

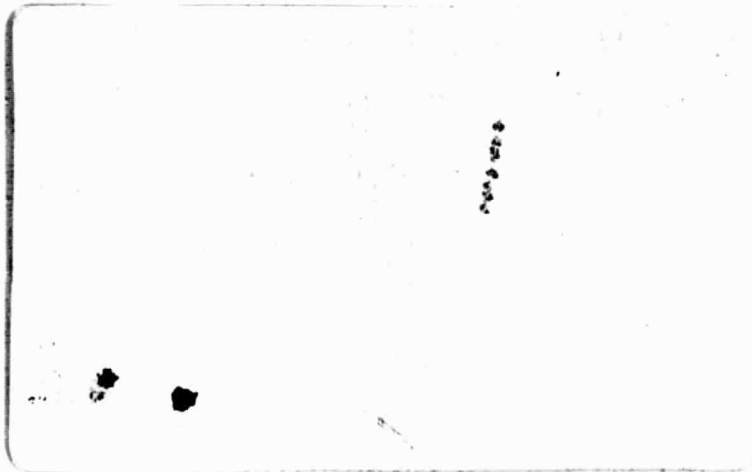
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On Io, All That Flickers Is Not Cold<sup>1</sup>

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ABSTRACT

It is shown that the 4.8- $\mu\text{m}$  flux from Io fluctuates with an amplitude of ~10 percent on time scales of every 28 seconds, 40 minutes, night-to-night, and perhaps year-to-year. Such behavior is found to be the result of random fluctuations for which the mean square fluctuation varies inversely with frequency for constant bandwidth measurement. It is suggested that the theory developed for thermionic emission from barium oxide cathodes in vacuum tubes is applicable to this situation. If this is so, the fluctuations in the flux from Io's volcanoes may be caused by diffusion of hot convective cells onto the surface of Io.

It is speculated that long-term fluctuations may furnish a means by which the Io volcanism can shut down and conserve energy. Thus the discrepancy that may exist between measurements of the current heat flow from Io and calculations of tidal dissipation may be resolved.

Tests for rapid flickering at 10  $\mu\text{m}$  showed no fluctuations greater than 1 percent. This agrees with the prediction of the author's "flow model" theory (Sinton, 1982) in which the 10  $\mu\text{m}$  volcanic thermal emission arises from cooling of "old" flows.

Running Head: Io Flickering

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## I. INTRODUCTION

The variability of the 5  $\mu\text{m}$  component on Io's thermally emitted radiation has been demonstrated in a number of papers (Witteborne et al., 1979, 1982; Sinton, 1980; Morrison and Telesco, 1980; Sinton et al., 1983). In the last named paper, it was demonstrated that at least some of the variability occurred in events or outbursts which have a duration of at least several hours. These occur somewhat at random, but seem to occur more frequently on the trailing hemisphere of Io and when Io is in the active sector of the Jovian magnetosphere. The cause of the variability is attributed to variability of thermal emission from volcanic eruptions.

In February 1981 while observing Io, I became aware that, according to the CRT display of the data acquisition system, I was spending fully as much time to achieve a specified S/N ratio on Io as I was on Ganymede and Callisto, even though Io is several times brighter at 4.8  $\mu\text{m}$  than the other two satellites. It became evident that Io was "flickering" or changing brightness in a time comparable to the 28 seconds required for an individual pair of integrations. Subsequently, data were taken in such a manner as to enhance the detection of rapid variations. After a determination of the spectrum of the rapid fluctuations, which showed that the spectrum was quite broad, it was realized that the previously reported night-to-night variability of Io and the rapid fluctuation might be part of the same phenomenon. The thesis of this paper is that the power spectrum of the fluctuations is closely fit by an inverse proportionality to frequency.

We discuss the nature of the rapid fluctuations in Section II. After an initial discussion of how the Io fluctuation can be separated from system noise, a statistical test of the reality of Io fluctuation is made, and this

is followed by a determination of the power spectrum of the rapidly sampled data. A cursory reader can skip this section entirely and go to Section III, but Section II lays the foundation for the statistical analysis of the rapid fluctuation.

In Section III, the fluctuation measurements at the various time scales are combined. It is found that an assumed reciprocal frequency dependence of the fluctuation power is in agreement with the observations. A physical model that predicts such a broad spectrum is suggested in Section IV. In Section V it is speculated that fluctuations of very long period may act to conserve the heat on Io and that the magnitude of the present volcanism is atypical. A negative test for rapid flickering at 10  $\mu$ m is reported in Section VI.

## II. STUDIES OF THE RAPID IO FLUCTUATION

### a. Determination of the Total Fluctuation

The measurement of the amount of fluctuation of a variable source in the presence of a nearly comparable measuring system noise is not an easy task. Only the total noise can be detected. To obtain an accurate value of the total noise a relatively large number of observations have to be made. The standard error estimate of a signal measurement is customarily determined from

$$\sigma = \left[ \frac{\sum_i S_i^2 - \frac{1}{n} (\sum_i S_i)^2}{(n-1)} \right]^{1/2} \quad (1)$$

where  $S_i$  is the  $i$ th measurement of the signal and  $n$  is the total number of data points. Normally one is primarily interested in the signal measurement and is only secondarily interested in the value of  $\sigma$  to set bounds on the

value of  $S$ . If, however, one wants to compare the value of  $\sigma$  for a fluctuating source with that for a nonvariable source, then an accurate value of  $\sigma$  is required. The estimate of the standard error of  $\sigma$  is (Cameron, 1960)

$$\sigma_{\sigma} = \left[ \frac{\sum S_i^2 - \frac{1}{n} (\sum S_i)^2}{2(n-1)^2} \right]^{1/2}, \quad (2)$$

and the ratio of the standard error estimate to its uncertainty is

$$\frac{\sigma}{\sigma_{\sigma}} = \sqrt{2(n-1)} \quad (3)$$

Thus, in order to know  $\sigma$  to 10 times its uncertainty, 51 measurements have to be made. Normally, at the 2.2-m telescope an integration requires  $10^8$  and a pair of integrations (the telescope is beam-switched by the data system between each) is required for a signal determination. Settling time requirements lead to a total interval of 28 seconds. Thus more than a  $1000^8$  are required for a measurement of  $\sigma$  good to 10 percent.

The problem is exacerbated in the presence of noise in the measuring system. Because the two sources of fluctuation are uncorrelated, the observed fluctuation is given by

$$\sigma^2 = \sigma_b^2 + \sigma_s^2 \quad (4)$$

and

$$\sigma_s = \sqrt{\sigma^2 - \sigma_b^2} \quad (5)$$

where  $\sigma_s$  is a measure of the fluctuation of the source and  $\sigma_b$  is the additional fluctuation of the system. The system noise can be determined by observing a star which is assumed not to fluctuate. If  $\sigma_b \cong \sigma_s$ , then by the usual method of combining errors, the ratio of  $\sigma_s$  to its uncertainty is  $[2(n-1)/5]^{1/2}$  if  $n$  measurements are made of both the fluctuating source and the system noise. For a 10 percent uncertainty,  $n$  must be 251 for each. Fortunately, in many cases the fluctuation of  $I_0$  was substantially larger than the observed fluctuation of stars.

