NASA CONTRACTOR REPORT 166583

(NASA-CR-166583) OPTIMATION OF COOLED SHIELDS IN INSULATIONS Final Report (Illinois Univ.) 89 p HC A05/MF A01 N84-27999
CSCL 20D Unclas
G3/34 13993

Optimation of Cooled Shields in Insulations

J. C. Chato
J. M. Khodadadi
J. Seyed-Yagoobi

JUL 1984
RECEIVED
NASA STI FACILITY
ACCESS DEPT.

CONTRACT NAG2-219
April 1984

NASA

REPRODUCED BY
U.S. DEPARTMENT OF COMMERCE
NATIONAL TECHNICAL
INFORMATION SERVICE
SPRINGFIELD, VA 22161

NASA CONTRACTOR REPORT 166583

Optimization of Cooled Shields in Insulations

J. C. Chato
J. M. Khodadadi
J. Seyed-Yagoobi
Department of Mechanical and Industrial Engineering
University of Illinois at Urbana-Ccampaign
Urbana, IL

Prepared for
Ames Research Center
under Grant NAG2-219

National Aeronautics and
Space Administration

Ames Research Center
Moffett Field California 94035

ABSTRACT

A relatively simple method has been developed to optimize the location, temperature, and heat dissipation rate of each cooled shield inside an insulation layer. The method is based on the minimization of the entropy production rate which is proportional to the heat leak across the insulation. The results show that the maximum number of shields to be used in most practical applications is three. However, cooled shields are useful only at low values of the overall, cold wall to hot wall absolute temperature ratio. The performance of the insulation system is relatively insensitive to deviations from the optimum values of temperature and location of the cooling shields.

Design curves are presented for rapid estimates of the locations and temperatures of cooling shields in various types of insulations, and an equation is given for calculating the cooling loads for the shields.

NOMENCLATURE

A	Area of heat flow, m^2
C_p	Specific heat of the boiloff vapor, kJ/kg•K
D	Functional defined by Eq. (14)
F	Functional defined by Eq. (13)
h_{fg}	Latent heat of vaporization of the boiloff liquid, kJ/kg
k	Thermal conductivity, W/m-K; with subscripts, coefficients in Eq. (1)
L	Overall thickness of insulation, m*
m,n	Exponents in conductivity function, Eq. (1)
P	T_S/T_C, temperature ratio
q	Heat flow rate, W
R	T_C/T_H, overall temperature ratio
s	Dimensionless entropy production rate defined by Eq. (5)
$\dot{S}$	Entropy production rate, W/K
t	Thickness between walls with single shield between, m*
T	Absolute temperature, K
x	Distance from cold wall, m*
x'	Distance from cold wall in a multi-shield configuration, m*
X	x/t, dimensionless distance*
X'	x'/L, dimensionless distance*
Y	Defined by Eq. (8)

Subscripts

C	Cold wall
H	Hot wall
i	i-th shield
min	Minimum
opt	Optimum
S	Shield

---

*For systems with single shield L = t, x = x', X = X'.

INTRODUCTION

The search for the ultimate, energy efficient insulation system has led in the past few years to a fascinating rediscovery and application of some fundamental concepts of thermodynamics: specifically, the second law and the use of entropy production rates and availability (or exergy) for design optimization purposes. The classical approach has been to minimize the heat flow between surfaces at different temperatures.

The concept of a single vapor-cooled shield in an insulation has been treated theoretically as far back as 1959 in Scott's classic textbook on cryogenics [1] and designs employing them were described not much later [2]. Paivanas, et al., obtained a patent [3] and later reported on the use of uniformly spaced multiple shields which were cooled by the boil-off from the insulated dewar [4]. Eyssa and Okasha [5] considered only radiative heat exchange between shields and minimized the total refrigeration power required. Hilal, et al., [6,7] used a similar minimization of refrigeration power as the design basis. Related works were reported by Bejan, et al., [8-11].

Recently, Bejan [12] proposed a new point of view, based on the second law of thermodynamics, which considers thermal insulations as dissipators of useful mechanical power (i.e. the availability or exergy) or, alternately, as generators of irreversibility or entropy. Thus, in this method, optimization of an insulation corresponds to minimization of either the entropy production rate or the irreversibility, or the decrease of availability. Various applications of this concept to insulation systems have been documented subsequently [13,14].

Our work grew out of an examination of Cunnington's paper [13] who utilized a numerical technique to find optimum temperatures at given locations for one and two shields for a thermal conductivity function of the form

$k_1T^{0.6}$. Although several equations seemed to be incorrectly printed we have found two of the design curves to be essentially correct. Thus, our purpose was

1. To develop a simple optimization technique;
2. To generalize the results to a broader class of insulations; and
3. To develop simple design methods for cooled shields.

The essentials of this report were already published [15].

ANALYSIS

We accept the previously developed concept that to optimize an insulation system is equivalent to minimizing the entropy production rate. In addition, we assume one-dimensional heat flow and that the heat capacity of the boil-off gas is adequate to do the cooling for all shields and does not impose a restriction on the optimization. In contrast to Bejan [9,11] who has developed a constrained optimization based on the heat capacity of the boiloff we employ the argument that in all practical systems the boil-off is generated by cooling of some equipment in addition to the heat leakage across the insulation.

Parallel heat paths, e.g. supports, have not been considered. However, each path can be optimized separately using its own thermal conductivity function. Then a design decision has to be made whether the two structures should be independently cooled at their respective optimum conditions.

We examine the general situation of an insulation where equivalent thermal conductivity, k, can be expressed as a two-term function of the absolute temperature

$$k = k_1 T^m + k_2 T^n \qquad (1)$$

where, typically, the first term represents actual conduction with $m \geq 1$ and the second term represents radiation with $n \geq 3$. In the following, m and n can be any value except -1.

The heat flow across a layer of insulation can be expressed in terms of Fourier's law

$$q\,dx = Ak\,dT \qquad (2)$$

Substituting k from Eq. (1) and integrating across a layer from one end at 1, to the other at 2, yields

$$q = \frac{A}{x_2 - x_1}\left[\frac{k_1}{m + 1}(T_2^{m+1} - T_1^{m+1}) + \frac{k_2}{n + 1}(T_2^{n+1} - T_1^{n+1})\right]. \qquad (3)$$

Now consider the insulation with a cooled shield at T_S located at x between a hot surface at T_H and a cold one at T_C, separated by the insulation thickness, t, as shown in Fig. 1a. The entropy production rate for the insulation can be determined from the heat flows and temperatures as follows

$$\dot{S} = -\frac{q_H}{T_H} + \frac{q_C}{T_C} + \frac{q_S}{T_S} \qquad (4)$$

where $q_S = q_H - q_C$.

The heat flow terms can be expressed in the form of Eq. (3) and the resulting expression can be non-dimensionalized using the following terms

$$s \equiv \frac{\dot{S}t}{Ak_H} \text{ where } k_H = k \text{ at } T_H, \qquad (5)$$

$$P \equiv \frac{T_S}{T_C}, \qquad (6)$$

$$R \equiv \frac{T_C}{T_H}, \qquad (7)$$

$$\gamma \equiv \frac{k_2(m + 1)}{k_1(n + 1)} T_H^{n-m}, \qquad (8)$$

and

$$X \equiv \frac{x}{t}. \qquad (9)$$

The resulting equation is

$$
\begin{aligned}
s(m + 1)(1 + \gamma \frac{n + 1}{m + 1}) \\
&= \frac{1}{1 - X} \{[(PR)^{m + 1} - (PR)^{m} - 1 + (PR)^{-1}] \\
&+ \gamma[(PR)^{n+1} - (PR)^{n} - 1 + (PR)^{-1}]\} \\
&+ \frac{1}{X} \{R^{m}[P^{m+1} - P^{m} - 1 + P^{-1}] \\
&+ \gamma\, R^{n}[P^{n+1} - P^{n} - 1 + P^{-1}]\} \qquad (10)
\end{aligned}
$$

Since R, the overall temperature ratio, is generally known, s is a function of P and X, and its extreme value can be found by differentiating it with respect to each variable separately and setting the results equal to zero. This procedure yields two equations to be solved simultaneously: $\partial s/\partial P = 0$ and $\partial s/\partial X = 0$. Because of the regular form of the expressions, one of the final two equations contains only a single unknown as follows:

$$
\frac{R^{m} F(m,P) + \gamma R^{n} F(n,P)}{[R^{m-1} D(m,P) + \gamma R^{n-1} D(n,P]^{2}}
= \frac{F(m,PR) + \gamma F(n,PR)}{[D(m,PR) + \gamma D(n,PR)]^{2}} \qquad (11)
$$

$$
\frac{X}{1-X} = - \frac{R^{m-1} D(m,P) + \gamma R^{n-1} D(n,P)}{D(m,PR) + \gamma D(n,PR)} \qquad (12)
$$

where the following functionals were used:

$$F(b,B) \equiv B^{b+1} - B^{b} - 1 + B^{-1} \tag{13}$$

$$D(b,B) \equiv (b + 1)\, B^{b} - bB^{b-1} - B^{-2}. \tag{14}$$

Thus, to find the optimum temperature and location for a shield, Eq. (11) can be solved for P, and then X can be calculated from Eq. (12). The heat to be removed by the shield, $q_S = q_H - q_C$, can be found, as before, from Eq. (3). In dimensionless form the equation becomes

$$\begin{aligned} &\frac{q_S t}{Ak_H\, T_H}(m+1)\left(1 + \gamma\,\frac{n+1}{m+1}\right) \\ &\qquad = \frac{1 - (PR)^{m+1} + \gamma[1 - (PR)^{n+1}]}{1 - X} \\ &\qquad - \frac{(PR)^{m+1} - R^{m+1} + \gamma[(PR)^{n+1} - R^{n+1}]}{X}. \end{aligned} \tag{15}$$

For multiple shields t_i represents the distance between the two surfaces surrounding the i-th shield on either side, $T_{H,i}$ and $T_{C,i}$ are the temperatures of these two surfaces, $X_i = x_i/t_i$ is the location of the shield relative to t_i, and x_i' is the location of the shield relative to the cold wall as shown in Fig. 1b. To determine the optimum temperatures and locations for multiple shields, first we assumed a temperature for the first shield next to the cold wall, then we used Eqs. (11) and (12) to find the temperature and location of the second shield. This process was repeated for the rest of the shields and the hot wall. Thus, each shield was optimized consecutively with respect to the two surfaces on either side. With given values of the overall temperatuare ratio, R, and of the number of shields, the process requires iterative solution.

To put the results into proper perspective, the entropy production rates can be compared to the thermodynamically minimum rate obtainable through spatially continuous cooling. According to Bejan [12], this rate is

$$\dot{S}_{min} = \frac{A}{\ell}\left[\int_{T_C}^{T_H} (k)^{1/2}\, T^{-1}\, dT\right]^2. \qquad (16)$$

This expression was evaluated analytically for the single-term functions of k, i.e. for $\gamma = 0$, and numerically otherwise.

RESULTS AND DISCUSSION

The first set of curves, Figs 2 through 9, show the relative entropy production rates for various thermal conductivity functions and for up to four optimally cooled shields as functions of the overall temperature ratio $R \equiv T_C/T_H$. The curves show that the entropy production rate increases with decreasing values of the temperature ratio, R, and with increasing values of the exponent, m and n. Adding shields, of course, reduces the entropy production rate; but for most of the practical temperature range, say $0.01 < R < 0.4$, only three shields contribute to significant decreases and adding a fourth shield can be considered unnecessary. No shields are useful at high values of R; but this "high" range is strongly dependent on the exponent of the temperature. The curves developed with $k = k_1 T^{0.6}$ for one and two shields were very close to those given by Cunnington [13], converted appropriately.

Study of the results of two-term conductivities reveals that the curves fall between those obtained for each of the two terms alone. If γ is small the first term, T^m, dominates; whereas if γ is large (>10), the second term, T^n, controls. Thus, general conclusions can be drawn from examining the results of the single-term conductivities.

The second set of curves, Figs. 10 through 31, show the optimum temperature ratios, T_S/T_H, and optimum locations, x'/L, of cooled shields as functions of the overall temperature ratio, T_C/T_H, for various thermal conductivity functions and with different number of cooled shields.

Figures 10 and 11 show the optimum single shield temperature ratios, $PR = T_S/T_H$, and locations, $X = x/L$, for five conductivity functions. Both of these functions generally decrease with decreasing R. The other figures in this set show shield temperatures and locations for systems with up to three shields and for both single-term and two-term conductivities. The results are strongly non-linear. For example, for k_1T^3 and R = 0.01, the optimum temperature ratios for three shields are about 0.09, 0.3, and 0.6 and the optimum locations are about 0.05, 0.2, and 0.5. As is to be expected, our unconstrained optimization yields a somewhat better performance per shield than Bejan's [9,11] constrained method.

The sensitivities of the entropy production rates to deviations from the optimum values of PR and X are demonstrated in the last set of curves, Figs. 32 through 35, for single shields. The sensitivity increases with the value of the exponents, m and n, but the curves are relatively flat near the minima. A ±20 percent change from optimum, for example, has negligible effect. Thus, the system is relatively tolerant of deviations from the optimum design conditions.

Calculations with two different conductivities on the two sides of a cooled shield show that using the better insulator on both sides always yields the optimum condition. However, if for some reason two types of insulations have to be used, then the better insulator should be placed on the warm side of the shield.

REFERENCES

1. Scott, R. B., Cryogenic Engineering, D. Van Nostrand Co., Princeton, N.J., 1959.

2. Murto, P. J., "A Gas-Shielded Storage and Transport Vessel for Liquid Helium," Advances in Cryogenic Engineering, Vol. 7, 1962, pp. 291-295.

3. Paivanas, J. A., A. W. Francis, and D. I-J. Wang, "US Patent No. 3, 133, 422, "Insulation Construction," May 19, 1964.

4. Paivanas, J. A., O. P. Roberts, and D. I-J. Wang, "Multishielding--An Advanced Superinsulation Technique," Advances in Cryogenic Engineering, Vol. 10, 1965, pp. 197-207.

5. Eyssa, Y. M., and O. Okasha, "Thermodynamic Optimization of Thermal Radiation Shields for a Cryogenic Apparatus," Cryogenics, Vol. 18, 1978, pp. 305-307.

6. Hilal, M. A. and R. W. Boom, "Optimization of Mechanical Supports for Large Superconductive Magnets," Advances in Cryogenic Engineering, Vol. 22, 1977, pp. 224-232.

7. Hilal, M. A., and Y. M. Eyssa, "Minimization of Refrigeration Power for Large Cryogenic Systems," Advances in Cryogenic Engineering, Vol. 25, 1980, pp. 350-357.

8. Bejan, A., and J. L. Smith, Jr., "Thermodynamic Optimization of Mechanical Supports for Cryogenic Apparatus," Cryogenics, Vol. 14, 1974, pp. 158-163.

9. Bejan, A., "Discrete Cooling of Low Heat Leak Supports to 4.2K," Cryogenics, Vol. 15, 1975, pp. 290-292.

10. Bejan, A., and J. L. Smith, Jr., "Heat Exchangers for Vapor-Cooled Conducting Supports of Cryostats," Advances in Cryogenic Engineering, Vol. 21, 1976, pp. 247-256.

11. Bejan, A., Entropy Generation through Heat and Fluid Flow, Wiley-Interscience, N.Y., 1982, Ch. 9.

12. Bejan, A. "A General Variational Principle for Thermal Insulation System Design," Int. J. Heat Mass Transfer, Vol. 22, 1979, pp. 219-227.

13. Cunnington, G. R., "Thermodynamic Optimization of a Cryogenic Storage System for Minimum Boiloff," No. AIAA-82-0075, AIAA 20th Aerospace Sciences Meeting, Orlando, Florida, 1982.

14. Schultz, W. and A. Bejan, "Exergy Conservation in Parallel Thermal Insulation Systems," Int. J. Heat Mass Transfer, Vol. 26, 1983, pp. 335-340.

15. Chato, J. C. and J. M. Khodadadi, "Optimization of Cooled Shields in Insulations," ASME Paper No. 83-WA/HT-57, 1983, to be published, Trans. ASME, J. Heat Transfer.

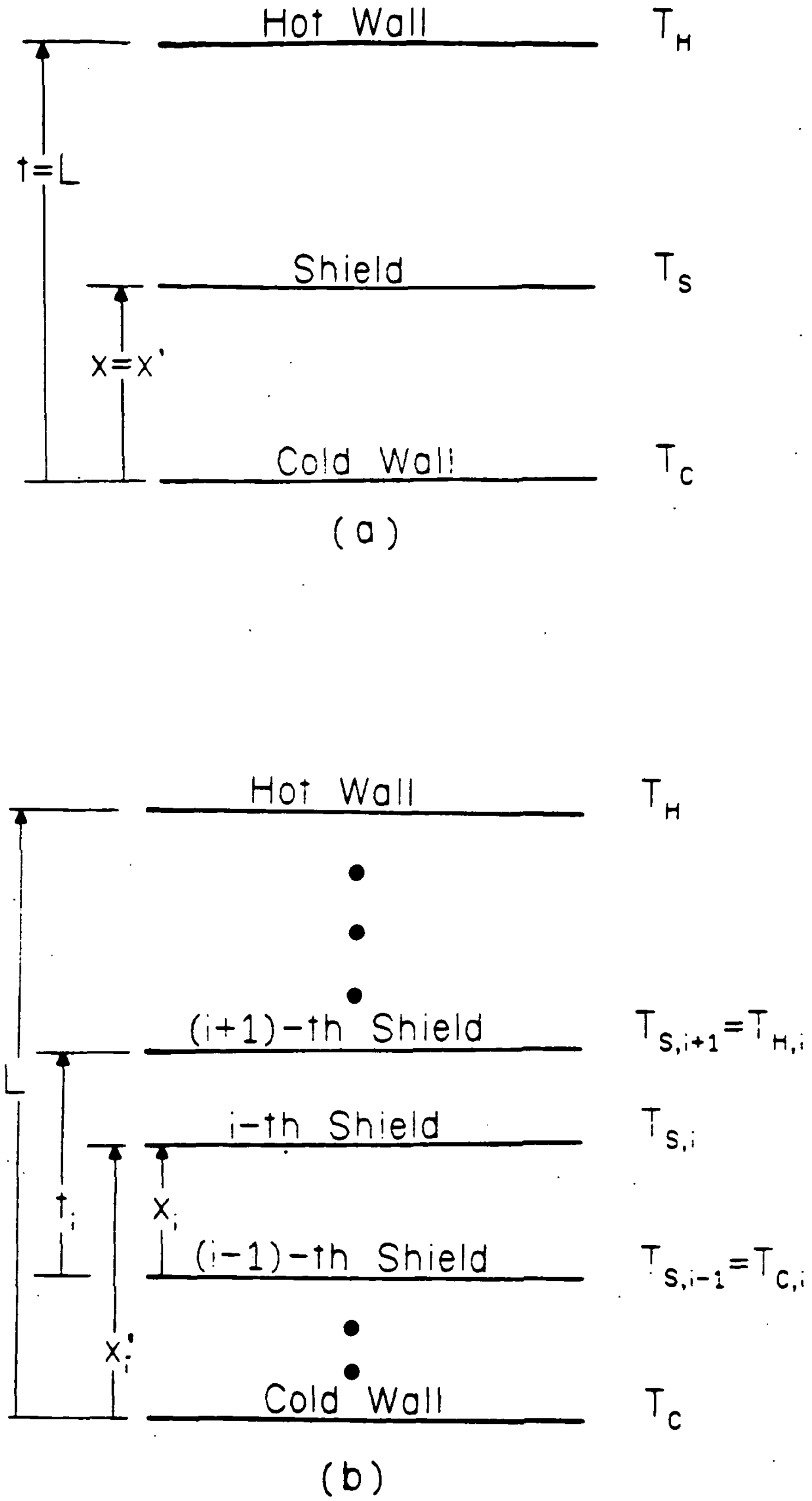


Figure 1 Schematic of the Nomenclature for (a) Single and (b) Multiple Shields

Curve Set 1: Figures 2 through 9

The effect of optimally cooled shields on

the entropy production rate for various thermal conductivities.

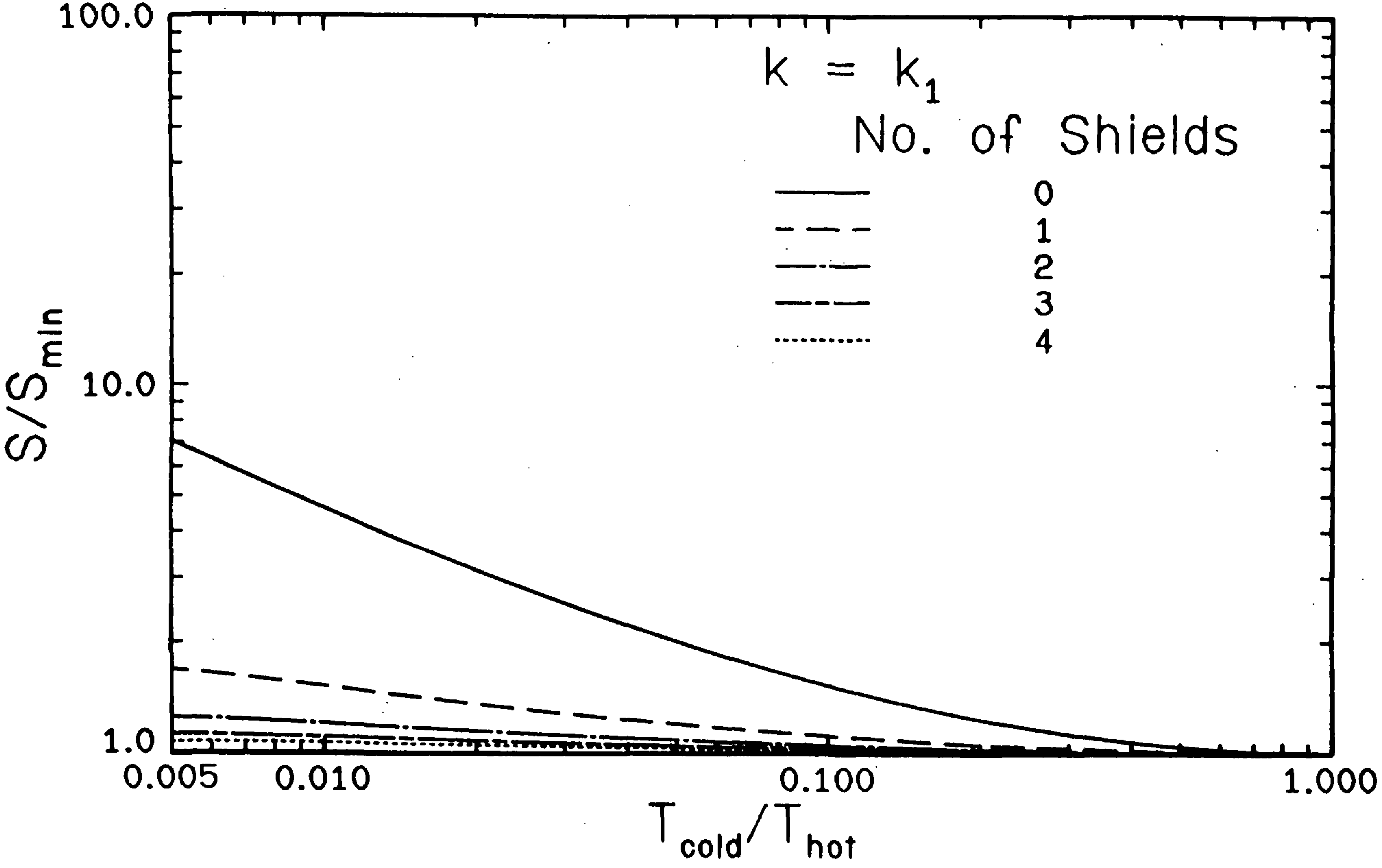

k = k_1
No. of Shields
0
1
2
3
4
100.0
10.0
1.0
S/S_min
0.005
0.010
0.100
1.000
T_cold/T_hot

Figure 2

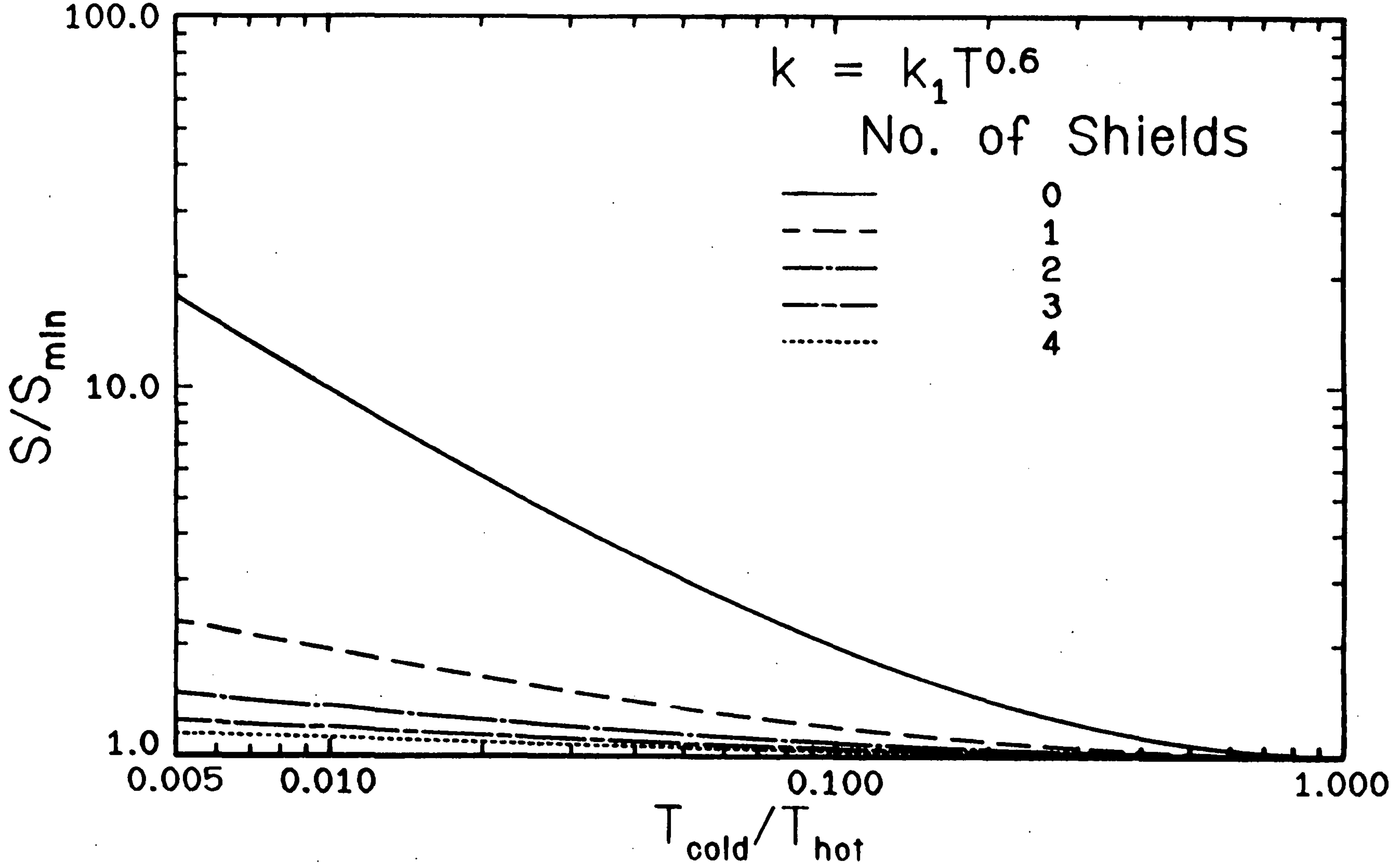

$k = k_1 T^{0.6}$
No. of Shields
0
1
2
3
4
100.0
10.0
1.0
S/S_{min}
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}

Figure 3

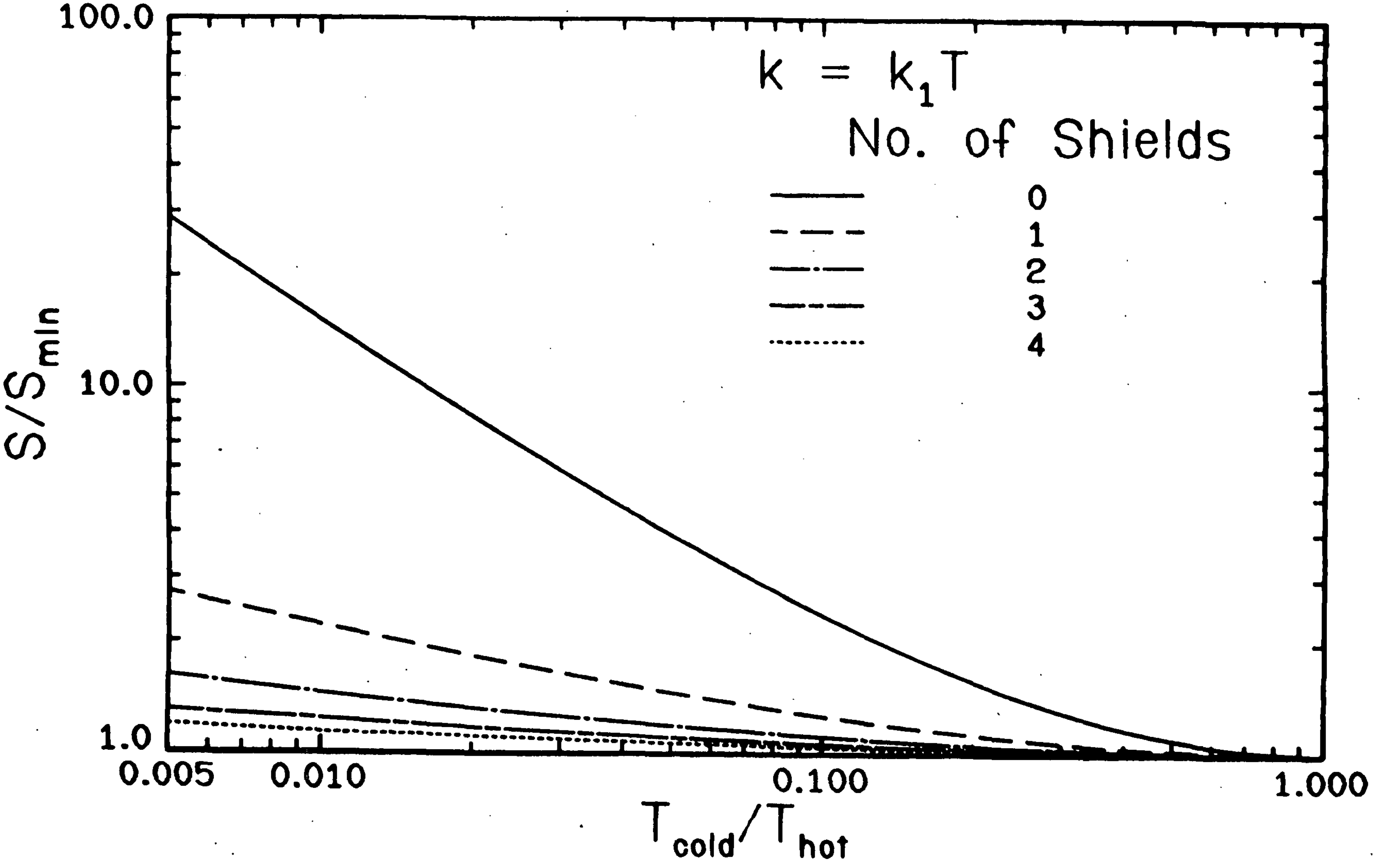

k = k_1T
No. of Shields
0
1
2
3
4
100.0
10.0
1.0
S/S_{min}
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}

