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# Acceleration of Runaway Electrons and Joule Heating in Solar Flares

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## ABSTRACT

The electric field acceleration of electrons out of a thermal plasma and the simultaneous Joule heating of the plasma are studied. Acceleration and heating timescales are derived and compared, and upper limits are obtained on the acceleration volume and the rate at which electrons can be accelerated. These upper limits, determined by the maximum magnetic field strength observed in flaring regions, place stringent restrictions upon the acceleration process. The role of the plasma resistivity in these processes is examined, and possible sources of anomalous resistivity are summarized. The implications of these results for the microwave and hard X-ray emission from solar flares are examined.

The major conclusions are:

- (1) The simple electric field acceleration of electrons is found, in agreement with Spicer (1983), to be incapable of producing a large enough electron flux to explain the bulk of the observed hard X-ray emission from solar flares as nonthermal bremsstrahlung. For the bulk of the X-ray emission to be nonthermal, at least  $10^4$  oppositely directed current channels are required, or an acceleration mechanism that does not result in a net current in the acceleration region is required.
- (2) If the bulk of the X-ray emission is thermal, a single current sheet can yield the required heating and acceleration timescales, and the required electron energies for the microwave emission. This is accomplished with an electric field that is much smaller than the Dreicer field ( $E_D/E \sim 10-50$ ).
- (3) The rise time of the nonthermal emission is determined by the time needed to generate the required number of runaway electrons, rather than the time

needed to accelerate the electrons to the required energies, which is generally a much shorter timescale.

(4) The acceleration of enough electrons to produce a microwave flare requires the resupply of electrons to both the current sheet and the runaway region of velocity space. The electrons may be supplied by either the inflow of electrons that are already in the runaway region, or by the collisional scattering of electrons into the runaway region from the bulk plasma. For the latter case, the number of accelerated electrons is sensitive to the plasma resistivity.

(5) To obtain the required heating timescale and electron energies, the resistivity in the current sheet must be much greater than the classical resistivity of a  $n=10^9 \text{ cm}^{-3}$ ,  $T_e=10^7 \text{ K}$  plasma. A plasma density  $\sim 10^{11} \text{ cm}^{-3}$  is required in the flaring region, or, if the density in the current sheet is less than  $10^{11} \text{ cm}^{-3}$ , the resistivity in the sheet must be anomalous.  $J_{\parallel}$ -driven anomalous resistivity cannot be responsible for the higher resistivity unless the plasma electrons can first be heated to a temperature that is at least an order of magnitude greater than the ion temperature.

(6) "Inertial resistivity", which was applied to flare heating by Duijveman, Hoyng, and Ionson (1981), is shown to always be negligible compared to the classical Coulomb resistivity.

(7) Whenever the current is inductively determined by the magnetic field through Ampere's law, the energy that goes into Joule heating will always be greater than the energy that goes into accelerated electrons.

Subject headings: particle acceleration — Sun: flares

## I. Introduction

The impulsive phase of solar flares is characterized by the production of energetic charged particles and the rapid heating of thermal plasma. DC electric fields provide the simplest and most direct means of accelerating electrons out of a thermal plasma. The energy released in flares is generally understood to derive from the energy stored in magnetic fields and their associated currents. Therefore, most solar flare models result in the production of DC electric fields or, at least, electric fields that are not rapidly alternating (see Heyvaerts 1981 for a review). Since there is a macroscopic current associated with the electric field ( $J = \sigma E$ ), there will also be simultaneous Joule heating of the thermal plasma.

Specific requirements for the number and energy of energetic electrons produced during a flare, and the timescales involved in accelerating them, are provided by microwave and hard X-ray observations of flares. The microwave emission from flares is understood to be gyrosynchrotron radiation from electrons with energies of 100 keV or greater. The hard X-ray emission ( $>25$  keV photons) can be interpreted as being either thick-target bremsstrahlung from nonthermal electrons (thin-target radiation may also contribute to the X-ray emission, but the process is less efficient), or thermal bremsstrahlung from hot, impulsively heated plasma. The relative contribution of thermal and nonthermal emission to the flare radiation is not well determined.

In view of the constraints from flare microwave and hard X-ray data, and the expectation that macroscopic electric fields and currents play an important role in solar flares, it is of interest to study the electric field acceleration of electrons and the Joule heating of flare plasma without

recourse to a specific model for the generation of the currents and electric fields. The basic physics of the electric field acceleration of "runaway" electrons is developed in Section II. Fundamental timescales for the acceleration of electrons out of a thermal plasma are derived and compared. Joule heating timescales are derived in Section III, and the coupling between Joule heating and electron acceleration is examined.

The resistivity (thermal collision frequency) of a plasma plays an important role in determining both the rate at which electrons are accelerated and the Joule heating rate. Hence, the role of anomalous resistivity in these processes is examined in Section IV. The requirements for simultaneous electron acceleration and Joule heating in solar flares are studied in Section V. The major results of the paper are discussed and summarized in Section VI.

Although the application in this paper is to solar flares, for which good microwave and X-ray spectral and timing data are available, it should be noted that most of the results presented here are directly applicable to any dynamic system of currents and magnetic fields where particle acceleration is apparent. In addition to the obvious application to stellar flares in general, runaway electron acceleration may be important in supernovae, close binary star systems, active galaxies, and other astrophysical systems where steep magnetic field gradients and, hence, significant DC electric fields can arise.

## II. Electron Acceleration

The response of a thermal plasma to an imposed electric field,  $\vec{E}$ , is to set up an electric current of current density  $\vec{J} = -en\vec{v}_d = \sigma\vec{E}$ , where  $n$  is the number density of thermal electrons,  $e$  is the magnitude of the electron charge,  $\vec{v}_d$  is the mean drift velocity of the current-carrying electrons, and  $\sigma$  is the conductivity of the plasma. A fraction of the current-carrying electrons, however, those with a velocity greater than a critical velocity,  $v_c$ , will be freely accelerated out of the thermal distribution (Dreicer 1959, 1960). This can be seen from the single-particle equation of motion for an electron in the plasma:

$$m \frac{d\vec{v}}{dt} = -e\vec{E} - m\vec{v}\nu(v), \quad (1)$$

where  $m$  is the electron mass and

$$\nu(v) = \frac{4\pi n e^4 \ln\Lambda}{m^2 v^3}, \quad (2)$$

$$\ln\Lambda = \ln(12\pi n \lambda_D^3) = 23.2 + \ln\left[\left(\frac{n}{10^9 \text{ cm}^{-3}}\right)^{-1/2} \left(\frac{T}{10^7 \text{ K}}\right)^{3/2}\right], \quad (2a)$$

is the classical Coulomb collision frequency (Spitzer 1962, Knoepfel and Spong 1979). (Collisions with  $Z > 1$  ions are neglected. See Knoepfel and Spong and Spitzer for a more precise determination of  $\nu$  and  $\ln\Lambda$ . Anomalous collision frequencies are discussed in Section IV.) The Debye length  $\lambda_D = v_e/\omega_e = (kT/4\pi n e^2)^{1/2}$ , where  $v_e = (kT/m)^{1/2}$  is the electron thermal velocity and  $\omega_e = (4\pi n e^2/m)^{1/2}$  is the electron plasma frequency.

The bulk of the electrons are accelerated by the electric field until the steady state  $eE = mv_d v_e$  (equation 1) is obtained, where

$$v_e \equiv v(v_e) = 3\omega \frac{\ln \Lambda}{e} = 10.0 \left( \frac{n}{10^9 \text{ cm}^{-3}} \right) \left( \frac{T}{10^7 \text{ K}} \right)^{-3/2} \left( \frac{\ln \Lambda}{23.2} \right) \text{ s}^{-1}. \quad (3)$$

(In using the thermal collision frequency,  $\nu_e$ , it is assumed that  $v_d \ll v_e$ .) Since  $J = nev_d$ , the resistivity of the plasma ( $\eta = 1/\sigma$ ) is found to be

$$\eta = \frac{mv_e}{ne^2} = \frac{4\pi\nu_e}{\omega_e^2}. \quad (4)$$

Note that the value of the resistivity given by equations 3 and 4 is a factor of 2.6 greater than the Spitzer-Harm resistivity, which averages over the electron distribution, accounting for the skewing of the distribution in the driving electric field, in addition to electron-ion and electron-electron collisions.