### b. Determination of the System Noise

In the previous analysis we assumed only one source of noise in the telescope-detector system. Actually there are several, but for our purpose they may be treated as only two: one is independent of the signal level and one proportional to the signal level. The first may comprise detector noise and fluctuation of the telescope background and sky emission. Note that the number of photons from the sky and telescope are many orders of magnitude greater than those from the object. Thus the term that is dependent on the square root of the signal is insignificant. The signal-dependent noise sources are composed of guiding errors, seeing excursions, and scintillation caused by the atmosphere. At the signal level of  $I_0$ , both the signal-dependent and signal-independent components are important. To measure these and to determine the total system noise,  $\sigma_b$ , at  $I_0$ 's signal level, a number of stars spanning a wide range of brightness have been measured. We did not have available a single star having exactly the brightness of  $I_0$ . For each observed star we have determined the noise-to-signal ratio,  $N/S$ , for a single pair of  $10^8$  integrations. This was generally determined from 16 pairs of integrations. The error of this quantity is, by (3),  $1/\sqrt{30}$  of its magnitude. For each observation, the total observed noise from the two components is given by the sum of their squares. This may be expressed as

$$N^2 = N_b^2 + C S^2 \quad (6)$$

where  $C^{1/2}$  is the proportionality constant for the signal ( $S$ ) dependent term.

Dividing by  $S^2$  gives us

$$\frac{N^2}{S^2} = \frac{N_b^2}{S^2} + C \quad (7)$$

It is convenient to express the signal in magnitudes on the right side of (7)

by putting  $S = 10^{-0.4 M}$  where  $M$  is the magnitude of the star. Extracting the square root gives us the most useful form

$$\frac{N}{S} = \sqrt{C + D \cdot 10^{0.8M}} \quad (8)$$

The constant  $D$  is then  $N_b^2$ . We found  $C$  and  $D$  for each night by applying linear regression to the data in the form of (7) and where the independent variable is  $1/S^2$ . However, the uncertainty of each  $N^2/S^2$  varies as the brightness of the star and so a regression analysis that was weighted according to the  $S/N$  was used. Since similar results were obtained for the several nights of each observing run, all of the data in each run were combined. Figure 1 shows the data in the form of (8) with the least-squares fitted curve. Error bars are not shown in these plots. With the exception of the discovery run, 22-25 February 1981,  $n$  was always 16 or an integer multiple of 16. Each data point plotted in panels b, c, and d of Fig. 1 represent 16 measurements and so by Eq. (2) the error bars extend  $(1/30)^{1/2}$  of the value of the point. The curves are all excellent fits to the data over a range as much as eight magnitudes. There are, however, differences from one observing run to another. Between February and March 1981 the background noise increased substantially, while better seeing apparently reduced the fluctuations of bright stars. A different detector system was introduced in 1982. Because of its better quantum efficiency and the use of 6-arcsecond instead of a 9-arcsecond aperture, the noise at the faint star end in the 1982 data is substantially reduced. The noise at the bright star end is the same or reduced despite the smaller aperture. The reason for this is apparently that the new system has a much more uniform response over the entrance aperture. The agreement at the faint end for the three runs in 1982 (see also Fig. 2) is most remarkable. Differences between runs are only a few percent.

c. Statistical Test of Io Fluctuation

The measurements of Io were made in exactly the same way as those of the stars. It is clear from the location of the Io points on the plots of Fig. 1 that a considerable number lie substantially above the curves from the star measurements. To see whether Io is really flickering we have applied the F-test for the significance of the ratio of the larger variance (Io) to the smaller (stars). We used the critical F value for 15 degrees of freedom ( $n-1$ ) for Io and ~400 degrees for the stars since typically the star fluctuation was determined from a combination of many observations. Using the double-tailed F critical value of 3.4 for 95 percent confidence, we have determined new curves in Figs. 1 and 2. If Io's N/S ratio lies above the appropriate curve, we have 95 percent confidence that Io was flickering during that measurement. We can also view the data in Figs. 1 and 2 in a different light. There are a total of 8 Io values above the 95 percent level. By random chance (since there are 91 Io determinations) we should expect less than 5. The stars appear to have approximately these numbers of high values.

It is clear that many Io points lie on the least-square curve or within a standard deviation. We conclude that while Io sometimes flickers, it does not always do so, as evidenced by the plot in Fig. 2.

Table I gives the part of the observed noise due to Io's fluctuation derived from (5). From the data in the table, we have formed the rms value. The rms is preferred over the straight average, not only because it is a more accurate appraisal of the fluctuations, but because theory, developed in a later section, requires it. In deriving the rms value, we have assumed that when the Io fluctuation is less or equal to that determined for stars, the

square root of the negative number in (5) is imaginary and its square reduces the sum. The final result of the  $I_o$  rms flickering amplitude from the 91 sets of measurements of Table I is  $0.060 \pm 0.005$  relative to its total signal.

#### d. Power Spectrum of the Rapid Fluctuation

We subsequently sought to measure the power spectrum of the flickering during one run that was available on the IRTF. Fig. 3 shows several selected time series of data in which we believe that  $I_o$  was flickering. Also shown in this figure are signal measurements of two stars whose brightness is very similar to that of  $I_o$ . Most of the IRTF measurements were made at an integration of two seconds in each of the two positions of the telescope (i.e., five times as fast as the 2.2- $\mu$ m data) since, at the time, we believed it was important to press toward the highest possible time resolution. As it turned out, most of the data do not have a fluctuation that is greater than the star data. Power spectra formed from the IRTF data of  $I_o$  did not show any repeatable peaks in the spectrum. They may, however, show a trend toward an increase at lower frequencies, but it is not at all clear that this trend is not also present in the stellar data.

We were more successful in determining the power spectrum from 2.2- $\mu$ m data we already had in hand and from some we subsequently obtained, albeit at longer sampling times. We selected 21 of the 91 data sets that have the highest F values so that we have a high degree of confidence that  $I_o$  was flickering while these data were taken. The mean power spectrum of these data is shown as the solid dots in Fig. 4. In computing this spectrum, we combined some contiguous sets of 16 or more data pairs. The computed power spectra

were "binned" into 10 frequencies of width 1.7 MHz beginning at 1.7 and extending to 17 MHz. The mean spectrum shows a marked decay toward higher frequencies, and the highest flicker power is found at the lowest frequency.

Because we selected data sets in which Io had greater than average flickering, the power spectrum derived by the above process is expected to have a greater than average flicker power at each frequency. To determine the average spectrum, the values in the various frequency bins have been scaled by the ratio of the mean variance of the 21 samples to the mean variance of the 91 samples of Table I. It is these scaled values that are shown in Fig. 4.

In performing any Fourier spectrum analysis one must always assure oneself that the spectrum is not aliased. The data samples that go into the spectrum of Fig. 4 are all from differencing two 10-second integrations spaced two seconds apart. Thus higher frequencies that would cause aliasing are removed by the integration process. The effective filtering corresponds to a sinc function attenuation of offending frequencies.

Since the peak of the flickering appears to be at still lower frequencies, we would like to press in that direction. It would be advantageous to spend entire nights observing Io continuously at  $4.8 \mu\text{m}$  in order to obtain nonaliased power spectra to 50 times lower frequencies. But such an undertaking would be at the expense of observing Io at other wavelengths and observing standard stars. With the data in hand, we are thwarted in doing a power spectrum by this problem of aliasing. One cannot numerically filter data that are incompletely sampled. In pressing toward the lowest frequencies that we shall discuss, we encounter numerous practical constraints that in fact prevent complete sampling. Such constraints are (i) the necessity of looking at the same hemisphere of Io, (ii) Io can only be observed when it is above the horizon, (iii) Jupiter and clouds interfere some



times, and (iv) telescope scheduling committees do not have unlimited time to award. Thus even a program that is optimally designed to reduce aliasing will not do appreciably better than the data in hand for the lowest frequencies at which we now know that  $I_0$  fluctuates.

In the next section we will show that it is possible to fit the available fluctuation data with a simple spectral dependence while using this spectral dependence to account for the aliasing inherent in our data sampling.