Figure 4

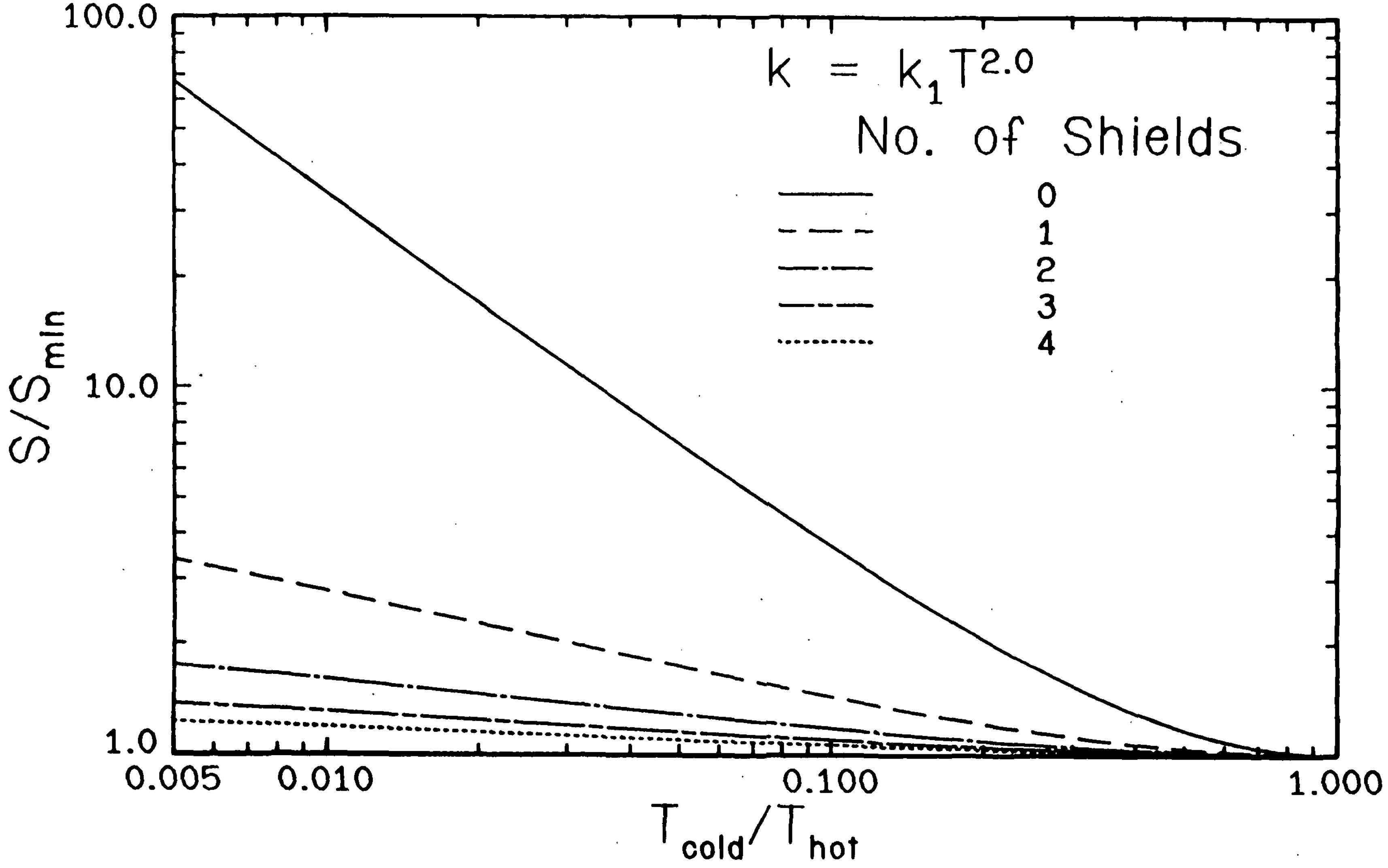

$k = k_1 T^{2.0}$
No. of Shields
0
1
2
3
4
100.0
10.0
1.0
S/S_{min}
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}

Figure 5

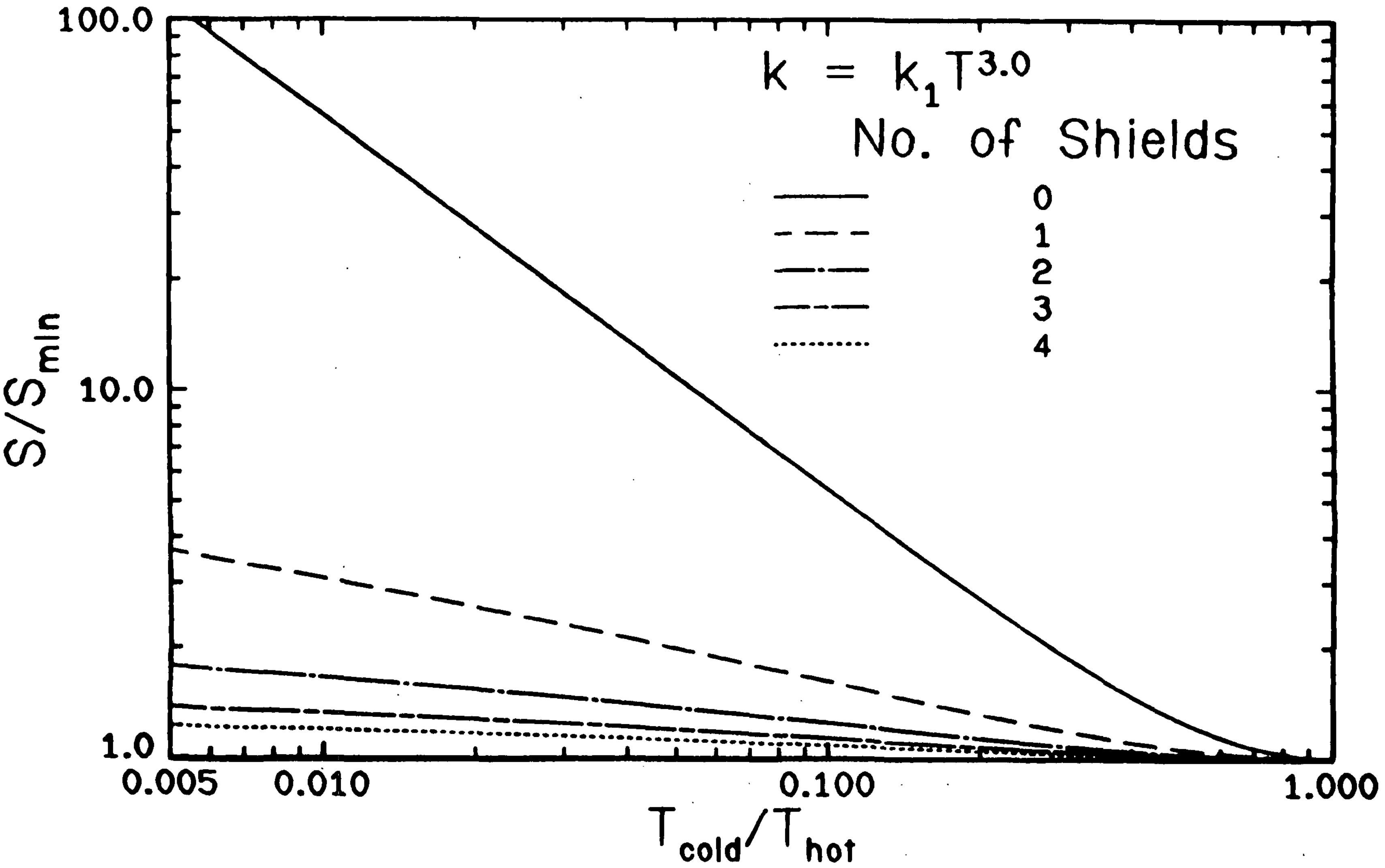

$k = k_1 T^{3.0}$
No. of Shields
0
1
2
3
4
100.0
10.0
1.0
S/S_{min}
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}

Figure 6

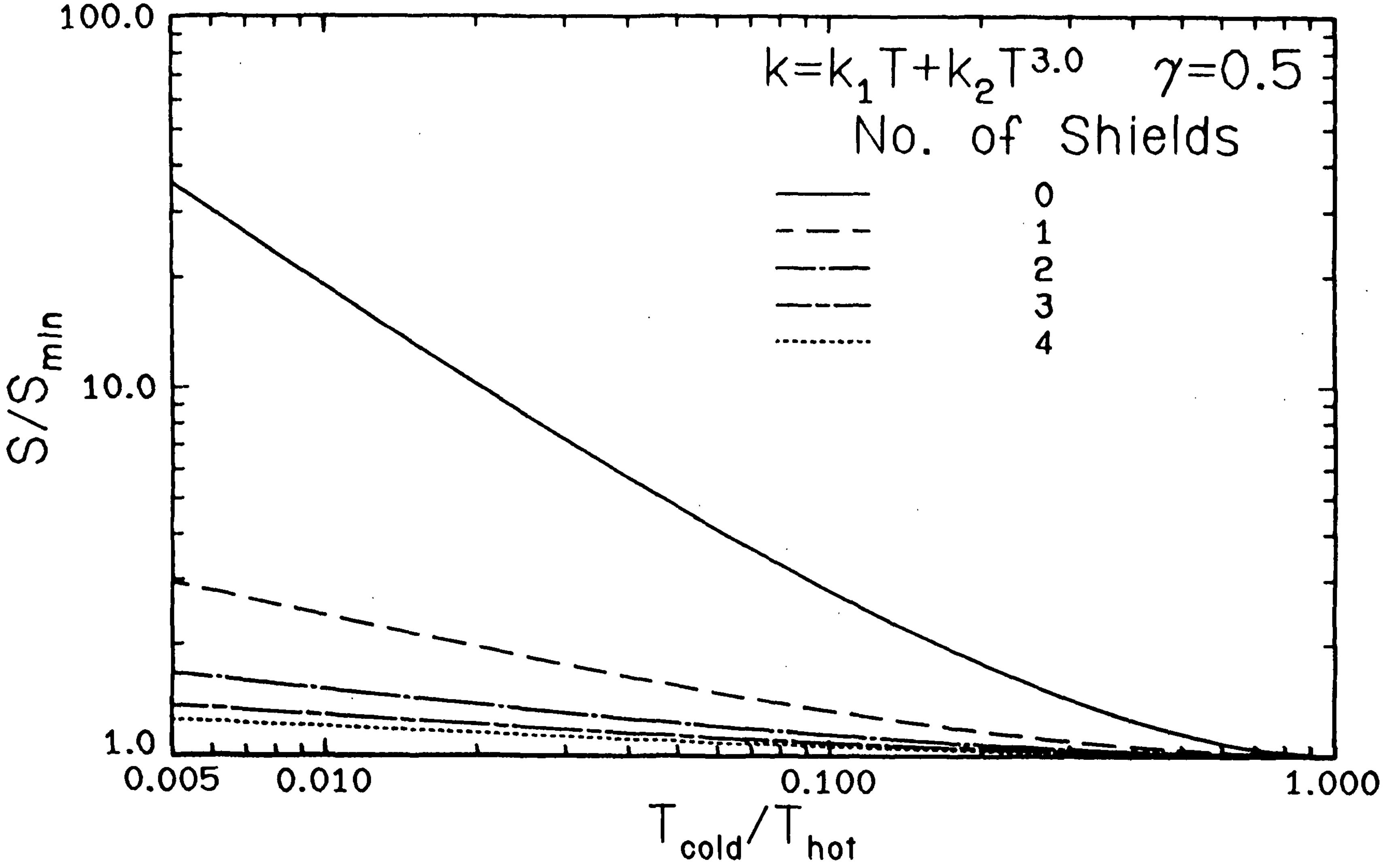

$k=k_1T+k_2T^{3.0}$ $\gamma=0.5$
No. of Shields
0
1
2
3
4
100.0
10.0
1.0
S/S_{min}
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}

Figure 7

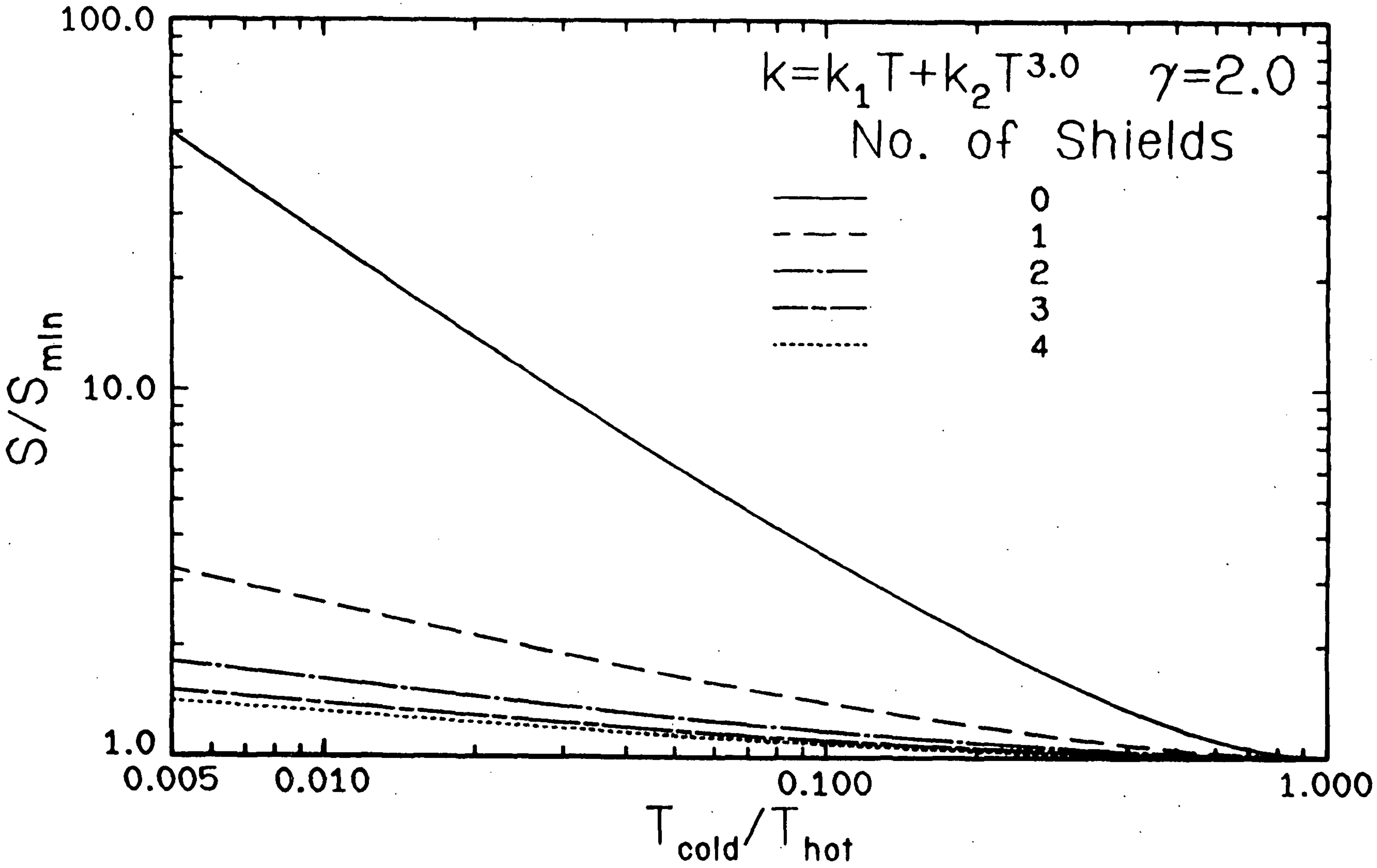

100.0
10.0
1.0
S/S_{min}
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}
$k=k_1T+k_2T^{3.0}$ $\gamma=2.0$
No. of Shields
0
1
2
3
4

Figure 8

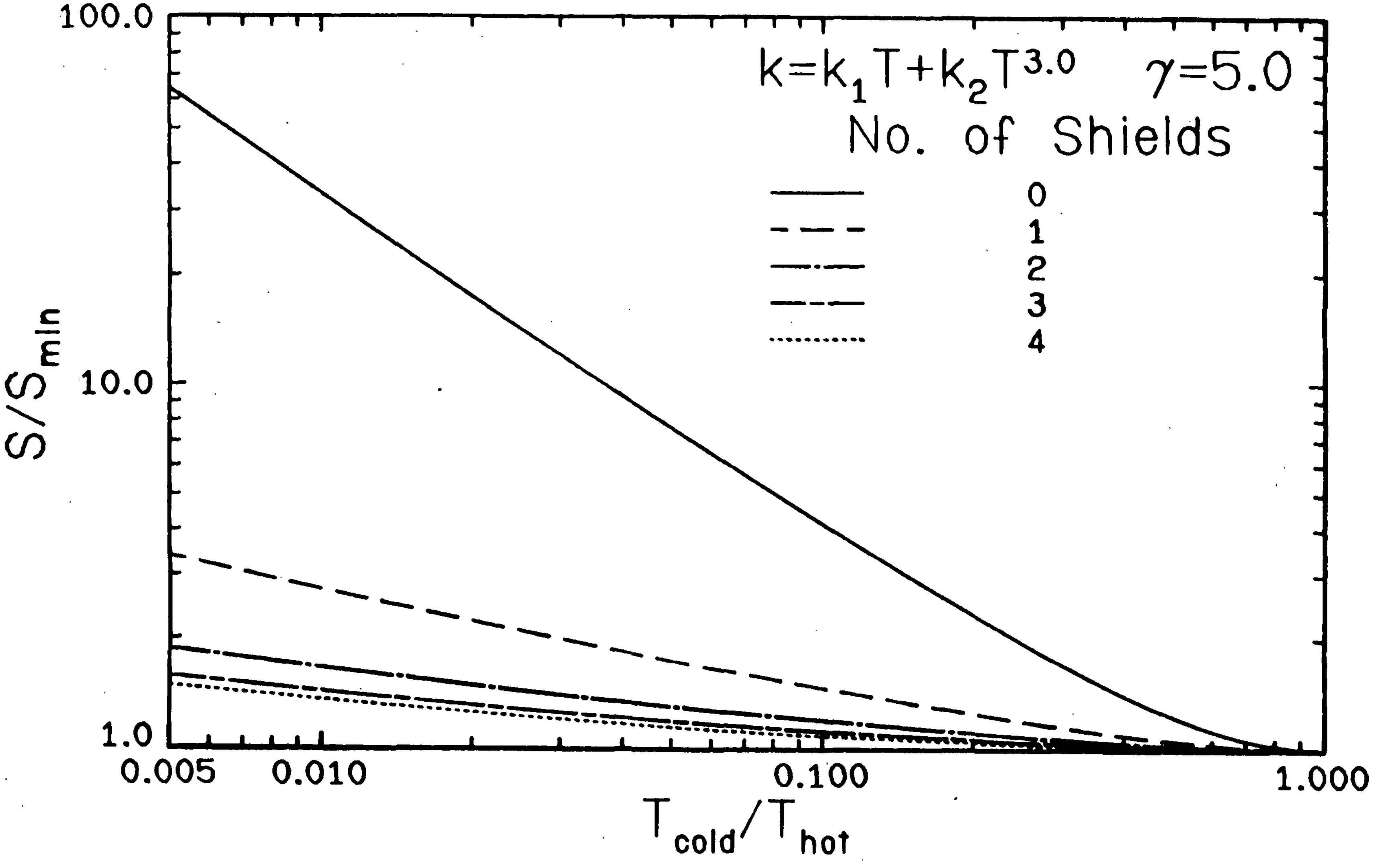

$k=k_1T+k_2T^{3.0}$ $\gamma=5.0$
No. of Shields
0
1
2
3
4
100.0
10.0
1.0
0.005
0.010
0.100
1.000
S/S_{min}
T_{cold}/T_{hot}

Figure 9

Curve Set 2: Figures 10 through 31

Optimal shield temperatures and locations for various thermal conductivity functions with different number of shields.

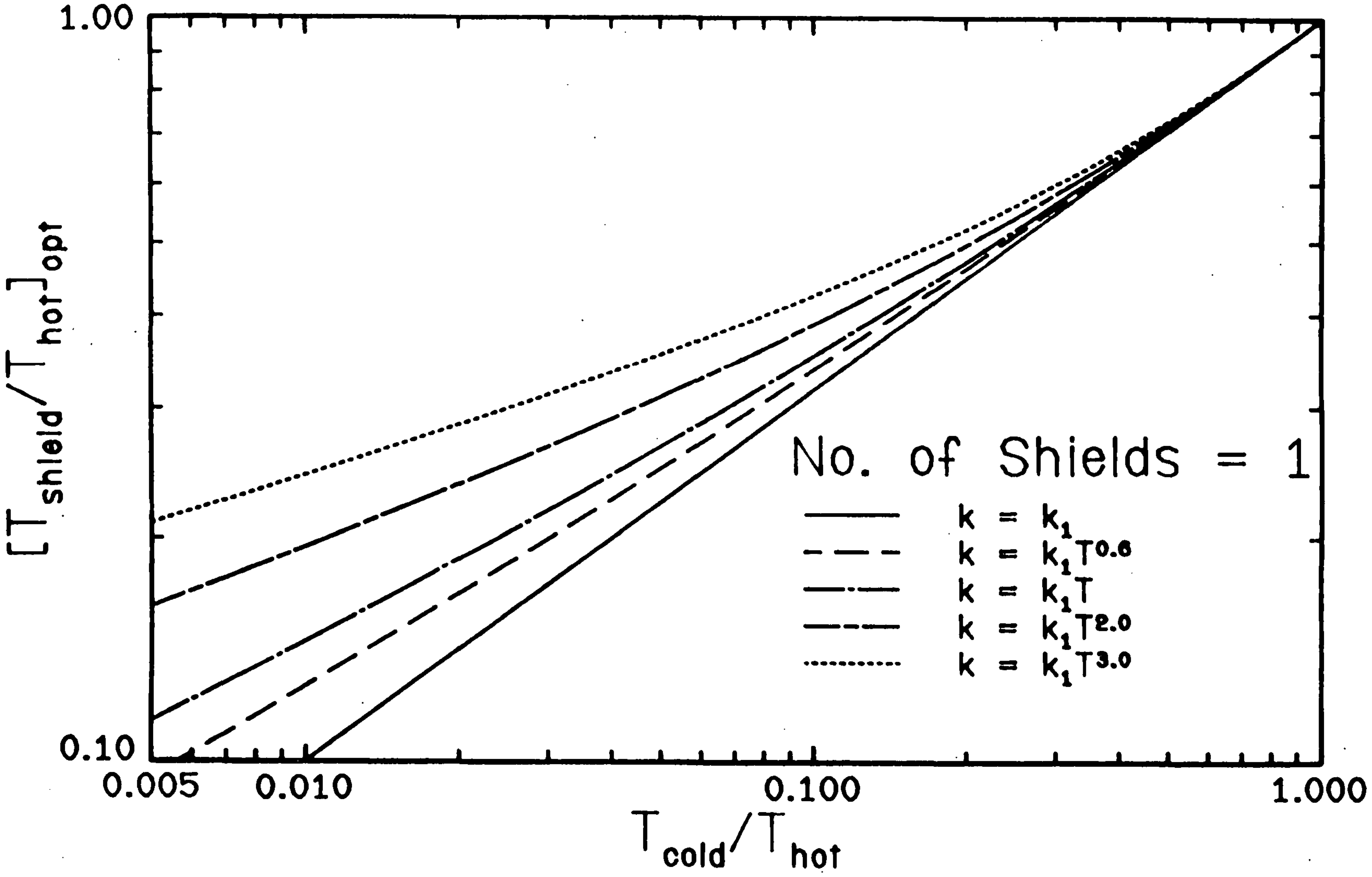

1.00
0.10
0.005
0.010
0.100
1.000
$[T_{shield}/T_{hot}]_{opt}$
T_{cold}/T_{hot}
No. of Shields = 1
$k = k_1$
$k = k_1T^{0.6}$
$k = k_1T$
$k = k_1T^{2.0}$
$k = k_1T^{3.0}$

Figure 10

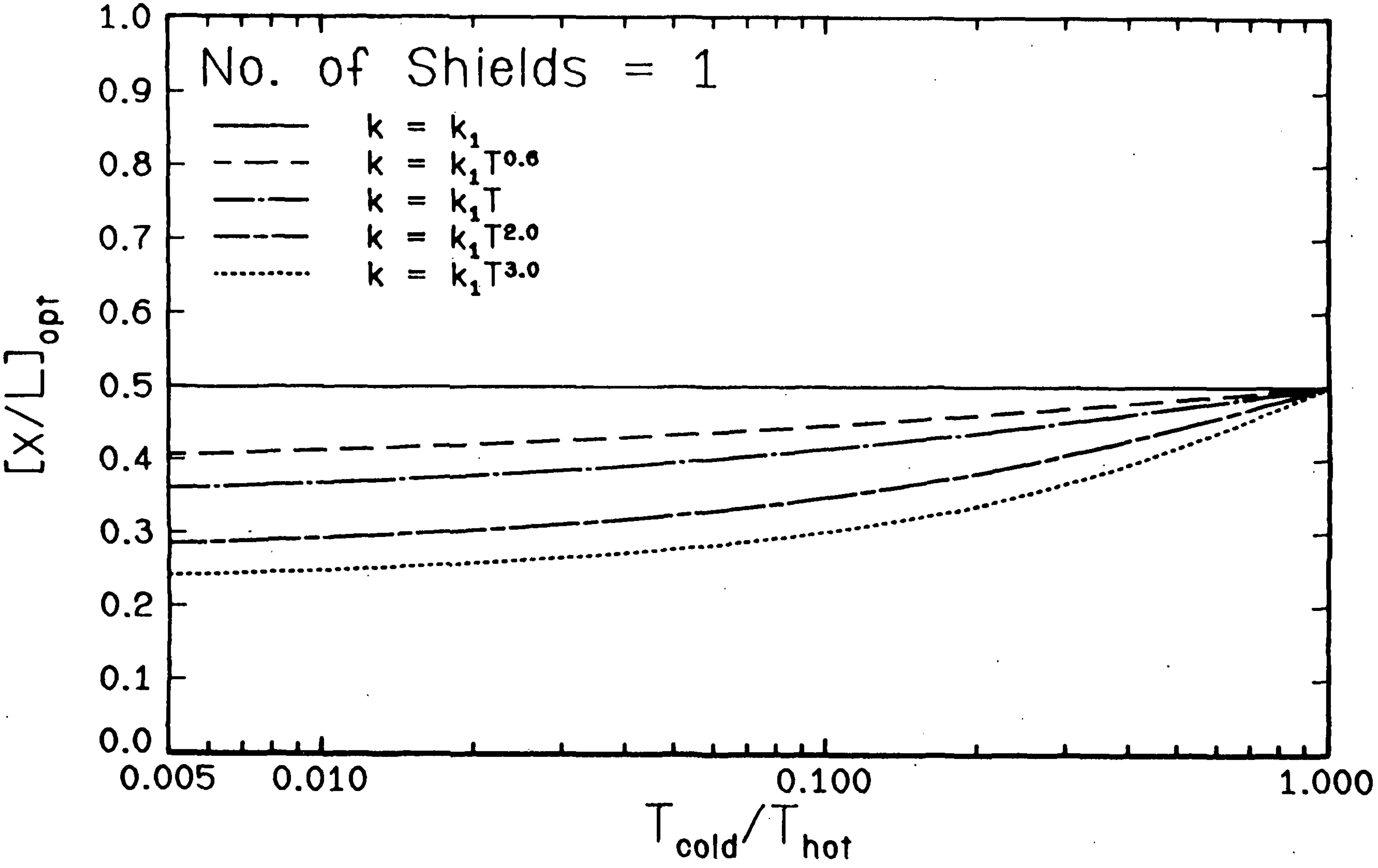

No. of Shields = 1
k = k$_1$
k = k$_1$T$^{0.6}$
k = k$_1$T
k = k$_1$T$^{2.0}$
k = k$_1$T$^{3.0}$
[X/L]$_{opt}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.005
0.010
0.100
1.000
T$_{cold}$/T$_{hot}$

Figure 11

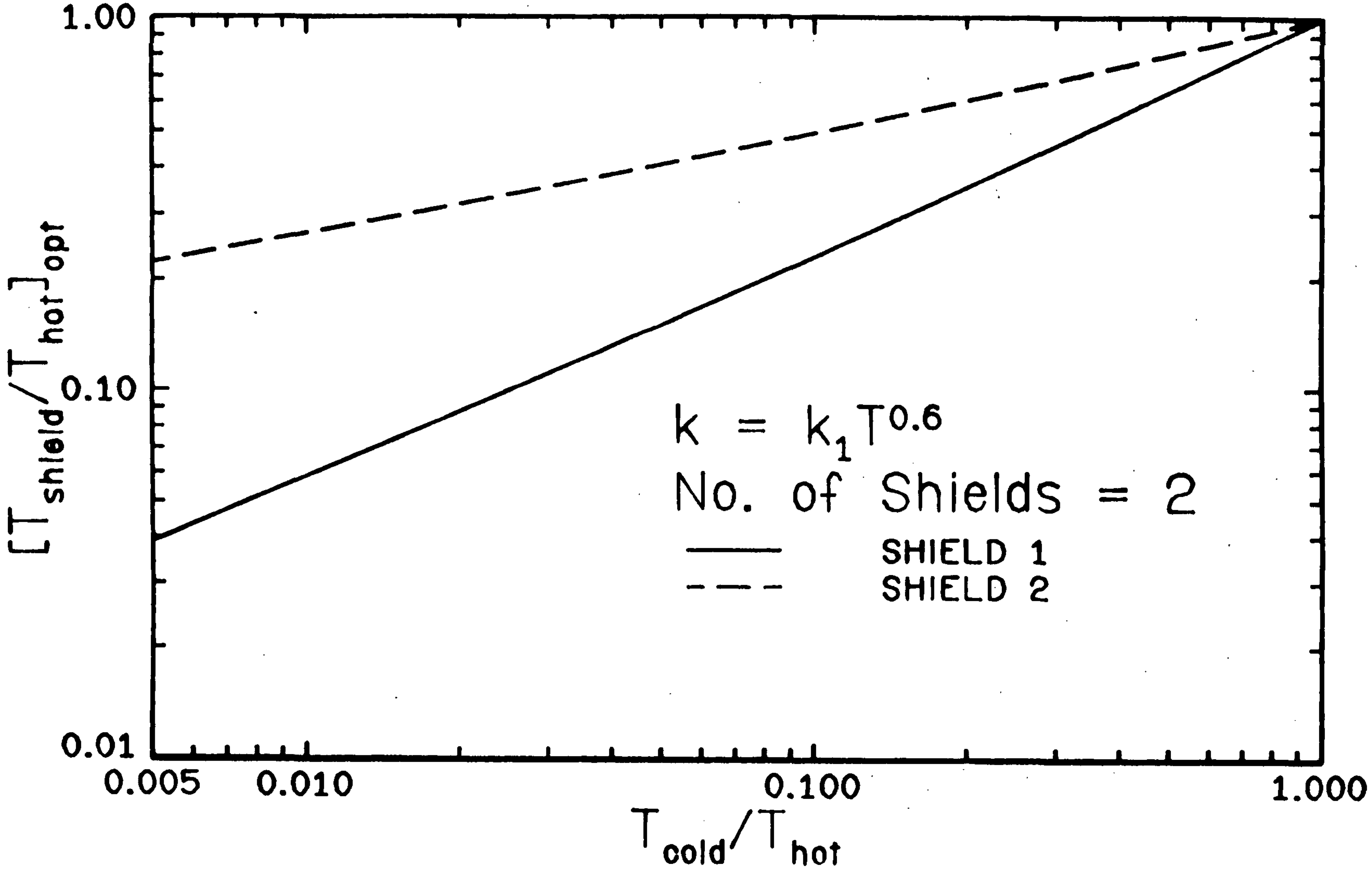

1.00
0.10
0.01
$[T_{shield}/T_{hot}]_{opt}$
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}
$k = k_1 T^{0.6}$
No. of Shields = 2
SHIELD 1
SHIELD 2

Figure 12

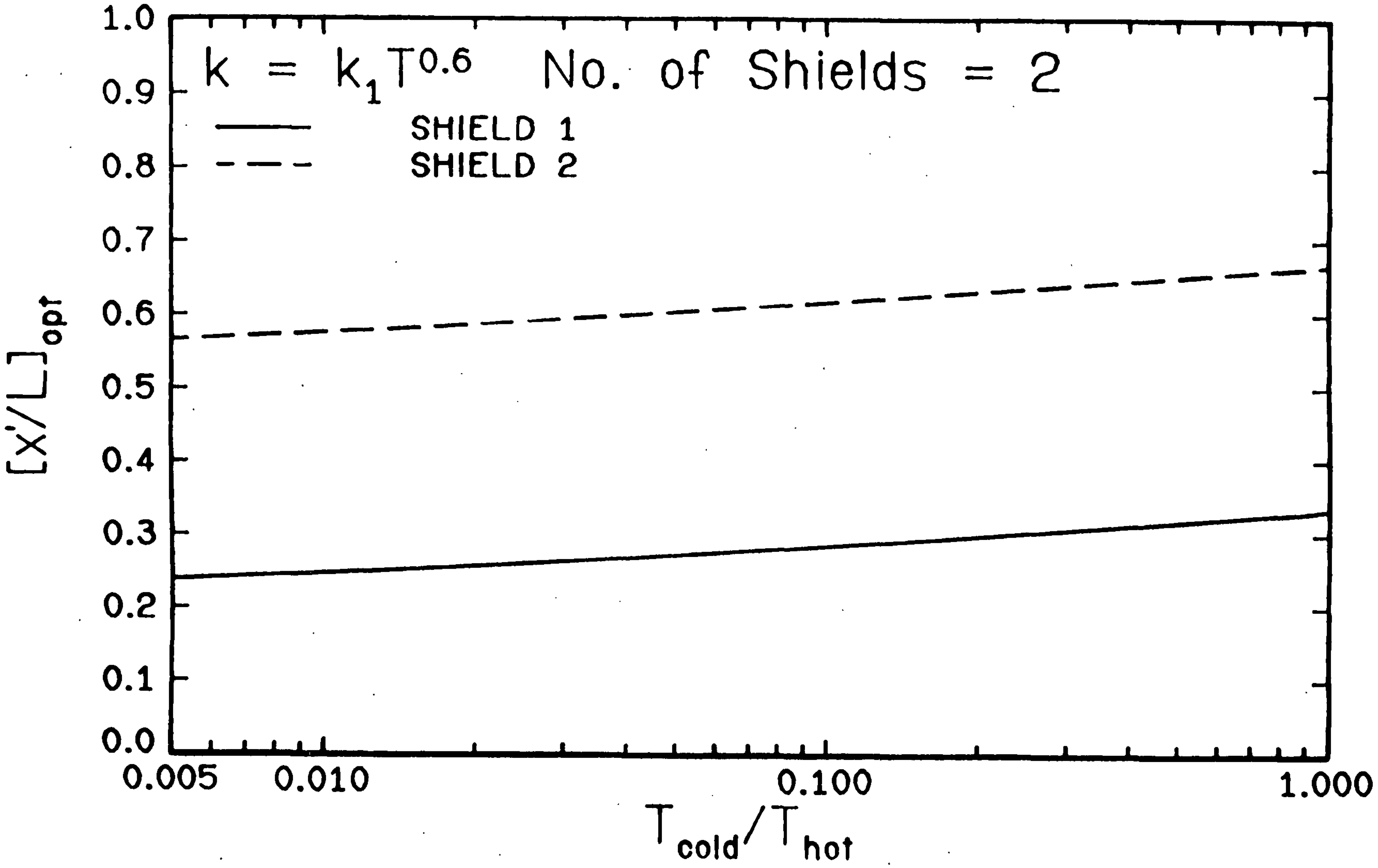

$k = k_1 T^{0.6}$ No. of Shields = 2
SHIELD 1
SHIELD 2
$[X'/L]_{opt}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}