As can be seen from equations 1 and 2, the frictional drag on the electrons decreases with increasing particle velocity. Hence, electrons in the initial thermal distribution with a high enough velocity will not be confined to the bulk current, but will be freely accelerated out of the thermal distribution. The condition for electrons to be in this "runaway" regime is, from equation 1,

$$eE > mv v(v) = mv_e \left( \frac{v_e}{v} \right)^3. \quad (5)$$

We see from equation 5 that a thermal electron of velocity  $v_e$  will run away if



the electric field strength is greater than

$$E_D = \frac{m}{e} v_e v_e = \frac{e \ln \Lambda}{\lambda_D^2} = 2.33 \times 10^{-8} \left( \frac{n}{10^9 \text{ cm}^{-3}} \right) \left( \frac{T}{10^7 \text{ K}} \right)^{-1} \left( \frac{\ln \Lambda}{23.2} \right) \text{ statvolt cm}^{-1}. \quad (6)$$

$E_D$  is called the Dreicer field. For smaller field strengths ( $E < E_D$ ), those electrons will run away that have velocities greater than

$$v_c = \left( \frac{m v_e v_e^3}{e E} \right)^{1/2} = \left( \frac{4 \pi n e^3 \ln \Lambda}{m E} \right)^{1/2} = \left( \frac{E_D}{E} \right)^{1/2} v_e. \quad (7)$$

It is easy to see from equations 4 and 6 that the Dreicer field and the electrical resistivity are related through the expression

$$\frac{E_D}{\eta} = n e v_e. \quad (8)$$

Since  $\eta = E/J = E/n e v_d$ , the following relationships are obtained:

$$\frac{v_d}{v_e} = \frac{E}{E_D} = \left( \frac{v_e}{v_c} \right)^2. \quad (9)$$

Hence, when  $E = E_D$ ,  $v_d = v_c = v_e$  and all of the thermal electrons are in the runaway regime.

A rough estimate of the fraction of the electrons in the initial thermal distribution that are in the runaway regime is

$$\frac{n_r}{n} = \exp \left[ - \frac{(v_c - v_d)^2}{2 v_e^2} \right] = \exp \left[ - \frac{1}{2} \left( \frac{v_c}{v_e} \right)^2 \right] \quad (10)$$

(Kaplan and Tsytovich 1973, Papadopoulos 1977). The timescale for the formation of the runaway tail is on the order of the collision time at the critical runaway velocity, or

$$\tau_T \equiv v^{-1}(v_c) = \left(\frac{v_c}{v_e}\right)^3 v_e^{-1}. \quad (11)$$

Although equation 10 estimates the initial number of electrons in the runaway regime, it does not give the total density or number of runaways produced, since, in addition to the acceleration of the initial thermal tail of electrons, new electrons are continuously scattered into the runaway region by collisions. The kinetic theory of runaway production yields the following result for the runaway rate (Kruskal and Bernstein 1964, Knoepfel and Spong 1979):

$$\gamma = 0.35 v_e \left(\frac{v_c}{v_e}\right)^{3/4} \exp\left[-2^{1/2} \left(\frac{v_c}{v_e}\right) - \frac{1}{4} \left(\frac{v_c}{v_e}\right)^2\right] s^{-1}. \quad (12)$$

This result is accurate for  $v_c \gg v_e$ . The rate at which runaway electrons are collisionally produced is, then,  $\dot{N}_{coll} = \gamma n V_J$ , where  $V_J$  is the volume of the current channel in which the electron acceleration is occurring (i.e., the volume of the acceleration region).

Before obtaining final expressions for  $\dot{N}_{coll}$  and the associated timescale, it is important to note that the volume  $V_J$  is not arbitrary, since the total allowable current in  $V_J$  is limited by the induction magnetic field associated with it. In other words, observations place an upper limit on the magnetic field strength that is present in active regions, and the magnetic

field associated with the total current  $I$  must not be stronger than this upper limit. This point was made earlier by Colgate, Audouze, and Fowler (1977) and by Colgate (1978), who argued that the hard x-ray emission from flares cannot be nonthermal bremsstrahlung from a directed beam of electrons, because the magnetic energy in the beam's self-generated field would exceed the magnetic energy available in the flare region by many orders of magnitude. It was realized by others, however, that the beam could induce a cospatial return current in the thermal plasma that would result in a negligible net current, thus avoiding the generation of a large induction field (Hoyng, Brown, and van Beek 1976; Knight and Sturrock 1977; Spicer and Sudan 1983). The current of interest here is driven by an associated electric field, however, as is required for the acceleration of runaway particles, and, therefore, there can be no cospatial return current (see also Hoyng 1977, Spicer 1983). Hence, in the acceleration region the induction magnetic field cannot be reduced by the presence of a return current.

For a current sheet of width  $w$ , thickness  $\delta r$  ( $\delta r \ll w$ ), and length  $L$ , with the current  $I$  flowing in the direction of  $L$ , the maximum value of the induction magnetic field is, from Ampere's law,  $B_J = (2\pi/c)(I/w)$ . With  $I = Jw\delta r = nev_d w\delta r$ ,  $B_J = (2\pi e/c)(nv_d \delta r)$ . Requiring  $B_J \leq B$ , the maximum magnetic field strength in the acceleration region, yields  $\delta r \leq (c/2\pi e)(B/nv_d)$ . Using equation 9 for  $v_d$ , the thickness of the current sheet is found to be

$$\delta r \leq 808 \left( \frac{B}{100 \text{ G}} \right) \left( \frac{n}{10^9 \text{ cm}^{-3}} \right)^{-1} \left( \frac{T}{10^7 \text{ K}} \right)^{-1/2} \left( \frac{v_c}{v_e} \right)^2 \text{ cm.} \quad (13a)$$

Since  $V_J = wL\delta r \equiv A\delta r$ , the acceleration volume is found to be

$$v_J \lesssim \frac{AB}{2\pi n e} \left(\frac{c}{v_e}\right) \left(\frac{v_c}{v_e}\right)^2 \quad (\text{sheet geometry}). \quad (13b)$$

The sheet area, A, is limited by the observed scale of the acceleration region. Note that, if the current is flowing on the surface of a cylinder, as is likely to be approximately the case in a magnetic flux tube, then  $A = \pi RL$ , where R is the radius of the cylinder ( $\delta r \ll R$ ).

It is interesting to compare equation 13b with the corresponding result for a filled cylindrical geometry (uniform current density filling a circular cross section - a current filament). For this case  $v_J = \pi R^2 L$  and  $B_J = (2/c)(I/R) = (2\pi e/c)(nv_d R)$ . Hence, requiring  $B_J \lesssim B$  gives  $R \lesssim (c/2\pi e)(B/nv_d)$  and

$$v_J \lesssim \frac{LB^2}{4\pi n^2 e^2} \left(\frac{c}{v_e}\right)^2 \left(\frac{v_c}{v_e}\right)^4 \quad (\text{filled cylindrical geometry}). \quad (14)$$

Since two spatial dimensions are constrained by the induction field, as compared to just one dimension for the sheet geometry, the constraint on  $v_J$  is more severe for a (filled) current filament.

