N84-28087

NASA Technical Memorandum 83721

Parameter Studies of Gear Cooling Using an Automatic Finite Element Mesh Generator

Lotfi E. El-Bayoumy and Lee S. Akin
California State University at Long Beach
Long Beach, California

and

Dennis P. Townsend
Lewis Research Center
Cleveland, Ohio

Prepared for the
Fourth International Power Transmission and Gearing Conference
sponsored by the American Society of Mechanical Engineers
Cambridge, Massachusetts, October 10–12, 1984

NASA

PARAMETER STUDIES OF GEAR COOLING USING AN AUTOMATIC FINITE ELEMENT MESH GENERATOR

Lotfi E. El-Bayoumy
California State University at Long Beach
Long Beach, California

Lee S. Akin
California State University at Long Beach
Long Beach, California

and

Dennis P. Townsend
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

SUMMARY

This work has investigated the range of accuracies achieved in the gear tooth temperature using an automatic finite element mesh generator. Gear web contribution to the gear cooling process is studied by introducing a varying size hole at the center of the gear because of the versatility of program TARG in allowing different heat transfer coefficients in different areas of the gear tooth. A study is carried out to evaluate the contribution of the loaded and unloaded faces as well as the top and bottom lands.

A general purpose two-dimensional finite element preprocessor ATOGEN has been developed for automatic generation of a finite element mesh over a pie-shaped sector of a gear. The program is used for facilitating the input to an upgraded version of a previously developed program for the thermal analysis of running gears (TARG). The latter program determined the steady state temperature distribution throughout the specified gear. The automatic mesh generator program includes a band width minimization routine for reducing computer cost.

E-2187

INTRODUCTION

The problem of predicting gear tooth temperature by means of a finite element technique has been recently treated by several authors (refs. 1 to 3). The finite element mesh used in these references, though applicable for any gear tooth proportions, including nonstandard teeth, was limited to a fixed number of elements along the tooth profile. Eight (8) divisions along the tooth appeared to be adequate at the time for predicting gear tooth temperature with reasonable accuracy. Because of the sharp temperature gradients observed in the results, it became necessary to develop a finite element preprocessor capable of descretizing the tooth into any required mesh. In addition to the customary element connections and nodal coordinate lists that an all purpose preprocessor would yield, boundary nodes on loaded and unloaded sides as well as top and bottom lands would be accordingly identified.

Parameters which are pertinent to the gear geometry such as all nodes on the involute profiles, radii of curvatures at the same, and adjacent nodes to the pie-shaped boundary constitute necessary output for executing the thermal analysis code. The Program ATOGEN has been developed for this purpose. The program optionally punches an output file for subsequent input to the Thermal Analysis of Running Gears program "TARG" which ultimately determines the steady state temperature distribution throughout the specified gear. Parametric studies are carried out to evaluate the effects on tooth temperatures of the mesh size (number of segments along tooth profile), the significance of gear blank size, and the individual contributions of the tooth boundaries to the convection cooling process.

DESCRIPTION OF PROGRAM ATOGEN

The program is partly based on methods developed in references 4 and 5. The pie-segment of a given gear is initially divided into a crude mesh made up of quadrilateral (four-sided) regions as shown in figure 1. The sides of each individual region are generally curvilinear[1] and a triangular region can be viewed as a quadrilateral by dividing one of the sides into two segments. The corners of the four sided regions in the tooth portion of the model identify significant points on the tooth profile. Starting from the outermost edge of the tooth these are the points on the top land, the pitch points, the lowest points of tooth contact and the points on the bottom land. Intermediate, or mid-side nodes are added on each side of the quadrilateral for achieving the closest approximation of the gear involute profile.