Figure 13

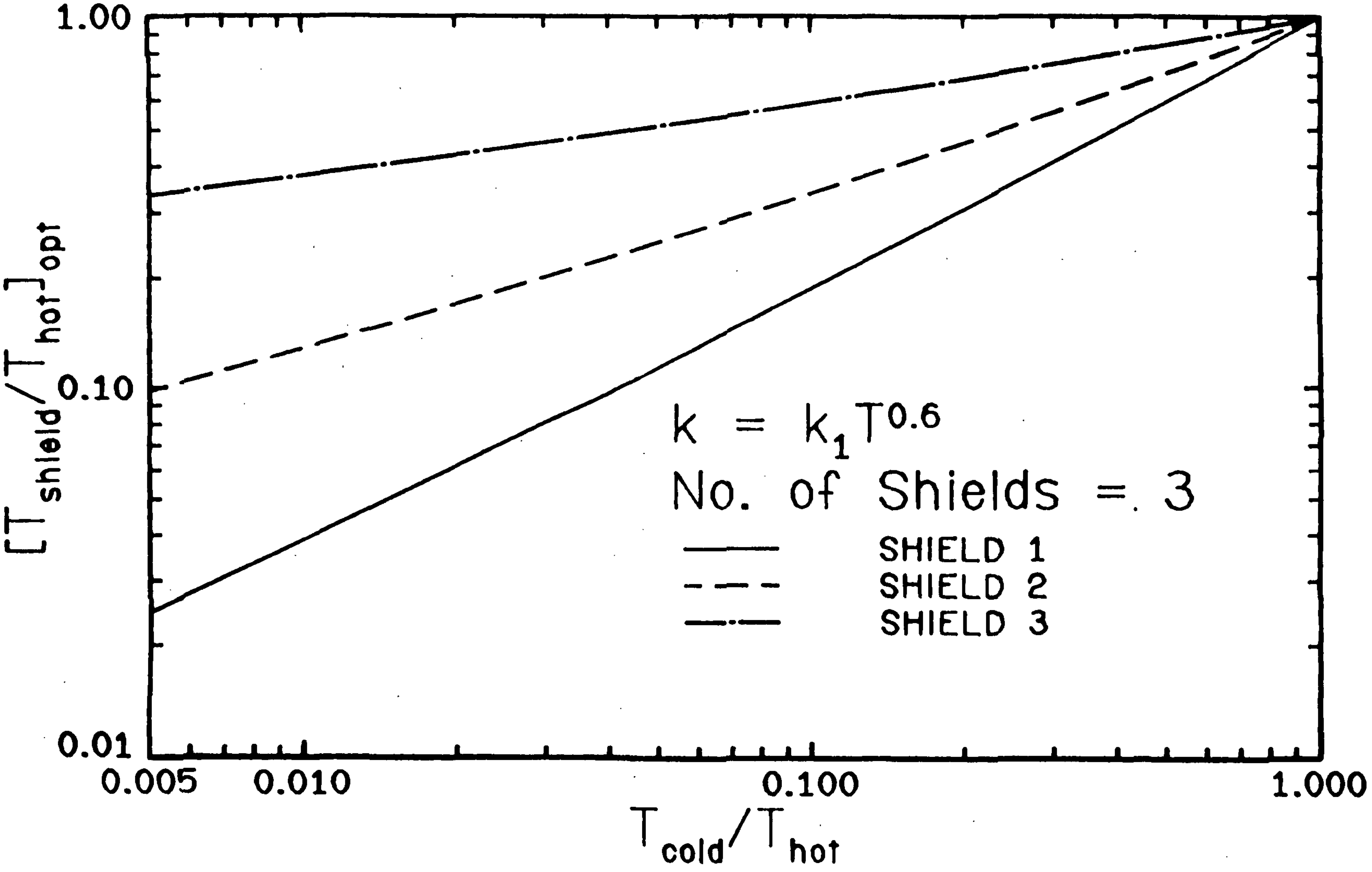

1.00
0.10
0.01
0.005
0.010
0.100
1.000
$[T_{shield}/T_{hot}]_{opt}$
T_{cold}/T_{hot}
$k = k_1 T^{0.6}$
No. of Shields = 3
SHIELD 1
SHIELD 2
SHIELD 3

Figure 14

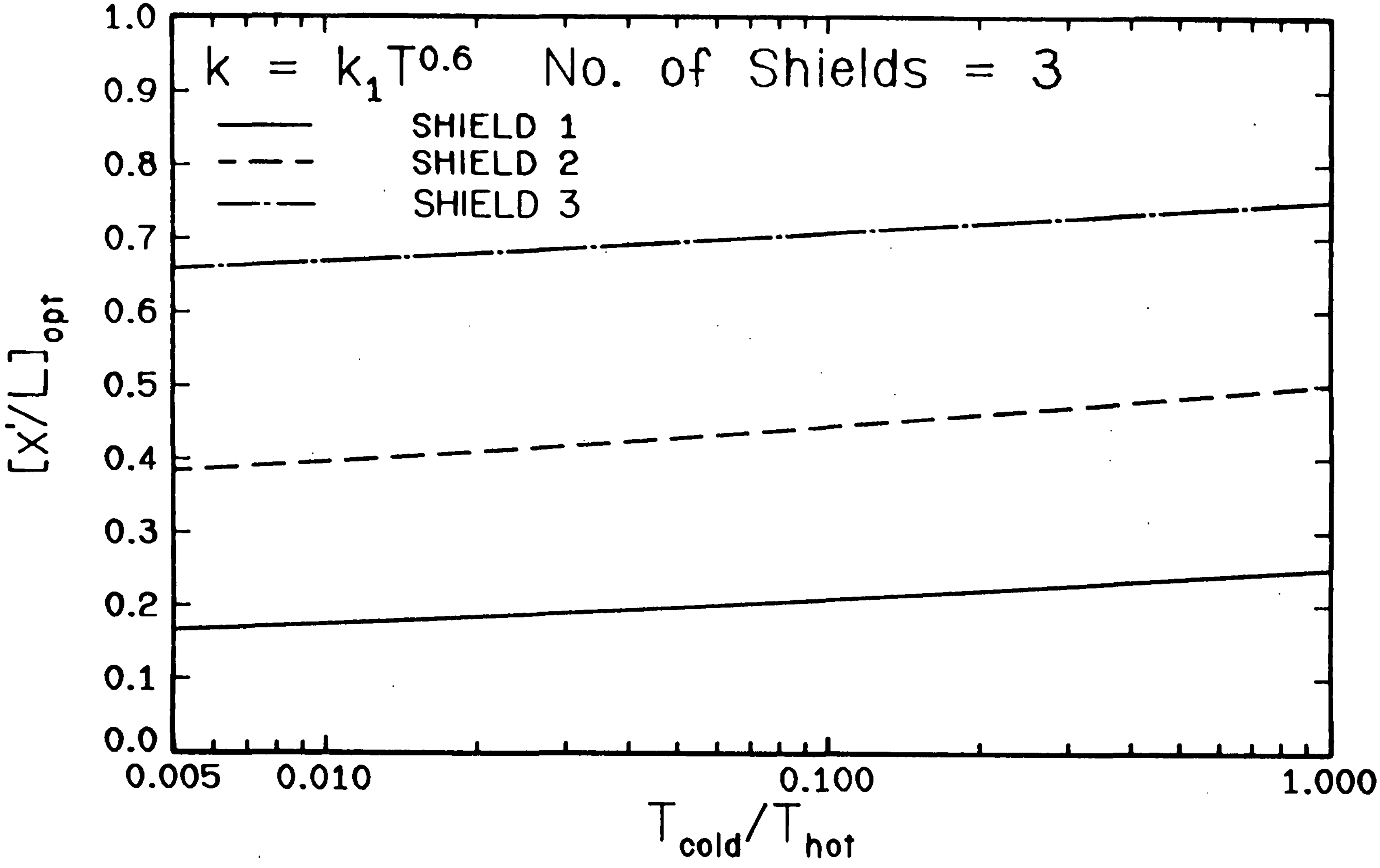

k = k₁T^0.6 No. of Shields = 3
SHIELD 1
SHIELD 2
SHIELD 3
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
[X'/L]opt
0.005
0.010
0.100
1.000
Tcold/Thot

Figure 15

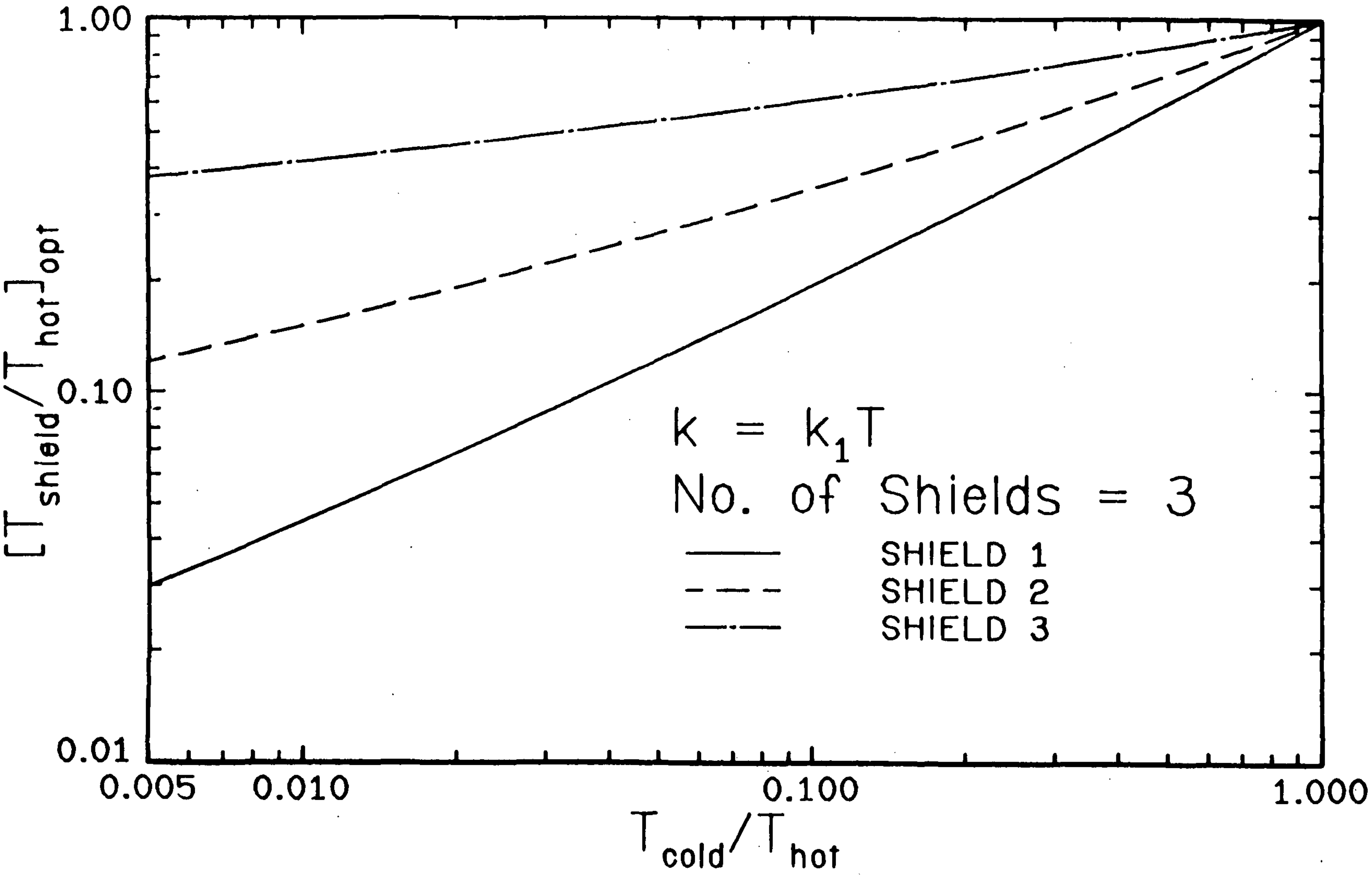

1.00
0.10
0.01
$[T_{shield}/T_{hot}]_{opt}$
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}
$k = k_1 T$
No. of Shields = 3
SHIELD 1
SHIELD 2
SHIELD 3

Figure 16

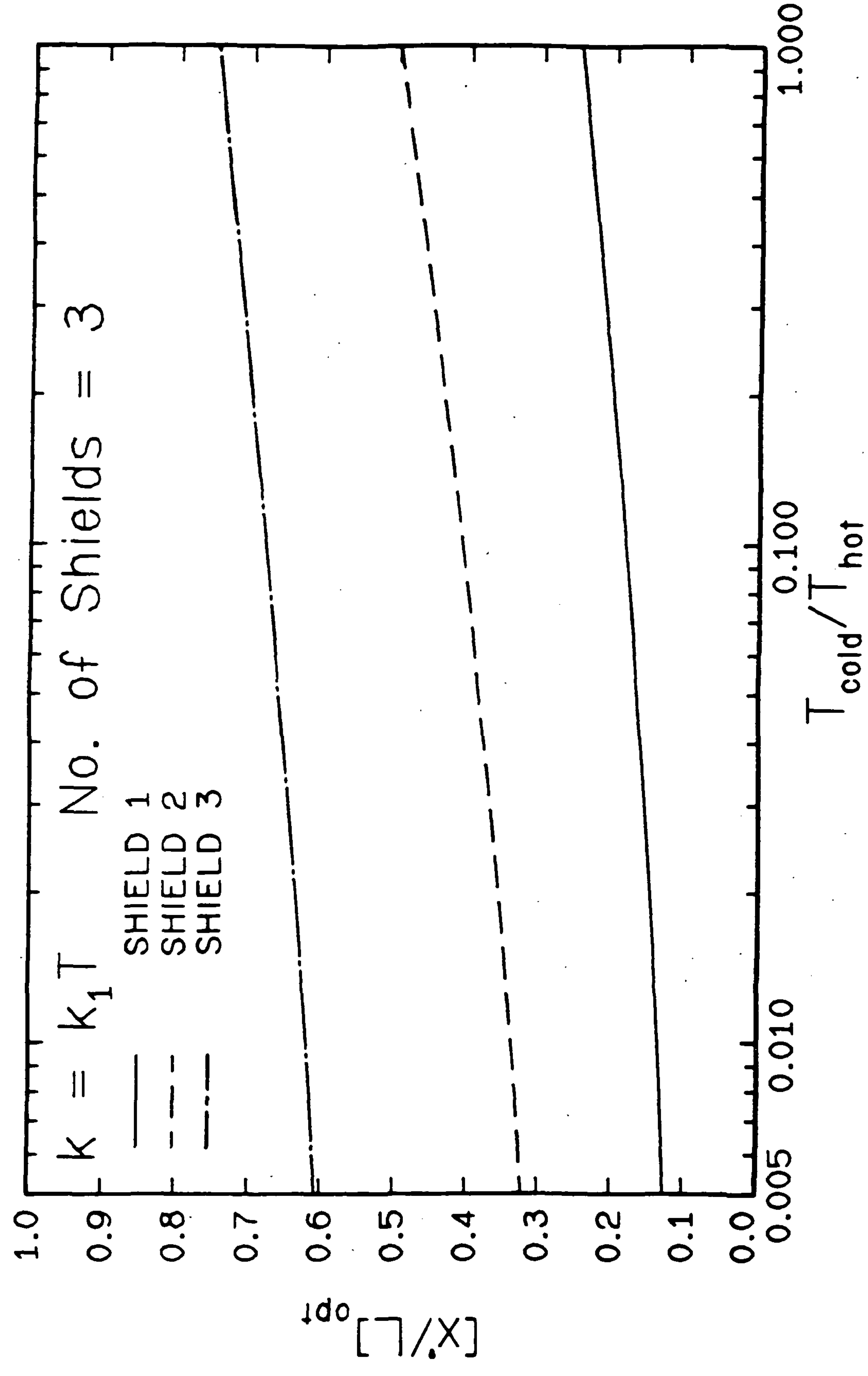

$k = k_1 T$
No. of Shields = 3
SHIELD 1
SHIELD 2
SHIELD 3
$[X'/L]_{opt}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}

Figure 17

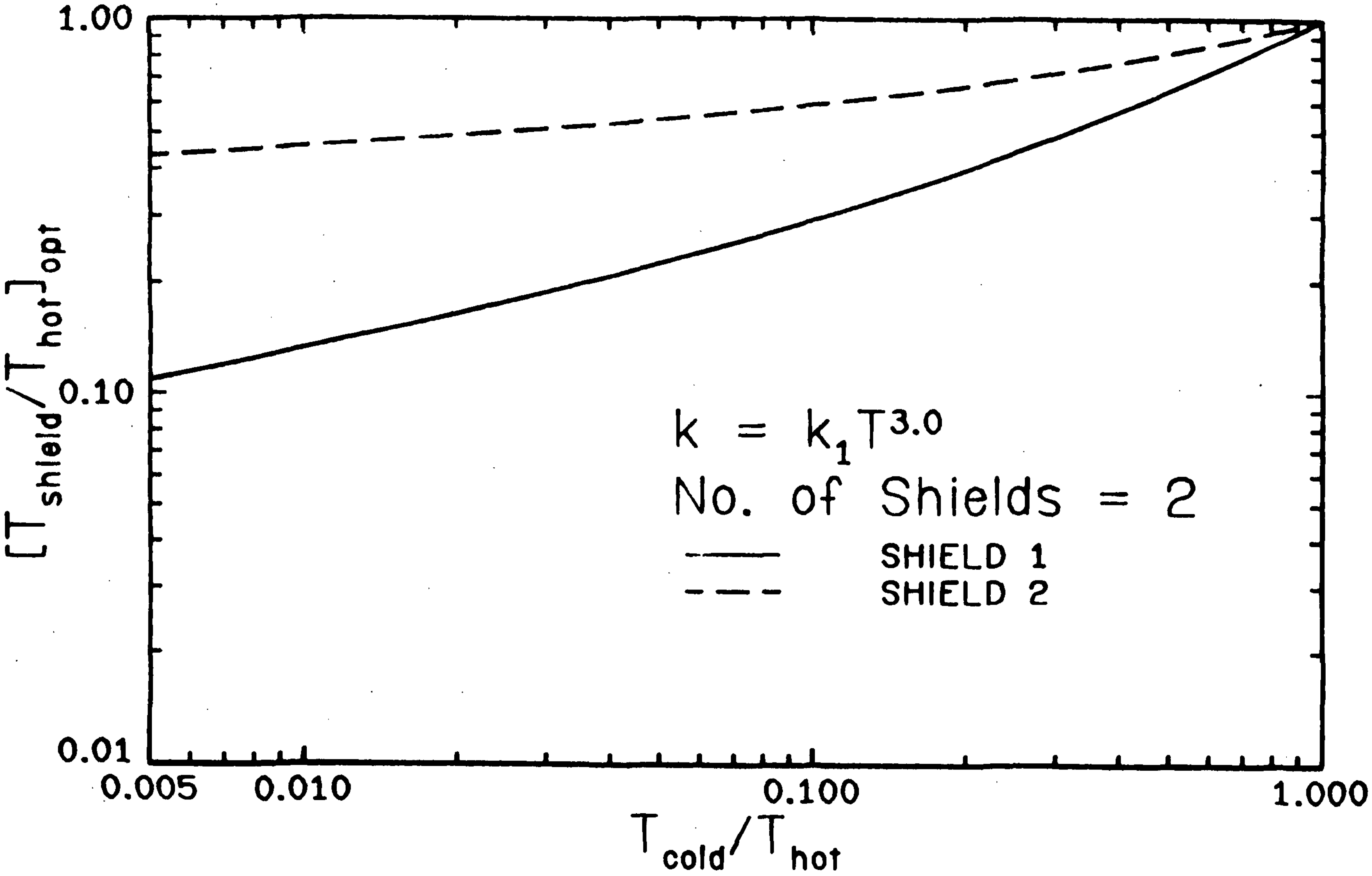

1.00
0.10
0.01
$[T_{shield}/T_{hot}]_{opt}$
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}
$k = k_1 T^{3.0}$
No. of Shields = 2
SHIELD 1
SHIELD 2

Figure 18

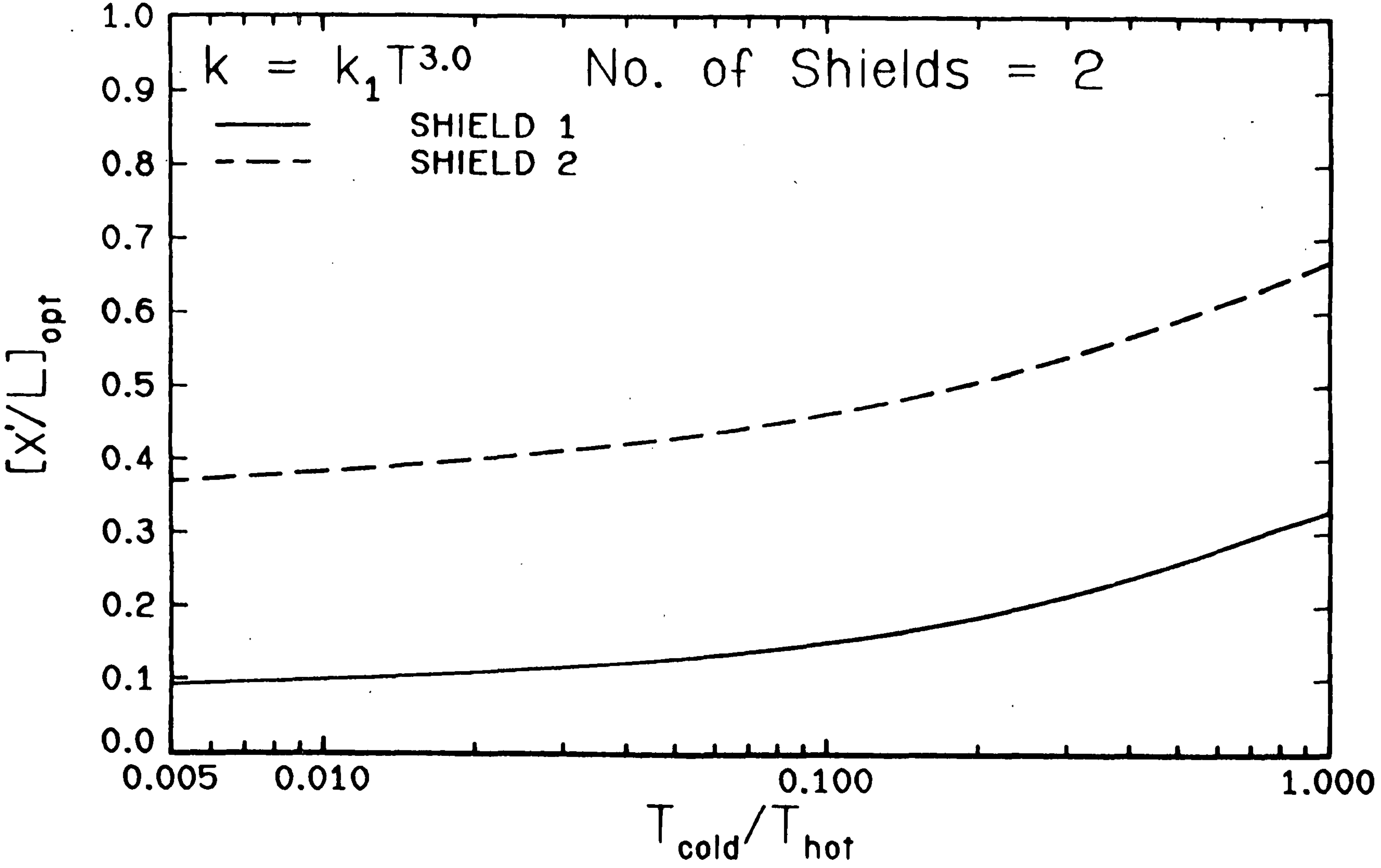

$k = k_1 T^{3.0}$
No. of Shields = 2
SHIELD 1
SHIELD 2
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$[X'/L]_{opt}$
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}

Figure 19

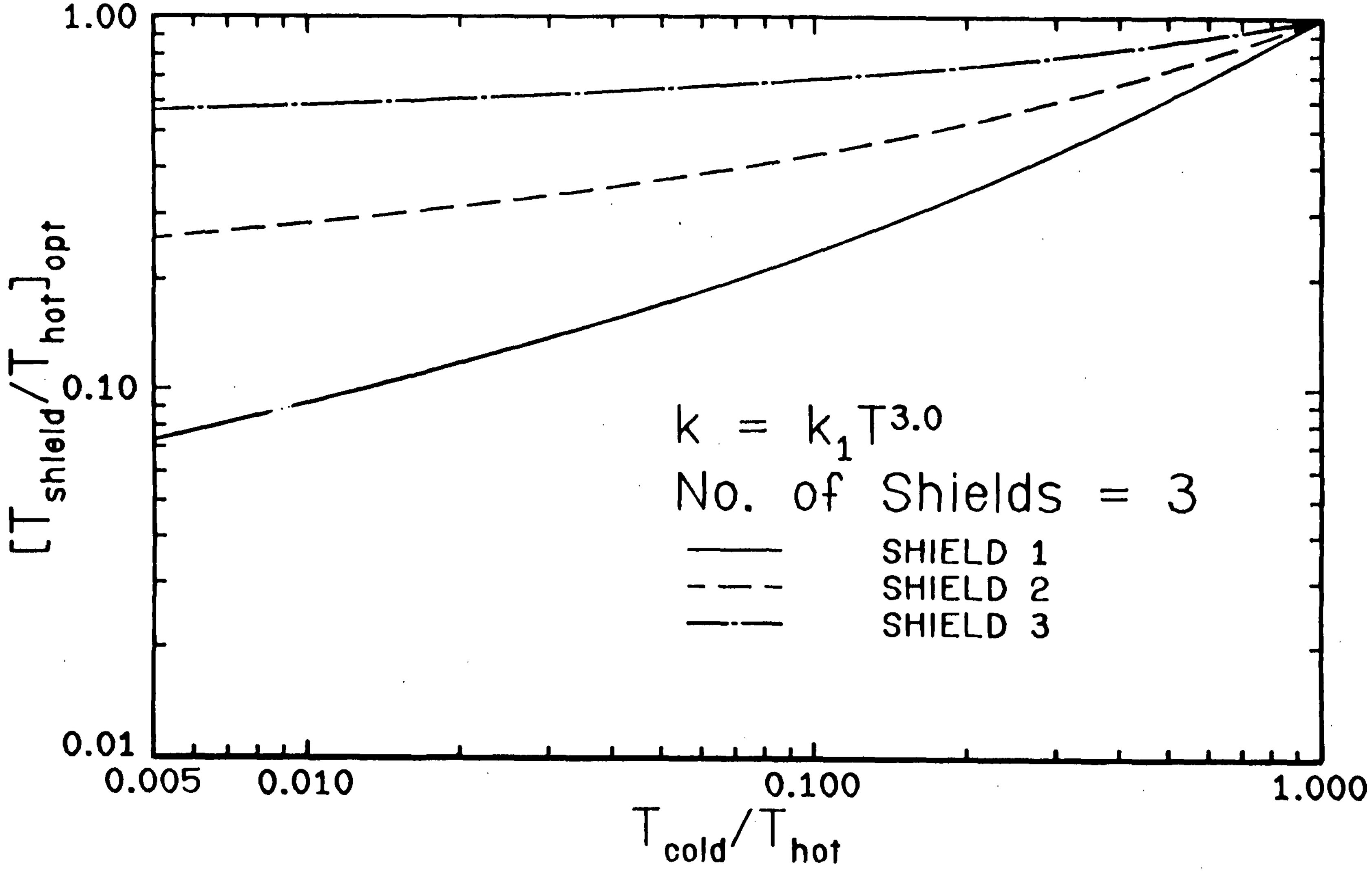

1.00
0.10
0.01
$[T_{shield}/T_{hot}]_{opt}$
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}
$k = k_1 T^{3.0}$
No. of Shields = 3
SHIELD 1
SHIELD 2
SHIELD 3

Figure 20

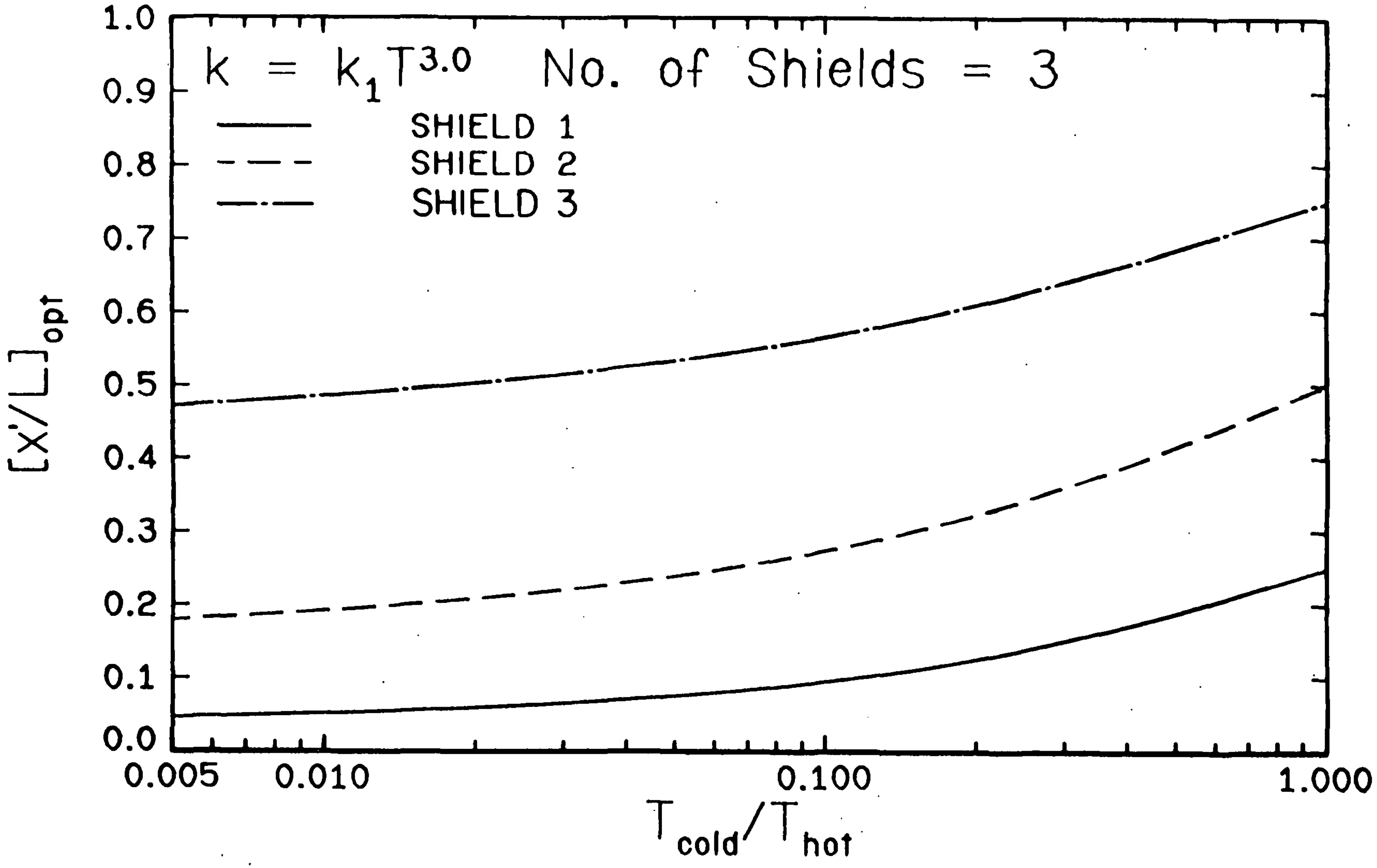

$k = k_1 T^{3.0}$ No. of Shields = 3
SHIELD 1
SHIELD 2
SHIELD 3
$[X'/L]_{opt}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}