Using equations 12 and 13b, the runaway production rate is found to be

$$\dot{N}_{\text{coll}} \lesssim 2.49 \times 10^7 AB v_e \left(\frac{c}{v_e}\right) f\left(\frac{v_c}{v_e}\right) \text{ electrons s}^{-1}, \quad (15)$$

where  $f(v_c/v_e) < 1$ , with  $f(v_c/v_e) = 1$  when  $v_c = 1.32v_e$ :

$$f\left(\frac{v_c}{v_e}\right) = 4.66 \left(\frac{v_c}{v_e}\right)^{11/4} \exp\left[-2^{1/2} \left(\frac{v_c}{v_e}\right) - \frac{1}{4} \left(\frac{v_c}{v_e}\right)^2\right]. \quad (16)$$

The timescale for the production of  $N$  runaway electrons is  $t_N \equiv N/\dot{N}$ , or, from equation 15,

$$t_N \gtrsim 165 \left(\frac{N}{10^{32}}\right) \left(\frac{A}{10^{18} \text{ cm}^2}\right)^{-1} \left(\frac{B}{100 \text{ G}}\right)^{-1} \left(\frac{T}{10^7 \text{ K}}\right)^{1/2} \left(\frac{v_e}{10 \text{ s}^{-1}}\right)^{-1} / f \left(\frac{v_c}{v_e}\right) \text{ s.} \quad (17)$$

In addition to the constraint on  $V_J$ , the induction field limit puts a strict upper limit on  $\dot{N}$ . The current associated with the runaway electrons is  $I_{\text{run}} = e\dot{N}$ . Since the induction field associated with this current is  $B_J = (2\pi/c)(I_{\text{run}}/w)$ , requiring  $B_J < B$  gives

$$\dot{N}_{\text{Max}} = \frac{cwB}{2\pi e} = 10^{30} \left(\frac{w}{10^9 \text{ cm}}\right) \left(\frac{B}{100 \text{ G}}\right) \text{ electrons s}^{-1}. \quad (18)$$

Note that this result is independent of the temperature, density, and resistivity of the plasma.  $I_{\text{run}}$  is a part of the total current,  $I$ , and will always be less than (or equal to)  $I$  if  $I$  is determined by  $B$  through Ampere's law (see Spicer 1983 for further discussion of this point). Hence, this maximum value of  $\dot{N}$  corresponds to the extreme case of  $I_{\text{run}} = I$ . This result for  $\dot{N}_{\text{Max}}$  leads to a minimum value for the timescale required to generate  $N$  runaway electrons:

$$t_N^{\text{min}} \equiv \frac{N}{\dot{N}_{\text{Max}}} = \frac{2\pi eN}{cwB} = 100 \left(\frac{N}{10^{32}}\right) \left(\frac{w}{10^9 \text{ cm}}\right)^{-1} \left(\frac{B}{100 \text{ G}}\right)^{-1} \text{ s.} \quad (19)$$

For a cylindrical sheet or filament geometry,  $\dot{N}_{\text{Max}}$  is larger, and  $t_N^{\text{min}}$  smaller, by a factor of  $\pi$  (with  $R$  replacing  $w$  in equations 18 and 19).

In addition to generating the runaway electrons, it is also necessary to accelerate them up to the required energies. Neglecting the collision term in equation 1, the time required to accelerate an electron from  $v_c$  to a velocity  $v$  is, for a constant electric field,

$$\tau_a = \frac{m(v-v_c)}{eE} = \left(\frac{E_D}{E}\right) \left(\frac{v}{v_e} - \frac{v_c}{v_e}\right) v_e^{-1} = \left(\frac{v_c}{v_e}\right)^2 \left(\frac{v}{v_e} - \frac{v_c}{v_e}\right) v_e^{-1}. \quad (20)$$

(If the final velocity is relativistic,  $v$  in equation 20 is replaced by  $\gamma v$ , where  $\gamma$  is the usual Lorentz factor. Fewer runaways are generated if  $v_c$  is relativistic, and none are generated if  $v_c > c$  - Connor and Hastie 1975.) An important constraint on the acceleration time is the distance over which the acceleration can occur. For the simplest case of a constant electric field, the acceleration distance is

$$x_a = \frac{1}{2} (v + v_c) \tau_a = \frac{1}{2} \left[ \left(\frac{v}{v_e}\right)^2 - \left(\frac{v_c}{v_e}\right)^2 \right] \left(\frac{v_c}{v_e}\right)^2 v_e v_e^{-1} = \frac{W_f - W_c}{eE}, \quad (21)$$

where  $W_f$  is the energy of the accelerated electron and  $W_c = mv_c^2/2$ . If the available distance over which the electrons can be accelerated is  $L$ , then requiring  $x_a < L$  yields an upper limit on  $\tau_a$ :

$$\tau_a \leq \frac{2L}{v+v_c} = 0.2 \left(\frac{L}{10^9 \text{ cm}}\right) \left(\frac{v+v_c}{10^{10} \text{ cm s}^{-1}}\right)^{-1} \text{ s}. \quad (22)$$

This requirement also leads to a lower limit on the electric field strength:

$$E \geq \frac{W_f - W_c}{eL} = 3.3 \times 10^{-7} \left(\frac{L}{10^9 \text{ cm}}\right)^{-1} \left(\frac{W_f - W_c}{100 \text{ keV}}\right) \text{ statvolt cm}^{-1}. \quad (23)$$

It is interesting that if we also require  $E < E_D$ , so that the entire particle distribution is not in the runaway regime, a lower limit on the collision frequency in the plasma is obtained (equations 6 and 23):

$$v_e > \frac{W_f - W_c}{m v_e L}, \quad \text{or} \quad \left( \frac{v_e}{10 \text{ s}^{-1}} \right) > 14 \left( \frac{W_f - W_c}{100 \text{ keV}} \right) \left( \frac{T}{10^7 \text{ K}} \right)^{-1/2} \left( \frac{L}{10^9 \text{ cm}} \right)^{-1}. \quad (24)$$

If the collision frequency is classical, this puts a constraint on the density and temperature in the current channel (equation 3). A time varying electric field is considered in Section III.

In closing this section, it is worthwhile to compare the runaway tail formation, particle generation, and acceleration timescales,  $\tau_T$ ,  $\tau_N$ , and  $\tau_a$ . The time required to accelerate  $N$  electrons out of a thermal plasma to a velocity  $v$  is determined by the longest of these timescales. It can be seen from equations 11 and 20 that  $\tau_a = [(v/v_c) - 1] \tau_T$ . therefore,  $\tau_a > \tau_T$  whenever  $v > 2v_c$ . Since  $v \gg v_c$  for most astrophysical systems, the timescale  $\tau_T$  is generally not important. Hence, the required time is determined by either  $\tau_a$  or  $\tau_N$ . The timescale  $\tau_a$  is restricted by an upper limit (equation 22), while a lower limit on  $\tau_N$  was found (equation 19). For the reference parameters used here in deriving the numerical coefficients, which are typical of a solar flare region,  $\tau_N$  is seen to exceed  $\tau_a$ . Hence,  $\tau_N$  is likely to be the dominant timescale for most astrophysical systems.

### III. Joule Heating

The purpose of this section is to derive an estimate of the time required

to heat a current-carrying region through current dissipation, rather than to develop a detailed model for specific conditions. Hence, an estimate of the time required to heat a volume  $V$  to a given temperature will be obtained, without considering the details of the dissipation process, or heat loss mechanisms that may eventually balance the Joule heating. This timescale will be compared with the runaway timescales derived in Section II. Models for the heating of non-flaring coronal loops through current dissipation have been developed by a number of authors (Tucker 1973, Rosner et al. 1978, Hinata 1980, Benford 1983). Joule heating in solar flares has been studied by Spicer (1981 a,b) and by Duijveman, Hoyng, and Ionson (1981).

Since the energy dissipated by a current density  $J$  is  $J \cdot E$  erg  $\text{cm}^{-3} \text{s}^{-1}$ , a Joule heating timescale  $\tau_J \equiv nkT/(J \cdot E)$  can be defined. Using  $J=E/\eta$  together with equations 4, 6, and 9 gives

$$\tau_J \equiv \frac{nkT}{J \cdot E} = \left(\frac{E_D}{E}\right)^2 v_e^{-1} = \left(\frac{v_c}{v_e}\right)^4 v_e^{-1}. \quad (25)$$

Note that  $\tau_J$  is always longer than the runaway tail formation timescale (equation 11) as long as  $v_c > v_e$ , since  $\tau_J/\tau_T = v_c/v_e$ . The timescale  $\tau_J$  is shorter than the runaway acceleration timescale,  $\tau_a$  (equation 20), whenever  $v > [(v_c/v_e)+1]v_c$ .