The coordinates of the grid points shown in figure 1 are computed in the program for any gear geometry having a modified involute tooth profile with a given number of teeth, diametral pitch, pressure angle, profile shift coefficient, whole depth, and topping. The x and y coordinates of grid points 22 through 28 are given by

$$\left.\begin{aligned} x_i &= r_i \sin \theta_i \\ y_i &= r_i \cos \theta_i \end{aligned}\right\} \quad i = 22, 23, \ldots, 28 \qquad (1)$$

where

$$r_i = \text{radius at grid point } i$$

$$\theta_i = \text{angle from the y-axis to the radius at } i$$

Using involutometry, the angle θ_i can be written as

---

[1]For simplicity, in the present version of the program, the sides are assumed as quadratic curves.

$$r_i = \text{inv}\ (D) +]/4Pd + Cp\ \tan D/Pd^2 - \text{inv}^{-1}\ (r_i/r_b)^2 - 1 \qquad (2)$$

where

$$\Phi = \text{pressure angle}$$

$$P_d = \text{diametral pitch}$$

$$C_p = \text{pressure shift coefficient}$$

$$r_b = \text{base radius}$$

A conformal mapping of the general quadrilateral in the $x - y$ plane into a square in the $\xi - \eta$ plane is accomplished using transformation

$$x = \sum_{i=1}^{N_r} N_i\ x_i,\ y = \sum_{i=1}^{N_r} N_i\ Y_i \qquad (3)$$

where,

$$N_r = \text{number of nodes around the quadrilateral}$$

For an eight-node quadrilateral, the shape functions N_i are given as (see ref. 6)

$$\left.\begin{aligned}
N_1 &= \frac{1}{4}(1 - \xi)(1 - \eta)(\xi + \eta + 1) \\
N_2 &= \frac{1}{2}(1 - \xi)(1 - \eta) \\
N_3 &= \frac{1}{4}(1 + \xi)(1 - \eta)(\xi - \eta - 1) \\
N_4 &= \frac{1}{2}(1 + \xi)(1 - \eta^2) \\
N_5 &= \frac{1}{4}(1 + \xi)(1 + \eta)(\xi + \eta - 1) \\
N_6 &= \frac{1}{2}(1 - \xi^2)(1 + \eta) \\
N_7 &= \frac{1}{4}(1 - \xi)(1 + \eta)(\eta - \xi - 1) \\
N_8 &= (1 - \xi)(1 - \eta^2)
\end{aligned}\right\} \qquad (4)$$

The pair (x, y) represents the coordinates of node i on the quadrilateral. The transformation is shown schematically in figure 2.

Once the transformation is accomplished, the square region in the $\xi - \eta$ space is divided equally into a number of rows (parallel to the ξ-axis and a number of columns. To avoid duplication of nodes, an equivalencing procedure is undertaken by assigning a connectivity matrix JT(I, J) to each region I with sides J as shown in figure 3. Nodes on adjoining sides are checked for duplicity via JT(I, J) and would acquire their original numbers if duplication is encountered. The node numbering follows a sequence that starts at $\xi = -1$, $\eta = +1$ and proceeds from left to right (ref. 4).

A boundary identification matrix JB(I, J) is assigned for each side J of a region I to associate the element boundaries with the original segments of the overall boundary. The purpose of this identification matrix is to distinguish areas along the boundary according to the convection heat transfer coefficient to be assumed along the exposed portion of the model boundary. Specifically two different heat transfer coefficients are assumed along the loaded side of the tooth one above the pitch points and one below it, one coefficient for the top land, and another one for the unloaded side of the tooth. These heat transfer coefficients are usually derived experimentally or from empirical formulas, (see ref. 1). Figure 4 shows the boundary identification numbers (chosen arbitrarily) for region sides that lie on the boundary. Sides which are not part of a boundary assume a zero JB value.

Each region is then divided into triangular elements by joining the shortest diagonal in the elementary quadrilateral. In addition to providing the capability of changing element size within a given region by assigning the number of rows and columns, it is possible to further refine the mesh by changing the relative locations of regional nodes on the line of symmetry. This is done because solution accuracy of stretched triangular elements is usually poor and one should strive to approach an equilateral triangle. Once the entire region has been descretized and node numbers, individually assigned, a new node renumbering scheme is used to minimize the conductivity matrix bandwidth.