### III. FLUCTUATIONS AT LOWER FREQUENCIES

#### a. The Broadband Nature of the Fluctuation

We are impressed by two facts: we are not able to find any repeatable maxima but rather find a trend toward lower frequencies in the power spectra, and the amplitude of flickering in the 2.2- $\mu$ m data ( $0.060 \pm 0.005$  fractionally) is of the same order as the large night-to-night fluctuations seen in 4.8- $\mu$ m photometry (Sinton et al., 1983). Could the flickering and the night-to-night fluctuations be part of the same broadband phenomenon? Thus it is important to determine the fluctuations at an intermediate time scale. To this end, on 9 and 10 June 1982 we acquired data in which we tried to make the 16 pairs of 10-second integrations (requiring 7.5 minutes with the required 2-second settling time) every half-hour, along with a short 3.8- and 2.2- $\mu$ m measurement and the same sequence on a standard star sandwiched inbetween. We also searched our available data for sequences that could be used. Note that by forming the mean of each 7.5-minute segment, we average over rapid variations, and at the same time, the standard deviation determination for each night

eliminates fluctuations which are very slow compared to the 4-5 hours during which the data is collected. The precise discussion of the effective frequency bandwidths is in a later section.

All together 12 nights of data are available for this analysis. These are shown in Table II. As before, we have to know the measured fluctuation of unvarying stars. This, determined from 34 sets of measurements of stars on the same nights, is 0.035 fractionally. The net rms fluctuation of Io is  $0.069 \pm 0.013$  of its total flux, which shows that Io varies on a time scale that averages 40.0 minutes between observations. The amplitude is comparable to that found with the fast flicker measurements, yet we have averaged each point over 7.5 minutes of data.

We can analyze our data for still lower frequencies. If, however, we compare data from adjacent nights, we are comparing different sides of Io which have known differences. Hence, we have compared nightly averages of data 48 hours apart, i.e., the same side of Io. There are six such pairs of nights listed in Table III. Longer runs did not occur generally because we request blocks of telescope time when Io is not close to Jupiter, and these blocks occur on a four-night cycle. We have assumed an average uncertainty in the relative calibration of the paired nights of 5 percent. Thus the reduced rms standard deviation of Io is  $0.139 \pm 0.036$ , also expressed as a fraction to the total flux. As expected, Io shows a marked fluctuation on this time scale. We may look at longer time scales and hence lower frequencies for fluctuations. We are, however, constrained by the sampling in the data in hand, and we cannot find longer repetitive intervals in the sampled data. The next longer amenable time scale maybe the yearly averages. In Table IV of Sinton et al. (1983) there is a difference of 6 percent for the leading hemisphere between the two years. Considering that each yearly value is an

average of ~80 measurements spread over ~20 nights, errors this large are not expected. However, the same standard stars have not been systematically used and there is the possibility of differences even among groups of the standards. We are presently seeking to reestablish standards and reduce this uncertainty. Nevertheless, it appears that Io is variable from year to year. The program is continuing and by next year we will have doubled the time base and will have tied together 45 standard stars of selected spectral types to improve the calibration. From 17 nights of observations obtained in 1982, the mean brightness of Io for both the leading and trailing hemispheres has dropped by ~20 percent since 1980. Thus there appears to be strong evidence of a year-to-year variation.

b. Test of Inverse Frequency Power Dependence

Obviously, the fluctuation spectrum of Io is very broad. Because of the aliasing, we also have very broad frequency response bands inherent in the sampling techniques that were employed in Tables I-III. First, we will make an estimate of the frequency dependence of the power spectrum. We will assume a spectral power dependence, and for each data set calculate how the aliasing will affect the response in a given frequency band. With the three different data sets we can then test if the assumed spectral dependence is consistent with the response that was corrected for aliasing.

The spectrum shown in Fig. 4 shows a definite tendency for the amplitude to increase with decreasing frequency. In reviewing the data acquired over various sampling intervals in Tables I-III, we are impressed by the fact that the standard deviation remains approximately 10 percent regardless of the sampling time. Despite our inability to determine the power spectrum that is free from the problems of aliasing for the longer period samplings, a little

reflection will show that the spectral amplitude must increase for frequencies below those of the power spectral analysis already discussed. Let's suppose that  $I_0$  had a white noise fluctuation, i.e., uniform power at all frequencies. Then as we increase the integration time,  $\tau$ , of a sample we would expect the estimate of the standard deviation of the mean to decrease as  $\tau^{-1/2}$ . The differences between samples must also decrease as the precision of the estimates improves. Yet it does not, even though the total integration times increases from ~20 seconds to thousands of seconds. In order for this to be true, the amount of the fluctuation of  $I_0$  must increase for the lower frequencies that are being sampled as the integration time and concomitantly the time between samples are increased.

From just the observed constancy of the standard deviation, it is possible to derive an estimate of the frequency spectrum of the fluctuation. As the total time for the array of measurements  $T$  is increased, the lowest frequency to cause a fluctuation between members of the array  $f$  decreases as  $1/T$ . If  $\tau$  always bears the same ratio to  $T$ , then the fluctuation for white noise would decrease as  $1/T^{1/2}$ . Since in fact the observed fluctuation stays constant, we guess that the noise in a constant bandwidth must increase as  $T^{1/2}$  or as  $f^{-1/2}$ .

It is customary to express noise measurements in terms of power. By power, the square of the amplitude of the fluctuation is meant. Thus this simple estimate from the near constancy of the fluctuation gives a spectral dependence for the noise power within a unit bandwidth that varies inversely as the frequency. It is shown in Appendix A that the variance is equal to the sum of the power over all of the frequencies that contribute to the fluctuation. As just discussed, the lowest frequency is  $1/T$ . For data that are uniformly sampled in time one can obtain the power spectrum of the

fundamental and all harmonics. Parseval's relation (Appendix A) tells us that the variance is equal to the total power at all frequencies,

$$\sum_{i=1}^n P_i,$$

where  $P_i$  is the power in the  $i$ th frequency. If we make the assumption that the power spectrum  $\propto f^{-1}$ , then the summation can be written

$$\sigma^2 = P_1 \sum_{i=1}^n i^{-1}, \quad (9)$$

where we have made use of the equivalence of variance and total power and where  $P_1$  is the power in the fundamental. It is well-known that the series in (9) does not converge as  $n \rightarrow \infty$ . However, the integration of signal over 28<sup>8</sup> in the case of Table I and 450<sup>8</sup> in the case of Tables II-III effectively attenuates frequencies that lie above 1/56 Hz for Table I and 1/900 Hz for Tables II-III because for these frequencies the flux from  $I_0$  was integrated over more than 1/2 cycle. Thus we can let  $n$  stop at an integer just greater than  $T/56$  for Table I and  $T/900$  for Tables II-III. These integers are 9, 16, and 192 respectively for Tables I-III, and the sums,  $S = \sum 1/i$ , in (9) are 2.829, 3.381, and 5.837. The bandwidth,  $\Delta f$  for each frequency is  $T^{-1}$ . Thus the power per frequency bandwidth for each fundamental period is given by

$$P_1 = \sigma T/S. \quad (10)$$

These values are plotted as open circles in Fig. 4. A line with the slope of -1 is shown in the logarithmic plot of this figure.

What we have done, then, is to assume for each set of data in Tables I-III that the noise power varies as  $f^{-1}$ . Using that assumption and the

equivalence of the variance to the total power, we have determined the amplitude of the lowest frequency for the data in each of the three tables. Comparison of these amplitudes in Fig. 4 to the line verifies the appropriateness of the  $f^{-1}$  assumption for the power spectrum. We have not shown that other spectral dependences cannot also work. A line with a slope as steep as  $-1.3$  would also fit within the error bars of the extreme points. A slope of  $-1$  is at the opposite extreme.