The timescale  $\tau_J$  is generally only a lower limit on the actual heating time,  $t_J$ , since the volume of plasma to be heated,  $V$ , is usually much larger than the volume of the current channel,  $V_J$ . The global heating of the plasma, neglecting losses, is determined by the equation (cf. Duijveman et al. 1981)



$$\frac{3}{2}nk\frac{dT_V}{dt} = (J \cdot E)V_J. \quad (26)$$

The heat deposited in the current channel may be conductively or convectively transported to the larger volume. It is implicitly assumed here that the heat is distributed throughout the larger volume on a timescale that is less than or on the order of the heating timescale. Otherwise the X-ray rise time will be determined by the heat transfer timescale rather than by the Joule heating timescale.

It is apparent from equation 26 that in order to correct for the larger volume to be heated,  $\tau_J$  should be divided by the factor  $\epsilon \equiv V_J/V$ , the fraction of the total volume occupied by the current channel. If the density in the current channel,  $n$ , is different from the average density in the volume  $V$ ,  $n_V$ , then  $\tau_J$  should also be multiplied by the factor  $n_V/n$ , where  $\tau_J$  is evaluated for the physical parameters within the current channel. (The density in an equilibrium current sheet may be larger than that in the surrounding medium, for example, so that pressure balance is maintained.) Using equation 13 for  $V_J$ , the heating timescale is found to be

$$\tau_J = \left(\frac{n_V}{n}\right) \tau_J / \epsilon \geq 1.24 \times 10^5 \left(\frac{A}{10^{18} \text{ cm}^2}\right)^{-1} \left(\frac{B}{100 \text{ G}}\right)^{-1} \left(\frac{n_V}{10^9 \text{ cm}^{-3}}\right) \left(\frac{V}{10^{27} \text{ cm}^3}\right) \left(\frac{T}{10^7 \text{ K}}\right)^{1/2} \left(\frac{v_e}{10 \text{ s}^{-1}}\right)^{-1} \left(\frac{v_c}{v_e}\right)^2 \text{ s}. \quad (27)$$

Since  $\epsilon$  is typically much less than unity, this timescale is generally much longer than  $\tau_J$ .

In order to obtain a better feeling for the derived timescales, the

remainder of this section will be devoted to examining some simple models for the time development of the Joule heating and electron acceleration. The plasma density, magnetic field, and the volume  $V$  will be taken to remain constant. The Joule heating term in equation 26 can be written as  $(\mathbf{J} \cdot \mathbf{E})V_J = V_J E^2 / n$ . If  $E$  and  $V_J$  both remain constant, this term is proportional to  $T^{3/2}$ , since  $n$  varies as  $T^{-3/2}$  (the slow variation of  $\ln \Lambda$  with  $T$  will be neglected). On the other hand, if the cross sectional area of the current channel is determined by the ambient magnetic field so that  $V_J$  scales with temperature as  $T^{-3/2}$  (equation 13), the Joule heating term is independent of  $T$ . For this case the total current,  $I$ , remains constant, while  $I$  is not conserved if  $V_J$  remains constant. If  $J$  instead of  $E$  remains constant, the Joule heating term  $(\mathbf{J} \cdot \mathbf{E})V_J = V_J n J^2$  is proportional to  $T^{-3/2}$ .  $V_J$  necessarily remains constant in this case, and the total current is conserved. Using equation 26, the thermal evolution of the plasma can be obtained for these three cases by solving the equation

$$\frac{d\tilde{T}}{dt} = \frac{2}{3t_J} \tilde{T}^s, \quad (28)$$

where  $\tilde{T} = T/T_0$ ,  $T_0$  is the temperature of the plasma at time  $t=0$ , and  $t_J$  is evaluated for  $T=T_0$  (equation 27). The solution to equation 28 is

$$T = T_0 \left[ 1 + \frac{2}{3}(1-s) \frac{t}{t_J} \right]^{1/(1-s)}. \quad (29)$$

For the case of  $E$  and  $V_J$  constant ( $s=3/2$ ), an increase in temperature by a factor of 10 requires  $t=2t_J$ . The temperature increases rapidly, within the

time  $t=3t_J$ , until the heating is balanced by losses. Since  $E$  is constant, the runaway acceleration time  $t_a=\tau_a$  (equation 20). The runaway production time,  $t_N$ , is given by equation 17 with  $T=T_0$  if  $t_N \ll t_J$ . The dominant effect of increasing the temperature on  $t_N$  is to increase  $f(v_c/v_e)$ , decreasing  $t_N$ . If  $t_N(T_0) \gg t_J$  and  $t_N(T_{Max}) \ll t_J$ , the runaway production time will be on the order of  $t_J$ . If  $t_N(T_0)$  and  $t_N(T_{Max})$  are both longer than  $t_J$ , the actual runaway production time will fall between  $t_N(T_{Max})$  and  $t_N(T_0)$ .

When  $E$  and  $I$  are held constant ( $s=0$ ), the temperature increase is more gradual. A factor of 10 increase in temperature requires  $t=13.5t_J$  and the temperature increases linearly. Hence,  $t_J(T_0)$  can significantly underestimate the actual heating time.

When  $J$  and  $I$  remain constant ( $s=-3/2$ ), the temperature increase is even more gradual. An order of magnitude increase in temperature requires  $t=189t_J$ . This slow heating occurs because of the  $T^{-3/2}$  dependence of the classical resistivity so that, as the temperature increases, the heating rate decreases. Since  $E=nJ$  is not constant, the acceleration time is also modified. Using equation 29 with  $s=-3/2$  for the time dependence of the temperature in  $E=nJ$ , and solving equation 1 without the frictional drag term, gives

$$t_a = \frac{3}{5} t_J \left[ \left( \frac{2\tau_a}{3t_J} + 1 \right)^{5/2} - 1 \right] . \quad (30)$$

When  $\tau_a \ll t_J$ ,  $t_a = \tau_a$ . If  $\tau_a \gg t_J$ ,  $t_a = 0.22(\tau_a/t_J)^{3/2} \tau_a$ . It is interesting that, unlike for the case of constant  $E$ ,  $t_N$  increases with increasing  $T$ .

The simple models presented here primarily indicate the care that must be

exercised in applying the derived timescales to astrophysical systems. They indicate how the actual physical times can vary from the estimated timescales under different physical conditions. Keeping these considerations in mind, however, they do indicate the conditions that are required for electric field acceleration and Joule heating to be applicable to a given astrophysical system.

#### IV. Anomalous Resistivity

The plasma resistivity can be much higher than the classical value (equations 3 and 4) if low-frequency electrostatic turbulence is present in the current channel. The current density  $J$  will induce the growth of such turbulence if its drift velocity ( $v_d$ ) exceeds a threshold velocity that is greater than the plasma ion sound speed,  $c_s = (kT_e/m_i)^{1/2}$ . The generation of anomalous resistivity by currents flowing parallel to the ambient magnetic field has been reviewed by Papadopoulos (1977). The generation of anomalous resistivity by currents flowing across the magnetic field has been reviewed by Papadopoulos (1980) and by Huba (1983). The growth of low-frequency electrostatic turbulence may also be stimulated by temperature or pressure gradients in the plasma (cf. Morrison and Ionson 1982), or by the presence of streaming, nonthermal electrons in the plasma (Papadopoulos 1977). The computed thresholds and wave levels for the  $J_{||}$ -driven instabilities, which are the better studied of the current driven instabilities, are summarized in this section.

For a current flowing along the ambient magnetic field, the instabilities that can lead to anomalous resistivity are the electrostatic ion cyclotron

(EIC), ion acoustic (IA), and Buneman instabilities. The Buneman instability is not of primary interest here, since its threshold velocity is given by  $v_d \gg 2v_e$  and, therefore, the instability only occurs when the entire thermal electron distribution is in the runaway regime. Computer simulations indicate that heating causes the instability to saturate when  $v_e = v_d$ . If the current is driven by a constant electric field the instability oscillates, with both  $v_d$  and  $v_e$  increasing in steps, keeping  $v_d = v_e$ .