Figure 21

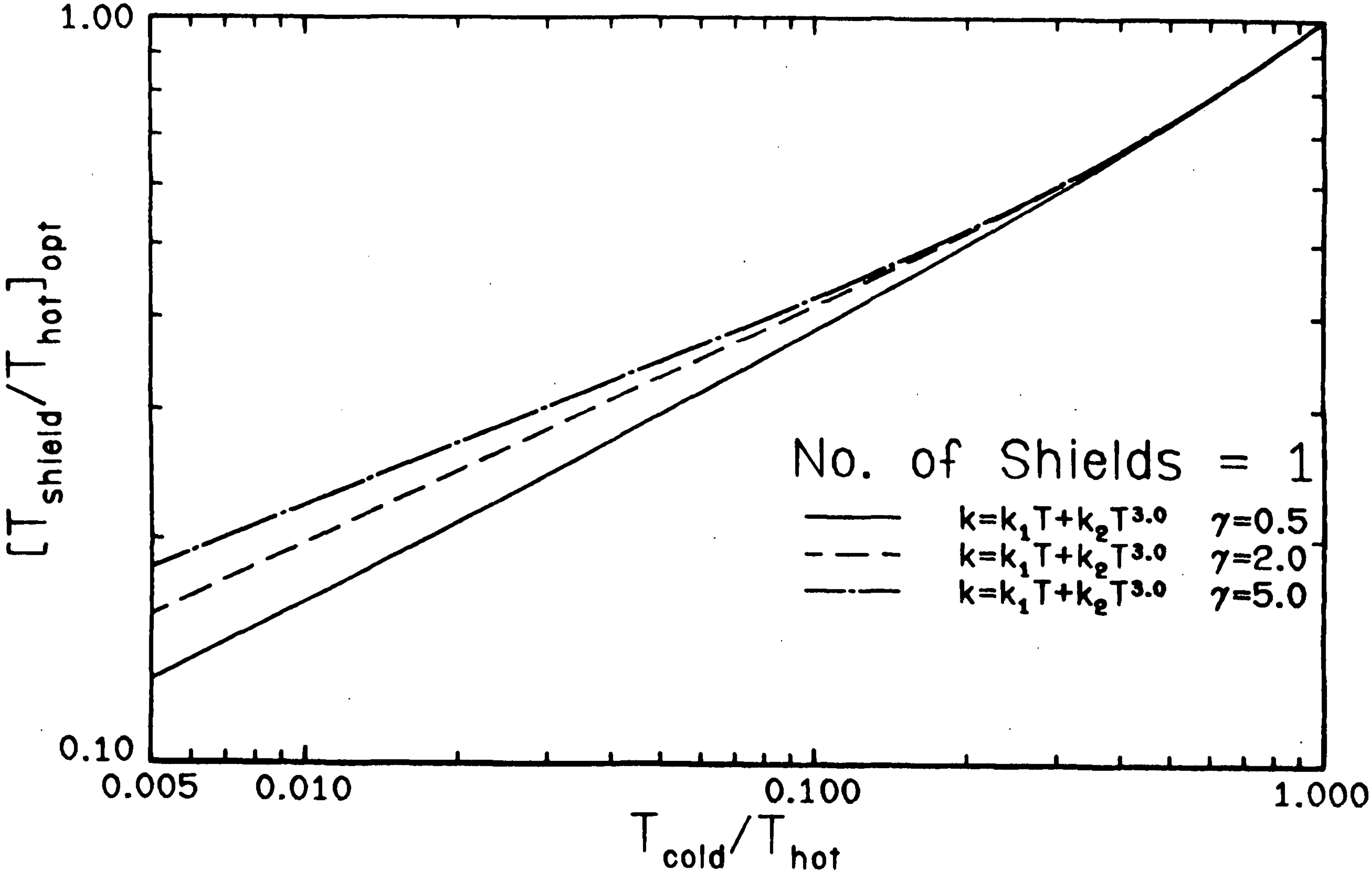

No. of Shields = 1
$k=k_1T+k_2T^{3.0}$ $\gamma=0.5$
$k=k_1T+k_2T^{3.0}$ $\gamma=2.0$
$k=k_1T+k_2T^{3.0}$ $\gamma=5.0$
$[T_{shield}/T_{hot}]_{opt}$
1.00
0.10
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}

Figure 22

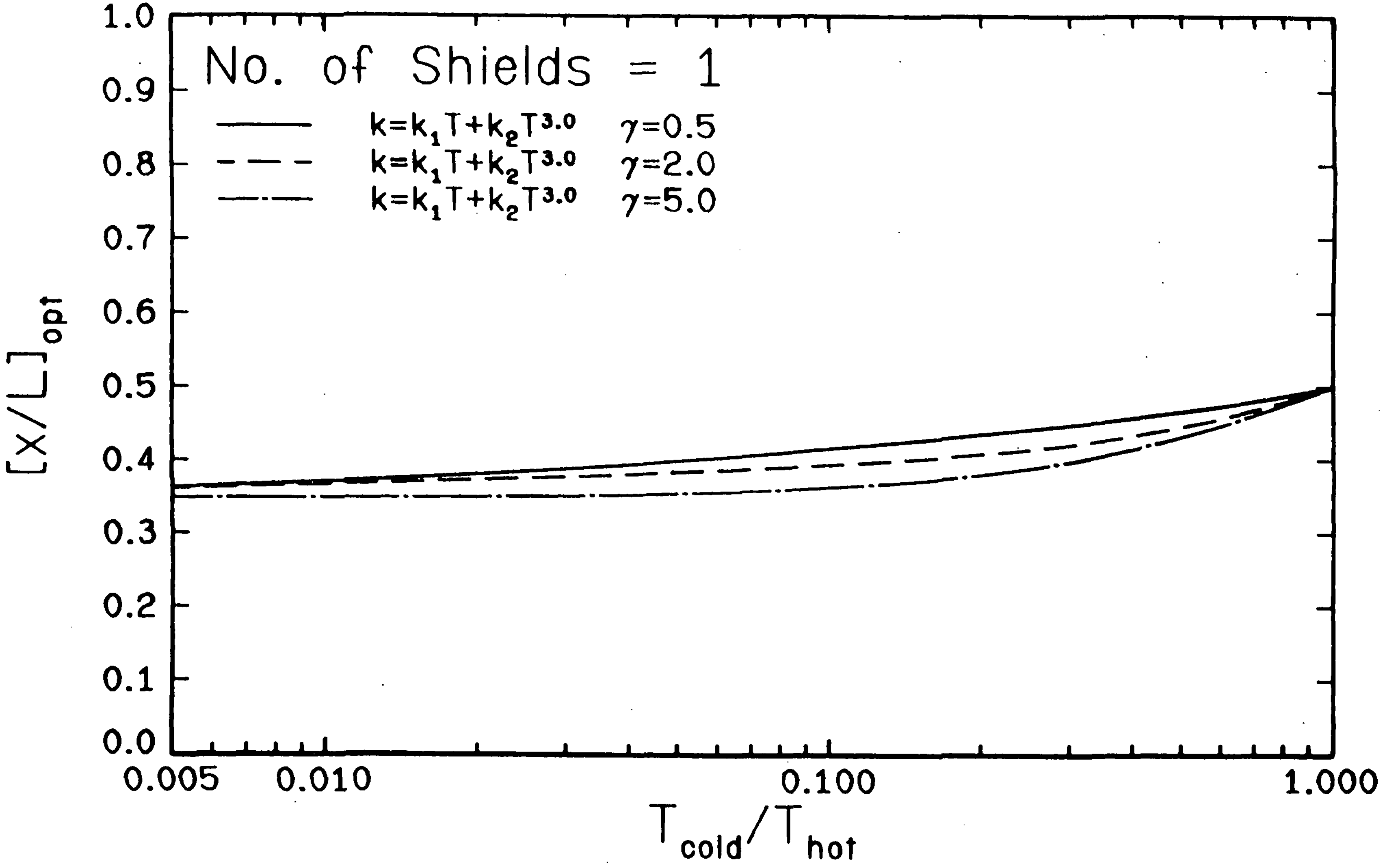

No. of Shields = 1
$k=k_1T+k_2T^{3.0}$ γ=0.5
$k=k_1T+k_2T^{3.0}$ γ=2.0
$k=k_1T+k_2T^{3.0}$ γ=5.0
$[X/L]_{opt}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}

Figure 23

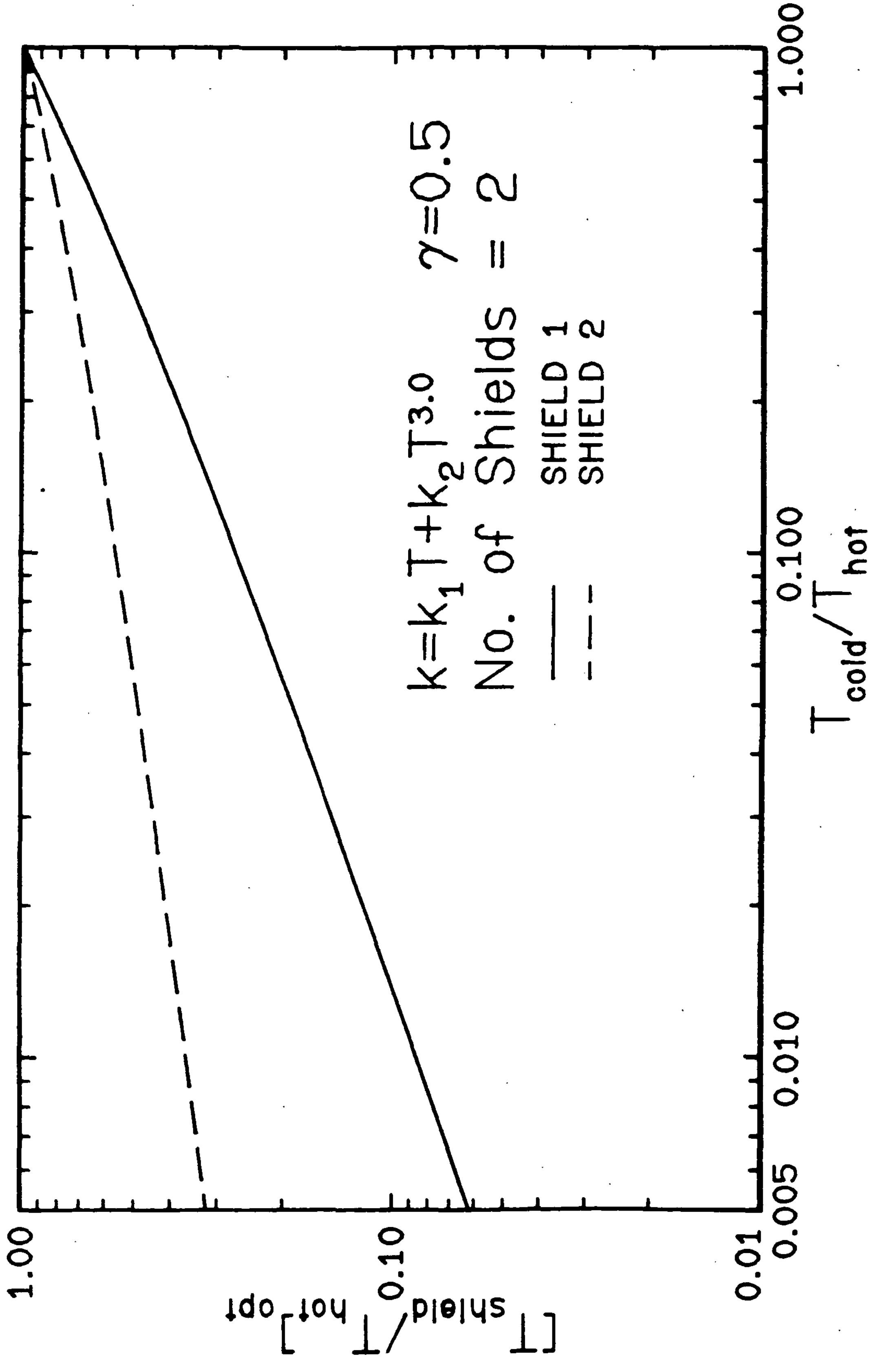

$k=k_1T+k_2T^{3.0}$ $\gamma=0.5$
No. of Shields = 2
SHIELD 1
SHIELD 2
1.00
0.10
0.01
$[T_{shield}/T_{hot}]_{opt}$
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}

Figure 24

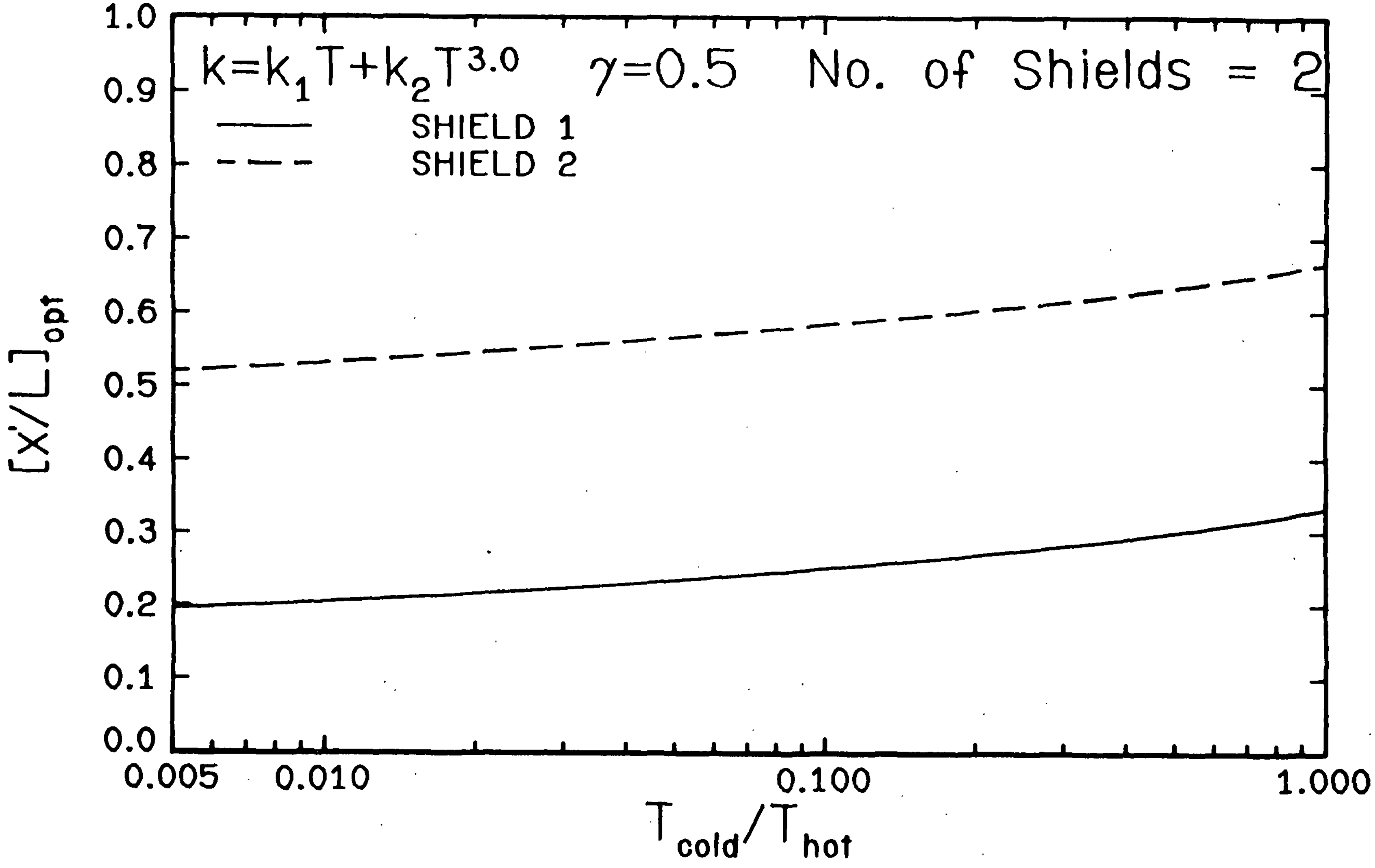

$k=k_1T+k_2T^{3.0}$
$\gamma=0.5$
No. of Shields = 2
SHIELD 1
SHIELD 2
$[X'/L]_{opt}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}

Figure 25

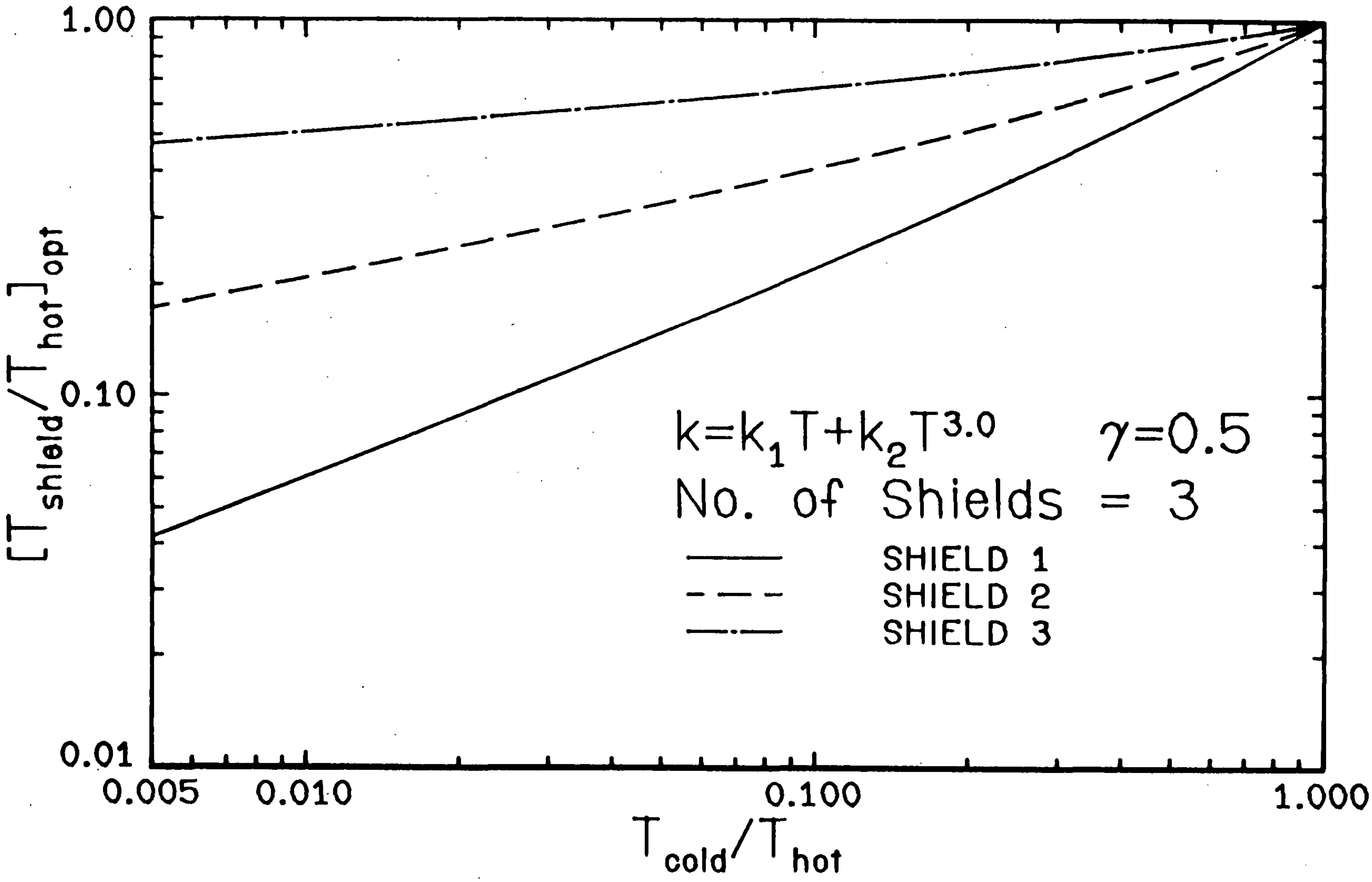

1.00
0.10
0.01
0.005
0.010
0.100
1.000
$[T_{shield}/T_{hot}]_{opt}$
T_{cold}/T_{hot}
$k=k_1T+k_2T^{3.0}$
$\gamma=0.5$
No. of Shields = 3
SHIELD 1
SHIELD 2
SHIELD 3

Figure 26

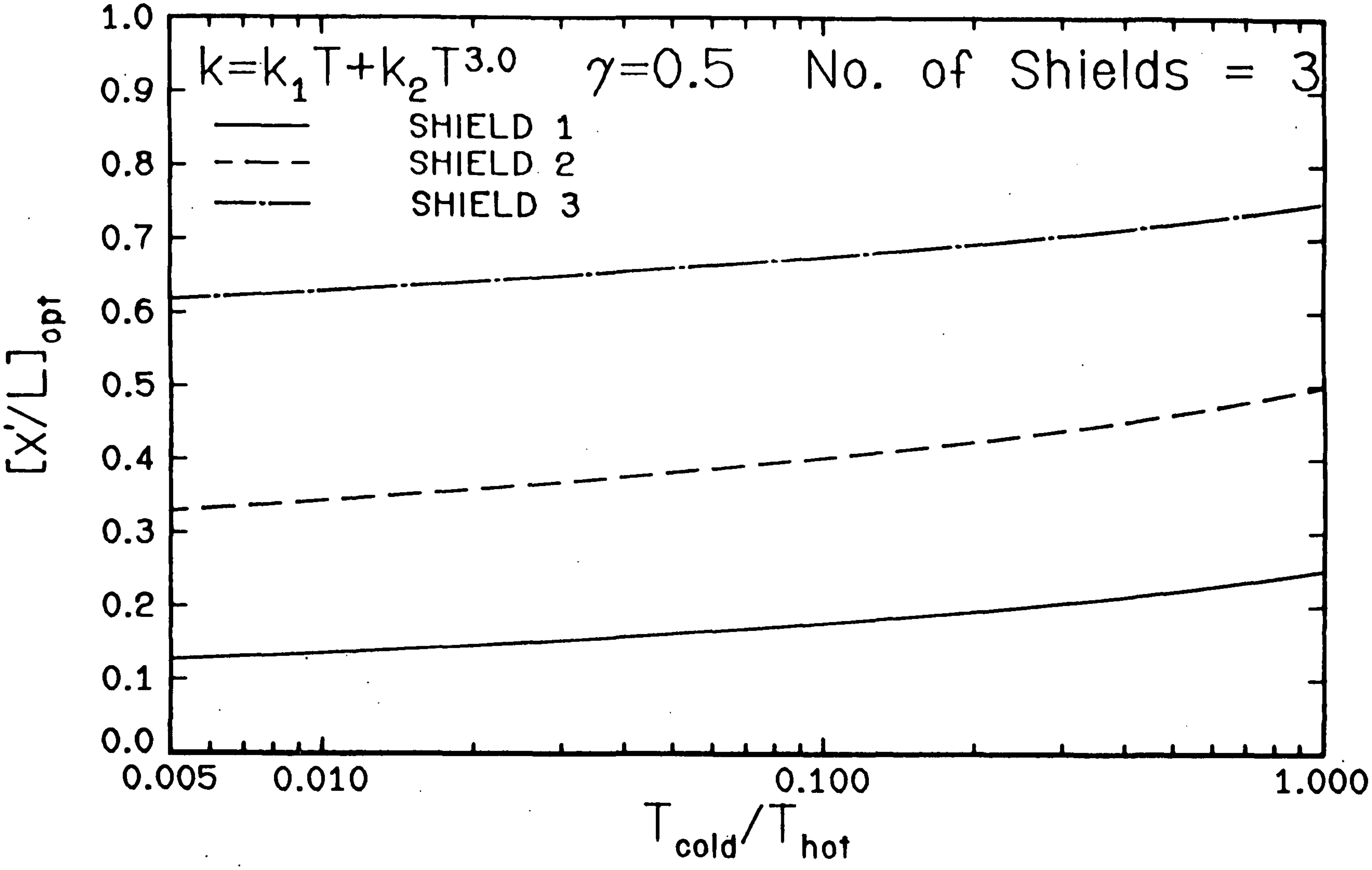

$k=k_1T+k_2T^{3.0}$ $\gamma=0.5$ No. of Shields = 3
SHIELD 1
SHIELD 2
SHIELD 3
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$[X'/L]_{opt}$
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}

Figure 27

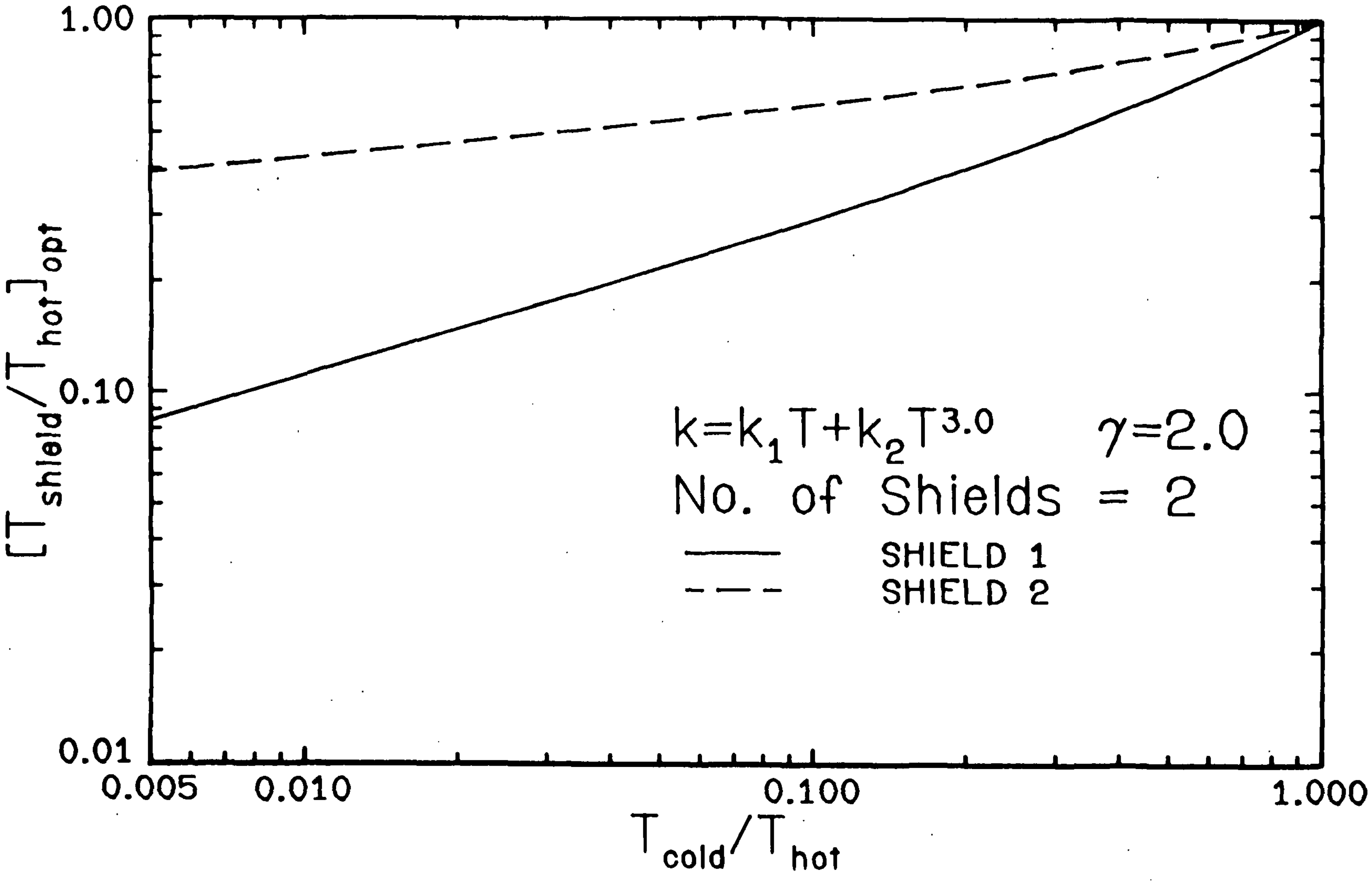

1.00
0.10
0.01
$[T_{shield}/T_{hot}]_{opt}$
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}
$k=k_1T+k_2T^{3.0}$
$\gamma=2.0$
No. of Shields = 2
SHIELD 1
SHIELD 2

Figure 28

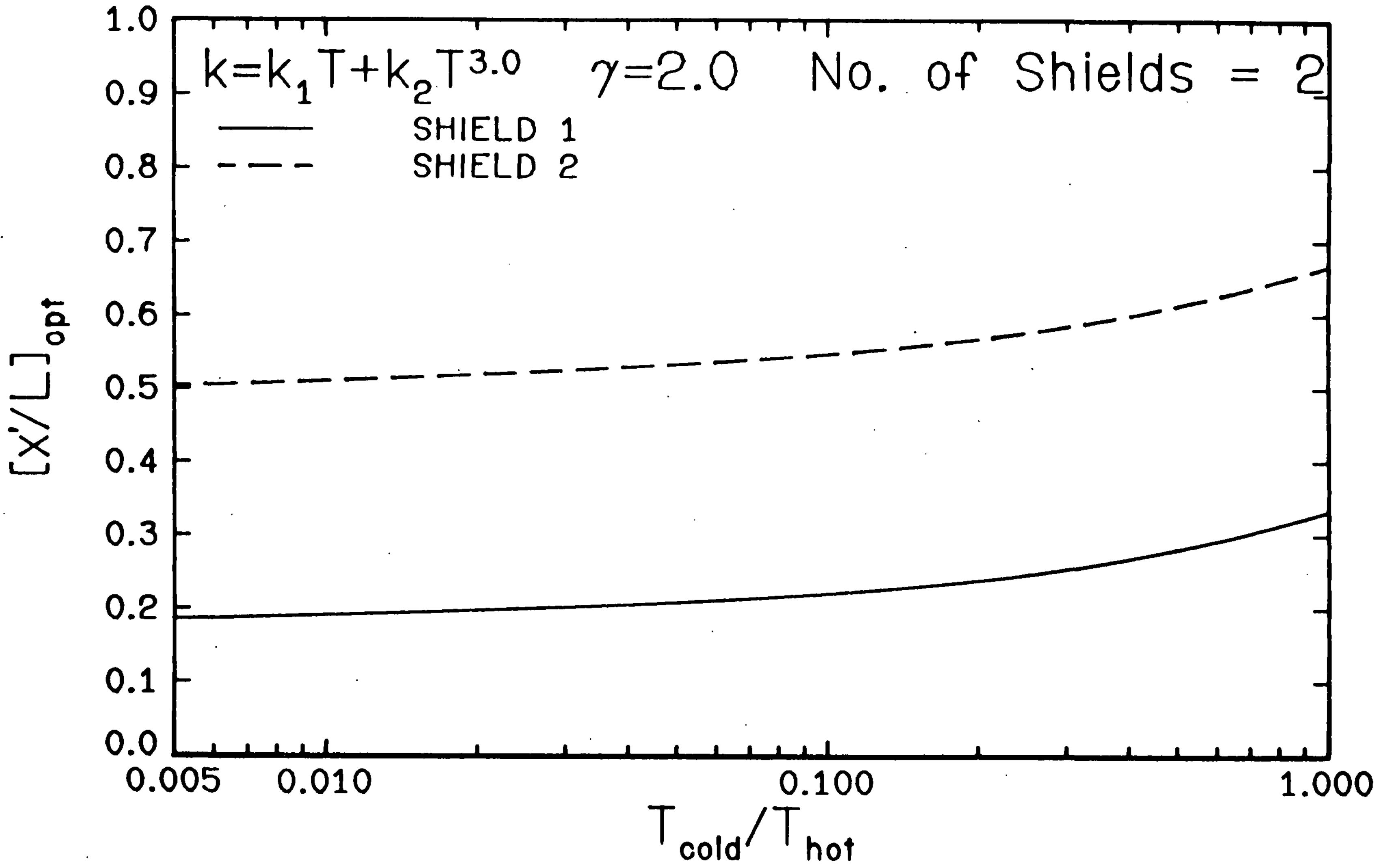

k=k₁T+k₂T^3.0 γ=2.0 No. of Shields = 2
SHIELD 1
SHIELD 2
[X'/L]opt
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.005
0.010
0.100
1.000
Tcold/Thot

Figure 29

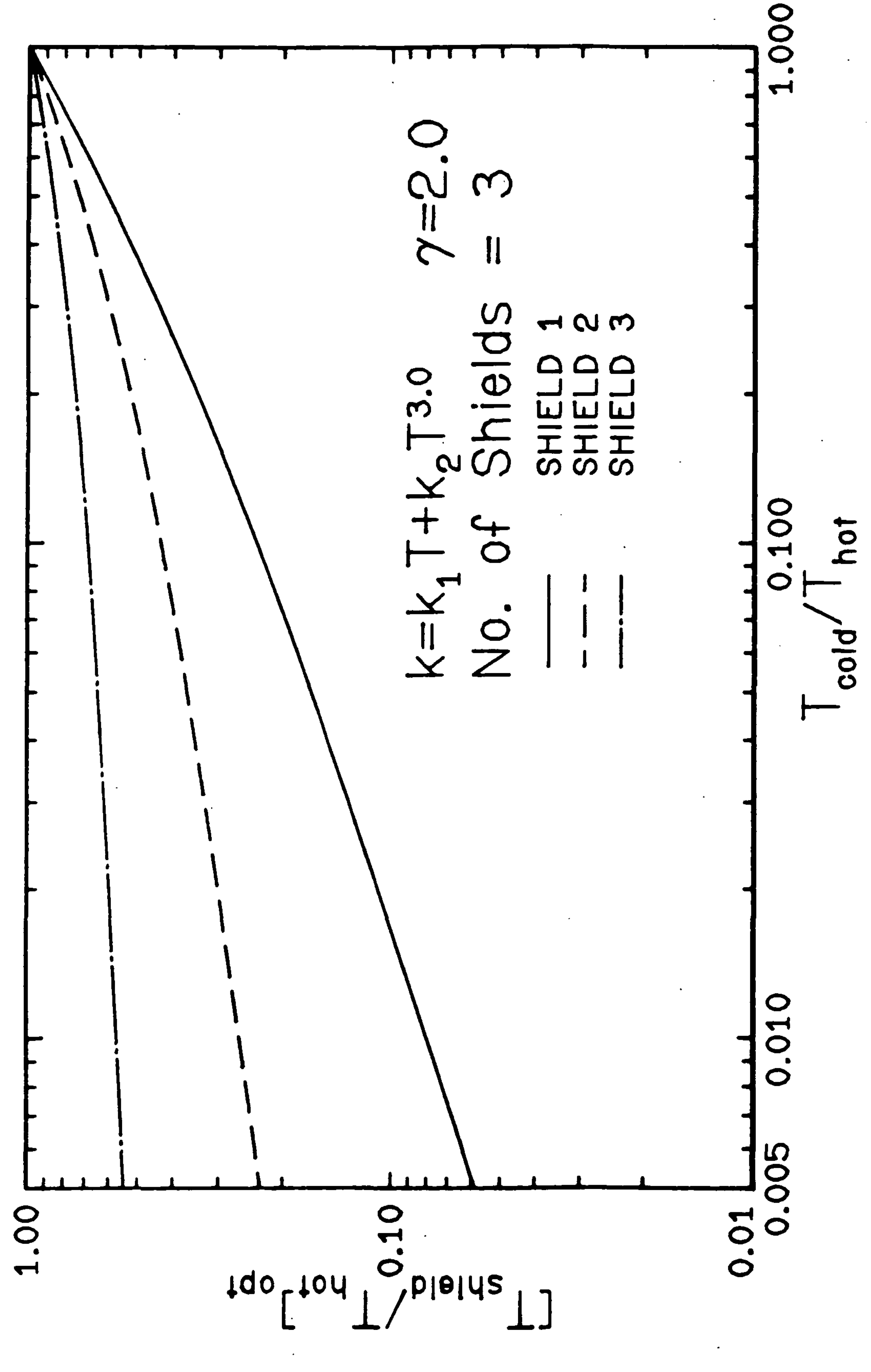

$k=k_1T+k_2T^{3.0}$ $\gamma=2.0$
No. of Shields = 3
SHIELD 1
SHIELD 2
SHIELD 3
T_{cold}/T_{hot}
$[T_{shield}/T_{hot}]_{opt}$
1.00
0.10
0.01
0.005
0.010
0.100
1.000