The routine scans for all nodes in the proximity of a given radius, proceeding from left to right. Using this approach it is found that a substantial reduction of the bandwidth is achieved leading to significant savings in computer time.

INFLUENCE OF MESH SIZE ON ACCURACY OF DETERMINED TOOTH TEMPERATURES

The test case evaluated here is the same one used and experimentally verified in reference 1. It consists of two identical eight pitch standard spur gears with 28 teeth each and 20° pressure angle. The face width of each gear is 0.25 in., and the root radius is 1.575 in.

Several meshes were generated using program ATOGEN for subsequent processing through the thermal analysis program TARG. Computer generated plots of the meshes in ascending order of number of segments (or levels) across the tooth are shown in figures 5 and 6. The coarsest mesh is similar to the model used in references 2 and 3 and consists of eight segments across the tooth profile. The finest mesh shown (enlarged for the tooth portion of the model for clarity) has 48 segments. The temperature contours (isotherms) are shown in figure 6 for two extreme meshes of 8 and 48 segments. Although little difference is apparent in the contour shapes of the two meshes, the average temperature could be off by as much as 12 percent when a coarse mesh is used. The trend is illustrated in a plot, (fig. 7), of temperature versus number segments. As expected, there is a number of segments beyond which no improvement in accuracy is discernible. Three temperature norms are chosen for this illustration: namely the maximum, average midplane, and average face temperatures. From the graph it is concluded that 24 segments will yield an accuracy better than 1.0 percent and with 18 segments the error in maximum temperature should not exceed 2.5 percent.

GEAR BLANK CONTRIBUTION TO THE COOLING PROCESS

Because of the option available in program ATOGEN of modelling a gear with an annulus, gear models are generated with various size holes to evaluate gear blank contribution to the cooling process. In addition to the baseline case with no holes, four other configurations are examined. The finite element models for the latter are shown in figure 8. The number of tooth segments used in all models is 18. A plot of the resulting temperatures versus hole size is shown in figure 9. The graph shows that the gear blank has very little influence on tooth temperatures and the heat transfer process is limited to the teeth regions. The steady state heat transfer problem in gearing can therefore be analyzed by considering only the tooth portion of the gear with little or no loss in accuracy.

BOUNDARY COOLING CONTRIBUTIONS

Included in the printout of a program TARG run is a heat balance check which assigns heat loss contributions of the different portions of the model. The convective heat transfer coefficients used in the present case are the same as those noted in reference 1 with nomenclature illustrated in figure 10. The percentage heat loss attributed to the lubricated flank of the tooth is 24.3 percent, that to the unlubricated flank is 43.1 percent, that to the gear transverse sides is 26.0 percent, and 6.6 percent to the top land of the tooth. Unfortunately these allocations are heavily dependent on the numerical value assumed for the heat transfer coefficients and would require experimental verification.

SUMMARY OF RESULTS

A brief description is given of the finite element preprocessor program ATOGEN developed for the automatic generation of finite element meshes of gears. The program is designed to produce input data necessary for running the steady state heat transfer analysis program TARG and includes in its code gears with modified involute profiles. In addition to the punch files, optional plots of the generated mesh can be produced by ATOGEN.

The advantages gained from having a finite element preprocessor are quite numerous. Some of these are self evident such as the time savings over manual preparation of element, nodal, and boundary data. Computer cost can be substantially reduced by judiciously assigning a finer mesh to areas of higher temperature gradient and using a coarse mesh elsewhere. Cost is also minimized through the use of a matrix bandwidth minimizer. These advantages were utilized in the present investigation with the following results obtained:

1. It was found that substantial errors in the predicted tooth temperatures could result from using a coarse mesh, such as the one used in reference 1. For the example shown the error in maximum temperature approached 12 percent. This error could be held to less than about 1 percent by using a mesh with 24 or more segments across the tooth.