#### IV. PHYSICAL MODEL FOR THE FREQUENCY DEPENDENCE

The reciprocal dependence on frequency of the square of the fluctuations brings to mind several cases in which similar noise is developed in electronic apparatus. These include many varieties of semiconductor devices and barium-oxide-coated cathodes of vacuum tubes. The theory of the latter has been elaborated (Schottky, 1926; Sproull, 1945; Macfarlane, 1947) and seems most analogous to the present case since electron micrographs of thermionic-emitting cathodes show that the emission occurs in evanescent patches that are called "hotspots." We can draw an analogy to the theory of the flicker effect developed by Macfarlane if we replace the thermionic current by the flux of thermal radiation. Note that in the electronics case, noise power is proportional to the square of the current fluctuations and that we find that the square of the thermal radiation fluctuations varies inversely as the frequency.

In the case of BaO cathodes, the theory supposes that clusters or patches of barium atoms diffuse to the surface, which Macfarlane termed "erupt," at a relatively small number of places. The presence of these atoms forms a polarized layer which locally reduces the work function for thermionic

emission. As electrons are emitted barium atoms are converted to ions, so the polarization of the layer and the thermionic emission decrease with time. In drawing the analogy with thermal emission from Io's volcanoes, we consider that convection in cells in calderas will bring hot magma up from beneath. As the magma radiates its heat away, the initially high flux of thermal radiation will decay with time. Thus we replace diffusion of clusters of Ba atoms to the surface with diffusion of hot convection cells of a magma to the surface. We can write an equation for the rate of change of excess energy per  $\text{cm}^2$ ,  $E$ , in the surface layer:

$$\frac{dE}{dt} = -F + \frac{D}{h} E \quad (11)$$

where  $F$  is the flux of radiant energy from the surface,  $D$  is the cellular diffusion coefficient, and  $h$  is the distance over which the diffusion takes place. This equation is of the same form as Eq. (1) of Macfarlane (1947). The other basic equation used by Macfarlane is Langmuir's equation which relates the current to the excess of atoms on the surface. The equivalent equation relating  $F$  to  $E$  can be found from the relation of the surface energy content to the specific heat,  $C$ , and temperature,  $T$ ,  $E = hCT$ ; and from the Stefan-Boltzmann law,  $F = \sigma T^4$ . Combining these gives

$$F = \sigma \frac{E^4}{h^4 C^4} \quad (12)$$

This does not have the same functional form as Langmuir's equation, which is exponential in the number of atoms. The author has solved the differential Eq. (11) when combined with (12) exactly, while Macfarlane found only an approximate solution to his equation. The two solutions, however, have approximately the same time dependence.

One additional assumption made by Macfarlane was that the number of excess atoms had a Gaussian distribution with an e-folding width of  $N$ , which would be analogous to a similar distribution in the initial temperature of the convection cells on Io. We will not repeat the lengthy mathematical development here. However, he shows that the dependence of noise power on current,  $j$ , and frequency follows the relationship

$$\langle \Delta j^2 \rangle \propto \frac{j^{1+x}}{f^x}, \quad (13)$$

where  $1 < x < 2$  for  $\infty > N > 0$ . The empirical value of  $x$  for cathodes is 1.25. In the previous section we found an empirical value of  $x$  for Io which lies between 1.0 and 1.3.

The similarity of the formulation of cathode flickering to what is known about the volcanism of Io and the spectrum of the fluctuation that is reported here make the development of the flicker theory for Io look very attractive. The author intends to make this application and investigate the implications of this theory for Io volcanism in a future paper.

#### V. SPECULATION ON LONG-TERM FLUCTUATIONS

Evidence was previously given that Io fluctuates from year to year on a global scale at  $4.8 \mu\text{m}$ . In this section we will speculate on the consequences of variability at even longer time scales, say 100 years or more. Eclipse measurements have shown that ~40 percent of Io's flux at  $4.8 \mu\text{m}$  is thermal emission from volcanoes (Sinton et al. 1980). There is already evidence that the flux varies by ~20 percent from year to year. Since these are similar, it



seems reasonable to suppose that excursions may reach 40 percent from the current value and that not only can the activity double, but it can also cease completely and in fact may do so for very long times. The times might be long enough that the present vents will freeze and thin places in the crust thicken. Thus Io may go from its present volcanically active state to an inactive one. Sinton (1982) discusses this possibility as an on-off thermostatic mechanism. The fluctuations discussed here and the apparently open-ended spectrum derived thus far constitute the first indication of how this might come about. Furthermore, the measurements of total volcanic heat flow (Matson et al., 1981; Sinton, 1981; Morrison and Telesco, 1980; Pearl and Sinton, 1982) when compared to estimates of tidal dissipation (Yoder, 1979; Cassen et al., 1982; Greenberg, 1982; Yoder and Peale, 1981) suggest that Io is currently losing more heat than it is gaining. Hence in the long term, activity may cease temporarily because Io may not be able to maintain the current level.

## VI. FLICKERING TESTS AT 10 $\mu\text{m}$

It seems obvious that the flickering at 4.8  $\mu\text{m}$  results from ongoing volcanic activity on Io. In an earlier paper (Sinton, 1982) we presented a steady-state model to explain the spectrum of Io observed from 2.2 to 30  $\mu\text{m}$  during an eclipse. In the model we ascribed the thermal emission at the short wavelengths (2.2-4.8  $\mu\text{m}$ ) to current eruptions, while the radiation at longer wavelengths was ascribed to the cooling of "old" flows. To test this idea, we reasoned that old, solidified flows would not flicker. Fig. 4 shows rapid time series measurements of Io at 10  $\mu\text{m}$  outside of eclipse. Flickering with an amplitude of 1 percent is not detectable at  $4\sigma$  in these data.

However, at this point the negative result for flickering should not be considered as definitive because  $I_0$  was not measurably flickering at  $4.8 \mu\text{m}$  either. More observations are deemed necessary for nonflickering to be considered as evidence for the flow model.

Eclipse observations at  $10 \mu\text{m}$  have thus far given quite consistent results. However, if the low flux at  $4.8 \mu\text{m}$  observed in 1982 is maintained, we should expect a decrease in the  $10 \mu\text{m}$  flux since, according to the flow model, the current activity as viewed at  $4.8 \mu\text{m}$  produces the flows that are subsequently seen at  $10 \mu\text{m}$ . Thus in future work we should cross-correlate the fluctuations observed at  $4.8$  with any variability measured at  $10 \mu\text{m}$  as a further test of the flow model.

## VII. CONCLUSIONS

It has been shown that the  $4.8\text{-}\mu\text{m}$  flux from  $I_0$  is variable at any time scale at which  $I_0$  has been observed. The spectrum of this fluctuation is such that the mean square fluctuation in a unit frequency interval varies inversely as the frequency within the errors of the determination over a frequency range of  $6 \mu\text{Hz}$  to  $17 \text{mHz}$ . This type of fluctuation or "noise" is similar to many sources of noise found in electronic equipment. A theory that has been developed for BaO cathodes appears to have many similarities to the volcanism on  $I_0$ , including the appearance of evanescent "hot spots" on the surface and the diffusion of clusters or cells which transport that which is to be emitted to the surface. In the case of cathodes, electrons stored in Ba atoms are transported; in the case of  $I_0$ , the heat stored in magma is transported. The

mathematical treatment is identical in the diffusion-conduction equation, but differs in the relation between storage and emission.

We speculate that the fluctuation of the volcanic activity on Io may extend to very long time periods, and there is evidence that it extends at least to intervals of several years. Such very low frequency variation may offer a mechanism whereby the volcanism can be intermittent as the author has suggested in a previous paper.

