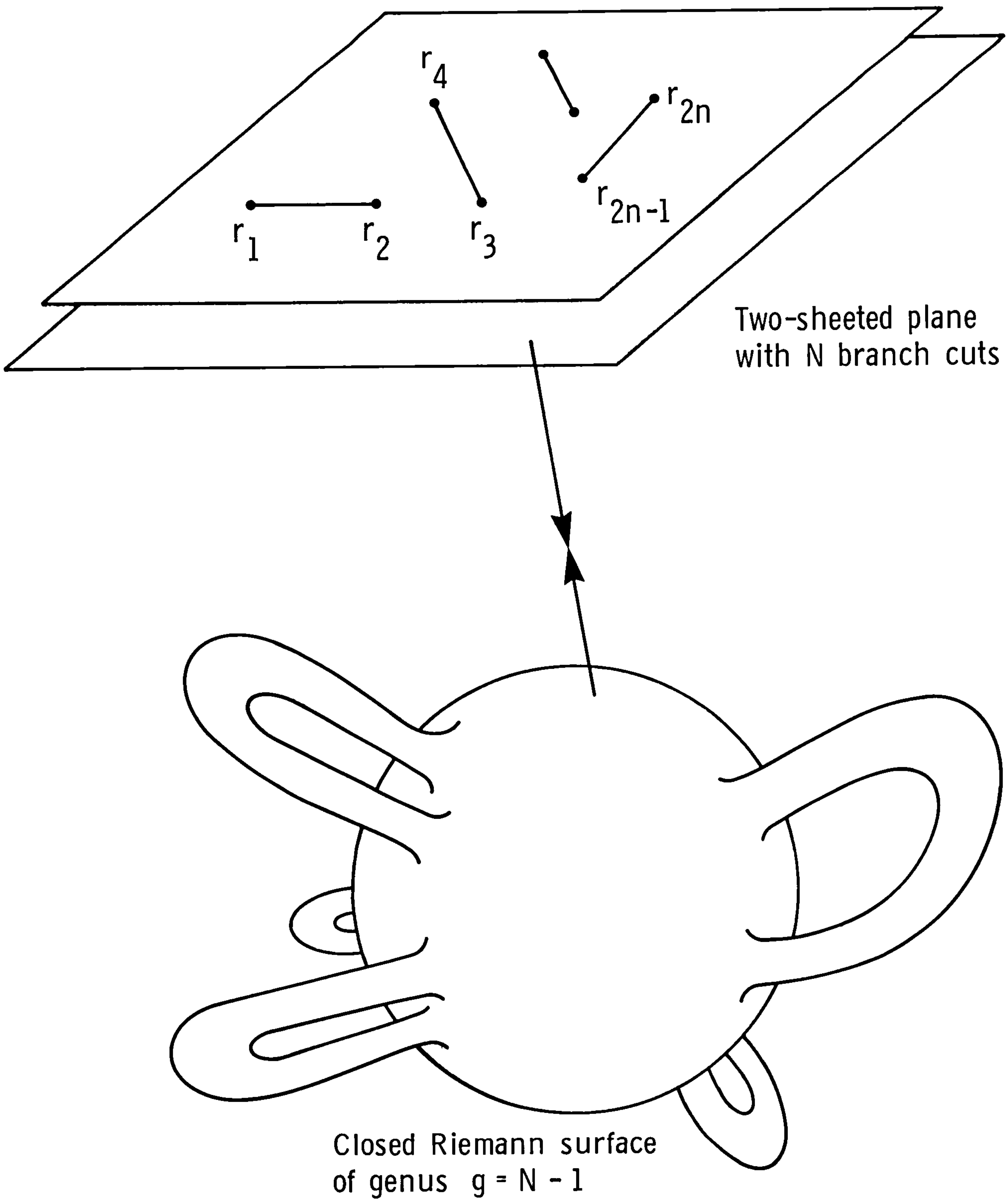
r_4
r_{2n}
r_{2n-1}
r_1
r_2
r_3
Two-sheeted plane
with N branch cuts
Closed Riemann surface
of genus $g = N - 1$