The instability thresholds for the EIC and IA instabilities have been computed by Kindel and Kennel (1971) and Morrison and Ionson (1982) and references therein. These thresholds are a sensitive function of the ratio of the electron temperature to the ion temperature,  $T_e/T_i$ . The thresholds are plotted in Figure 1, based upon the results of Morrison and Ionson. These authors have accounted for the skewing of the electron distribution function in the driving electric field, and their (steady-state current) velocity thresholds are somewhat higher (by a factor of approximately 2) than those of Kindel and Kennel and others. ( $T_e$  is the temperature of the unperturbed Maxwellian distribution function.) The effect of runaway electrons has not been included, which is likely to further increase the threshold velocities. For  $0.1 < T_e/T_i < 8$  the EIC instability has the lowest threshold (cf. Kindel and Kennel). Otherwise, the IA instability is the first to go unstable. When  $T_e = T_i$ ,  $v_{thr}(EIC) = 0.8v_e$  ( $\approx 35c_s$ ). When  $T_e = 8T_i$ ,  $v_{thr}(EIC, IA) = 8c_s$ . For  $v_{thr} = c_s$ ,  $T_e$  must be more than an order of magnitude larger than  $T_i$ , with the IA mode being the unstable mode.

Since the wave turbulence has the effect of increasing the collision frequency in the plasma, both the Joule heating and the runaways are

affected. If the effective collision frequency due to the wave turbulence,  $\nu_{\text{eff}}(v)$ , simply scales with particle velocity in the same way as the classical collision frequency, so that  $\nu_{\text{eff}}(v) = \nu_{\text{eff}}(v_e/v)^3$ , the results of the previous sections carry over to the case of a turbulent plasma by simply replacing  $v_e$  with  $\nu_{\text{eff}}$ . Such a velocity dependence is expected for IA turbulence (Kaplan and Tsytovich 1973, Papadopoulos 1977). The turbulent collision frequency will be assumed here to always have the simple  $v^{-3}$  velocity dependence. Hence, in the turbulent plasma,  $\nu_e \rightarrow \nu_{\text{eff}}$ ,  $n_{\text{eff}} = 4\pi\nu_{\text{eff}}/\omega_e^2$ ,  $E_D^{\text{eff}} = (m/e)\nu_e\nu_{\text{eff}}$ ,  $\nu_c^{\text{eff}} = (m\nu_{\text{eff}}\nu_e^3/eE)^{1/2}$ , etc.

The approximate relationship between the anomalous collision frequency and the energy density in low-frequency IA turbulence,  $W^{\text{IA}}$ , is

$$\nu_{\text{eff}}^{\text{IA}} = \left( \frac{W^{\text{IA}}}{nkT_e} \right) \omega_e. \quad (31)$$

The actual turbulence level that will be reached for a given instability is not well established, and depends upon the details of the system. Simulations of the IA instability typically yield an effective collision frequency that is on the order of

$$\nu_{\text{eff}}^{\text{IA}} = 10^{-2} \omega_e = 10^6 \left( \frac{n}{10^9 \text{ cm}^{-3}} \right)^{1/2} \left( \frac{v_e}{10^8 \text{ s}^{-1}} \right)^{-1} v_e \text{ s}^{-1}. \quad (32)$$

Hence, an effective resistivity that is as much as a million times greater than the classical resistivity may be obtained if the drift velocity of the primary current exceeds the threshold for the IA instability.

The anomalous resistivity that can result from the EIC instability is

less well established than the result for the IA instability. Quoted here is the result of Ionson (1976), which has been found to be consistent with a computer simulation of the instability (Pritchett, Ashour-Abdalla, and Dawson 1981):

$$v_{\text{eff}}^{\text{EIC}} = 0.2\Omega_i = 2 \times 10^4 \left( \frac{B}{100 \text{ G}} \right) \left( \frac{v_e}{10 \text{ s}^{-1}} \right)^{-1} v_e \text{ s}^{-1}, \quad (33)$$

where  $\Omega_i$  is the ion gyrofrequency. If both the EIC and the IA modes are present, the IA mode will generally provide the largest contribution to the effective resistivity.

If the primary current is stable to the EIC and IA instabilities, it may be possible for a significant level of low-frequency IA turbulence to be generated by the runaway electrons. If the runaway electron distribution develops a positive slope so that it is unstable to the growth of Langmuir waves, and the energy density in these waves (normalized to  $nkT$ ) reaches a level on the order of  $(v_e/v_r)^2$ , where  $v_r$  is the mean streaming speed of the runaway electrons, the waves can nonlinearly collapse to shorter wavelengths, generating ion density fluctuations in the process (the oscillating two-stream instability - OTS). The effective collision frequency resulting from the presence of these density fluctuations has been estimated by Papadopoulos and Coffey (1974) to be

$$v_{\text{eff}}^{\text{OTS}} = 0.3\omega_e \left( \frac{n_r}{n} \right) \left( \frac{v_r}{v_e} \right)^2 \left( \frac{\Delta v_r}{v_r} \right), \quad (34)$$

where  $\Delta v_r$  is the velocity spread in the runaway electron beam. For  $\Delta v_r/v_r =$

1/3,  $v_r/v_e = 10$ , and  $n_r = 10^{-3}n$ , for example,  $v_{\text{eff}}^{\text{OTS}} = 10^{-2}\omega_e$ . Hence, it may be possible to generate an anomalous resistivity that is comparable to that estimated for the IA instability (equation 33). The generation of anomalous resistivity by streaming relativistic electrons has been studied by Scott et al. (1980), with application to the heating of gas in clusters of galaxies by cosmic ray electrons.

An additional form of resistivity, "inertial resistivity" (Speiser 1970), has been applied to the heating of plasma in solar flares by Duijveman, Hoyng, and Ionson (1981). The argument for inertial resistivity is that if the time required for thermal electrons to cross the current sheet,  $\sim \delta r/v_e$ , where  $\delta r$  is the thickness of the sheet (cf. equation 13), is less than a collision time,  $v_e^{-1}$ , the electrons will be accelerated by the electric field while they are within the sheet and a net current density  $J_{\text{IN}} = (ne^2/m)(\delta r/v_e)E$  will flow. Since this current is thermalized in a few collision times, there is a contribution to the Joule heating rate with an effective collision frequency of  $v_{\text{eff}}^{\text{IN}} = v_e/\delta r$ . (This assumes, of course, that the crossing of the sheet by the thermal electrons is not suppressed by the ambient magnetic field.) Since  $\delta r \lesssim 1$  km for solar flare conditions (see equation 13a and Section V.a.), the ratio  $v_{\text{eff}}^{\text{IN}}/v_e$  can be quite large. This result is misleading, however, since  $v_{\text{eff}}^{\text{IN}}$  is derived on the basis of the current density  $J_{\text{IN}}$ , which is quite small when compared with the steady-state current density  $J = E/\eta_e$ , where  $\eta_e$  is the classical (Coulomb) resistivity. It is easy to see that the ratio of these current densities is  $J_{\text{IN}}/J = v_e \delta r/v_e$ , and the ratio of the corresponding Joule heating rates per unit volume is  $v_e J_{\text{IN}}^2/v_e J^2 = (v_e \delta r/v_e)^2$ . Since the ratio of the volume containing  $J_{\text{IN}}$  to the volume containing  $J$  is  $V_{\text{IN}}/V_J = v_e/v_e \delta r$ , the



ratio of the total heating rates is  $v_e J_{IN}^2 v_{IN} / v_e J^2 v_J = v_e \delta r / v_e$ . The original condition for inertial resistivity to be important was  $v_e \delta r / v_e < 1$ , however. Hence, heating due to inertial resistivity will never be significant compared to the heating by the primary current. The timescale for  $J_{IN}$  to be thermalized is the same as the timescale for the steady-state current density  $J$  to be established (a few collision times). Therefore, even if a steady-state current density ( $J$ ) has not yet been achieved, inertial effects cannot contribute to the heating of the plasma until the larger current density  $J$  is established.