2. The amount of heat dissipation through the gear blank was fond to be negligible for the cases studied. The error incurred in the temperatures by considering only the tooth portion of the gear is less than 3 percent.

REFERENCES

1. Townsend, D. P. and Akin, L. S., "Analytical and Experimental Spur Gear Tooth Temperatures Affected by Operating Variables," Journal of Mechanical Design, Vol. 103, No. 4, Jan. 1981, pp. 219-226.

2. Wang, K. L. and Cheng, H. S., "A Numerical Solution to the Dynamic Load, Film Thickness, and Surface Temperatures in Spur Gears, Part II - Results," Journal of Mechanical Design, Vol. 103, No. 4, Jan. 1981, pp. 188-194.

3. Patir, N. and Cheng, H. S., "Prediction of Bulk Temperature in Spur Gears Based on Finite Element Temperature Analysis," ASLE Preprint No. 77-LC-3B-2, Oct. 1977.

4. Segerlind, L. J., Applied Finite Element Analysis, Wiley, New York, 1976.

5. Durocher, L. L., "A Versatile Two-Dimensional Mesh Generator With Automatic Bandwidth Reduction," Computers Structures, Vol. 10, Aug. 1979, pp. 561-575.

6. Zienkiewicz, O. C., The Finite Element Method, Third Edition, McGraw-Hill, New York, 1977, pp. 148-210.

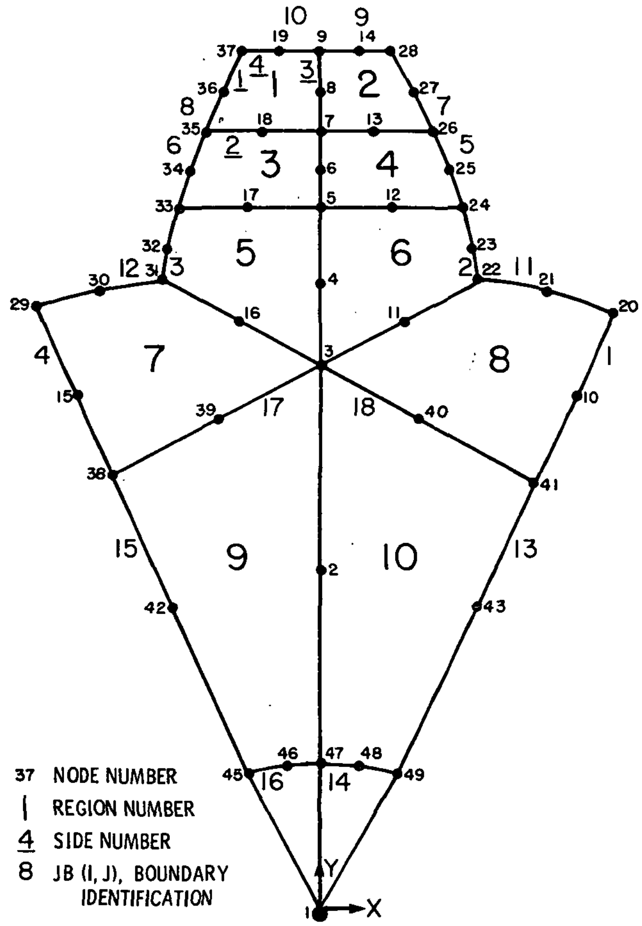


Figure 1. - Regions, sides, nodes and boundary identification.

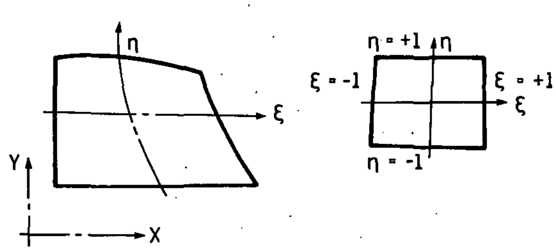


Figure 2. - Transformation coordinates.