Figure 30

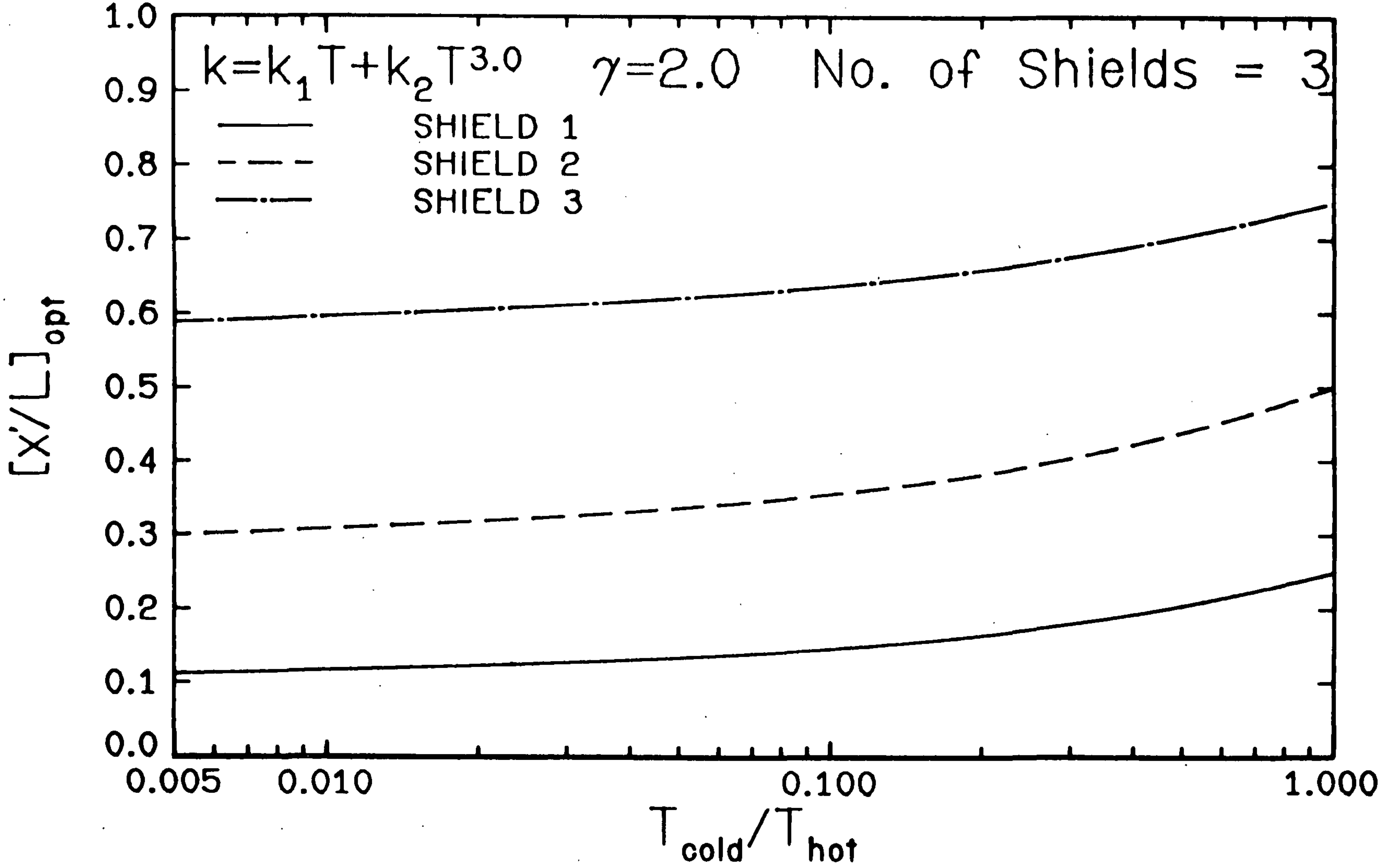

$k=k_1T+k_2T^{3.0}$ $\gamma=2.0$ No. of Shields = 3
SHIELD 1
SHIELD 2
SHIELD 3
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
$[X'/L]_{opt}$
0.005
0.010
0.100
1.000
T_{cold}/T_{hot}

Figure 31

Curve Set 3: Figures 32 through 35

System sensitivity to deviations from the optimum shield
temperatures and locations for two overall temperature ratios
with one cooled shield

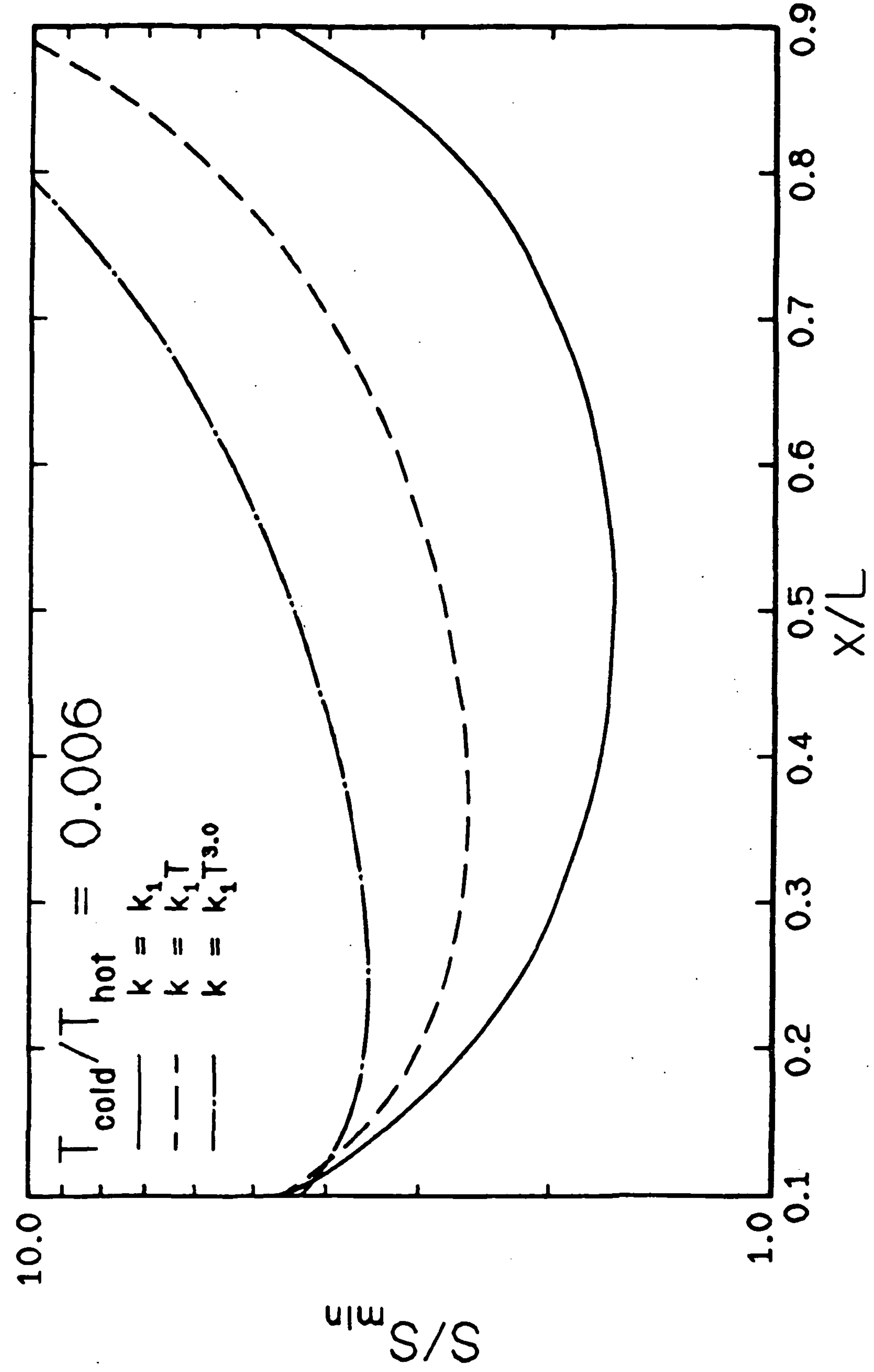
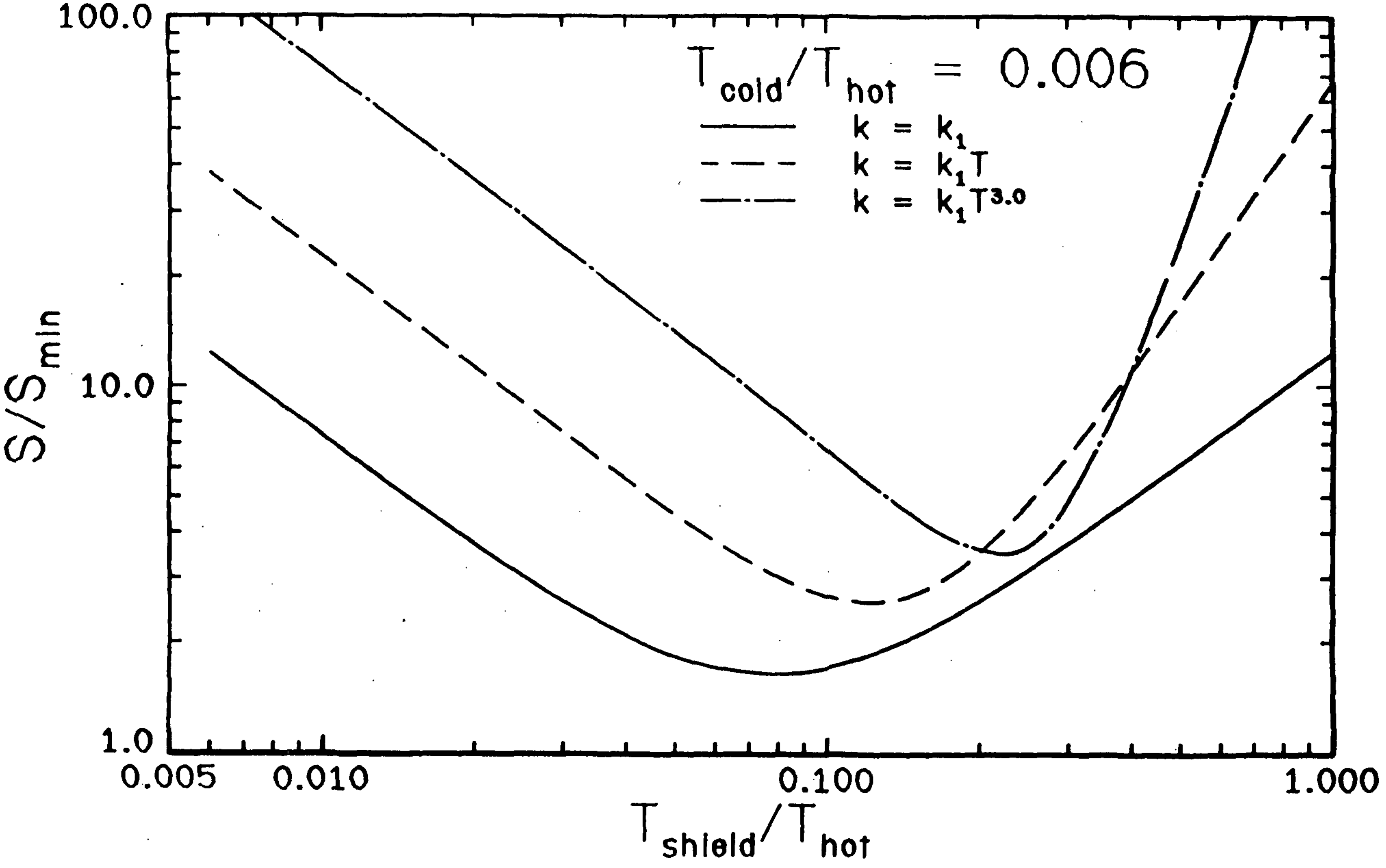

$T_{cold}/T_{hot} = 0.006$
$k = k_1$
$k = k_1T$
$k = k_1T^{3.0}$
S/S$_{min}$
10.0
1.0
X/L
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

Figure 32



$T_{cold}/T_{hot} = 0.006$
$k = k_1$
$k = k_1 T$
$k = k_1 T^{3.0}$
S/S_{min}
100.0
10.0
1.0
0.005
0.010
0.100
1.000
T_{shield}/T_{hot}

Figure 33

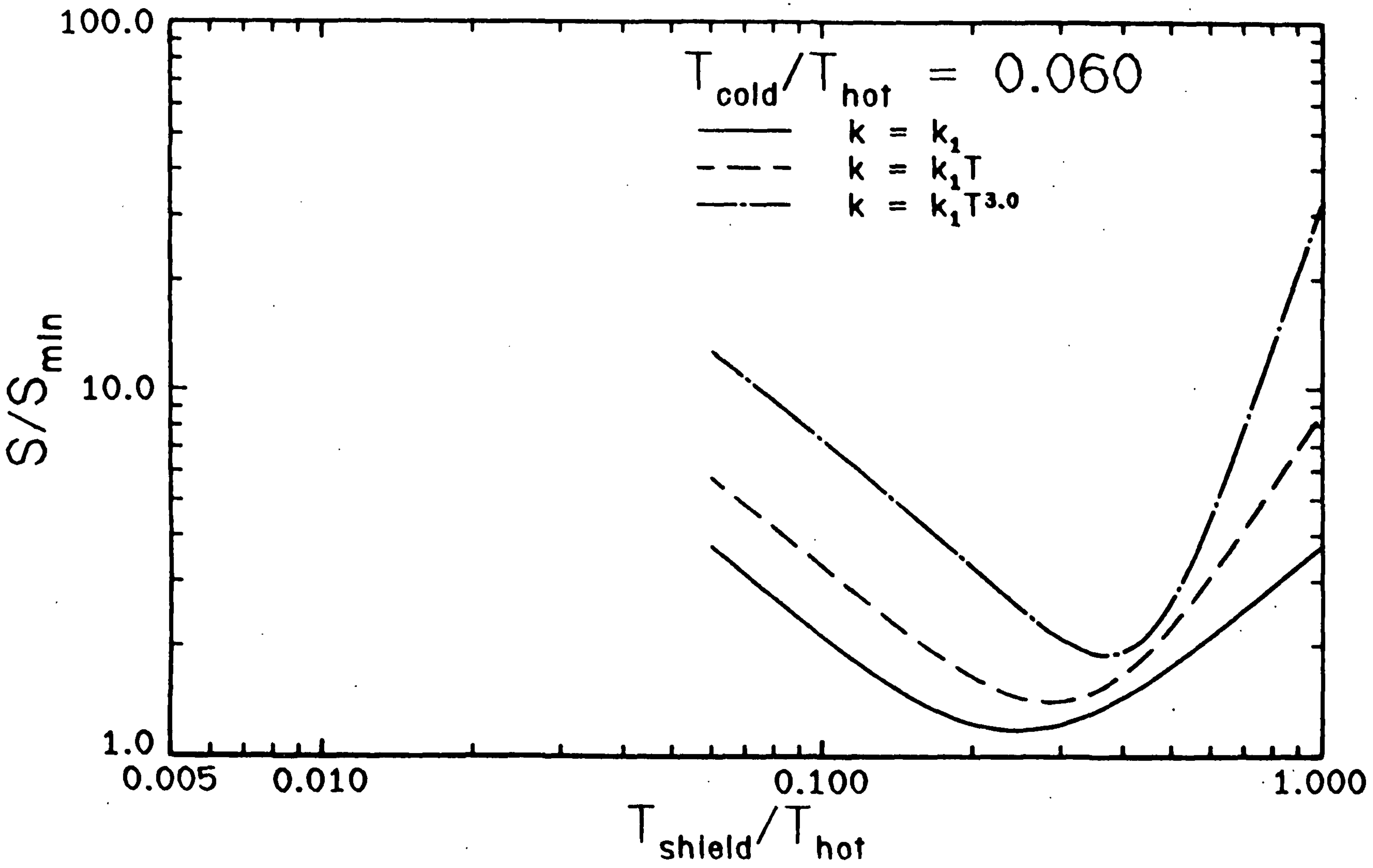

T_{cold}/T_{hot} = 0.060
$k = k_1$
$k = k_1T$
$k = k_1T^{3.0}$
S/S_{min}
100.0
10.0
1.0
0.005
0.010
0.100
1.000
T_{shield}/T_{hot}

Figure 34

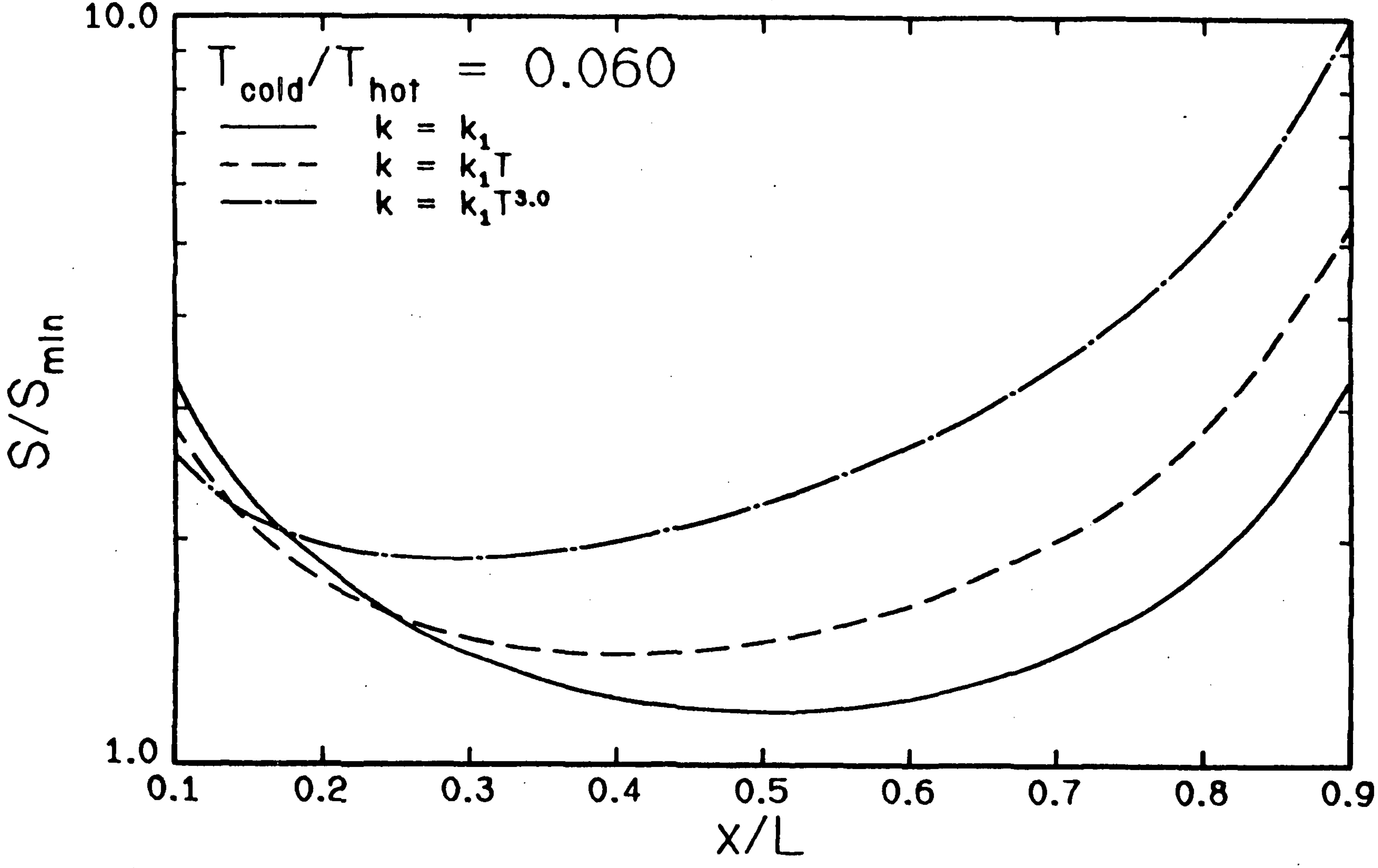

T_{cold}/T_{hot} = 0.060
$k = k_1$
$k = k_1T$
$k = k_1T^{3.0}$
S/S_{min}
10.0
1.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
X/L

Figure 35

APPENDIX

COMPUTER PROGRAMS

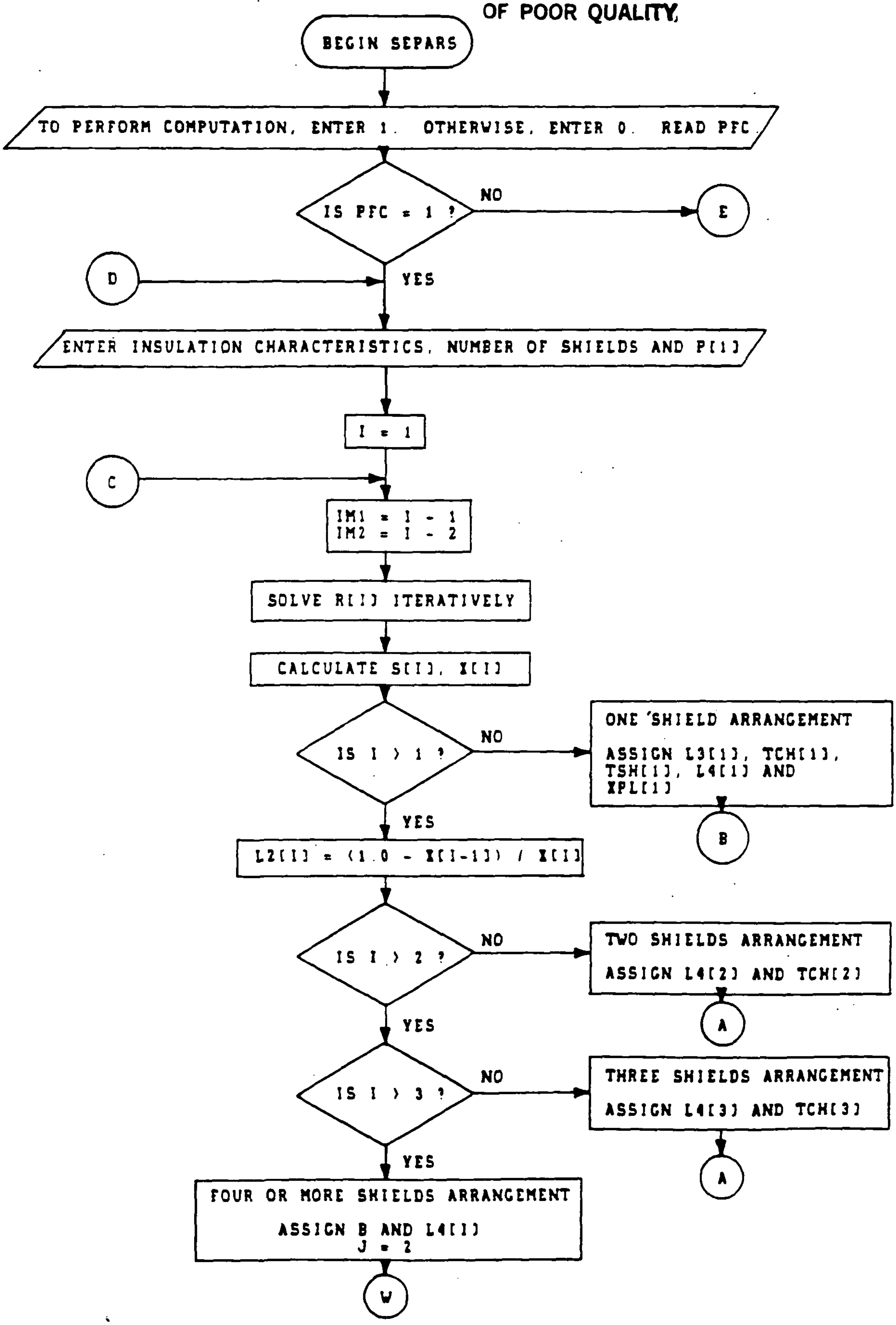
SEPARS and SHIELD

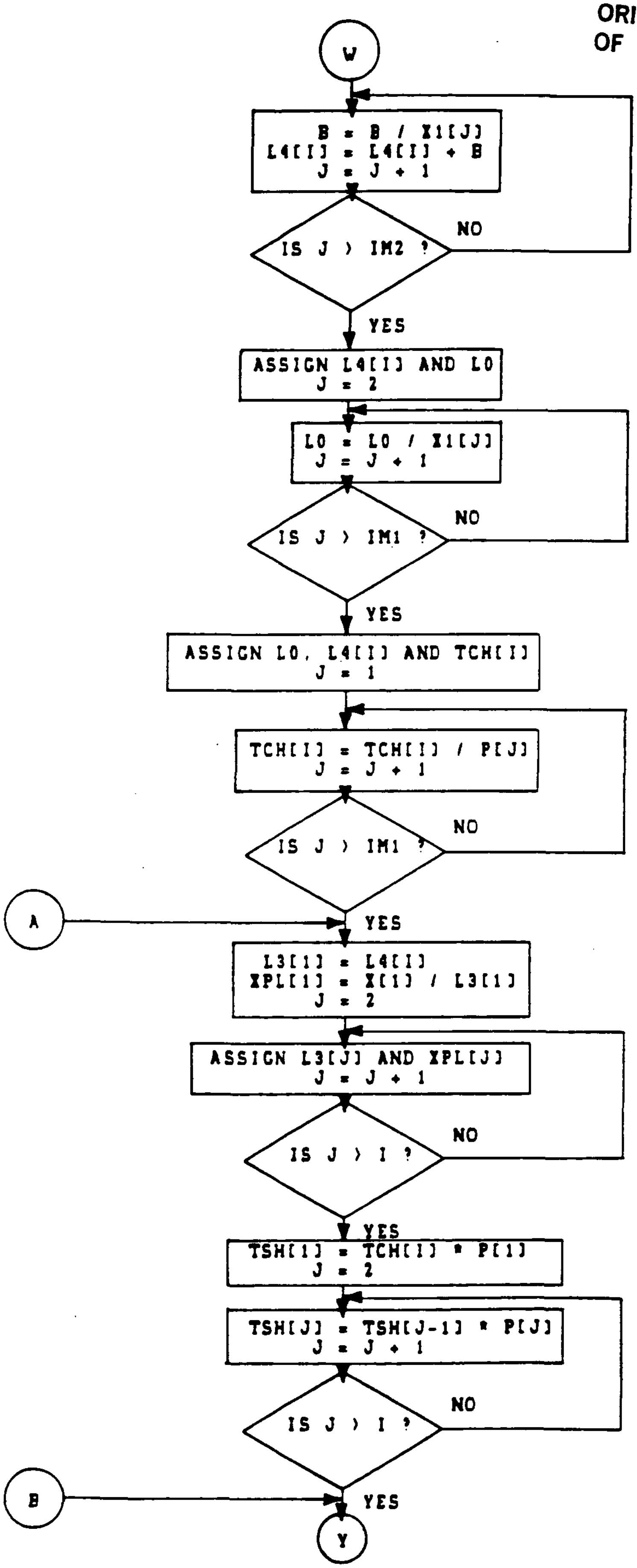
These two programs are essentially identical, but SEPARS is written in PASCAL whereas SHIELD is in BASIC.

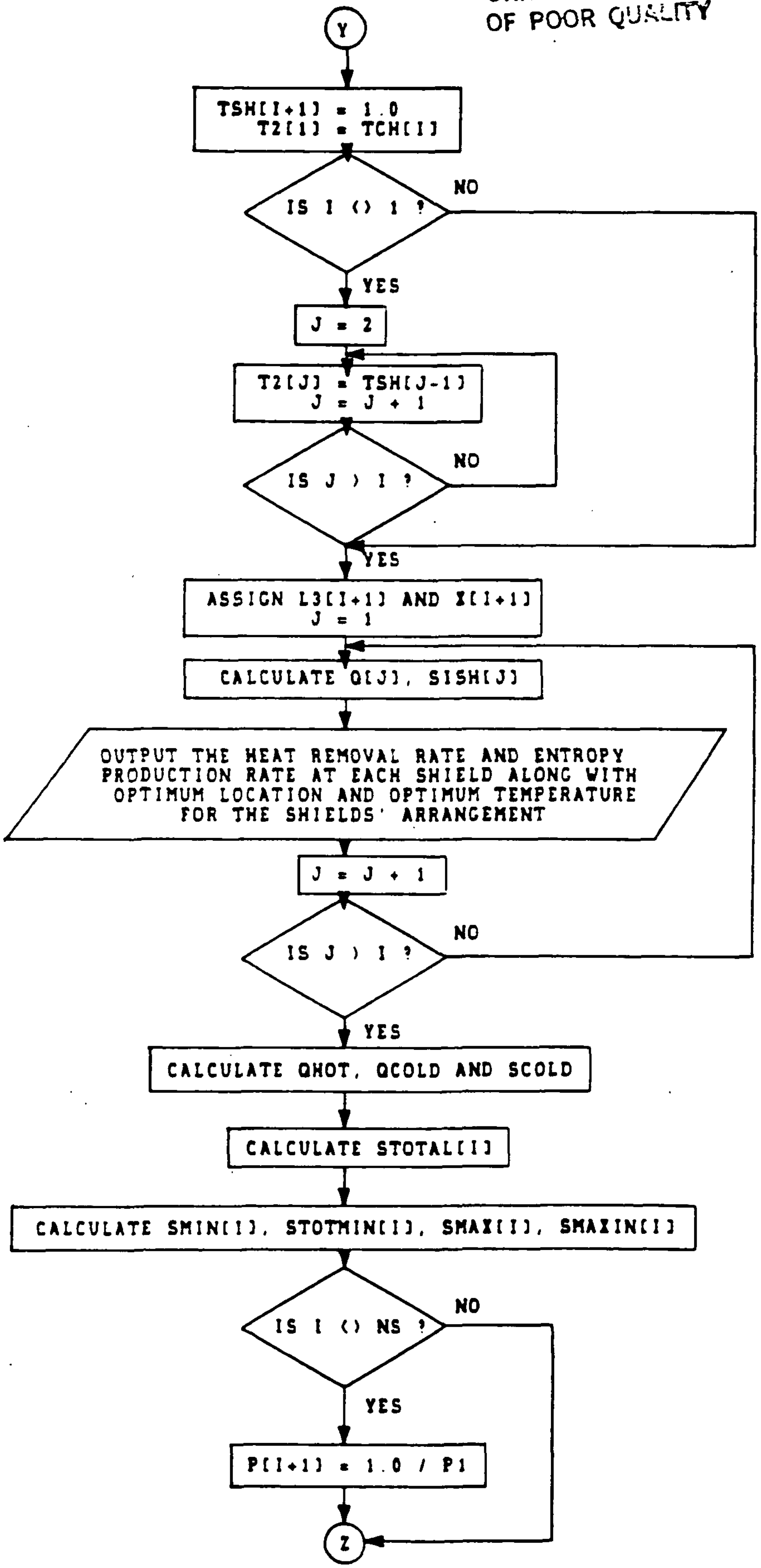
To allow for consecutive calculations of different systems, the program always recycles to the starting point. Consequently, the first input requested is either a 1, if a calculation is to be performed, or a 0, if no more work is to be done.