#### V. Application to Solar Flares

A typical microwave flare requires that  $10^{31} - 10^{32}$  electrons be accelerated to energies on the order of 100 keV. It is interesting to compare this number of electrons with the number that can be supplied by the original distribution of thermal electrons in the current channel,  $N = n(v > v_c) V_J$ , without considering the resupply of particles to the system. The maximum number of electrons that can be accelerated is obtained when  $v_d = v_c = v_e$  and  $n(v > v_c) = n$ , giving (equation 13b)

$$N < \frac{BA}{2\pi e} \left(\frac{c}{v_e}\right) = 8.1 \times 10^{29} \left(\frac{B}{100 \text{ G}}\right) \left(\frac{A}{10^{18} \text{ cm}^2}\right) \left(\frac{T}{10^7 \text{ K}}\right)^{-1/2} \text{ electrons.} \quad (35)$$

This result, which is independent of the density and the resistivity of the plasma, corresponds to accelerating all of the electrons in the current channel. A sheet area of  $10^{18} \text{ cm}^2$ , if spread out in a single sheet parallel to the solar surface, corresponds to a 14" x 14" sheet, as seen from the earth.

The number of accelerated electrons decreases rapidly with an increase in the ratio  $v_c/v_e$  (equation 10). Note that a much smaller limit on  $N$  is obtained if the current is assumed to be confined to a cylindrical volume (equation 14), rather than to a sheet geometry.

The maximum observed magnetic field strengths in flaring regions are on the order of 1000 G. It is interesting that under plausible, although somewhat extreme conditions it is possible for enough electrons to be accelerated from the original plasma in the channel to produce a microwave flare. In general, however, additional time is required for more electrons to be supplied from the thermal plasma. These particles must be obtained from plasma outside of the acceleration region. The rate at which runaways are produced is then determined by the collisional rate,  $\dot{N}_{coll}$  (equation 15), or the rate at which new thermal electrons with  $v > v_c$  flow into the current channel, whichever is greater. The current density  $J$  can supply electrons with  $v > v_c$  to the acceleration region at the rate  $\dot{N}_{RJ} = n(v > v_c)v_d w \delta r$ . If plasma is flowing into the current sheet from the sides, as in a region where magnetic field merging is occurring, electrons in the runaway regime can be supplied at the rate  $\dot{N}_{RA} = 2n_o(v > v_c)v_{IN}A$ , where  $n_o$  is the density of the plasma outside of the current sheet and  $v_{IN}$  is the inflow speed of the plasma. For a steady-state magnetic neutral sheet, for example,  $v_{IN}$  is estimated to be  $\sim 0.01-0.1v_A$ , where  $v_A = B/(4\pi n_o)^{1/2}$  is the Alfvén speed (Priest 1981).  $\dot{N}_{coll}$  is generally larger than  $\dot{N}_{RJ}$ . The ratio of  $\dot{N}_{coll}$  to  $\dot{N}_{RA}$  is

$$\frac{\dot{N}_{coll}}{\dot{N}_{RA}} = 1.4 \times 10^{-5} \left( \frac{B}{100 \text{ G}} \right) \left( \frac{v_e}{10 \text{ s}^{-1}} \right) \left( \frac{n_o}{10^9 \text{ cm}^{-3}} \right)^{-1} \left( \frac{v_{IN}}{10^8 \text{ cm s}^{-1}} \right)^{-1} \left( \frac{T}{10^7 \text{ K}} \right)^{1/2}$$

$$\cdot \left(\frac{v_c}{v_e}\right)^{11/4} \exp\left[-\frac{1}{2}\left(\frac{v_c}{v_e}\right) + \frac{1}{4}\left(\frac{v_c}{v_e}\right)^2\right] , \quad (36)$$

where equation 10 has been used for  $n_0(v > v_c)$ . For  $v_c/v_e = 5$  and the other parameters as shown, for example,  $\dot{N}_{\text{coll}}/\dot{N}_{\text{RA}} = 5 \times 10^{-4}$ . Hence,  $\dot{N}_{\text{RA}}$  is greater than  $\dot{N}_{\text{coll}}$  unless  $v_{\text{IN}}$  is small or the thermal collision frequency in the channel is much greater than  $10 \text{ s}^{-1}$ . Irrespective of whichever process determines the runaway production rate, it cannot exceed the upper limit given by equation 18, however.

When  $\dot{N}_{\text{RA}}$  does not exceed  $\dot{N}_{\text{coll}}$ , an additional requirement for the runaway rate to be determined by  $\dot{N}_{\text{coll}}$  is that thermal electrons be flowing into the channel at a rate that is greater than  $\dot{N}_{\text{coll}}$ . Hence, either  $\dot{N}_J = n v_d w d r$  or  $\dot{N}_A = 2 n_0 v_{\text{IN}} A$  must exceed  $\dot{N}_{\text{coll}}$ . Multiplying equation 36 by the factor  $\exp[-(v_c/v_e)^2/2]$  gives the ratio  $\dot{N}_{\text{coll}}/\dot{N}_A$ . It is easily seen that, unless there is little or no inflow from the sides of the channel,  $\dot{N}_A$  does exceed  $\dot{N}_{\text{coll}}$ . The ratio  $\dot{N}_{\text{coll}}/\dot{N}_J$  is equal to simply  $\gamma L/v_d$ . The timescale  $\gamma^{-1}$  represents the time required for all of the thermal electrons entering the current sheet to be scattered into the runaway regime, while  $L/v_d$  is the time required for an average thermal electron in the current density  $J$  to traverse the length of the sheet.  $\dot{N}_{\text{coll}} < \dot{N}_J$  when  $L/v_d < \gamma^{-1}$ . It is interesting that when  $\dot{N}_{\text{coll}} \gg \dot{N}_J$ , the current is dominated by runaway electrons so that there is primarily particle acceleration and minimal Joule heating.  $\dot{N}_{\text{coll}}$  can exceed  $\dot{N}_J$  when the resistivity in the channel is high and  $v_c/v_e$  is small. As can be seen from equations 13 and 18, however,  $\dot{N}_J = \dot{N}_{\text{Max}}$  if  $B_J = B$ . In this case  $\dot{N}_{\text{coll}}$  will always be less than or equal to  $\dot{N}_J$ , since  $\dot{N}_{\text{coll}}$  cannot exceed  $\dot{N}_{\text{Max}}$ . Hence, as has been emphasized by Spicer (1983), the primary current will always exceed the

runaway current if  $J$  is determined by  $B$  through Ampere's law and, therefore, the production of runaways will be maintained by  $\dot{N}_J$ .

It has been noted (cf. Norman and Smith 1978) that since the fraction of electrons in the runaway regime depends only upon  $v_c/v_e$ , and, therefore,  $v_d/v_e$  (equations 9 and 10), if  $v_d$  remains constant and there is a change in the plasma resistivity, there is no change in the number of accelerated electrons. Since the number of accelerated electrons depends in general upon the runaway production rate,  $\dot{N}$ , rather than simply upon the fraction of particles initially in the runaway regime, however, this conclusion is not universally valid. If the runaway rate is determined by  $\dot{N}_{RA}$  and  $v_{IN}$  is not sensitive to the resistivity in the channel, this conclusion remains valid. If the runaway rate is determined by  $\dot{N}_{coll}$ , however, the number of accelerated particles is sensitive to the resistivity in the current channel.

The upper limit on  $\dot{N}$  is particularly important for interpreting the hard X-ray emission from flares. Interpreting the  $\gtrsim 25$  keV X-ray emission from a flare to be thick-target, nonthermal bremsstrahlung requires a minimum of  $10^{35}$  electrons  $s^{-1}$  to be accelerated. Since observed magnetic field strengths are limited to  $B \lesssim 1000$  G, equation 18 shows that such a high particle flux cannot be achieved. Hence, the simple electric field acceleration of runaway electrons cannot produce a high enough electron flux to explain the bulk of the observed X-ray emission as non-thermal bremsstrahlung. This result agrees with a similar result obtained by Spicer (1983; see also Hoyng 1977). The hard X-ray emission could all be non-thermal if the flaring region is filamented into many small, oppositely directed (so that the net current is small or vanishes) current channels (with each channel conforming to the upper

limit on  $\dot{N}$ ). This requires at least  $10^4$  individual current channels to produce a ( $>25$  keV) hard X-ray burst, however. The plausibility of producing this situation, and its stability, requires further study. Alternatively, an efficient statistical acceleration process, which does not require or result in a net current in the acceleration region, may be able to generate the required flux of nonthermal electrons. Otherwise, although the highest energy X-rays may be nonthermal, the bulk of the hard X-ray emission must be thermal.