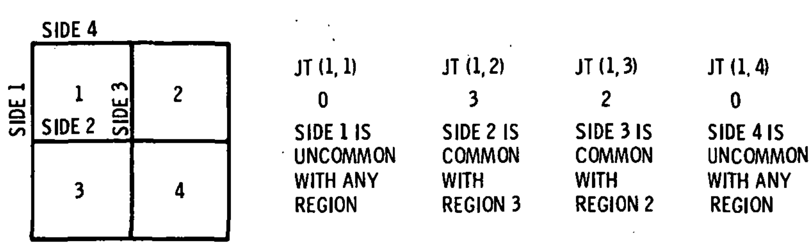


Figure 3. - Regional connectivity matrix layout.

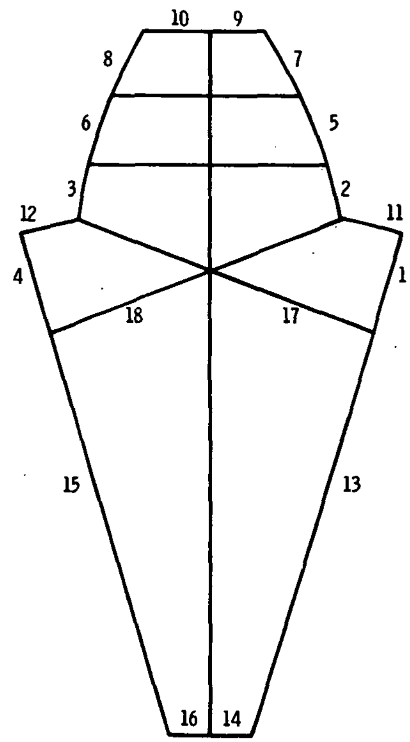


Figure 4. - JB(I, J) boundary identification numbers.

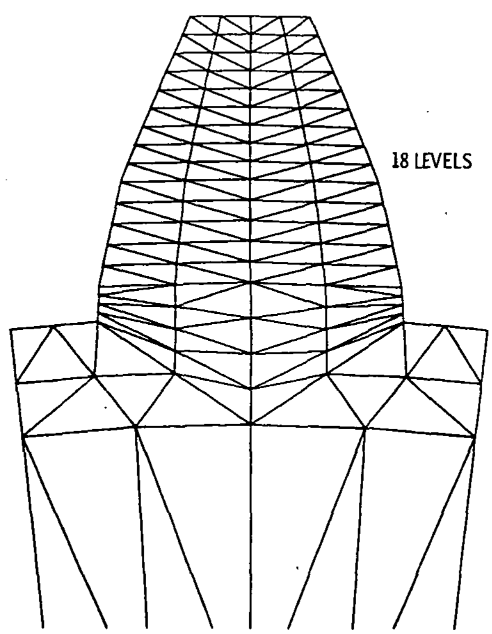
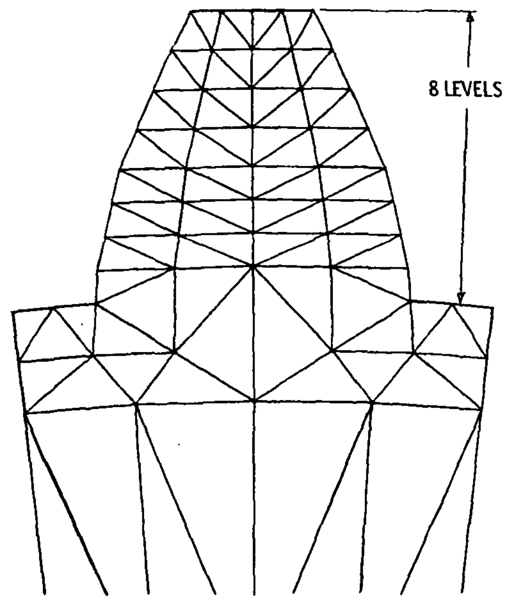
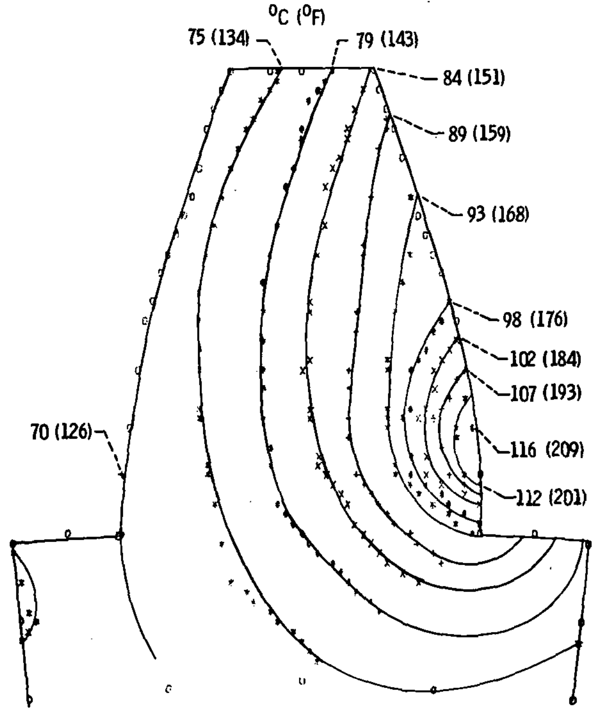