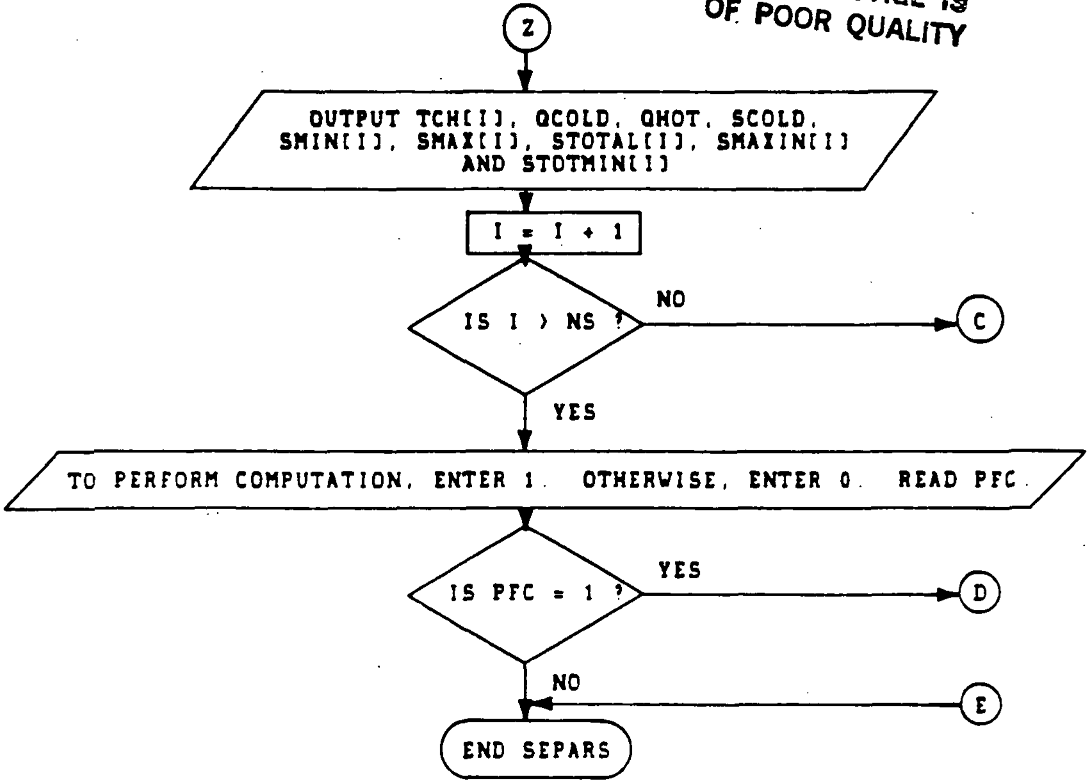
Next the program requests input of the insulation's characteristics, specifically, the two exponents of the temperatures in the two-term conductivity function, the maximum number of cooled shields (<10) to evaluate, the value of γ, and the temperature ratio of the first shield to the cold wall, $P(1) = T_{S1}/T_C$. The program calculates and presents the characteristics of all optimal systems of cooled shields from one shield to the maximum number specified in the input.

The flow chart and a program sample follows.


BEGIN SEPARS
TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0. READ PFC
IS PFC = 1 ?
NO
E
D
YES
ENTER INSULATION CHARACTERISTICS, NUMBER OF SHIELDS AND P[I]
I = 1
C
IM1 = I - 1
IM2 = I - 2
SOLVE R[I] ITERATIVELY
CALCULATE S[I], X[I]
IS I > 1 ?
NO
ONE SHIELD ARRANGEMENT
ASSIGN L3[1], TCH[1], TSH[1], L4[1] AND XPL[1]
B
YES
L2[I] = (1.0 - X[I-1]) / X[I]
IS I > 2 ?
NO
TWO SHIELDS ARRANGEMENT
ASSIGN L4[2] AND TCH[2]
A
YES
IS I > 3 ?
NO
THREE SHIELDS ARRANGEMENT
ASSIGN L4[3] AND TCH[3]
A
YES
FOUR OR MORE SHIELDS ARRANGEMENT
ASSIGN B AND L4[I]
J = 2
W


W
B = B / X1[J]
L4[I] = L4[I] + B
J = J + 1
IS J > IM2 ?
NO
YES
ASSIGN L4[I] AND L0
J = 2
L0 = L0 / X1[J]
J = J + 1
IS J > IM1 ?
NO
YES
ASSIGN L0, L4[I] AND TCH[I]
J = 1
TCH[I] = TCH[I] / P[J]
J = J + 1
IS J > IM1 ?
NO
A
YES
L3[1] = L4[I]
XPL[1] = X[1] / L3[1]
J = 2
ASSIGN L3[J] AND XPL[J]
J = J + 1
IS J > I ?
NO
YES
TSH[1] = TCH[I] * P[1]
J = 2
TSH[J] = TSH[J-1] * P[J]
J = J + 1
IS J > I ?
NO
B
YES
Y


Y
TSH[I+1] = 1.0
T2[1] = TCH[I]
IS I <> 1 ?
NO
YES
J = 2
T2[J] = TSH[J-1]
J = J + 1
IS J > I ?
NO
YES
ASSIGN L3[I+1] AND X[I+1]
J = 1
CALCULATE Q[J], SISH[J]
OUTPUT THE HEAT REMOVAL RATE AND ENTROPY PRODUCTION RATE AT EACH SHIELD ALONG WITH OPTIMUM LOCATION AND OPTIMUM TEMPERATURE FOR THE SHIELDS' ARRANGEMENT
J = J + 1
IS J > I ?
NO
YES
CALCULATE QHOT, QCOLD AND SCOLD
CALCULATE STOTAL[I]
CALCULATE SMIN[I], STOTMIN[I], SMAX[I], SMAXIN[I]
IS I <> NS ?
NO
YES
P[I+1] = 1.0 / P1
Z


Z
OUTPUT TCH[I], QCOLD, QHOT, SCOLD, SMIN[I], SMAX[I], STOTAL[I], SMAXIN[I] AND STOTMIN[I]
I = I + 1
IS I > NS ?
NO
C
YES
TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0. READ PFC.
IS PFC = 1 ?
YES
D
NO
E
END SEPARS

```
PROGRAM SHIELDS(INPUT/,OUTPUT,JMK),



(***************************************************************************************************************)
(***************************************************************************************************************)
(***************************************************************************************************************)
(***************************************************************************************************************)
(**********************************                                          ***********************************)
(**********************************   -----------)    SEPARS   (-----------    ***********************************)
(**********************************                                          ***********************************)
(**********************************      J  C  CHATO  &  J  M  KHODADADI     ***********************************)
(********************************** DEPT  OF MECHANICAL & INDUSTRIAL ENGRS   ***********************************)
(**********************************  UNIV  OF ILLINOIS AT URBANA-CHAMPAIGN    ***********************************)
(**********************************          1206 W  GREEN STREET             ***********************************)
(**********************************           URBANA. IL  61801               ***********************************)
(**********************************                                          ***********************************)
(**********************************               JULY  1983                 ***********************************)
(**********************************                                          ***********************************)
(***************************************************************************************************************)
(***************************************************************************************************************)
(***************************************************************************************************************)
(***************************************************************************************************************)
                           (********************************************************)
                           (*                                                      *)
                           (*   THIS PASCAL PROGRAM WAS DEVELOPED TO OPTIMIZE THE   *)
                           (*   LOCATION, TEMPERATURE AND HEAT DISSIPATION RATE     *)
                           (*   OF EACH COOLED SHIELD INSIDE AN INSULATION LAYER    *)
                           (*   THE THERMAL CONDUCTIVITY OF THE INSULATION HAS      *)
                           (*   THE GENERAL FORM,                                   *)
                           (*                                                      *)
                           (*             K = K1*(T**M) + K2*(T**N)                *)
                           (*                                                      *)
                           (*   THE METHOD IS BASED ON THE MINIMIZATION OF THE      *)
                           (*   ENTROPY PRODUCTION RATE WHICH IS PROPORTIONAL TO    *)
                           (*   THE HEAT LEAK ACROSS THE INSULATION                 *)
                           (*                                                      *)
                           (********************************************************)
                           (********************************************************)


LABEL   100
LABEL   200,

TYPE
        ARRAYS=ARRAY[1..10] OF REAL.            (*  THE SIZE OF ARRAYS DETERMINES THE MAXIMUM NUMBER OF SHIELDS  *)
        ARRAYP=ARRAY[1..11] OF REAL.            (*  THE SIZE OF ARRAYP IS EQUAL TO NS+1  *)

                                                   (**************************************************)
VAR                                                (*  --)  LET THICK[I] REPRESENT THE SPACING  (--  *)
                                                   (*  --)  BETWEEN (I-1)-TH & (I+1)-TH SHIELDS (--  *)
                                                   (**************************************************)

        L2                   ARRAYS.            (*  THICK[I] / THICK[I-1]  *)
        L3                   ARRAYP.            (*  DUMMY VARIABLE  *)
        L4                   ARRAYS.            (*  OVERALL INSULATION THICKNESS / THICK[I] RATIO  *)
        P                    ARRAYS.            (*  I-TH SHIELD / LOCAL COLD TEMPERATURE RATIO. ALWAYS ) 1  *)
        Q                    ARRAYS.            (*  I-TH SHIELD DIMENSIONLESS HEAT REMOVAL RATE  *)
        R                    ARRAYS.            (*  I-TH LOCAL COLD / LOCAL HOT TEMPERATURE RATIO. ALWAYS ( 1  *)
        S                    ARRAYS.            (*  DIMENSIONLESS ENTROPY PRODUCTION RATE FOR I-TH LAYER  *)
        SISH                 ARRAYS.            (*  I-TH SHIELD DIMENSIONLESS ENTROPY PRODUCTION RATE  *)
        SMAX                 ARRAYS.            (*  MAXIMUM DIMENSIONLESS ENTROPY PRODUCTION RATE  *)
        SMIN                 ARRAYS.            (*  MINIMUM DIMENSIONLESS ENTROPY PRODUCTION RATE  *)
        SMAXIN               ARRAYS.            (*  SMAX[I] / SMIN[I]  *)
        STOTAL               ARRAYS.            (*  TOTAL DIMENSIONLESS ENTROPY PRODUCTION RATE  *)
        STOTMIN              ARRAYS.            (*  STOTAL[I] / SMIN[I]  *)
        TCH                  ARRAYS.            (*  COLD WALL / HOT WALL TEMPERATURE RATIO. ALWAYS ( 1  *)
        TSH                  ARRAYP.            (*  I-TH SHIELD / HOT WALL TEMPERATURE RATIO. ALWAYS ( 1  *)
        T1                   ARRAYS.            (*  DUMMY VARIABLE  *)
        X                    ARRAYP.            (*  DISTANCE FROM LOCAL COLD SHIELD / THICK[I] RATIO  *)
        XPL                  ARRAYS.            (*  DISTANCE FROM COLD WALL / OVERALL INSULATION THICKNESS  *)
        X1                   ARRAYS.            (*  X[I] / (1.0-X[I])  *)
```

```
            B                   : REAL;             (*  DUMMY VARIABLE  *)
            CC                  : REAL;             (*  DUMMY VARIABLE  *)
            COUNT               : INTEGER;          (*  NUMBER OF ITERATIONS NEEDED TO DETERMINE R[1]  *)
            DD                  : REAL;             (*  DUMMY VARIABLE  *)
            G,G1                : REAL;             (*  DUMMY VARIABLES  *)
            GAMA                : REAL;             (*  [K2*(M+1)]/[K1*(N+1)] * THOT**(N-M) , ALWAYS >= 0  *)
                                                    (*     WHERE THOT IS THE HOT WALL TEMPERATURE [K]  *)
            I,IM1,IM2,J         : INTEGER;          (*  INDICES FOR LOOPS  *)
            JMK                 : TEXT;             (*  OUTPUT FILE TO BE USED IF DESIRED  *)
            L0                  : REAL;             (*  DUMMY VARIABLE  *)
            M                   : REAL;             (*  1ST  POWER IN THE THERMAL CONDUCTIVITY EQUATION  *)
            MP1                 : REAL;             (*  EQUALS M+1  *)
            N                   : REAL;             (*  2ND  POWER IN THE THERMAL CONDUCTIVITY EQUATION  *)
            NP1                 : REAL;             (*  EQUALS N+1  *)
            NS                  : INTEGER;          (*  NUMBER OF SHIELDS  *)
            PFC                 : INTEGER;          (*  PROGRAM FLOW CONTROLLER  *)
            P1                  : REAL;             (*  I-TH SHIELD / LOCAL HOT TEMPERATURE RATIO, ALWAYS < 1  *)
            QCOLD               : REAL;             (*  HEAT OUT AT COLD WALL  *)
            QHOT                : REAL;             (*  HEAT IN AT HOT WALL  *)
            SCOLD               : REAL;             (*  ENTROPY PRODUCTION RATE AT COLD WALL  *)
            U,V                 : REAL;             (*  DUMMY VARIABLES  *)
            W1,W2,W3            : REAL;             (*  DUMMY VARIABLES  *)
            Z1,Z2               : REAL;             (*  DUMMY VARIABLES  *)




    PROCEDURE INPUTH;
     BEGIN              (*  INPUT OF DATA HEADING  *)
      WRITELN;
      WRITELN('         ENTER    ---->    M    N    NS   GAMA    P[1]    <----');
      WRITELN('   ');
      WRITELN('         WHERE    M ----- 1ST  POWER IN THE THERMAL CONDUCTIVITY EQUATION');
      WRITELN('                  N ----- 2ND  POWER IN THE THERMAL CONDUCTIVITY EQUATION');
      WRITELN('                  NS ---- NUMBER OF SHIELDS');
      WRITELN('                  GAMA -- =0 IF USING ONE TERM THERMAL CONDUCTIVITY EQUATION');
      WRITELN('                          >0 IF USING TWO TERM THERMAL CONDUCTIVITY EQUATION');
      WRITELN('                  P[1] -- 1ST  SHIELD / COLD WALL TEMPERATURE RATIO, ALWAYS > 1');
      WRITELN('   ')
     END;               (*  INPUT OF DATA HEADING  *)



    PROCEDURE PFCH;
     BEGIN              (*  PFCH  *)
      WRITELN;
      WRITELN('         TO PERFORM COMPUTATION, ENTER 1.  OTHERWISE, ENTER 0.');
      WRITELN
     END;               (*  PFCH  *)



    PROCEDURE SINGLESPACE;
     BEGIN              (*  SINGLE SPACE IN OUTPUT  *)
      WRITELN('  ')
     END;               (*  SINGLE SPACE IN OUTPUT  *)



    FUNCTION PWR(XX,E:REAL):REAL;
     VAR
           A                    : REAL;
     BEGIN              (*  COMPUTE XX**E  *)
      A :=E*LN(XX);
      PWR :=EXP(A)
     END;               (*  COMPUTE XX**E  *)



    FUNCTION D(E,XX:REAL):REAL;
     BEGIN              (*  FUNCTIONAL D  *)
      D :=(E+1.0)*PWR(XX,E)-E/(PWR(XX,(1.0-E)))-(1.0/SQR(XX))
     END;               (*  FUNCTIONAL D  *)
```

```
FUNCTION F(E,XX:REAL):REAL;
 BEGIN                (*  FUNCTIONAL F  *)
  F :=(PWR(XX,(E+1.0)))-PWR(XX,E)-1.0+(1.0/XX)
 END.                 (*  FUNCTIONAL F  *)



FUNCTION SIMPSON(TCHR REAL):REAL.
TYPE
        ARR=ARRAY[1  101] OF REAL.

VAR
        C,Y                ARR.
        DELTAT             . REAL.
        H                  . REAL.
        K,L                  INTEGER.

BEGIN                 (*  COMPUTE MINIMUM ENTROPY PRODUCTION RATE USING SIMPSON'S NUMERICAL INTEGRATION SCHEME  *)
 DELTAT :=(1 0-TCHR)/100 0,
 FOR L :=1 TO 101 DO
  BEGIN
   C[L] :=TCHR+DELTAT*(L-1);
   Y[L] :=PWR((PWR(C[L],M)+GAMA*NP1/MP1*PWR(C[L],N)),0.5)/C[L]
  END.
 H :=Y[1]+Y[101].
 FOR K :=2 TO 100 DO
  BEGIN
   IF K=((K DIV 2)*2) THEN
    H :=H+4 0*Y[K]
   ELSE
    H :=H+2 0*Y[K]
  END.
 SIMPSON :=(SQR(DELTAT/3 0*H))/(1 0+GAMA*NP1/MP1)
END.                  (*  COMPUTE MINIMUM ENTROPY PRODUCTION RATE USING SIMPSON'S NUMERICAL INTEGRATION SCHEME  *)







             (*  MAIN PROGRAM BODY  *)

BEGIN
 PTCH.
 READLN.
 READ(PFC).
 WHILE PFC=1 DO
 BEGIN

             (*  THIS BLOCK IS USED TO INPUT THE INSULATION THERMAL CONDUCTIVITY. NUMBER  *)
             (*  OF SHIELDS AND 1ST. SHIELD / COLD WALL TEMPERATURE RATIO                 *)

  INPUTH
  READLN.
  READ(M.N.NS.GAMA.P[1]).
  SINGLESPACE.
  IF GAMA=0 0 THEN
   WRITELN('        THERMAL CONDUCTIVITY OF THE INSULATION IS K = K1*T**',M.3.1)
  ELSE
   BEGIN
    WRITELN('        THERMAL CONDUCTIVITY OF THE INSULATION IS K = K1*T**',M:3 1,' + K2*T**',N.3 1),
    WRITELN('                  [K2*(M+1)]/[K1*(N+1)]*THOT**(N-M) = ',GAMA 9 2)
   END.
  SINGLESPACE.
  SINGLESPACE.


  MP1 :=M+1 0.
  NP1 :=N+1 0.
  FOR I :=1 TO NS DO
   BEGIN
    IM1 :=I-1.
    IM2 :=I-2.
    B[I] :=0 000001.
    CC :=0 1;
    DD :=1.0.
    COUNT :=0.

```

```
        (*  THIS BLOCK CALCULATES R[I] ITERATIVELY  *)

REPEAT
 P1 :=P[I]*R[I];
 V1 :=PWR(R[I],M)*F(M,P[I])+GAMA*PWR(R[I],N)*F(N,P[I]);
 V2 :=SQR(PWR(R[I],(M-1.0))*D(M,P[I])+GAMA*PWR(R[I],(N-1.0))*D(N,P[I]));
 V3 :=-SQR(D(M,P1)+GAMA*D(N,P1))/(F(M,P1)+GAMA*F(N,P1));
 G :=(V2/V1)+V3;
 G1 :=G*DD;
 IF G1<0.0  THEN GOTO 100;
 IF G1=0.0  THEN GOTO 200;
 CC :=(-0.1)*CC;
 IF ABS(CC)<0.000001  THEN GOTO 200;
 DD :=-DD;
 100 R[I] :=R[I]+CC;
 IF (R[I]>0.999999) OR (R[I]<0.000001) THEN
  BEGIN
   R[I] :=R[I]-0.9*CC;
   CC :=0.1*CC
  END;
 200 COUNT :=COUNT+1;
UNTIL (G1=0.0) OR (ABS(CC)<0.000001);


U :=-(PWR(R[I],(M-1.0))*D(M,P[I])+GAMA*PWR(R[I],(N-1.0))*D(N,P[I]));
X1[I] :=U/(D(M,P1)+GAMA*D(N,P1));
X[I] :=X1[I]/(1.0+X1[I]);
V :=(F(M,P1)+GAMA*F(N,P1))/(1.0-X[I]);
S[I] :=V+(PWR(R[I],M)*F(M,P[I])+GAMA*PWR(R[I],N)*F(N,P[I]))/X[I];
S[I] :=S[I]/(1.0+GAMA*NP1/MP1)/MP1;

        (*  IN THIS BLOCK VARIABLES ARE ASSIGNED FOR DIFFERENT SHIELD CONFIGURATIONS  *)

IF I>1 THEN
 BEGIN
  L2[I] :=(1.0-X[I-1])/X[I];
  IF I>2 THEN
   IF I>3 THEN
    BEGIN
     B :=1.0;
     L4[I] :=0.0;
     FOR J :=2 TO IM2 DO
      BEGIN
       B :=B/X1[J];
       L4[I] :=L4[I]+B
      END;
     L4[I] :=L4[I]*(1.0-X[1])+1.0;
     L0 :=1.0-X[1];
     FOR J :=2 TO IM1 DO L0 :=L0/X1[J];
     L0 :=L0/X[I];
     L4[I] :=L4[I]+L0;
     TCH[I] :=R[I];
     FOR J :=1 TO IM1 DO TCH[I] :=TCH[I]/P[J];
    END
   ELSE
    BEGIN
     L4[3] :=1.0+(1.0-X[1])*(1.0-X[2])/(X[2]*X[3]);
     TCH[3] :=R[3]/(P[1]*P[2])
    END
  ELSE
   BEGIN
    L4[2] :=X[1]+(1.0-X[1])/X[2];
    TCH[2] :=R[2]/P[1]
   END;
   L3[1] :=L4[I];
   XPL[1] :=X[1]/L3[1];
   FOR J :=2 TO I DO
    BEGIN
     L3[J] :=L3[J-1]/L2[J];
     XPL[J] :=XPL[J-1]+X[J]/L3[J]
    END;
   TSH[1] :=TCH[I]*P[1];
   FOR J :=2 TO I DO TSH[J] :=TSH[J-1]*P[J]
 END
ELSE
```

```
          BEGIN
           L3[I] :=1.0;
           TCH[I] :=R[I];
           TSH[I] :=TCH[I]*P[I];
           L4[I] :=1.0;
           XPL[I] :=X[I]
          END;
        TSH[I+1] :=1.0;
        T2[1] :=TCH[I];


        SINGLESPACE;
        WRITELN('        NUMBER OF SHIELDS    = ',I:2);
        WRITELN('        NUMBER OF ITERATIONS = ',COUNT:3);
        SINGLESPACE;
        SINGLESPACE;
        WRITELN('                        HEAT REMOVAL    ENTROPY PRODUCTION    OPTIMUM      OPTIMUM');
        WRITELN('                            RATE              RATE           LOCATION    TEMPERATURE');
        WRITELN('                        ------------    ------------------    --------    -----------');
        SINGLESPACE;
        IF I<>1 THEN
         FOR J :=2 TO I DO T2[J] :=TSH[J-1];
        L3[I+1] :=L3[I];
        X[I+1] :=1.0-X[I];

                (*  IN THIS BLOCK DIMENSIONLESS HEAT REMOVAL AND ENTROPY PRODUCTION RATES  *)
                (*  ARE CALCULATED FOR EACH SHIELD                                         *)

        FOR J :=1 TO I DO
         BEGIN
          Z1 :=((PWR(TSH[J+1],MP1)-PWR(TSH[J],MP1))*L3[J+1]/X[J+1]-(PWR(TSH[J],MP1)-PWR(T2[J],MP1))*L3[J]/X[J])/MP1;
          Z2 :=((PWR(TSH[J+1],NP1)-PWR(TSH[J],NP1))*L3[J+1]/X[J+1]-(PWR(TSH[J],NP1)-PWR(T2[J],NP1))*L3[J]/X[J])/MP1;
          Q[J] :=(Z1+GAMA*Z2)/(1.0+GAMA*NP1/MP1);
          SISH[J] :=Q[J]/TSH[J];
          WRITELN('        SHIELD ',J:2,' ':5,Q[J]:9:5,' ':11,SISH[J]:9:5,' ':9,XPL[J]:9:5,' ':5,TSH[J]:9:5);
         END;

                (*  FINALLY, OTHER QUANTITIES OF INTEREST ARE CALCULATED IN THIS BLOCK  *)

        SINGLESPACE;
        QHOT :=((1.0-PWR(TSH[1],MP1)+GAMA-GAMA*PWR(TSH[1],NP1))*L3[I]/(X[I+1]*MP1))/(1.0+GAMA*NP1/MP1);
        QCOLD :=(PWR(TSH[I],MP1)-PWR(TCH[I],MP1)+GAMA*PWR(TSH[I],NP1)-GAMA*PWR(TCH[I],NP1))*L3[I]/(X[I]*MP1);
        QCOLD :=QCOLD/(1.0+GAMA*NP1/MP1);
        SCOLD :=QCOLD/TCH[I];
        STOTAL[I] :=SCOLD-QHOT;
        FOR J :=1 TO I DO STOTAL[I] :=STOTAL[I]+SISH[J];
        SMIN[I] :=SIMPSON(TCH[I]);
        STOTMIN[I] :=STOTAL[I]/SMIN[I];
        SMAX[I] :=((1.0-PWR(TCH[I],MP1)+GAMA-GAMA*PWR(TCH[I],NP1))*(1.0/TCH[I]-1.0)/MP1)/(1.0+GAMA*NP1/MP1);
        SMAXMIN[I] :=SMAX[I]/SMIN[I];


        IF I<>NS THEN P[I+1] :=1.0/P[I];
        SINGLESPACE;
        WRITELN('        COLD WALL / HOT WALL TEMPERATURE RATIO     = ',TCH[I]:14:6);
        WRITELN('        HEAT OUT AT COLD WALL                      = ',QCOLD:14:6);
        WRITELN('        HEAT IN AT HOT WALL                        = ',QHOT:14:6);
        WRITELN('        ENTROPY PRODUCTION RATE AT COLD WALL       = ',SCOLD:14:6);
        WRITELN('        ENTROPY PRODUCTION RATE AT HOT WALL        = ',-QHOT:14:6);
        WRITELN('        MINIMUM ENTROPY PRODUCTION RATE            = ',SMIN[I]:14:6);
        WRITELN('        MAXIMUM ENTROPY PRODUCTION RATE            = ',SMAX[I]:14:6);
        WRITELN('        TOTAL ENTROPY PROD. RATE WITH ',I:2,' SHIELDS   = ',STOTAL[I]:14:6);
        WRITELN('        MAXIMUM / MINIMUM ENTROPY PRODUCTION RATIO = ',SMAXMIN[I]:14:6);
        WRITELN('        TOTAL / MINIMUM ENTROPY PRODUCTION RATIO   = ',STOTMIN[I]:14:6);
        SINGLESPACE;
        SINGLESPACE;
        SINGLESPACE

       END;
      PFCH;
      READLN;
      READ(PFC)
     END
    END
/EOP
??
```

```
        TO PERFORM COMPUTATION, ENTER 1.  OTHERWISE, ENTER 0.

? 1

        ENTER     ---->    M    N    NS   GAMA    P[1]    <----

        WHERE:    M ----- 1ST. POWER IN THE THERMAL CONDUCTIVITY EQUATION
                  N ----- 2ND. POWER IN THE THERMAL CONDUCTIVITY EQUATION
                  NS ---- NUMBER OF SHIELDS
                  GAMA -- =0 IF USING ONE TERM THERMAL CONDUCTIVITY EQUATION
                          >0 IF USING TWO TERM THERMAL CONDUCTIVITY EQUATION
                  P[1] -- 1ST. SHIELD / COLD WALL TEMPERATURE RATIO, ALWAYS > 1

?      1.0     3.0     1      2.5     15.0

        THERMAL CONDUCTIVITY OF THE INSULATION IS K = K1*T**1.0 + K2*T**3.0
                  [K2*(M+1)]/[K1*(N+1)]*THOT**(N-M) =       2.50


        NUMBER OF SHIELDS     =  1
        NUMBER OF ITERATIONS = 35

                       HEAT REMOVAL      ENTROPY PRODUCTION      OPTIMUM        OPTIMUM
                           RATE                 RATE             LOCATION     TEMPERATURE
                       ------------      ------------------      --------     -----------

        SHIELD  1        0.43837               1.85659            0.36744       0.23611

        COLD WALL / HOT WALL TEMPERATURE RATIO      =        0.015741
        HEAT OUT AT COLD WALL                       =        0.014350
        HEAT IN AT HOT WALL                         =        0.452719
        ENTROPY PRODUCTION RATE AT COLD WALL        =        0.911631
        ENTROPY PRODUCTION RATE AT HOT WALL         =       -0.452719
        MINIMUM ENTROPY PRODUCTION RATE             =        1.000503
        MAXIMUM ENTROPY PRODUCTION RATE             =       18.236148
        TOTAL ENTROPY PROD. RATE WITH  1 SHIELDS    =        2.315503
        MAXIMUM / MINIMUM ENTROPY PRODUCTION RATIO =        18.226982
        TOTAL / MINIMUM ENTROPY PRODUCTION RATIO    =        2.314340


        TO PERFORM COMPUTATION, ENTER 1.  OTHERWISE, ENTER 0.

? 1

        ENTER     ---->    M    N    NS   GAMA    P[1]    <----

        WHERE:    M ----- 1ST. POWER IN THE THERMAL CONDUCTIVITY EQUATION
                  N ----- 2ND. POWER IN THE THERMAL CONDUCTIVITY EQUATION
                  NS ---- NUMBER OF SHIELDS
                  GAMA -- =0 IF USING ONE TERM THERMAL CONDUCTIVITY EQUATION
                          >0 IF USING TWO TERM THERMAL CONDUCTIVITY EQUATION
                  P[1] -- 1ST. SHIELD / COLD WALL TEMPERATURE RATIO, ALWAYS > 1

?      1.0 . 090     2      0.0     25.0

        THERMAL CONDUCTIVITY OF THE INSULATION IS K = K1*T**1.0


        NUMBER OF SHIELDS     =  1
        NUMBER OF ITERATIONS = 23
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
                    HEAT REMOVAL      ENTROPY PRODUCTION     OPTIMUM       OPTIMUM
                        RATE                 RATE            LOCATION    TEMPERATURE
                    ------------      ------------------     --------    -----------

   SHIELD  1          0.75466              7.03151            0.35870      0.10732


   COLD WALL / HOT WALL TEMPERATURE RATIO     =        0.004293
   HEAT OUT AT COLD WALL                      =        0.016030
   HEAT IN AT HOT WALL                        =        0.770687
   ENTROPY PRODUCTION RATE AT COLD WALL       =        3.734070
   ENTROPY PRODUCTION RATE AT HOT WALL        =       -0.770687
   MINIMUM ENTROPY PRODUCTION RATE            =        3.504633
   MAXIMUM ENTROPY PRODUCTION RATE            =      115.966533
   TOTAL ENTROPY PROD. RATE WITH  1 SHIELDS   =        9.994893
   MAXIMUM / MINIMUM ENTROPY PRODUCTION RATIO =       33.089491
   TOTAL / MINIMUM ENTROPY PRODUCTION RATIO   =        2.851908



   NUMBER OF SHIELDS    =  2
   NUMBER OF ITERATIONS = 36

                    HEAT REMOVAL      ENTROPY PRODUCTION     OPTIMUM       OPTIMUM
                        RATE                 RATE            LOCATION    TEMPERATURE
                    ------------      ------------------     --------    -----------

   SHIELD  1          0.05470              2.71297            0.17465      0.02016
   SHIELD  2          0.88421              4.70678            0.48690      0.18786

   COLD WALL / HOT WALL TEMPERATURE RATIO     =        0.000806
   HEAT OUT AT COLD WALL                      =        0.001162
   HEAT IN AT HOT WALL                        =        0.940073
   ENTROPY PRODUCTION RATE AT COLD WALL       =        1.440716
   ENTROPY PRODUCTION RATE AT HOT WALL        =       -0.940073
   MINIMUM ENTROPY PRODUCTION RATE            =        3.921467
   MAXIMUM ENTROPY PRODUCTION RATE            =      619.477774
   TOTAL ENTROPY PROD. RATE WITH  2 SHIELDS   =        7.920388
   MAXIMUM / MINIMUM ENTROPY PRODUCTION RATIO =      157.970919
   TOTAL / MINIMUM ENTROPY PRODUCTION RATIO   =        2.019751



   TO PERFORM COMPUTATION, ENTER 1.  OTHERWISE, ENTER 0.

? 0
    0.175 CP SECS,  12415B CM USED.
```