Figure 4

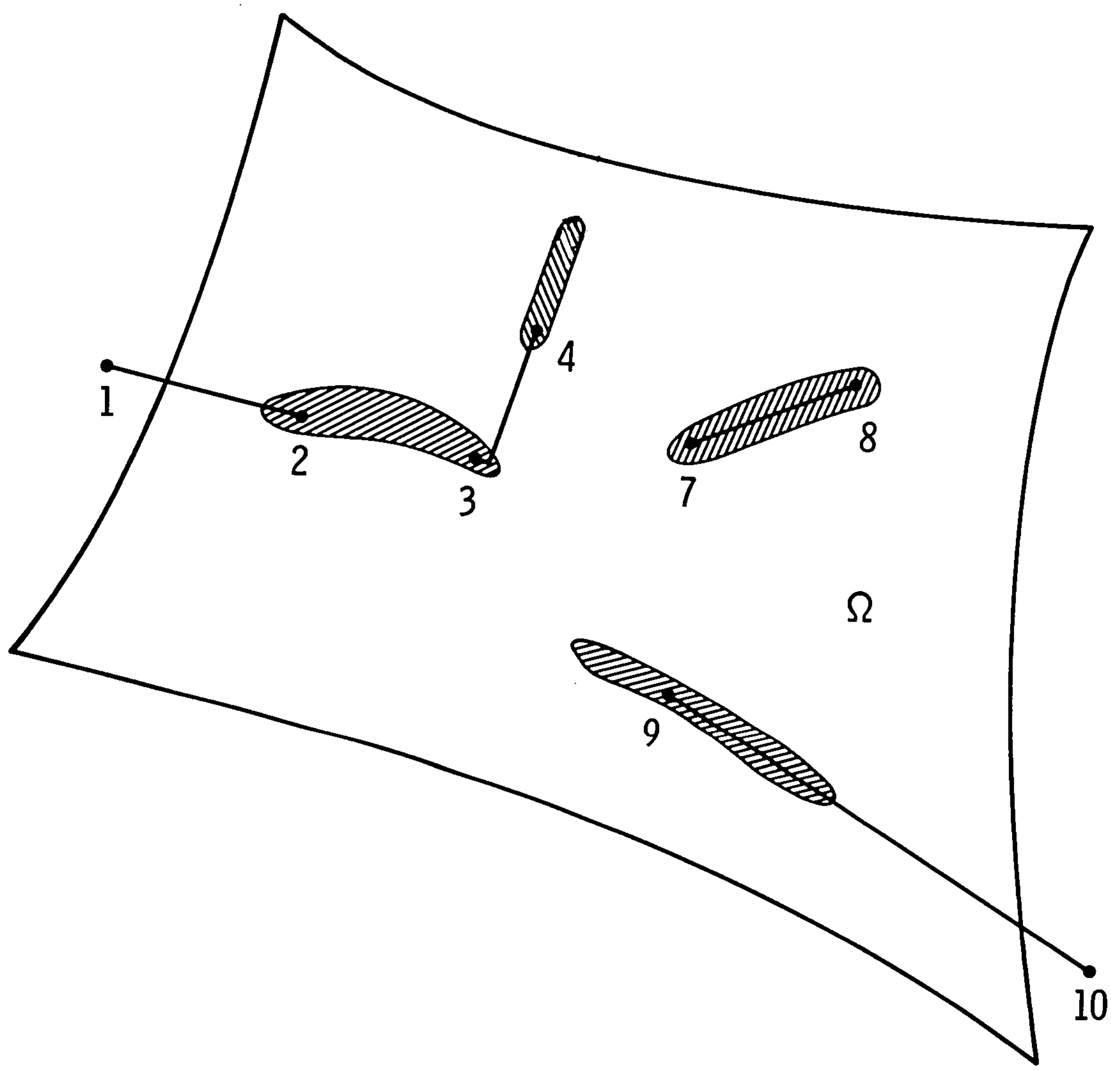

Figure 5

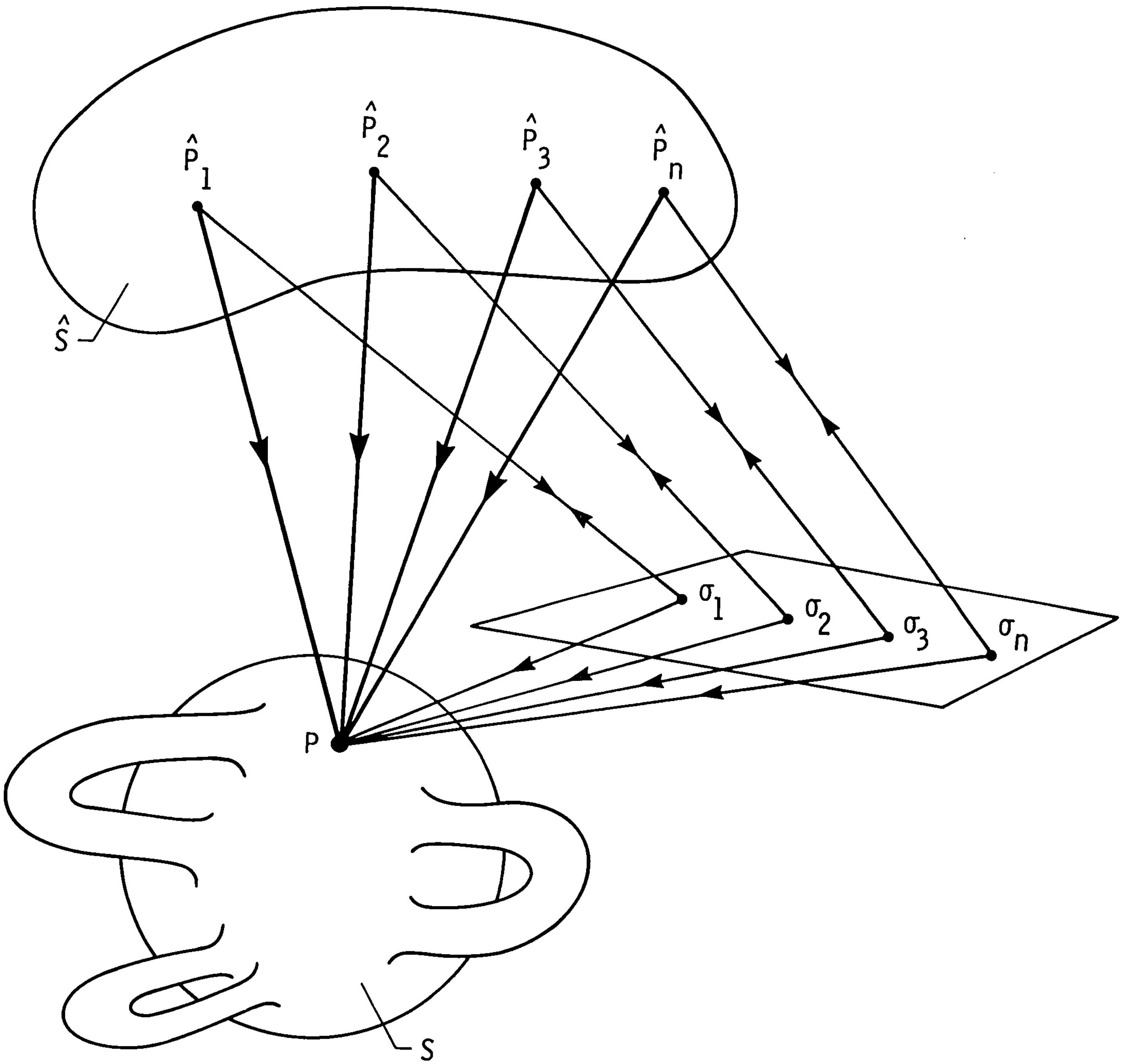

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Figure 6

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16. Abstract

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