Lastly, it was found that on one night in which tests were made, Io did not flicker at 10  $\mu$ m. This tentative result is in agreement with predictions of the author's "flow model," but further tests are required.

APPENDIX A

RELATIONSHIP BETWEEN THE TOTAL SPECTRAL POWER AND THE VARIANCE

We have not been able to find this relationship in any popular textbook probably because most textbooks treat Fourier series separately from statistics. Parseval's relation for Fourier series gives

$$\frac{a_0^2}{2} + \sum_{i=1}^{\infty} c_i^2 = \frac{2}{T} \int_0^T [f(t)]^2 dt$$

where  $a_0 = \frac{2}{T} \int_0^T f(t) dt$

$$\begin{aligned} \text{and } c_i^2 = a_i^2 + b_i^2 = & \left[ \frac{2}{T} \int_0^T f(t) \cos \left( \frac{2\pi i}{T} t \right) dt \right]^2 \\ & + \left[ \frac{2}{T} \int_0^T f(t) \sin \left( \frac{2\pi i}{T} t \right) dt \right]^2 . \end{aligned}$$

Since  $c_i$  is the amplitude of the sinusoidal wave of harmonic  $i$ , the power in the harmonic is

$$P_i = \frac{c_i^2}{2} ,$$

where the factor of 1/2 arises from taking the average power over a sinusoidal wave. The constant term,  $a_0/2$ , is the average value of  $f(t)$  over the interval  $0 < t < T$ . If the function is sampled at  $n$  equally spaced points over the interval we may write

$$\frac{a_n}{2} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n f\left(\frac{jT}{n}\right) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n f_j$$

Approximating the integral by a sum using the trapezoidal rule, we have

$$\frac{2}{T} \int_0^T [f(t)]^2 dt = \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{j=1}^n [f(\frac{jT}{n})]^2 = \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{j=1}^n f_j^2$$

Thus we have:

$$\sum_{i=1}^n P_i = \lim_{n \rightarrow \infty} \frac{\sum_{j=1}^n f_j^2 - \frac{1}{n} \left( \sum_{j=1}^n f_j \right)^2}{n}$$

In the limit as  $n \rightarrow \infty$  the left side is the total power in all the harmonics while the right side is the variance. It may be seen that in an electrical circuit the left side expresses the total power in all the ac harmonics while the right side expresses the total power as proportional to the mean square voltage or current variations. Conservation of energy requires that the equality hold.

#### ACKNOWLEDGMENTS

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TABLE I

Fluctuation of Signal Relative to Total Signal at 28-Second Intervals

Date	$I_0$ $\sigma$	Star $\sigma_b$	$I_0$ $\sigma_s$	Date	$I_0$ $\sigma$	Star $\sigma_b$	$I_0$ $\sigma_s$
23 Feb 81	0.069	0.065	0.023	25 Feb 81	0.059	0.050	0.031
	0.026	0.065	0.060 <sup>b</sup>		0.062	0.050	0.037
	0.070	0.065	0.026		0.057	0.050	0.027
	0.085 <sup>a</sup>	0.065	0.055		0.067 <sup>a</sup>	0.050	0.045
	0.121 <sup>a</sup>	0.065	0.102		0.055	0.053	0.015
	0.053	0.065	0.038 <sup>b</sup>		0.087 <sup>a</sup>	0.053	0.069
	0.058	0.065	0.029 <sup>b</sup>		0.182 <sup>a</sup>	0.123	0.134
	0.131 <sup>a</sup>	0.065	0.114		0.168 <sup>a</sup>	0.069	0.153
	0.064	0.065	0.011 <sup>b</sup>		0.193 <sup>a</sup>	0.120	0.151
	0.067	0.065	0.016		0.091	0.069	0.059
	0.059	0.065	0.027 <sup>b</sup>		0.156	0.123	0.096
	0.062	0.065	0.020 <sup>b</sup>		0.186 <sup>a</sup>	0.123	0.140
24 Feb 81	0.114 <sup>a</sup>	0.055	0.100		0.163	0.123	0.107
	0.134 <sup>a</sup>	0.048	0.125	19 Mar 81	0.107	0.064	0.082
	0.035	0.046	0.030 <sup>b</sup>		0.067	0.069	0.016 <sup>b</sup>
	0.095	0.050	0.083		0.089	0.069	0.056
	0.037	0.046	0.027 <sup>b</sup>		0.069	0.069	0.000
	0.046	0.046	0.000		0.086	0.069	0.051
	0.047	0.046	0.017		0.115 <sup>a</sup>	0.069	0.092
	0.070	0.046	0.053		0.128	0.080	0.100
25 Feb 81	0.044	0.048	0.019 <sup>b</sup>		0.164 <sup>a</sup>	0.069	0.149
	0.053	0.053	0.000	2 Jan 82	0.138 <sup>a</sup>	0.055	0.127
	0.061	0.055	0.040 <sup>b</sup>		0.148 <sup>a</sup>	0.058	0.136
23 Feb 81	0.048 <sup>a</sup>	0.053	0.022 <sup>b</sup>	4 Jan 82	0.080 <sup>a</sup>	0.051	0.062



Table I (continued)

Date	I <sub>o</sub> σ	Star σ <sub>b</sub>	I <sub>o</sub> σ <sub>s</sub>	Date	I <sub>o</sub> σ	Star σ <sub>b</sub>	I <sub>o</sub> σ <sub>s</sub>
4 Jan 82	0.048	0.051	0.017 <sup>b</sup>	4 Feb 82	0.050	0.034	0.037
	0.087 <sup>a</sup>	0.051	0.077		0.035	0.034	0.008
	0.092 <sup>a</sup>	0.051	0.077		0.030	0.034	0.016 <sup>b</sup>
	0.097 <sup>a</sup>	0.051	0.083		0.051	0.034	0.038
	0.097 <sup>a</sup>	0.051	0.083		0.048	0.034	0.034
	0.091 <sup>a</sup>	0.051	0.075		0.028	0.034	0.019 <sup>b</sup>
	0.090	0.051	0.074		0.030	0.034	0.016 <sup>b</sup>
5 Jan 82	0.063	0.051	0.037		0.042	0.034	0.025
	0.065	0.051	0.040		0.028	0.034	0.019 <sup>b</sup>
	0.055	0.051	0.021		0.038	0.042	0.018 <sup>b</sup>
	0.059	0.051	0.030		0.041	0.042	0.009 <sup>b</sup>
	0.064	0.051	0.039	5 Feb 82	0.048	0.042	0.023
	0.049	0.051	0.014 <sup>b</sup>		0.033	0.042	0.026 <sup>b</sup>
	0.093 <sup>a</sup>	0.051	0.078		0.028	0.042	0.031 <sup>b</sup>
	0.055	0.051	0.021		0.042	0.042	0.000
	0.065	0.051	0.040		0.050	0.042	0.027
11 Jan 82	0.069	0.049	0.049		0.040	0.042	0.013 <sup>b</sup>
	0.056	0.049	0.027		0.048	0.042	0.023
	0.051	0.049	0.014		0.039	0.042	0.016 <sup>b</sup>
	0.093 <sup>a</sup>	0.049	0.079		0.042	0.042	0.000
	0.055	0.049	0.025		0.033	0.042	0.026 <sup>b</sup>
					0.049	0.042	0.025

Root-mean-square I<sub>o</sub> fast flicker, N/S = 0.060 ± 0.005. Flicker in magnitudes = 0.062 ± 0.005.

<sup>a</sup> These data sets were used for the computation of the power spectrum.

<sup>b</sup> These values are imaginary and are included in the calculation of the root-mean-square flicker.