PROGRAM SHIELD

```
00010 REM THIS IS A "BASIC" PROGRAM TO CALCULATE OPTIMUM TEMPERATURES.
00020 REM LOCATIONS, AND COOLING LOADS FOR COOLED SHIELDS IN A CRYOGENIC
00030 REM INSULATION SYSTEM WHOSE THERMAL CONDUCTIVITY FOLLOWS THE RELATION
00040 REM      K=C1*T^M0 + C2*T^N0
00045 REM MODIFIED IN LATE NOV. 1982.
00050 REM
00060 REM DEFINITION OF SYMBOLS USED
00070 REM
00080 REM COLD-SIDE WALL TEMPERATURE  T0
00090 REM WARM-SIDE WALL TEMPERATURE  T9
00100 REM SPACING BETWEEN SHIELDS AT I+1 AND I-1  L1(I)
00110 REM OVERALL THICKNESS OF INSULATION  L
00120 REM LOCAL SPACING RATIO, L1(I)/L1(I-1). L2(I)
00130 REM OVERALL SPACING RATIO, L/L1(1). L4(I)
00140 REM (DISTANCE FROM COLD WALL)/L  L5(I)
00150 REM I-TH SHIELD TEMPERATURE  T(I)
00160 REM I-TH SHIELD POSITION RATIO  X(I)
00170 REM I-TH SHIELD TEMPERATURE RATIO  P(I) (ALWAYS >1)
00180 REM I-TH COLD-WARM TEMPERATURE RATIO  R(I) (ALWAYS <1)
00190 REM I-TH DIMENSIONLESS ENTROPY PRODUCTION RATE  S(I)
00195 REM I-TH DIMENSIONLESS HEAT REMOVAL RATE  Q(I)
00210 REM TOTAL DIMENSIONLESS ENTROPY PROD. RATE. S2(I)
00220 REM MINIMUM ENTROPY PRODUCTION RATE. S0(I)
00230 REM ENTROPY PROD. RATE WITHOUT SHIELDS  S9(I)
00240 REM ENTROPY PROD. RATE RATIOS  S3=S2/S0 AND S4=S9/S0
00250 REM NUMBER OF SHIELDS  M <= OR <10)
00260 REM
00265 DIM X(101),Y(101)
00270 PRINT" "
00280 PRINT "INPUT 1 IF MORE WORK IS TO BE DONE, 0 IF FINISHED"
00290 INPUT A
00300 IF A=0 THEN 01350
00310 PRINT "INPUT  M0,N0,M,GAMMA & P(1)"
00320 INPUT M0,N0,M,G0,P(1)
00325 DEF FND(Y)=(M0+1)*Y^M0-M0/(Y^(1-M0))-1/(Y*Y)
00330 DEF FNE(Y)=(N0+1)*Y^N0-N0/(Y^(1-N0))-1/(Y*Y)
00335 DEF FNF(Y)=Y^(M0+1)-Y^M0+1/Y-1
00340 DEF FNG(Y)=Y^(N0+1)-Y^N0+1/Y-1
00350 PRINT " EXPONENT M0=",M0,"   EXPONENT N0=",N0,"  GAMMA=",G0
00355 M1=M0+1
00358 N1=N0+1
00360 FOR I=1 TO M
00370 I1=I-1
00380 I2=I-2
00390 R(I)=.000001
00400 C=.1
00410 D=1
00420 P1=P(I)*R(I)
00430 W1=(R(I)^M0*FNF(P(I))+G0*R(I)^N0*FNG(P(I)))
00435 W2=((R(I)^(M0-1)*FND(P(I))+G0*R(I)^(N0-1)*FNE(P(I)))^2)
00438 W3=-((FND(P1)+G0*FNE(P1))^2)/(FNF(P1)+G0*FNG(P1))
00439 G=W2/W1+W3
00440 G1=G*D
00450 IF G1<0 THEN 00500
00460 IF G1=0 THEN 00570
00470 C=-.1*C
00480 IF ABS(C)<.000001 THEN 00570
00490 D=-D
00500 R(I)=R(I)+C
00510 IF R(I)>0.999999 THEN 00540
00520 IF R(I)<0.000001 THEN 00540
00530 GOTO 00420
00540 R(I)=R(I)-.9*C
00550 C=.1*C
00560 GOTO 00420
00570 U=-(R(I)^(M0-1)*FND(P(I))+G0*R(I)^(N0-1)*FNE(P(I)))
00575 X1(I)=U/(FND(P1)+G0*FNE(P1))
00580 X(I)=X1(I)/(1+X1(I))
00590 V=(FNF(P1)+G0*FNG(P1))/(1-X(I))
00595 S(I)=V+(R(I)^M0*FNF(P(I))+G0*R(I)^N0*FNG(P(I)))/X(I)
00596 S(I)=S(I)/(1+G0*N1/M1)/M1
00600 IF I>1 THEN 00670
00610 L3(1)=1
00620 R0(1)=R(1)
```

```
00630 T1(1)=R0(I)*P(1)
00640 L4(1)=1
00650 L5(1)=X(1)
00660 GOTO 01030
00670 L2(I)=(1-X(I-1))/X(I)
00680 IF I>2 THEN 00720
00690 L4(2)=X(1)+(1-X(1))/X(2)
00700 R0(2)=R(2)/P(1)
00710 GOTO 00930
00720 IF I>3 THEN 00760
00730 L4(3)=1+(1-X(1))*(1-X(2))/(X(2)*X(3))
00740 R0(3)=R(3)/(P(1)*P(2))
00750 GOTO 00930
00760 B=1
00770 L4(I)=0
00780 FOR J=2 TO I2
00790 B=B/X(J)
00800 L4(I)=L4(I)+B
00810 NEXT J
00820 L4(I)=L4(I)*(1-X(1))+1
00830 L0=1-X(1)
00840 FOR J=2 TO I1
00850 L0=L0/X(J)
00860 NEXT J
00870 L0=L0^X(I)
00880 L4(I)=L4(I)+L0
00890 R0(I)=R(I)
00900 FOR J=1 TO I1
00910 R0(I)=R0(I)/P(J)
00920 NEXT J
00930 L3(1)=L4(I)
00940 L5(1)=X(1)/L3(1)
00950 FOR J=2 TO I
00960 L3(J)=L3(J-1)/L2(J)
00970 L5(J)=L5(J-1)+X(J)/L3(J)
00980 NEXT J
00990 T1(1)=R0(I)*P(1)
01000 FOR J=2 TO I
01010 T1(J)=T1(J-1)*P(J)
01020 NEXT J
01030 T1(I+1)=1
01040 PRINT" "
01050 PRINT" I=",I
01060 PRINT" "
01070 T2(1)=R0(I)
01080 FOR J=2 TO I
01090 T2(J)=T1(J-1)
01100 NEXT J
01103 L3(I+1)=L3(I)
01104 X(I+1)=1-X(I)
01110 FOR J=1 TO I
WIDE LINE
01120 Z1=((T1(J+1)^M1-T1(J)^M1)*L3(J+1)/X(J+1)-(T1(J)^M1-T2(J)^M1)*L3(J)/X(J))/M1
WIDE LINE
01125 Z2=((T1(J+1)^N1-T1(J)^N1)*L3(J+1)/X(J+1)-(T1(J)^N1-T2(J)^N1)*L3(J)/X(J))/M1
01128 Q(J)=(Z1+G0*Z2)/(1+G0*N1/M1)
01130 S1(J)=Q(J)/T1(J)
01140 PRINT"    J=",J,"  Q=";Q(J),"  S1=",S1(J);"  L5=",L5(J),"  T/T9=",T1(J)
01150 NEXT J
01152 Q9=(1-T1(I)^M1+G0-G0*T1(I)^N1)*L3(I)/(X(I+1)*M1)
01153 Q9=Q9/(1+G0*N1/M1)
01154 Q0=(T1(1)^M1-R0(I)^M1+G0*T1(1)^N1-G0*R0(I)^N1)*L3(1)/(X(1)*M1)
01155 Q0=Q0/(1+G0*N1/M1)
01154 S8=Q0/R0(I)
01160 S2(I)=S8-Q9
01162 REM CALCULATING DATA TO GET SMIN
01163 D=(1-R0(I))/100
01164 FOR L=1 TO 101
01165 C(L)=R0(I)+D*(L-1)
01166 Y(L)=(C(L)^M0+G0*N1/M1*C(L)^N0)^0.5/C(L)
01167 NEXT L
01170 FOR J=1 TO I
01180 S2(I)=S2(I)+S1(J)
01200 NEXT J
01201 REM OBTAIN SMIN USING SIMPSON'S RULE
01202 H=Y(1)+Y(101)
```

```
01203 FOR K=2 TO 100
01204 IF K/2=INT(K/2) THEN 01207
01205 H=H+2*Y(K)
01206 GO TO 01208
01207 H=H+4*Y(K)
01208 NEXT K
01210 S0(I)=((D/3*H)^2)/(1+G0*N1/M1)
01220 S3(I)=S2(I)/S0(I)
01230 S9(I)=((1-R0(I)^M1+G0-G0*R0(I)^N1)*(1/R0(I)-1)/M1)/(1+G0*N1/M1)
01240 S4(I)=S9(I)/S0(I)
01250 IF I=M THEN 01270
01260 P(I+1)=1/P1
01270 PRINT" "
01280 PRINT"     P=",P(I),"  R=",R(I),"  X=",X(I),"  X1=",X1(I),"  S=",S(I)
01290 PRINT"     L2=",L2(I),"  L4=",L4(I)
01291 PRINT" "
01292 PRINT" COLD WALL/HOT WALL TEMPERATURE RATIO, T0/T9=",R0(I)
01293 PRINT" HEAT OUT AT COLD WALL=",Q0,"   HEAT IN AT WARM WALL=",Q9
01295 PRINT" ENTROPY PRODUCTION RATE AT COLD WALL=",S8
01297 PRINT" ENTROPY PRODUCTION RATE AT WARM WALL=",(-Q9)
01300 PRINT" MINIMUM ENTROPY PRODUCTION RATE, S0=",S0(I)
01302 PRINT" ENTROPY PRODUCTION RATE FOR",I,"SHIELDS, S2=",S2(I)
01304 PRINT" MAXIMUM ENTROPY PRODUCTION RATE, S9=",S9(I)
01310 PRINT" ENTROPY PRODUCTION RATE RATIOS, S3=S2/S0 AND S4=S9/S0"
01320 PRINT"        S3=",S3(I),"  S4=",S4(I)
01330 NEXT I
01340 GOTO 00270
01350 END
/EOP
```

NEWRAF

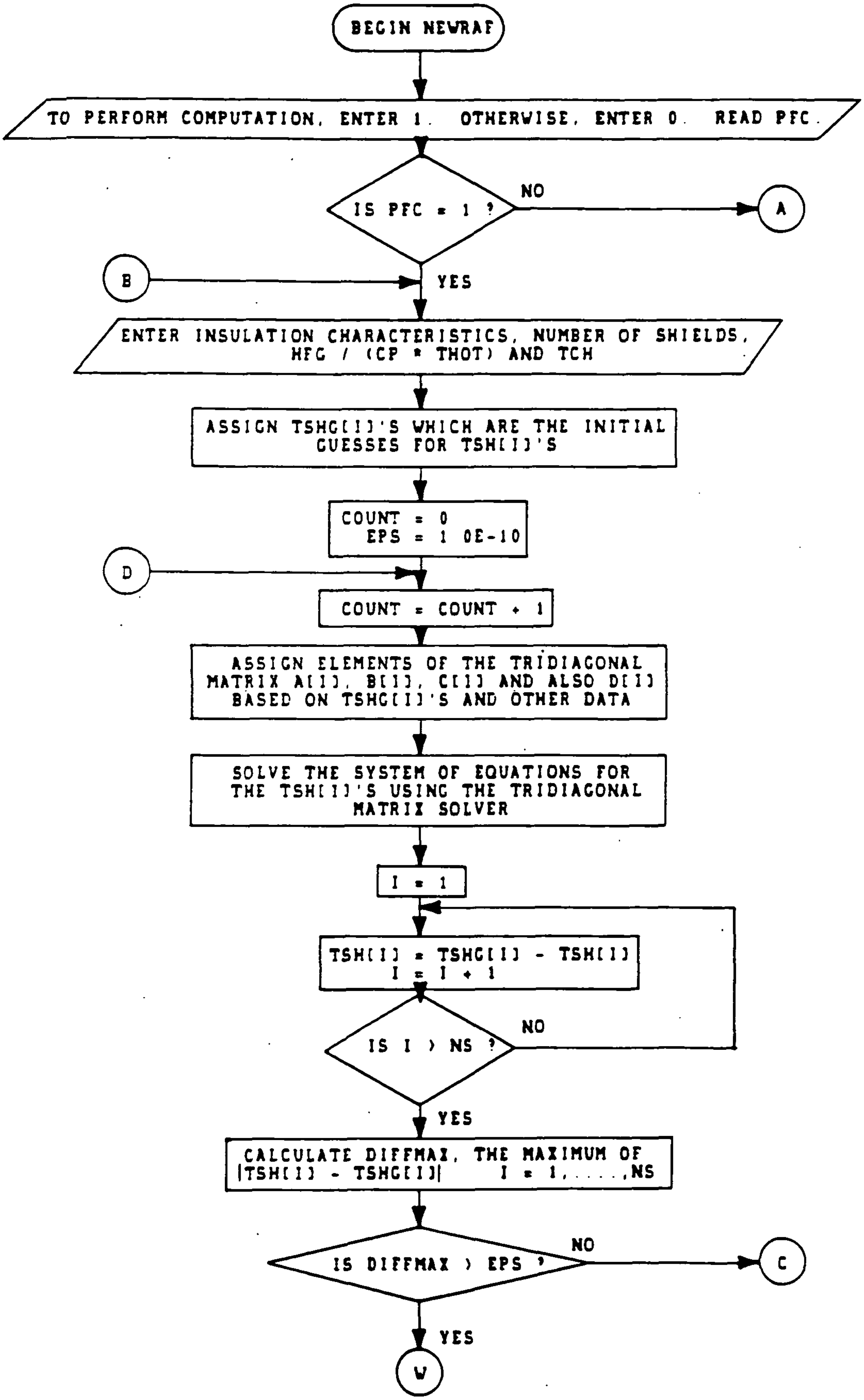
This program solves the original, complete, constrained optimization equations developed in Ref. [9] without the simplifying assumption suggested there which eliminated the dimensionless parameter, h_{fg}/C_pT_H. Only single-term thermal conductivity functions were considered in this analysis.

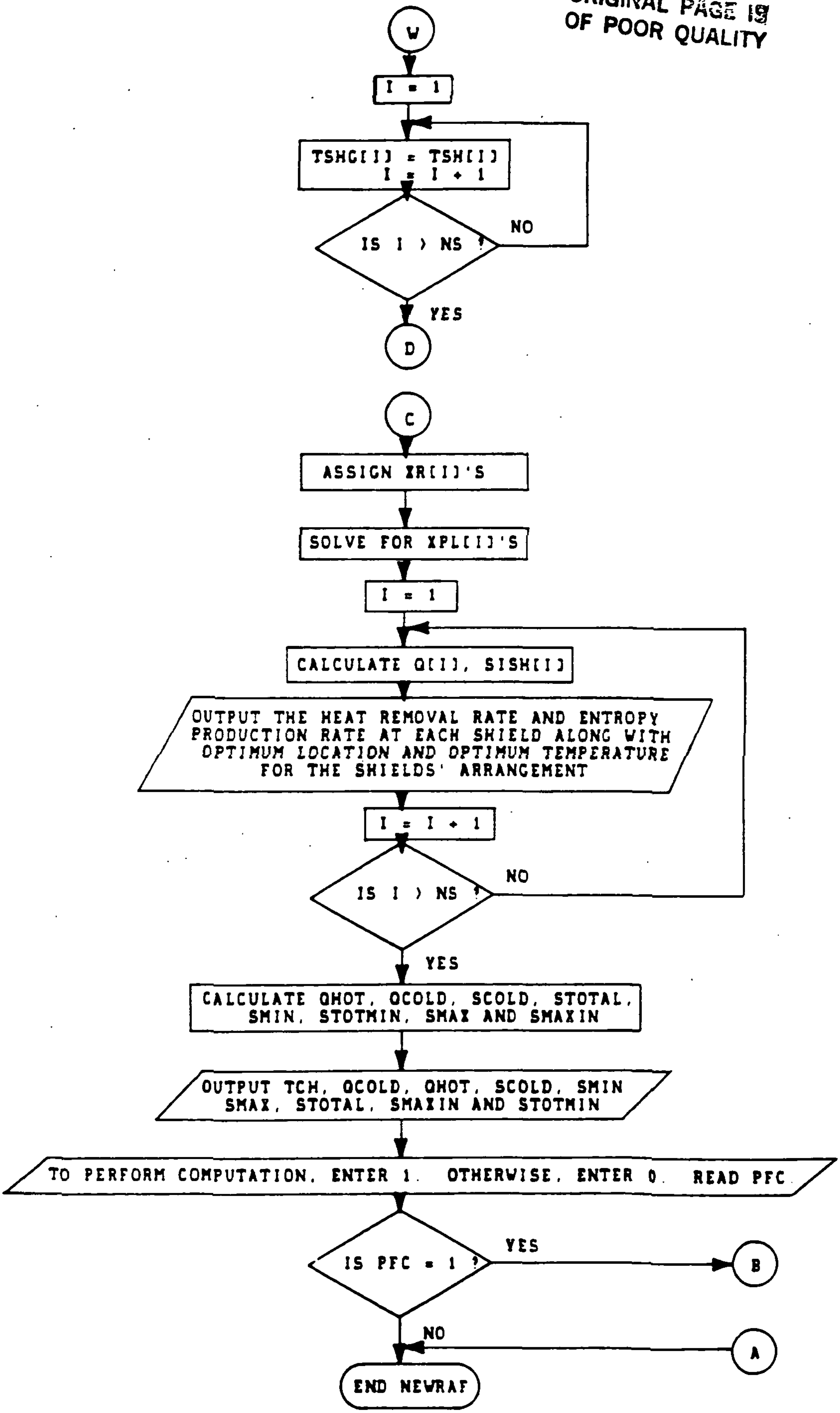
This program also recycles to the starting point. Consequently, the first input is either a 1, if a calculation is to be performed, or a 0, if no more work is to be done.

Next the program requests input of the insulation's characteristics, specifically, the exponent of temperature in the thermal conductivity function, the number of cooled shields, the dimensionless parameter h_{fg}/C_pT_H for the boiloff from the insulated container, and $R = T_C/T_H$.

The output specifies the optimal characteristics of the given number of shields with the constraint that the cooling capacity is limited to the boiloff of the liquid due only to the heat leak through the insulation itself.

The flow chart and a program sample follows.


BEGIN NEWRAF
TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0. READ PFC.
IS PFC = 1 ?
NO
A
B
YES
ENTER INSULATION CHARACTERISTICS, NUMBER OF SHIELDS, HFG / (CP * THOT) AND TCH
ASSIGN TSHG[I]'S WHICH ARE THE INITIAL GUESSES FOR TSH[I]'S
COUNT = 0
EPS = 1.0E-10
D
COUNT = COUNT + 1
ASSIGN ELEMENTS OF THE TRIDIAGONAL MATRIX A[I], B[I], C[I] AND ALSO D[I] BASED ON TSHG[I]'S AND OTHER DATA
SOLVE THE SYSTEM OF EQUATIONS FOR THE TSH[I]'S USING THE TRIDIAGONAL MATRIX SOLVER
I = 1
TSH[I] = TSHG[I] - TSH[I]
I = I + 1
IS I > NS ?
NO
YES
CALCULATE DIFFMAX, THE MAXIMUM OF |TSH[I] - TSHG[I]| I = 1,....,NS
IS DIFFMAX > EPS ?
NO
C
YES
W


W
I = 1
TSHC[I] = TSH[I]
I = I + 1
IS I > NS ?
NO
YES
D
C
ASSIGN XR[I]'S
SOLVE FOR XPL[I]'S
I = 1
CALCULATE Q[I], SISH[I]
OUTPUT THE HEAT REMOVAL RATE AND ENTROPY PRODUCTION RATE AT EACH SHIELD ALONG WITH OPTIMUM LOCATION AND OPTIMUM TEMPERATURE FOR THE SHIELDS' ARRANGEMENT
I = I + 1
IS I > NS ?
NO
YES
CALCULATE QHOT, QCOLD, SCOLD, STOTAL, SMIN, STOTMIN, SMAX AND SMAXIN
OUTPUT TCH, QCOLD, QHOT, SCOLD, SMIN SMAX, STOTAL, SMAXIN AND STOTMIN
TO PERFORM COMPUTATION, ENTER 1. OTHERWISE, ENTER 0. READ PFC
IS PFC = 1 ?
YES
B
NO
A
END NEWRAF

```
PROGRAM NEWRAPH(INPUT/,OUTPUT,AKM);



(********************************************************************************)
(********************************************************************************)
(********************************************************************************)
(********************************************************************************)
(*********************************                    ***************************)
(*********************************  -----------)  NEWRAF  (-----------  *********)
(*********************************                    ***************************)
(*********************************     J. C. CHATO  &  J. M. KHODADADI    ********)
(********************************* DEPT  OF MECHANICAL & INDUSTRIAL ENGRG. ******)
(********************************* UNIV  OF ILLINOIS AT URBANA-CHAMPAIGN  ********)
(*********************************         1206 W  GREEN STREET          ********)
(*********************************         URBANA, IL   61801            ********)
(*********************************                    ***************************)
(*********************************             JULY  1983                ********)
(*********************************                    ***************************)
(********************************************************************************)
(********************************************************************************)
(********************************************************************************)
(********************************************************************************)
                              (******************************************************)
                              (*                                                    *)
                              (*   THIS PASCAL PROGRAM WAS DEVELOPED TO OPTIMIZE THE *)
                              (*   LOCATION, TEMPERATURE AND HEAT DISSIPATION RATE  *)
                              (*   OF EACH COOLED SHIELD INSIDE AN INSULATION LAYER *)
                              (*   THE THERMAL CONDUCTIVITY OF THE INSULATION HAS   *)
                              (*   THE GENERAL FORM,                                *)
                              (*                                                    *)
                              (*                  K = K1*(T**M)                     *)
                              (*                                                    *)
                              (*   THE OBJECTIVE HAS BEEN TO SOLVE THE SET OF 2*NS+1 *)
                              (*   NON-LINEAR EQUATIONS OBTAINED BY BEJAN, A. "DIS- *)
                              (*   CRETE COOLING OF LOW HEAT LEAK SUPPORTS TO 4.2 K," *)
                              (*   CRYOGENICS, VOL  15, 1975, PP.290-292.           *)
                              (*                                                    *)
                              (*   SOLUTION IS BASED ON THE NEWTON-RAPHSON TECHNIQUE *)
                              (*   DISCUSSED BY STOECKER, W. F., DESIGN OF THERMAL  *)
                              (*   SYSTEMS,2ND  EDITION, SECTION 6-11, PP  117-119,  *)
                              (*   MCGRAW-HILL BOOK CO., NY, 1980                   *)
                              (*                                                    *)
                              (******************************************************)
                              (******************************************************)


LABEL    100;

TYPE
        ARRAYS=ARRAY[1..10] OF REAL;      (*  THE SIZE OF ARRAYS DETERMINES THE MAXIMUM NUMBER OF SHIELDS  *)
        ARRAYP=ARRAY[1..11] OF REAL;      (*  THE SIZE OF ARRAYP IS NS+1  *)
        ARRAYT=ARRAY[1..20] OF REAL;      (*  THE SIZE OF ARRAYT SHOULD BE TWICE THE NUMBER OF SHIELDS  *)

VAR

        A                  : ARRAYS;      (*  LOWER-DIAGONAL ELEMENTS OF THE TRIDIAGONAL MATRIX  *)
        B                  : ARRAYS;      (*  DIAGONAL ELEMENTS OF THE TRIDIAGONAL MATRIX  *)
        C                  : ARRAYS;      (*  UPPER-DIAGONAL ELEMENTS OF THE TRIDIAGONAL MATRIX  *)
        D                  : ARRAYS;      (*  RIGHT-HAND SIDE OF THE SET OF EQUATIONS DURING ITERATIONS  *)
        Q                  : ARRAYS;      (*  I-TH DIMENSIONLESS HEAT REMOVAL RATE  *)
        QL                 : ARRAYP;      (*  DIMENSIONLESS HEAT TRANSFER BETWEEN SHIELDS  *)
        S                  : ARRAYS;      (*  DIMENSIONLESS ENTROPY PRODUCTION RATE FOR I-TH LAYER  *)
        SMAX               : REAL;        (*  MAXIMUM DIMENSIONLESS ENTROPY PRODUCTION RATE  *)
        SMIN               : REAL;        (*  MINIMUM DIMENSIONLESS ENTROPY PRODUCTION RATE  *)
        SMAXMIN            : REAL;        (*  SMAX / SMIN  *)
        STOTAL             : REAL;        (*  TOTAL DIMENSIONLESS ENTROPY PRODUCTION RATE  *)
        STOTMIN            : REAL;        (*  STOTAL / SMIN  *)
        SISH               : ARRAYS;      (*  I-TH DIMENSIONLESS ENTROPY PRODUCTION RATE  *)
        TSH                : ARRAYS;      (*  I-TH SHIELD / HOT WALL TEMPERATURE RATIO, ALWAYS < 1  *)
        TSHG               : ARRAYS;      (*  GUESSED I-TH SHIELD / HOT WALL TEMPERATURE RATIO, ALWAYS < 1  *)
        WORK               : ARRAYT;      (*  DUMMY VARIABLES  *)
        X                  : ARRAYP;      (*  SPACING BETWEEN NEIGHBORING SHIELDS / INSULATION THICKNESS  *)
        XPL                : ARRAYS;      (*  DISTANCE FROM COLD WALL / INSULATION THICKNESS  *)
        XR                 : ARRAYS;      (*  X[I] / X[I-1])  *)


        AKM                : TEXT;        (*  OUTPUT FILE TO BE USED IF DESIRED  *)
        BETA               : REAL;        (*  PARAMETER DEFINED IN PROCEDURE INPUTH  *)
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
          BOLD              : REAL;          (* DUMMY VARIABLE USED IN SOLVING THE TRIDIAGONAL MATRIX  *)
          COUNT             : INTEGER;       (* NUMBER OF ITERATIONS NEEDED TO DETERMINE TSH[I]'S  *)
          DELTATG,DEN       : REAL;          (* DUMMY VARIABLES  *)
          DIFF,DIFFMAX      : REAL;          (* DUMMY VARIABLES USED IN CHECKING CONVERGENCE  *)
          DIV,DMAX,DMIN     : REAL;          (* DUMMY VARIABLES USED IN SOLVING THE TRIDIAGONAL MATRIX  *)
          EPS               : REAL;          (* A SMALL VALUE USED TO OBSERVE IF CONVERGENCE IS OBTAINED  *)
          GI,GIM1,GIP1      : REAL;          (* DUMMY VARIABLES  *)
          GOLD              : REAL;          (* DUMMY VARIABLE USED IN SOLVING THE TRIDIAGONAL MATRIX  *)
          I,J               : INTEGER;       (* INDICES FOR LOOPS  *)
          ITERIN            : INTEGER;       (* INDEX USED TO TERMINATE ITERATIONS  *)
          M                 : REAL;          (* POWER OF THE THERMAL CONDUCTIVITY EQUATION  *)
          MM1               : REAL;          (* EQUALS M-1  *)
          MP1               : REAL;          (* EQUALS M+1  *)
          NS                : INTEGER;       (* NUMBER OF SHIELDS  *)
          NSP1              : INTEGER;       (* EQUALS NS+1  *)
          PFC               : INTEGER;       (* PROGRAM FLOW CONTROLLER  *)
          TCH               : REAL;          (* COLD WALL / HOT WALL TEMPERATURE RATIO, ALWAYS < 1  *)
          TI,TIM1           : REAL;          (* DUMMY VARIABLES  *)
          QCOLD             : REAL;          (* HEAT OUT AT COLD WALL  *)
          QHOT              : REAL;          (* HEAT IN AT HOT WALL  *)
          SCOLD             : REAL;          (* ENTROPY PRODUCTION RATE AT COLD WALL  *)
          XTOTAL            : REAL;          (* SUM OF X[I]'S; SHOULD EQUAL 1 AFTER SUCCESSFUL COMPUTATION  *)




   PROCEDURE INPUTH;
    BEGIN               (*  INPUT OF DATA HEADING  *)
     WRITELN;
     WRITELN('         ENTER    ---->    M    NS    BETA    TCH    <----');
     WRITELN('   ');
     WRITELN('         WHERE    M ----- POWER IN THE THERMAL CONDUCTIVITY EQUATION');
     WRITELN('                  NS ---- NUMBER OF SHIELDS');
     WRITELN('                  BETA -- HFG / (CP*THOT)');
     WRITELN('                           HFG --- HEAT OF VAPORIZATION [J/KG]');
     WRITELN('                           CP ---- SPECIFIC HEAT AT CONSTANT PRESSURE [J/KG K]');
     WRITELN('                           THOT -- HOT WALL TEMPERATURE [K]');
     WRITELN('                  TCH --- COLD WALL / HOT WALL TEMPERATURE RATIO, ALWAYS < 1');
     WRITELN('   ')
    END;                (*  INPUT OF DATA HEADING  *)



   PROCEDURE PFCH;
    BEGIN               (*  PFCH  *)
     WRITELN;
     WRITELN('         TO PERFORM COMPUTATION, ENTER 1.  OTHERWISE, ENTER 0 ');
     WRITELN
    END;                (*  PFCH  *)



   PROCEDURE SINGLESPACE;
    BEGIN               (*  SINGLE SPACE IN OUTPUT  *)
     WRITELN('   ')
    END;                (*  SINGLE SPACE IN OUTPUT  *)



   FUNCTION PWR(XX,E:REAL):REAL;
    VAR
          A                 : REAL;
    BEGIN               (*  COMPUTE XX**E  *)
     A:=E*LN(XX);
     PWR:=EXP(A)
    END;                (*  COMPUTE XX**E  *)



   FUNCTION MAXOF2(NO1,NO2:REAL):REAL;
    BEGIN              (*  DETERMINES THE LARGEST OF THE TWO GIVEN NUMBERS  *)
     IF NO1<>NO2 THEN
      IF NO1>NO2 THEN
       MAXOF2:=NO1
      ELSE
       MAXOF2:=NO2
     ELSE
      MAXOF2:=NO1
    END;               (*  DETERMINES THE LARGEST OF THE TWO GIVEN NUMBERS  *)