In view of the preceding results, it is of interest to determine what conditions are required for the production of an impulsive hard X-ray burst through Joule heating, and an accompanying microwave burst through the acceleration of runaway electrons when a single current sheet is present. The microwave and hard X-ray rise times in solar flares range from seconds to hundreds of seconds (cf. Wiehl et al. 1983). For concreteness, a timescale of 30 sec will be taken here as typical of both the microwave and hard X-ray rise times. The hard X-ray rise time is related to  $t_J$  and, considering the discussion at the end of Section II, the microwave rise time is expected to be related to  $t_N = N/\dot{N}$ . The exact relationship between  $t_J$  and  $t_N$  and the rise times depends upon the individual flare, but for the purposes of this analysis it is reasonable to take  $t_J = t_N = 30$  sec. In equation 27 for  $t_J$ ,  $n_V$  and  $V$  are not independent, since they are related through the (observed) emission measure of the thermal X-rays,  $EM = n_V^2 V$ . It is apparent from inspection of equation 27 that  $t_J$  can only be of the right magnitude if  $v_e$  is much greater than  $10 \text{ s}^{-1}$  in the current channel, or if the X-ray emission volume is much smaller than  $10^{27} \text{ cm}^3$  (and, therefore,  $n_V$  is larger than  $10^9 \text{ cm}^{-3}$ ). Note also that  $B$  must be somewhat larger than 100 G, or  $w$  larger than  $10^9$  cm, if the

lower limit on  $t_N$  (equation 19) is to be satisfied when  $N=10^{32}$  electrons.

Taking the runaway production rate to be determined by  $\dot{N}_{\text{coll}}$ , so that  $t_N$  is given by equation 17, estimates for the required values of the collision frequency and  $v_c/v_e$  can be obtained. The value of  $v_c/v_e$  depends only upon the ratio of  $t_J$  to  $t_N$  and the value of  $n_V V/N$ . For  $t_J=t_N$  and  $n_V=10^9 \text{ cm}^{-3}$ ,  $V=10^{27} \text{ cm}^3$ , and  $N=10^{32}$ , so that  $n_V V/N = 10^4$  (Case I),  $v_c/v_e=6$  and, using equation 9,  $v_d=1.2c_s$ . When  $n_V=10^{11} \text{ cm}^{-3}$  and  $V=10^{23} \text{ cm}^3$ , so that  $n_V V/N = 10^2$  (Case II),  $v_c=4v_e$  and  $v_d=2.5c_s$ . Similar results are obtained for  $v_c$  if  $t_N$  is determined by  $\dot{N}_{\text{RA}}$  with  $t_N=30 \text{ sec}$  and  $v_{\text{IN}}=10^8 \text{ cm s}^{-1}$ , rather than by  $\dot{N}_{\text{coll}}$ .

From equation 27 with  $t_J=30 \text{ sec}$ , the required collision frequency in the current channel for Case I is found to be  $\sim 10^5$  times greater than the classical collision frequency when  $n=10^9 \text{ cm}^{-3}$  and  $T=10^7 \text{ K}$  (this factor decreases somewhat if A or B is increased). Therefore, either the density in the current channel must be  $\sim 10^{14} \text{ cm}^{-3}$  ( $n$  can be smaller if  $T$  is lower), or the resistivity in the channel must be anomalous. For Case II with  $n=n_V$ , the required collision frequency is only  $\sim 10$  times greater than the classical value. Densities as high as  $10^{14} \text{ cm}^{-3}$  (but with  $T \sim 10^5 \text{ K}$ ) have been predicted for pre-flare current sheets (Syrovatskii 1976). Densities of  $10^{11} \text{ cm}^{-3}$  are commonly deduced from UV observations of flare loops (Dere et al. 1979 - it is not at present clear whether these densities are representative of the impulsive hard X-ray region, however). Hence, the impulsive microwave and hard X-ray emissions can be generated without the presence of anomalous resistivity within the current sheet. This is most easily accomplished with a plasma density  $\sim 10^{11} \text{ cm}^{-3}$  in both the current sheet and the X-ray emitting region. For a density of  $10^{14} \text{ cm}^{-3}$  and  $T \sim 10^7 \text{ K}$ , the thickness of the current

sheet is  $\leq 1$  cm (equation 13a) and is less than or on the order of the gyroradius of a 100 keV electron (depending upon the value of B). It is also worth noting that for this case, with  $T \sim 10^7$  K the bremsstrahlung cooling time in the sheet is on the order of the Joule heating time. If the density in the current channel is  $\sim 10^9 - 10^{10}$  cm $^{-3}$ , anomalous resistivity is required. For  $J_{||}$ -generated anomalous resistivity, Figure 1 and the derived values for  $v_d$  indicate that the ion acoustic instability will dominate. A high value of  $T_e/T_i$  ( $> 10$ ) is required for the instability, however.

Although it is beyond the scope of this paper to consider heat transport mechanisms in flares, it is desirable to check the plausibility of the assumption made in Section III that the heat transfer timescale is less than or on the order of the Joule heating timescale. Taking the heat to be transported primarily in the direction perpendicular to the plane of the current sheet, the effective speed at which the heat must be transported is on the order of  $V/2At_J$ . This gives  $\sim 100$  km s $^{-1}$  for Case I and  $\sim 1,000$  cm s $^{-1}$  for Case II. Both velocities are well below the Alfvén speed, and the highest (Case I) is on the order of the ion sound speed. Hence, it is reasonable to take  $t_J$  to be the dominant timescale. It is certainly possible that in some flares the X-ray rise time is determined by the heat transport timescale, rather than by  $t_J$ . In the absence of an additional heating mechanism, however, these flares will be characterized by a declining temperature during the rise phase of the flare, as the heat energy is spread over the larger volume. In this case, the results of the preceding paragraph are minimum requirements for the Joule heating to be accomplished with a single current sheet.

The time required to accelerate an electron from  $v_c$  to an energy of 100 keV ( $v = 1.64 \times 10^{10} \text{ cm s}^{-1}$ ) is given by equation 20. For Case I  $\tau_a \sim 10^{-4}$  sec, while for Case II  $\tau_a \sim 10^{-2}$  sec. These acceleration times are much less than  $\tau_N$ , as expected. The energy that can be gained by an electron in a current channel of length  $L$  is found from equations 21, 9, and 6 to be

$$W_f - W_c = 7.0 \left( \frac{T}{10^7 \text{ K}} \right)^{1/2} \left( \frac{v_e}{10 \text{ s}^{-1}} \right) \left( \frac{L}{10^9 \text{ cm}} \right) \left( \frac{v_c}{v_e} \right)^{-2} \text{ keV.} \quad (37)$$

For Case II with  $T=10^7$  K and  $v_c/v_e \approx 4$ ,  $W_c \approx 8$  keV. Electron energies  $\sim 400$  keV can be attained in a distance of  $10^9$  cm for Case II. Much higher energies,  $\sim 20$  MeV, are possible for Case I. If the resistivity is due to a high density rather than to anomalous resistivity, however, the small thickness of the current sheet is likely to limit the electron energy that is attainable. It is interesting that, as was found for the Joule heating and runaway production timescales, the acceleration of electrons to an energy of 100 keV requires a collision frequency that is much greater than  $10 \text{ s}^{-1}$ .

## VI. Discussion and Conclusions

The rate at which the current  $I$  puts energy into Joule heating is  $\sim IEL$ , while the rate at which energy goes into runaway electrons is  $I_{\text{run}}E$  integrated over the length of the current sheet. Therefore, the energy that goes into particle acceleration will always be less than the energy that goes into heating if  $I_{\text{run}} < I$  or, equivalently, if  $\dot{N} < \dot{N}_J$ . If  $I$  is determined inductively with  $B_J = B$ , then, as was seen in Section V,  $\dot{N}_J = \dot{N}_{\text{Max}}$  and  $\dot{N} < \dot{N}_J$ . Hence, the energy that goes into accelerated electrons will always be less than the



energy that goes into heating the plasma for such an inductively generated flare. This general result is in agreement with the conclusions reached by Smith (1980) from his analysis of specific inductive acceleration mechanisms.