TABLE II  
 Fluctuation of Signal Relative to Total Signal  
 at Nominal 40-Minute Intervals

Date	Number	Average Spacing (min)	$\sigma$	$\sigma_s$
23 Feb 81	9	37.5	0.067	0.056
24	6	36.0	0.151	0.146
25	9	39.5	0.049	0.035
18 Mar 81	8	58.6	0.137	0.131
19 81	7	62.0	0.089	0.081
4 Jan 82	5	33.5	0.024	0.025 <sup>a</sup>
5 82	5	46.2	0.039	0.019
11 82	3	31.5	0.058	0.045
4 Feb 82	7	33.2	0.014	0.032 <sup>a</sup>
5 82	6	32.2	0.062	0.056
9 Jun 82	9	32.0	0.070	0.060
10 82	5	38.5	0.039	0.019

Measured fluctuations of stars on the same nights from 34 sets of  
 2 to 6 measurements 0.035 mag.

<sup>a</sup>These numbers are imaginary and are included in the calculation of the  
 root-mean-square fluctuation.

Root-mean-square Io fluctuation  $0.069 \pm 0.013$ .

TABLE III

Observations of the Same Hemisphere Separated by 48 Hours

Hemisphere	Date UT	Duration min.	Number	Mean 2.5 log p <sup>a</sup> mag.	s.d. mag.	Corrected for Measure- ment error mag.
Leading	23 Feb 81	308	13	-0.096	0.172	0.165
	25	329	12	0.147		
	2 Jan 82	19	2	-0.250	0.089	0.074
	4 82	134	5	-0.124		
Trailing	13 Dec 79	301	7	0.250	0.271	0.266
	15	178	6	-0.133		
	20	249	5	0.110	0.125	0.115
	22	346	13	-0.120		
	24	—	1	0.08		
Perijovian	22 Apr 81	44	3	-0.020	0.071	0.050
	24	63	6	-0.120		
Apojovian	21 Apr 81	98	3	-0.203	0.057	0.027
	23 81	286	12	-0.123		

<sup>a</sup>The quantity p is the quasi-geometric albedo as defined in Sinton et. al (1983).

Net rms variation of I<sub>o</sub> = 0.141 ± 0.038 mag.

Variation reduced to fraction of signal = 0.139 ± 0.036.

FIGURE CAPTIONS

Fig. 1. The noise-to-signal ratios, N/S, for stars (points) and Io (open circles) determined during various observing runs at  $4.8 \mu\text{m}$ . The curves are weighted linear regression fits to the stellar data in the form of Eq. (8). The observing runs for several panels are (a) 22-25 Feb 81, (b) 18-19 Mar 81, (c) 2-5 Jan 82, (d) 10-12 Jan 82. The error bars for all points except some of those of panel (a) are  $(1/30)^{1/2}$  of the value. For Io measurements that lie above the dotted curve, there is 95 percent confidence that Io was flickering.

Fig. 2. The noise-to-signal ratio for the observing run 3-5 Feb 82 for stars (open circles) and Io (11 points in the speckled box). During this run Io flickered very little.

Fig. 3. Rapid changes in Io's flux at  $4.8 \mu\text{m}$ , 21 Apr 81 (upper) two traces. The lower traces of stars show the IRTF bolometer system noise. All measurements were made with the same gain and the stars have nearly the same brightness as Io. The breaks in the traces were to check centering.

Fig. 4. The power spectrum of Io's fluctuations. The solid points are from the mean power spectrum derived from the selected 21 time series. The plotted points have been reduced so that selection effect is eliminated, and their average power is the same as for the entire 91 rapid time series. The open circles are spectral points derived by the method described in the text from sequences every  $\sim 40$  minutes (middle point), every 48 hours (left point), and the 91 rapid time series (right point). The line has a slope of  $-1$ .

Fig. 5. Time series of measurements of Io's flux at  $10 \mu\text{m}$ , 24 Apr 81 (upper three traces). The star observed in the lower trace was about three times brighter than Io, but the data are shown at the same gain. The drift evident in the star's trace is caused by a slight tracking error.