Figure 5. - Computer generated mesh. Enlarged plot (FAC = 3, shift 3000).

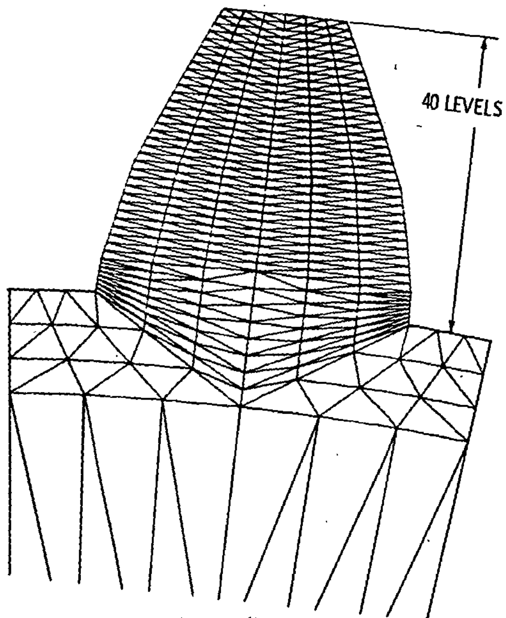


(a) Mesh.

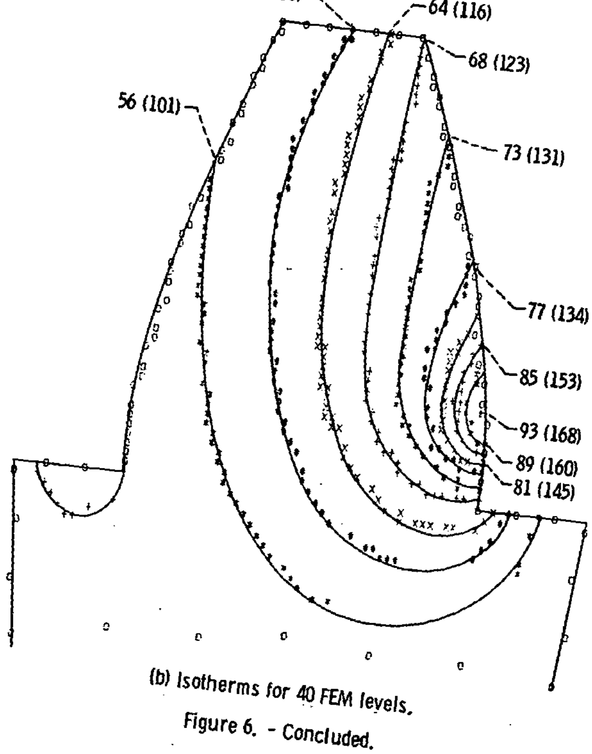


(b) Isotherms for 8 FEM levels.