```

```


FUNCTION MINOF2(NO1,NO2.REAL):REAL;
 BEGIN              (*  DETERMINES THE SMALLEST OF THE TWO GIVEN NUMBERS  *)
  IF NO1()NO2 THEN
   IF NO1)NO2 THEN
    MINOF2.=NO2
   ELSE
    MINOF2 =NO1
  ELSE
   MINOF2.=NO1
 END,               (*  DETERMINES THE SMALLEST OF THE TWO GIVEN NUMBERS  *)








               (*  MAIN PROGRAM BODY  *)

BEGIN
 PECH.
 READLN.
 READ(PFC),
 WHILE PFC=1 DO
 BEGIN

               (*  THIS BLOCK IS USED TO INPUT THE INSULATION THERMAL CONDUCTIVITY, NUMBER  *)
               (*  OF SHIELDS, HFG/(CP*THOT) AND COLD WALL / HOT WALL TEMPERATURE RATIO     *)

  INPUTH
  READLN
  READ(M.NS.BETA.TCH);
  SINGLESPACE.
  WRITELN('        THERMAL CONDUCTIVITY OF THE INSULATION IS K = K1*T**',M 3 1),
  WRITELN                                  HFG / (CP*THOT) = ',BETA.9 5),
  SINGLESPACE
  SINGLESPACE.


  MP1 =M+1 0,
  MM1 =M-1 0.

               (*  INITIAL GUESSED VALUES FOR TSH[I]'S ARE ENTERED  *)

  DELTATG =(1 0-TCH)/(NS+1 0),
  FOR J =1 TO NS DO TSHG[J] =J*DELTATG+TCH,

               (*  VARIABLE USED TO CHECK CONVERGENCE CRITERION IS SET AND THE ITERATIVE PROCEDURE  *)
               (*  OF NEWTON-RAPHSON METHOD IS STARTED  *)

  EPS =1 0E-10,
  COUNT =0
  ITERIN =0,
  REPEAT
   COUNT =COUNT+1,
   FOR I =1 TO NS DO
    BEGIN
     GI =TSHG[I].
     IF NS()1 THEN
      IF I()1 THEN
       IF I()NS THEN
        BEGIN
         GIM1 =TSHG[I-1],
         GIP1 =TSHG[I+1]
        END
       ELSE
        BEGIN
         GIM1 =TSHG[I-1],
         GIP1 =1 0
        END
      ELSE
       BEGIN
        GIM1 =TCH.
        GIP1 =TSHG[I+1]
       END
     ELSE
      BEGIN
       GIM1 =TCH.
```

```
      CIP1 =1.0
     END,

           (*  ELEMENTS OF THE TRIDIAGONAL MATRIX ARE COMPUTED  *)

     A[I] =PWR(CIP1,MP1)-PWR(CI,M)*(-M*CI+MP1*(TCH-BETA));
     B[I] =MP1*PWR(CI,MM1)*(M*CIM1*(BETA-TCH+CI)+CI*(-2.0*(BETA-TCH))),
     B[I] =B[I]+MP1*PWR(CI,MM1)*CI*(-M*(BETA-TCH)-CI*(M+2.0)),
     C[I] =MP1*PWR(CIP1,M)*(BETA-TCH+CIM1),
     D[I] =CIM1*(PWR(CIP1,MP1)-PWR(CI,MP1)+BETA*MP1*PWR(CI,M)-TCH*MP1*PWR(CI,M)+MP1*PWR(CI,MP1)),
     D[I] =D[I]+CI*(PWR(CI,M)*(TCH-BETA)-BETA*MP1*PWR(CI,M)+TCH*MP1*PWR(CI,M)-MP1*PWR(CI,MP1));
     D[I] =D[I]+CIP1*(BETA*PWR(CIP1,M)-TCH*PWR(CIP1,M));
    END,
   A[1] =0.0,
   C[NS] =0.0,

           (*  THE TRIDIAGONAL MATRIX SOLVER IS SHOWN IN THIS BLOCK   *)
           (*  SEE WESTLAKE, J. R., A HANDBOOK OF NUMERICAL MATRIX    *)
           (*  INVERSION AND SOLUTION OF LINEAR EQUATIONS, SECTION    *)
           (*  2.7, PP. 34-35, JOHN WILEY & SONS, INC., NY, 1968      *)

   IF B[1]=0.0 THEN GOTO 100,
   BOLD =C[1]/B[1]
   GOLD =D[1]/B[1],
   WORK[1] =GOLD
   WORK[NS+1] =BOLD,
   DMAX =ABS(B[1]),
   DMIN =ABS(B[1]),
   FOR I =2 TO NS DO
    BEGIN
     DIV =B[I]-A[I]*BOLD,
     IF DIV=0.0 THEN GOTO 100,
     DMAX =MAXOF2(DMAX,ABS(DIV)),
     DMIN =MINOF2(DMIN,ABS(DIV)),
     GOLD =(D[I]-A[I]*GOLD)/DIV,
     WORK[I] =GOLD,
     BOLD =C[I]/DIV
     WORK[I+NS] =BOLD
    END
   TSH[NS] =GOLD
   J =NS
   FOR I =2 TO NS DO
    BEGIN
     J =J-1,
     GOLD =WORK[J]-WORK[J+NS]*GOLD,
     TSH[J] =GOLD
    END,

           (*  NEWLY CALCULATED VALUES OF TSH[I]'S ARE COMPUTED  *)

   FOR I =1 TO NS DO TSH[I] =TSHG[I]-TSH[I],

           (*  CONVERGENCE IS CHECKED. IF THE CRITERION IS SATISFIED, THE ITERATION IS  *)
           (*  TERMINATED, OTHERWISE THE NEWLY CALCULATED TSH[I]'S ARE USED AS NEW      *)
           (*  GUESSES FOR ANOTHER ROUND OF ITERATION                                   *)

   DIFFMAX =1.0E-15,
   FOR I =1 TO NS DO
    BEGIN
     DIFF =ABS(TSH[I]-TSHG[I]),
     DIFFMAX =MAXOF2(DIFF,DIFFMAX)
    END,
   IF DIFFMAX<=EPS THEN
    ITERIN =1
   ELSE
    FOR I =1 TO NS DO TSHG[I] =TSH[I],
  UNTIL ITERIN=1,

           (*  IN THIS BLOCK QUANTITIES USED IN DETERMINING THE SHIELDS' SPACINGS ARE COMPUTED  *)

  FOR I =1 TO NS DO
   BEGIN
    TI =TSH[I],
    IF NS<>1 THEN
     IF I<>1 THEN
      IF I<>NS THEN
       TIP1 =TSH[I-1]
      ELSE
       TIM1 =TSH[I-1]
     ELSE
```

```
    TIM1:=TCH
   ELSE
    TIM1:=TCH;
   XR[I]:=(MP1*PWR(TI,M)*(TI-TIM1))/(PWR(TI,MP1)-PWR(TIM1,MP1));
  END;
 DEN:=1.0;
 FOR I:=1 TO NS DO DEN:=DEN*XR[NS-I+1]+1.0;
 NSP1:=NS+1;

          (* FINALLY, SPACINGS BETWEEN SHIELDS AND OTHER QUANTITIES OF INTEREST ARE CALCULATED  *)

 X[1]:=1.0/DEN;
 XPL[1]:=X[1];
 XTOTAL:=X[1];
 FOR I:=2 TO NSP1 DO
  BEGIN
   X[I]:=X[I-1]*XR[I-1];
   IF I<>NSP1 THEN XPL[I]:=XPL[I-1]+X[I];
   XTOTAL:=XTOTAL+X[I]
  END;
 IF (ABS(XTOTAL-1.0) > 1.0E-5) THEN GOTO 100;
 QL[1]:=(PWR(TSH[1],MP1)-PWR(TCH,MP1))/(X[1]*MP1);
 QL[NSP1]:=(1.0-PWR(TSH[NS],MP1))/(X[NSP1]*MP1);
 FOR I:=2 TO NS DO QL[I]:=(PWR(TSH[I],MP1)-PWR(TSH[I-1],MP1))/(X[I]*MP1);
 SINGLESPACE;
 WRITELN('          NUMBER OF SHIELDS    = ',NS:2);
 WRITELN('          NUMBER OF ITERATIONS = ',COUNT:2);
 SINGLESPACE;
 SINGLESPACE;
 WRITELN('                            HEAT REMOVAL     ENTROPY PRODUCTION     OPTIMUM       OPTIMUM');
 WRITELN('                                RATE                RATE           LOCATION    TEMPERATURE');
 WRITELN('                            ------------     ------------------     --------    -----------');
 SINGLESPACE;
 FOR I:=1 TO NS DO
  BEGIN
   Q[I]:=QL[I+1]-QL[I];
   SISH[I]:=Q[I]/TSH[I];
   WRITELN('          SHIELD ',I:2,'   ',Q[I]:9:5,' ':11,SISH[I]:9:5,' ':9,XPL[I]:9:5,' ':5,TSH[I]:9:5)
  END;
 SINGLESPACE;
 SINGLESPACE;
 QHOT:=QL[NSP1];
 QCOLD:=QL[1];
 SCOLD:=QCOLD/TCH;
 STOTAL:=SCOLD-QHOT;
 FOR J:=1 TO NS DO STOTAL:=STOTAL+SISH[J];
 IF M<>0.0 THEN
  SMIN:=SQR((1.0-PWR(TCH,(M/2.0)))/(M/2.0))
 ELSE
  SMIN:=SQR(LN(1.0/TCH));
 STOTMIN:=STOTAL/SMIN;
 SMAX:=(1.0-PWR(TCH,MP1))*(1.0/TCH-1.0)/MP1;
 SMAXIN:=SMAX/SMIN;
 SINGLESPACE;
 WRITELN('          COLD WALL / HOT WALL TEMPERATURE RATIO     = ',TCH:14:6);
 WRITELN('          HEAT OUT AT COLD WALL                      = ',QCOLD:14:6);
 WRITELN('          HEAT IN AT HOT WALL                        = ',QHOT:14:6);
 WRITELN('          ENTROPY PRODUCTION RATE AT COLD WALL       = ',SCOLD:14:6);
 WRITELN('          ENTROPY PRODUCTION RATE AT HOT WALL        = ',-QHOT:14:6);
 WRITELN('          MINIMUM ENTROPY PRODUCTION RATE            = ',SMIN:14:6);
 WRITELN('          MAXIMUM ENTROPY PRODUCTION RATE            = ',SMAX:14:6);
 WRITELN('          TOTAL ENTROPY PROD. RATE WITH ',NS:2,' SHIELDS    = ',STOTAL:14:6);
 WRITELN('          MAXIMUM / MINIMUM ENTROPY PRODUCTION RATIO = ',SMAXIN:14:6);
 WRITELN('          TOTAL / MINIMUM ENTROPY PRODUCTION RATIO   = ',STOTMIN:14:6);
 100 SINGLESPACE;
 IF (DIV=0.0) OR (B[1]=0.0) THEN
  BEGIN
   SINGLESPACE;
   WRITELN('          --->  CHECK THE ASSEMBLY OF COEFFICIENTS TO BE USED IN TRIDIAGONAL MATRIX   <---');
   WRITELN('          --->                   CHECK THE TRIDIAGONAL MATRIX SOLVER                  <---')
  END;
 IF (ABS(XTOTAL-1.0) > 1.0E-5) THEN
  BEGIN
   SINGLESPACE;
   WRITELN('          --->   XTOTAL IS NOT EQUAL TO 1.0    <---');
   WRITELN('          --->  COMPUTATIONS ARE NOT CORRECT   <---')
  END;
 SINGLESPACE;
 SINGLESPACE;
```

```
           PFCH,
           READLN.
           READ(PFC)
         END
       END
   /EOP.
```

```
        TO PERFORM COMPUTATION, ENTER 1.  OTHERWISE, ENTER 0.

 ? 1

        ENTER     ---->    M    NS    BETA    TCH    <----

        WHERE:    M ----- POWER IN THE THERMAL CONDUCTIVITY EQUATION
                  NS ---- NUMBER OF SHIELDS
                  BETA -- HFG / (CP*THOT)
                            HFG --- HEAT OF VAPORIZATION [J/KG]
                            CP ---- SPECIFIC HEAT AT CONSTANT PRESSURE [J/KG K]
                            THOT -- HOT WALL TEMPERATURE [K]
                  TCH --- COLD WALL / HOT WALL TEMPERATURE RATIO, ALWAYS < 1

 ?      1.0      3      0.0145    0.001

        THERMAL CONDUCTIVITY OF THE INSULATION IS K = K1*T**1.0
                                        HFG / (CP*THOT) =    0.01450


        NUMBER OF SHIELDS     =  3
        NUMBER OF ITERATIONS =  9


                     HEAT REMOVAL      ENTROPY PRODUCTION      OPTIMUM       OPTIMUM
                        RATE                 RATE              LOCATION     TEMPERATURE
                     ------------      ------------------      --------     -----------

        SHIELD  1      0.10438              1.56143             0.09719       0.06685
        SHIELD  2      0.25983              1.12595             0.28870       0.23076
        SHIELD  3      0.47781              0.89782             0.58568       0.53219


        COLD WALL / HOT WALL TEMPERATURE RATIO      =        0.001000
        HEAT OUT AT COLD WALL                       =        0.022985
        HEAT IN AT HOT WALL                         =        0.864998
        ENTROPY PRODUCTION RATE AT COLD WALL        =       22.984544
        ENTROPY PRODUCTION RATE AT HOT WALL         =       -0.864998
        MINIMUM ENTROPY PRODUCTION RATE             =        3.751018
        MAXIMUM ENTROPY PRODUCTION RATE             =      499.499501
        TOTAL ENTROPY PROD. RATE WITH  3 SHIELDS    =       25.704743
        MAXIMUM / MINIMUM ENTROPY PRODUCTION RATIO =      133.163725
        TOTAL / MINIMUM ENTROPY PRODUCTION RATIO    =        6.852738


        TO PERFORM COMPUTATION, ENTER 1.  OTHERWISE, ENTER 0.

 ? 1

        ENTER     ---->    M    NS    BETA    TCH    <----

        WHERE:    M ----- POWER IN THE THERMAL CONDUCTIVITY EQUATION
                  NS ---- NUMBER OF SHIELDS
                  BETA -- HFG / (CP*THOT)
                            HFG --- HEAT OF VAPORIZATION [J/KG]
                            CP ---- SPECIFIC HEAT AT CONSTANT PRESSURE [J/KG K]
                            THOT -- HOT WALL TEMPERATURE [K]
                  TCH --- COLD WALL / HOT WALL TEMPERATURE RATIO, ALWAYS < 1

 ?       1.0     2    0.0154     0.000806
```

```
        THERMAL CONDUCTIVITY OF THE INSULATION IS K = K1*T**1.0
                                    HFG / (CP*THOT) =   0.01540


        NUMBER OF SHIELDS     =  2
        NUMBER OF ITERATIONS =  8

                     HEAT REMOVAL     ENTROPY PRODUCTION      OPTIMUM        OPTIMUM
                         RATE                RATE             LOCATION     TEMPERATURE
                     ------------     ------------------      --------     -----------

        SHIELD  1      0.19732              1.97595            0.16252       0.09986
        SHIELD  2      0.59037              1.48999            0.48495       0.39623


        COLD WALL / HOT WALL TEMPERATURE RATIO     =        0.000806
        HEAT OUT AT COLD WALL                      =        0.030677
        HEAT IN AT HOT WALL                        =        0.818366
        ENTROPY PRODUCTION RATE AT COLD WALL       =       38.061092
        ENTROPY PRODUCTION RATE AT HOT WALL        =       -0.818366
        MINIMUM ENTROPY PRODUCTION RATE            =        3.776103
        MAXIMUM ENTROPY PRODUCTION RATE            =      619.846992
        TOTAL ENTROPY PROD. RATE WITH  2 SHIELDS   =       40.708665
        MAXIMUM / MINIMUM ENTROPY PRODUCTION RATIO =      164.149921
        TOTAL / MINIMUM ENTROPY PRODUCTION RATIO   =       10.780603


        TO PERFORM COMPUTATION, ENTER 1.  OTHERWISE, ENTER 0.

? 0
    0.072 CP SECS,  11471B CM USED.
/BYE

3KMUFTC  COSTS:    255.028 SRUS AT $.0059 = $1.50
```

DESINS

This program optimizes the characteristics of a single cooled shield with different insulations on the two sides. Only one-term thermal conductivity functions are considered.

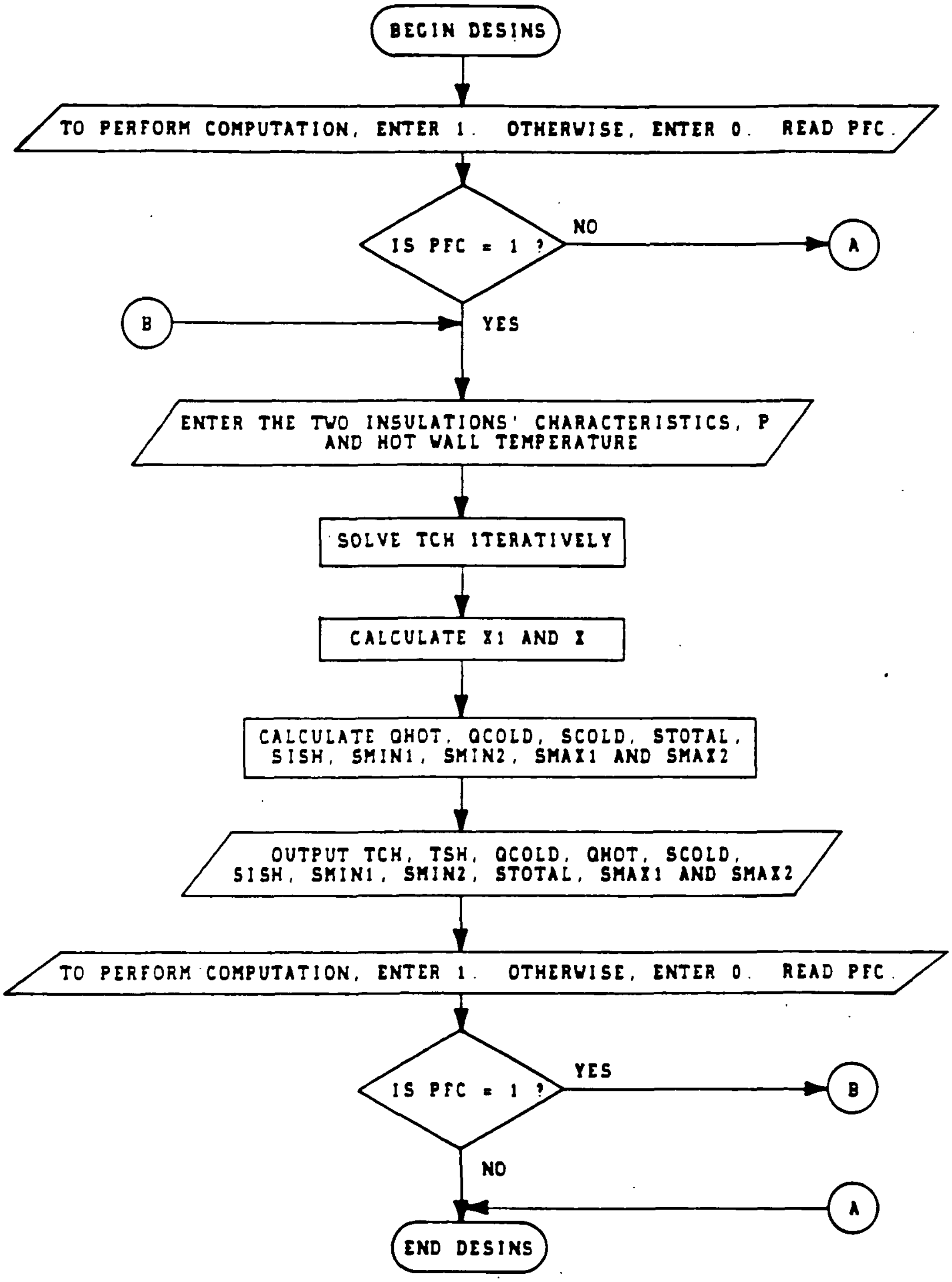
This program also recycles to the starting point; thus the first input is 1, if a calculation is to be performed, or 0 if no more work is to be done.

Next inputs are the characteristics of the two insulations, specifically, the exponents of temperature in the thermal conductivity functions on the hot and cold sides of the shield, a coefficient ratio ALFA (defined in the program), the shield to cold wall temperature ratio, $P = T_S/T_C$, and the hot wall temperature, T_H.

The output specifies the optimal characteristics of the cooled shield as well as other, related information.

The flow diagram and a program sample follows.

ORIGINAL PAGE IS
OF POOR QUALITY



ORIGINAL PAGE IS
OF POOR QUALITY

```
PROGRAM DIFFCOND(INPUT/,OUTPUT,SKM);



(***************************************************************************************************************)
(***************************************************************************************************************)
(***************************************************************************************************************)
(***************************************************************************************************************)
(**********************************                                           **********************************)
(**********************************   -----------)  DESINS  (-----------      **********************************)
(**********************************                                           **********************************)
(**********************************      J  C. CHATO  &  J. M  KHODADADI      **********************************)
(********************************** DEPT  OF MECHANICAL & INDUSTRIAL ENGRG    **********************************)
(**********************************  UNIV  OF ILLINOIS AT URBANA-CHAMPAIGN    **********************************)
(**********************************          1206 W  GREEN STREET             **********************************)
(**********************************           URBANA, IL   61801              **********************************)
(**********************************                                           **********************************)
(**********************************               JULY  1983                  **********************************)
(**********************************                                           **********************************)
(***************************************************************************************************************)
(***************************************************************************************************************)
(***************************************************************************************************************)
(***************************************************************************************************************)
                           (********************************************************)
                           (*                                                      *)
                           (*   THIS PASCAL PROGRAM WAS DEVELOPED TO OPTIMIZE THE  *)
                           (*   LOCATION, TEMPERATURE AND HEAT DISSIPATION RATE    *)
                           (*   FOR A COOLED SHIELD IN A CRYOGENIC INSULATION      *)
                           (*   SYSTEM WHOSE THERMAL CONDUCTIVITY HAS THE FORM,    *)
                           (*                                                      *)
                           (*              K = K1*(T**M)     ON THE HOT SIDE,      *)
                           (*              K = K2*(T**N)     ON THE COLD SIDE      *)
                           (*                                                      *)
                           (*   THE METHOD IS BASED ON THE MINIMIZATION OF THE     *)
                           (*   ENTROPY PRODUCTION RATE WHICH IS PROPORTIONAL TO   *)
                           (*   THE HEAT LEAK ACROSS THE INSULATION.               *)
                           (*                                                      *)
                           (********************************************************)
                           (********************************************************)


LABEL    100;
LABEL    200;
LABEL    300;

VAR


        P                  : REAL;             (*  SHIELD / COLD WALL TEMPERATURE RATIO, ALWAYS > 1  *)
        SMAX1              : REAL;             (*  MAXIMUM ENTROPY PRODUCTION RATE BASED ON K1*T**M  *)
        SMAX2              : REAL;             (*  MAXIMUM ENTROPY PRODUCTION RATE BASED ON K2*T**N  *)
        SMIN1              : REAL;             (*  MINIMUM ENTROPY PRODUCTION RATE BASED ON K1*T**M  *)
        SMIN2              : REAL;             (*  MINIMUM ENTROPY PRODUCTION RATE BASED ON K2*T**N  *)
        STOTAL             : REAL;             (*  TOTAL DIMENSIONLESS ENTROPY PRODUCTION RATE  *)
        S1SH               : REAL;             (*  DIMENSIONLESS ENTROPY PRODUCTION RATE AT SHIELD  *)
        TCH                : REAL;             (*  COLD WALL / HOT WALL TEMPERATURE RATIO, ALWAYS < 1  *)
        TSH                : REAL;             (*  SHIELD / HOT WALL TEMPERATURE RATIO, ALWAYS < 1  *)
        X                  : REAL;             (*  DISTANCE FROM COLD WALL / THICKNESS RATIO  *)
        X1                 : REAL;             (*  X / (1.0-X)  *)


        CC                 : REAL;             (*  DUMMY VARIABLE  *)
        COUNT              : INTEGER;          (*  NUMBER OF ITERATIONS NEEDED TO DETERMINE TCH  *)
        DD                 : REAL;             (*  DUMMY VARIABLE  *)
        ALFA               : REAL;             (*  [K2*(N+1)]/[K1*(M+1)]  *)
        G,G1               : REAL;             (*  DUMMY VARIABLES  *)
        IND                : INTEGER;          (*  INDEX TO TERMINATE THE SEARCH FOR TCH  *)
        M                  : REAL;             (*  POWER OF THE THERMAL CONDUCTIVITY ON HOT SIDE  *)
        MP1                : REAL;             (*  EQUALS M+1  *)
        N                  : REAL;             (*  POWER OF THE THERMAL CONDUCTIVITY ON COLD SIDE  *)
        NP1                : REAL;             (*  EQUALS N+1  *)
        PFC                : INTEGER;          (*  PROGRAM FLOW CONTROLLER  *)
        QCOLD              : REAL;             (*  HEAT OUT AT COLD WALL  *)
        QHOT               : REAL;             (*  HEAT IN AT HOT WALL  *)
        SCOLD              : REAL;             (*  ENTROPY PRODUCTION RATE AT COLD WALL  *)
        SKM                : TEXT;             (*  OUTPUT FILE TO BE USED IF DESIRED  *)
        THOT               : REAL;             (*  HOT WALL TEMPERATURE [K]  *)
```

```


PROCEDURE INPUTH;
 BEGIN                 (*  INPUT OF DATA HEADING  *)
  WRITELN;
  WRITELN('         ENTER    ----)    M    N    ALFA    P    THOT    (----');
  WRITELN('  '),
  WRITELN('         WHERE.   M ----- POWER OF THE THERMAL CONDUCTIVITY EQUATION ON THE HOT SIDE'),
  WRITELN('                  N ----- POWER OF THE THERMAL CONDUCTIVITY EQUATION ON THE COLD SIDE'),
  WRITELN('                  ALFA -- [K2*(N+1)]/[K1*(M+1)]'),
  WRITELN('                  P ----- SHIELD / COLD WALL TEMPERATURE RATIO, ALWAYS > 1'),
  WRITELN('                  THOT -- HOT WALL TEMPERATURE (K)'),
  WRITELN('  ')
 END,                  (*  INPUT OF DATA HEADING  *)



PROCEDURE PFCH,
 BEGIN                 (*  PFCH  *)
  WRITELN,
  WRITELN('         TO PERFORM COMPUTATION, ENTER 1   OTHERWISE, ENTER 0 '),
  WRITELN
 END.                  (*  PFCH  *)



PROCEDURE SINGLESPACE.
 BEGIN                 (*  SINGLE SPACE IN OUTPUT  *)
  WRITELN('  ')
 END.                  (*  SINGLE SPACE IN OUTPUT  *)



FUNCTION PWR(XX,E REAL) REAL,
 VAR
       A                   REAL.
 BEGIN                 (*  COMPUTE XX**E  *)
  A =E*LN(XX).
  PWR =EXP(A)
 END.                  (*  COMPUTE XX**E  *)



FUNCTION D(E,XX REAL) REAL,
 BEGIN                 (*  FUNCTIONAL D  *)
  D =(E+1 0)*PWR(XX,E)-E/(PWR(XX,(1 0-E)))-(1 0/SQR(XX))
 END.                  (*  FUNCTIONAL D  *)



FUNCTION F(E,XX REAL) REAL,
 BEGIN                 (*  FUNCTIONAL F  *)
  F =(PWR(XX,(E+1 0)))-PWR(XX,E)-1 0+(1 0/XX)
 END.                  (*  FUNCTIONAL F  *)







           *  MAIN PROGRAM BODY  *)

BEGIN
 PFCH.
 READLN,
 READ(PFC),
 WHILE PFC=1 DO
 BEGIN

          (*  THIS BLOCK IS USED TO INPUT THE TWO INSULATION THERMAL CONDUCTIVITIES,  *)
          (*  SHIELD / COLD WALL TEMPERATURE RATIO AND HOT WALL TEMPERATURE           *)

  INPUTH.
  READLN.
  READ(M,N,ALFA,P,THOT),
  SINGLESPACE,
  WRITELN('         THERMAL CONDUCTIVITY OF THE INSULATION ON THE HOT SIDE IS K  = K1*(T**',M 3 1,').');
  WRITELN('         THERMAL CONDUCTIVITY OF THE INSULATION ON THE COLD SIDE IS K = K2*(T**',N 3 1,').'),
  WRITELN('                                               [K2*(N+1)]/[K1*(M+1)] = ',ALFA 9 2);
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
WRITELN('                                    HOT WALL TEMPERATURE = ',THOT:9:2,' [K]');
SINGLESPACE,
SINGLESPACE,


MP1 .=M+1.0;
NP1 .=N+1.0,
  TCH =0 000001,
  CC.=0.1;.
  DD =1 0,
  COUNT =0,

          (*  THIS BLOCK CALCULATES TCH ITERATIVELY  *)

  REPEAT
   TSH =P*TCH.
   G =D(N,P)*D(N,P)/F(N,P)-PWR(TCH,(2.0-N))*D(M,TSH)*D(M,TSH)/F(M,TSH)/ALFA,
   G1 =G*DD.
   IF G1(0.0  THEN GOTO 100;
   IF G1=0 0  THEN GOTO 200,
   CC =(-0 1)*CC;
   IF ABS(CC)(0.000001  THEN GOTO 200,
   DD =-DD.
   100 TCH =TCH+CC,
   IF (TCH:0 999999) OR (TCH(0 000001) THEN
    BEGIN
     TCH =TCH-0 9*CC,
     CC =0 1*CC,
     IF ABS(CC)(0 000001 THEN IND =1
    END.
   200 COUNT =COUNT+1,
  UNTIL (G1=0 0) OR (ABS(CC)(0 000001) OR (IND=1),


  IF IND=1 THEN
   BEGIN
    SINGLESPACE.
    SINGLESPACE.
    SINGLESPACE.
    WRITELN('        ---)        OPTIMUM CRITERION CANNOT BE SATISFIED       (--- '),
    WRITELN('        ---) USE SINGLE INSULATION WITH THE LOWER CONDUCTIVITY (---'),
    GOTO 300
   END

          (*  OTHER QUANTITIES OF INTEREST ARE COMPUTED IN THIS SECTION  *)

  X1 =-ALFA*PWR(TCH,(N-1 0))*D(N,P)/D(M,TSH).
  X =X1/(1 0+X1),
  QHOT =(1.0-PWR(TSH,MP1))/(((1.0-X)*MP1);
  QCOLD =ALFA*PWR(TCH,NP1)*(PWR(P,NP1)-1.0)/(PWR(THOT,(M-N))*X*MP1),
  SCOLD =QCOLD/TCH.
  STOTAL =(F(M,TSH)/(1 0-X)+ALFA*PWR(TCH,N)*F(N,P)/X)/MP1,
  SISH = (QHOT-QCOLD)/TSH,
  IF M=0 0 THEN
    SMIN1 =SQR(LN(1 0/TCH))
  ELSE
    SMIN1 =SQR((1 0-PWR(TCH,(M/2.0)))/(M/2.0)),
  IF N=0 0 THEN
    SMIN2 =SQR(LN(1 0/TCH))
  ELSE
    SMIN2 =SQR((1 0-PWR(TCH,(N/2 0)))/(N/2 0)),
  SMAX1 =F(M,TCH)/MP1,
  SMAX2 =F(N,TCH)/NP1,


  SINGLESPACE,
  WRITELN('         NUMBER OF ITERATIONS                            = ',COUNT 8);
  WRITELN('         COLD WALL / HOT WALL TEMPERATURE RATIO          = ',TCH 14 6),
  WRITELN('         SHIELD / HOT WALL TEMPERATURE RATIO             = ',TSH 14.6),
  WRITELN('         SHIELD LOCATION                                 = ',X:14 6),
  WRITELN('         HEAT OUT AT SHIELD                              = ',QHOT-QCOLD 14.6),
  WRITELN('         HEAT OUT AT COLD WALL                           = ',QCOLD 14 6),
  WRITELN('         HEAT IN AT HOT WALL                             = ',QHOT 14:6),
  WRITELN('         ENTROPY PRODUCTION RATE AT COLD WALL            = ',SCOLD 14 6),
  WRITELN('         ENTROPY PRODUCTION RATE AT HOT WALL             = ',-QHOT 14 6),
  WRITELN('         ENTROPY PRODUCTION RATE AT SHIELD               = ',SISH 14 6),
  WRITELN('         MINIMUM ENTROPY PRODUCTION RATE BASED ON K1*T**M = ',SMIN1 14 6),
  WRITELN('         MINIMUM ENTROPY PRODUCTION RATE BASED ON K2*T**N = ',SMIN2 14 6),
  WRITELN('         TOTAL ENTROPY PRODUCTION RATE                   = ',STOTAL 14 6),
  WRITELN('         ENTROPY PROD. W/O SHIELD BASED ON K1*T**M       = ',SMAX1 14 6),
```

```
        WRITELN('        ENTROPY PROD  W/O SHIELD BASED ON K2*T**N          = ',SMAX2 14 6),
        300 SINGLESPACE,
        SINGLESPACE,
        SINGLESPACE,
      PFCH,
      READLN,
      READ(PFC)
     END
    END
```

```
        TO PERFORM COMPUTATION, ENTER 1.  OTHERWISE, ENTER 0.

? 1

        ENTER     ---->     M    N    ALFA    P    THOT    <----

        WHERE:    M ----- POWER OF THE THERMAL CONDUCTIVITY EQUATION ON THE HOT SIDE
                  N ----- POWER OF THE THERMAL CONDUCTIVITY EQUATION ON THE COLD SIDE
                  ALFA -- [K2*(M+1)]/[K1*(N+1)]
                  P ----- SHIELD / COLD WALL TEMPERATURE RATIO, ALWAYS > 1
                  THOT -- HOT WALL TEMPERATURE [K]

?     1.0   0.0    20.0    4.5    300.0

        THERMAL CONDUCTIVITY OF THE INSULATION ON THE HOT SIDE IS K  = K1*(T**1.0).
        THERMAL CONDUCTIVITY OF THE INSULATION ON THE COLD SIDE IS K = K2*(T**0.0).
                                                 [K2*(M+1)]/[K1*(N+1)] =     20.00
                                                  HOT WALL TEMPERATURE =    300.00 [K]


        NUMBER OF ITERATIONS                                 =        36
        COLD WALL / HOT WALL TEMPERATURE RATIO               =        0.001666
        SHIELD / HOT WALL TEMPERATURE RATIO                  =        0.007497
        SHIELD LOCATION                                      =        0.390755
        HEAT OUT AT SHIELD                                   =        0.820144
        HEAT OUT AT COLD WALL                                =        0.000497
        HEAT IN AT HOT WALL                                  =        0.820641
        ENTROPY PRODUCTION RATE AT COLD WALL                 =        0.298568
        ENTROPY PRODUCTION RATE AT HOT WALL                  =       -0.820641
        ENTROPY PRODUCTION RATE AT SHIELD                    =      109.396253
        MINIMUM ENTROPY PRODUCTION RATE BASED ON K1*T**M =        3.680131
        MINIMUM ENTROPY PRODUCTION RATE BASED ON K2*T**N =       40.925828
        TOTAL ENTROPY PRODUCTION RATE                        =      178.307751
        ENTROPY PROD. W/O SHIELD BASED ON K1*T**M            =      299.619216
        ENTROPY PROD. W/O SHIELD BASED ON K2*T**N            =      598.241762


        TO PERFORM COMPUTATION, ENTER 1.  OTHERWISE, ENTER 0.

? 0
     0.044 CP SECS,   10233B CM USED.
```

1. Report No.
166583

2. Government Accession No.

3. Recipient's Catalog No.

4. Title and Subtitle
Optimation of Cooled Shields in Insulations

5. Report Date
April 1984

6. Performing Organization Code

7. Author(s)
J.C.Chato, J.M.Khodadadi and J.Seyed-Yagoobi

8. Performing Organization Report No.
UILU ENG-84-4004

9. Performing Organization Name and Address
Department of Mechanical and Industrial Enrineering
University of Illinois at Urbana-Champaign
Urbana. IL 61801

10. Work Unit No.
T6590

11. Contract or Grant No.
NAG2-219

12. Sponsoring Agency Name and Address
National Aeronautics and Space Administration
Washington, D.C. 20546

13. Type of Report and Period Covered
Final Report

14. Sponsoring Agency Code
506-54-21

15. Supplementary Notes
Point of Contact: Technical Monitor, Peter Kittel, MS 244-7
Ames Research Center, Moffett Field, CA 94035
(415) 965-6525 or FTS 448-6525

16. Abstract

A relatively simple method has been developed to optimize the location, temperature, and heat dissipation rate of each cooled shield inside an insulation layer. The method is based on the minimization of the entropy production rate which is proportional to the heat leak across the insulation. The results show that the maximum number of shields to be used in most practical applications is three. However, cooled shields are useful only at low values of the overall, cold wall to hot wall absolute temperature ratio. The performance of the insulation system is relatively insensitive to deviations from the optimum values of the temperature and location of the cooling shields.

Design curves are presented for rapid estimates of the locations and temperatures of cooling shields in various types of insulations, and an equation is given for calculating the cooling loads for the shields.

17. Key Words (Suggested by Author(s))
Vapor Cooled Shields
Insulation
Dewars

18. Distribution Statement
Unclassified-Unlimited

STAR Category 34

19. Security Classif. (of this report)
Unclassified

20. Security Classif. (of this page)
Unclassified

21. No. of Pages
87

22. Price*

*For sale by the National Technical Information Service, Springfield, Virginia 22161

NTIS does not permit return of items for credit or refund. A replacement will be provided if an error is made in filling your order, if the item was received in damaged condition, or if the item is defective.

Reproduced by NTIS

National Technical Information Service
Springfield, VA 22161

This report was printed specifically for your order from nearly 3 million titles available in our collection.

For economy and efficiency, NTIS does not maintain stock of its vast collection of technical reports. Rather, most documents are printed for each order. Documents that are not in electronic format are reproduced from master archival copies and are the best possible reproductions available. If you have any questions concerning this document or any order you have placed with NTIS, please call our Customer Service Department at (703) 487-4660.

About NTIS

NTIS collects scientific, technical, engineering, and business related information — then organizes, maintains, and disseminates that information in a variety of formats — from microfiche to online services. The NTIS collection of nearly 3 million titles includes reports describing research conducted or sponsored by federal agencies and their contractors; statistical and business information; U.S. military publications; audiovisual products; computer software and electronic databases developed by federal agencies; training tools; and technical reports prepared by research organizations worldwide. Approximately 100,000 *new* titles are added and indexed into the NTIS collection annually.

For more information about NTIS products and services, call NTIS at (703) 487-4650 and request the free *NTIS Catalog of Products and Services*, PR-827LPG, or visit the NTIS Web site **http://www.ntis.gov**.

NTIS
Your indispensable resource for government-sponsored information—U.S. and worldwide