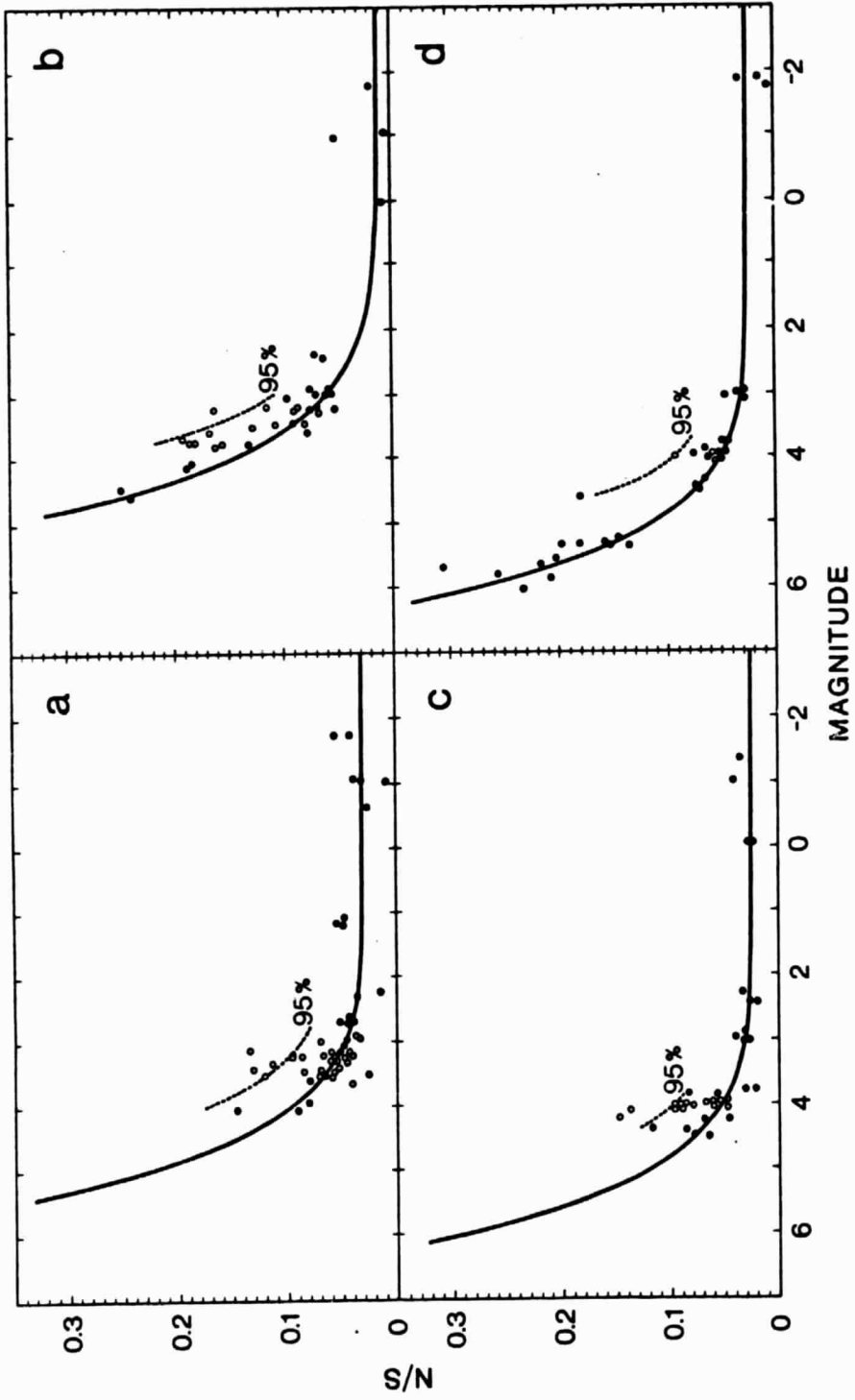


Figure 1

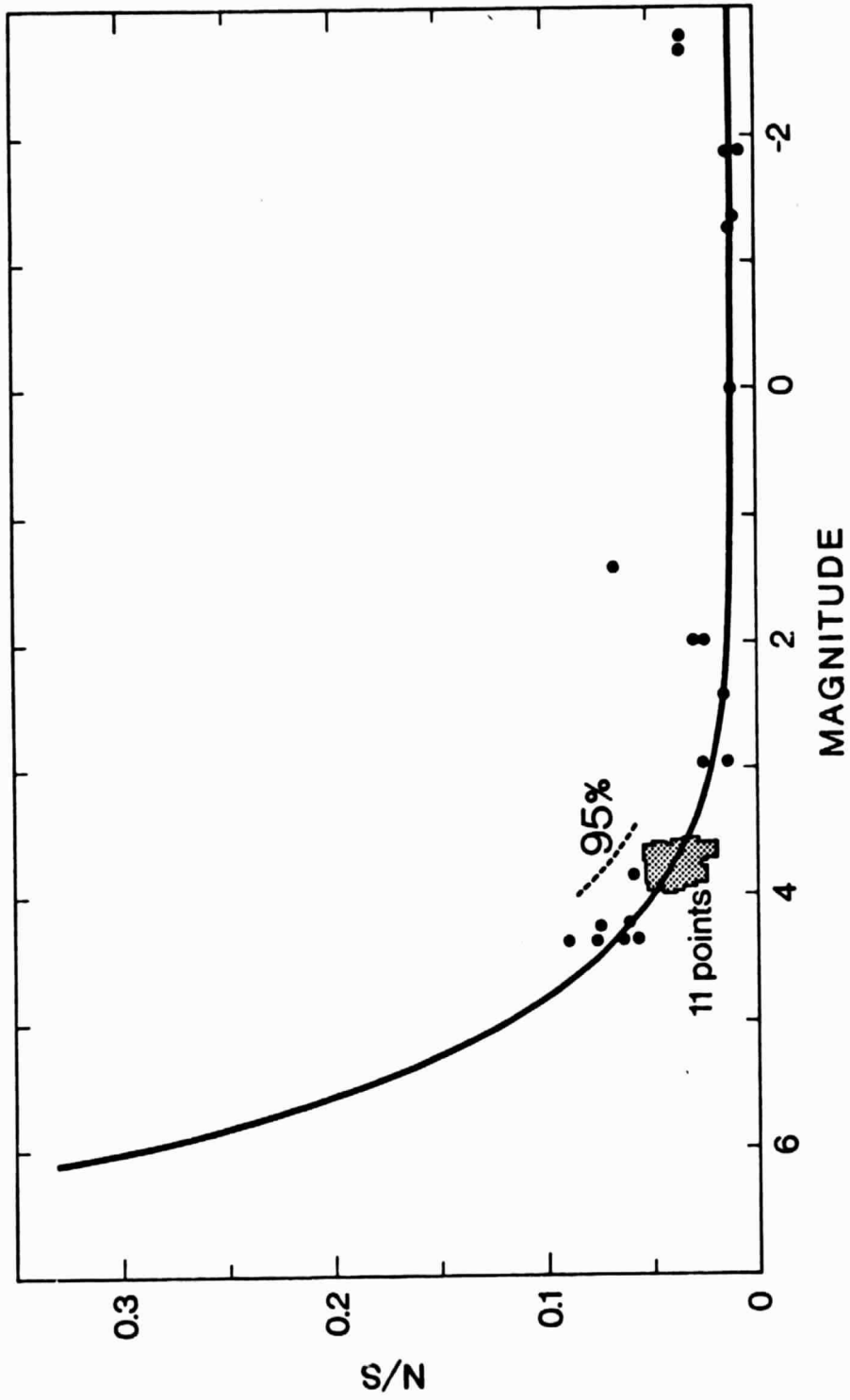


Figure 2

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OF POOR QUALITY