The runaway production rate that is required for a microwave flare is  $\dot{N} \sim (10^{32} \text{ electrons}) / (30 \text{ sec}) = 3 \times 10^{30} \text{ electrons s}^{-1}$ . Since this is on the order of  $\dot{N}_{\text{Max}}$  (equation 18), solar flares will have  $I_{\text{run}} \sim I$  when the electrons exit the acceleration region. The fact that the rate of electron acceleration required for a microwave flare is on the order of  $\dot{N}_{\text{Max}}$  may be an indication that an inductive flare model, with  $I$  determined by  $B$  through Ampere's law, is indeed correct.

The direct acceleration of electrons by an electric field has the important consequence that the electron flux from one or a small number of current channels cannot be large enough to explain the bulk of the observed hard X-ray emission from flares as nonthermal bremsstrahlung. Hence, an important test of flare models involving the directed acceleration of electrons is to determine whether or not the lowest energy hard X-rays (20 to  $\sim 100$  keV) from a given flare must be primarily nonthermal in origin. The current that is required to produce a typical hard X-ray burst has an induction magnetic field of at least  $10^6$  G associated with it. Since such fields are not observed on the sun, if nonthermal electrons are required for these lowest energy X-rays, either the acceleration region must contain at least  $10^4$  individual, oppositely directed current channels, or the acceleration mechanism must not require a directed current of electrons in the acceleration region.

If the bulk of the hard X-ray emission is thermal, a single current sheet

is able to generate the required microwave and hard X-ray emissions. A current sheet with dimensions  $\sim 10^9 \times 10^9$  cm that is inductively generated by a magnetic field of a few hundred Gauss can accelerate the required number of runaway electrons and deposit the required amount of heat on a timescale of a few tens of seconds if the density in the sheet and in the more extended X-ray emitting volume is  $\sim 10^{11}$  cm $^{-3}$ . The thickness of the current sheet will be  $\sim 400$  cm, and the volume of gas to be heated is  $\sim 200$  times larger than the volume of the current channel. Electrons can be accelerated up to energies  $\sim 400$  keV in a timescale  $\sim 10^{-2}$  sec. If the density in the current sheet is less than  $\sim 10^{11}$  cm $^{-3}$ , the resistivity in the sheet must be anomalous. A large collision frequency decreases the Joule heating timescale and the runaway production and acceleration timescales, and increases the maximum energy of the accelerated electrons. For an effective collision frequency  $\sim 10^6$  s $^{-1}$ , electron energies up to  $\sim 20$  MeV are obtained. The required drift velocity of the current-carrying thermal electrons in the sheet is not high enough to expect  $J_{\parallel}$ -driven anomalous resistivity unless the thermal electrons are first heated to a temperature that is well above  $10T_i$ . If such an electron temperature can be attained, the ion acoustic instability will dominate.

In a recent study of the hard X-ray and microwave emissions from a flare that occurred on June 25, 1980, it was concluded that the emissions at the time of the peak 6 cm microwave radiation were most likely nonthermal (Holman, Kundu, and Dennis 1984). Since the power flux of electrons required for the hard X-ray emission (assumed to be thick-target bremsstrahlung) did not increase at this time, however, it was also concluded that either the acceleration mechanism must have only operated on already energetic particles

at this point in the flare, or two populations of energetic electrons were present. In view of the results obtained here, the best interpretation is that the bulk of the hard X-ray emission was thermal bremsstrahlung, while the secondary peak at the time of the microwave peak (the X-ray spectrum flattened at this time) corresponds to the acceleration of electrons to higher energies. The total electron power flux did not increase at this time because, although electrons were accelerated to high energies, the total energy in accelerated electrons was small compared to the energy in the hot thermal plasma.

Some of the results presented in this paper have been summarized elsewhere (Holman 1984). These results will be applied to a specific flare in a subsequent publication (Holman and Kundu, in preparation; see also Holman 1984).

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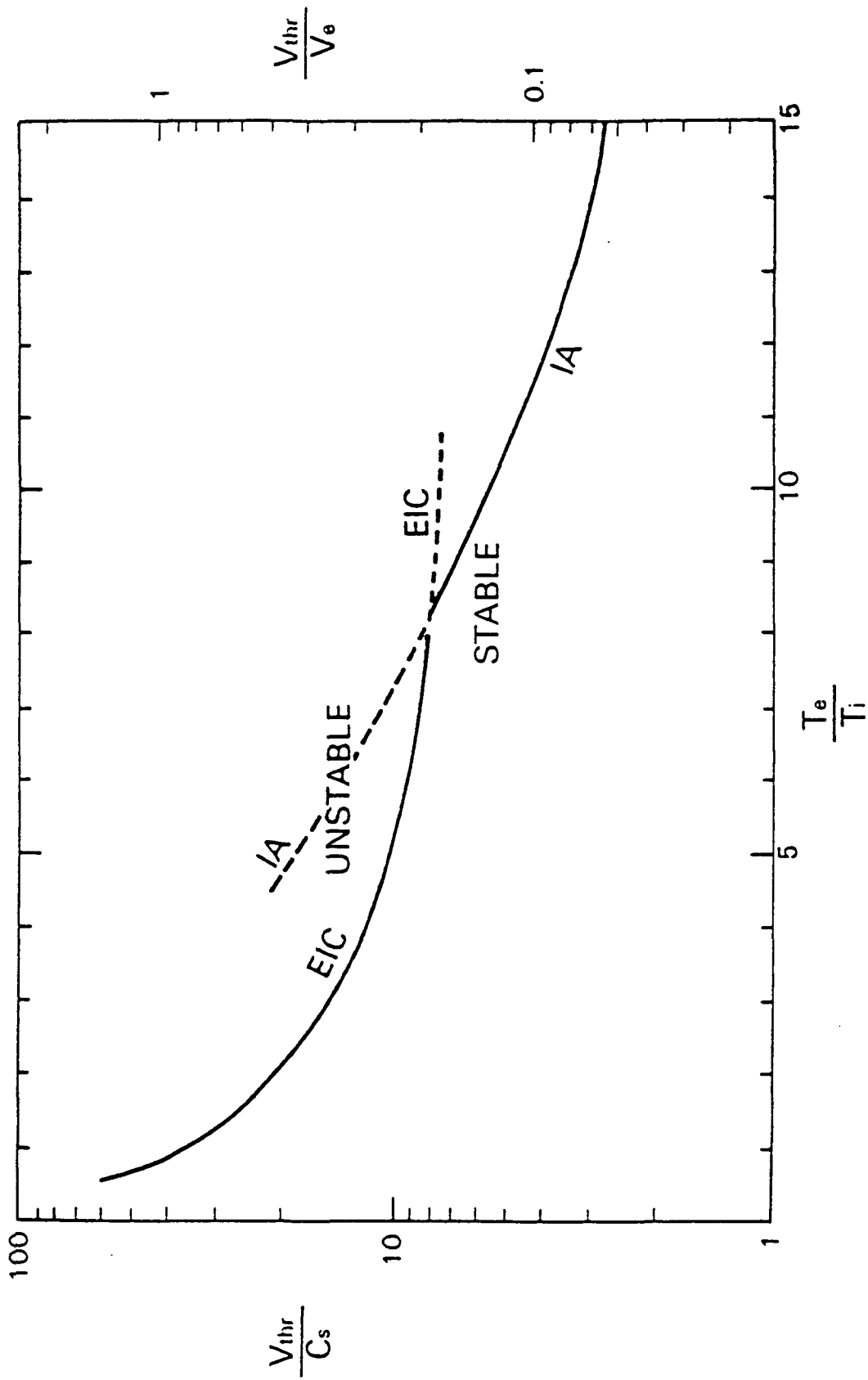


Figure 1. The instability thresholds for the electrostatic ion cyclotron and ion acoustic instabilities, where  $c_s = (kT_e/m_i)^{1/2}$  and  $v_e = (kT_e/m_e)^{1/2}$  (after Figure 2 of Morrison and Ionson, 1982). A current is unstable if  $v_d > v_{thr}$ .

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