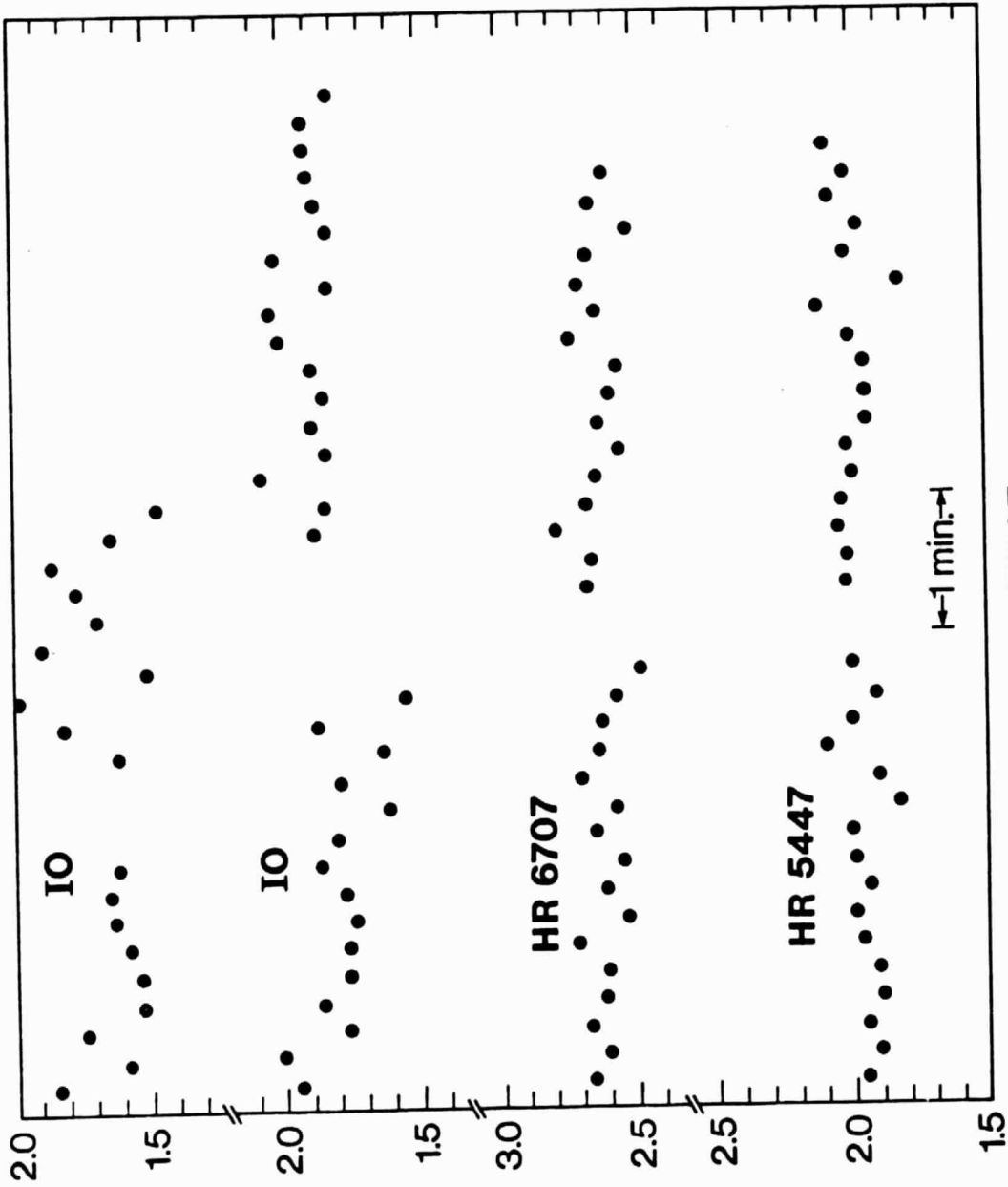


Figure 3

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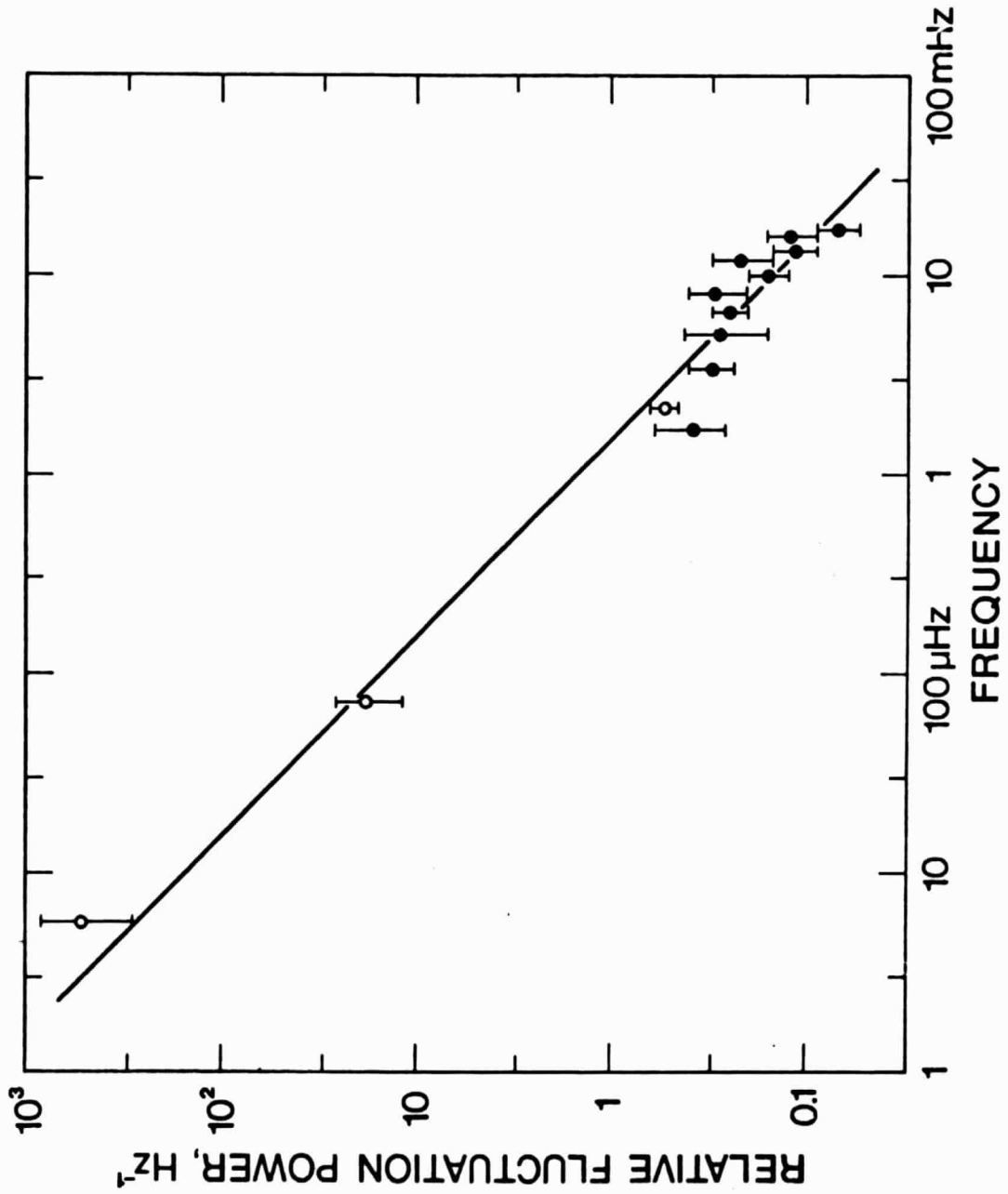


Figure 4



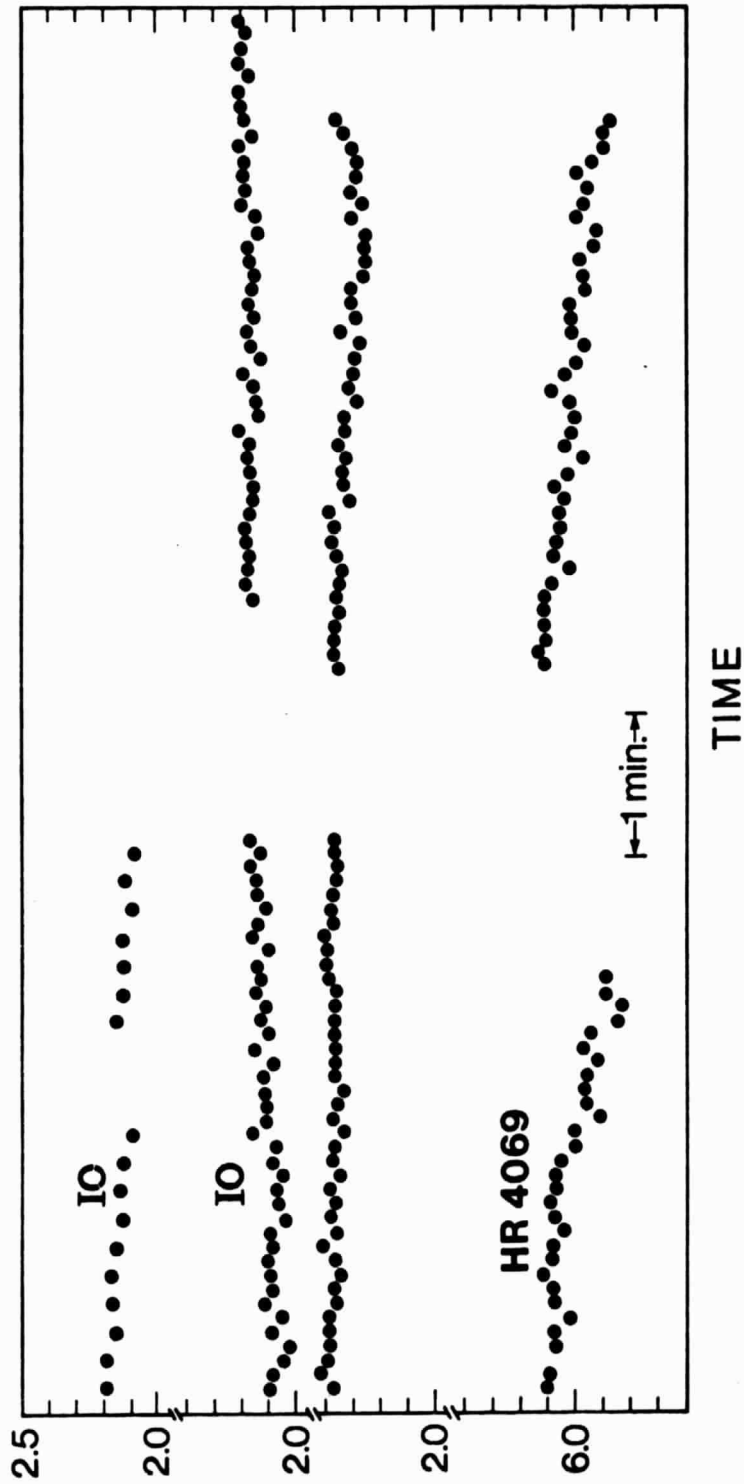


Figure 5