Figure 6. - Effect of different FEM levels.



(c) Mesh.



(b) Isotherms for 40 FEM levels.

Figure 6. - Concluded.

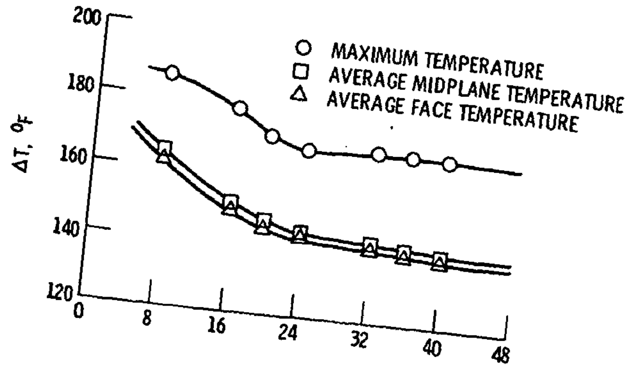


Figure 7. - Number of finite element segments.

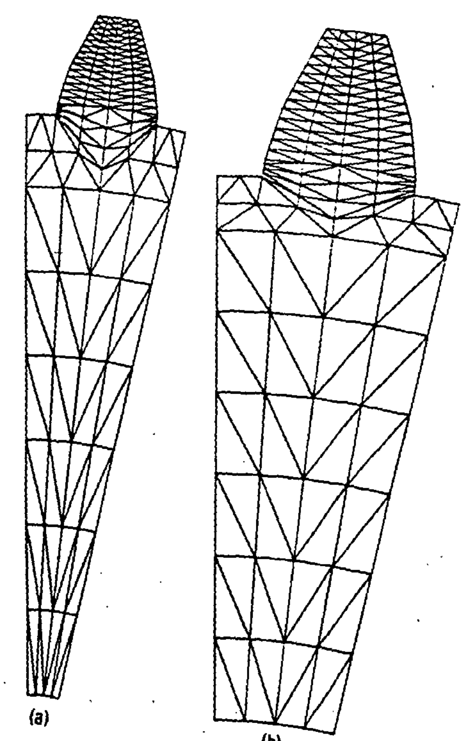
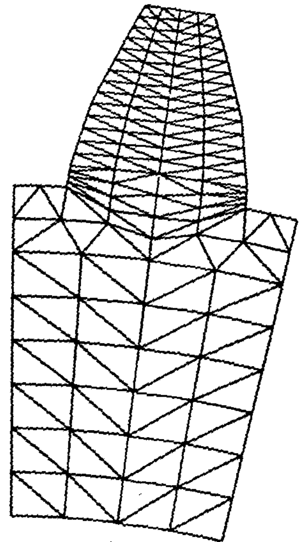
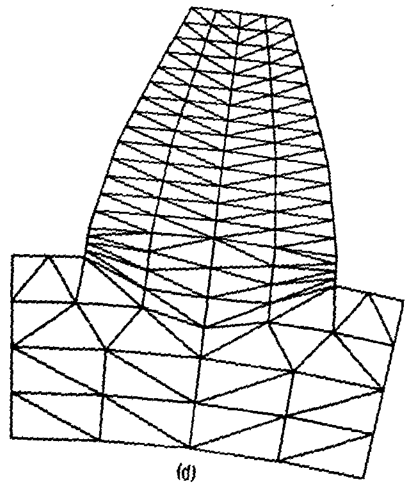


Figure 8. - Gear blank configurations.



(c)



(d)

Figure 8. - Concluded.

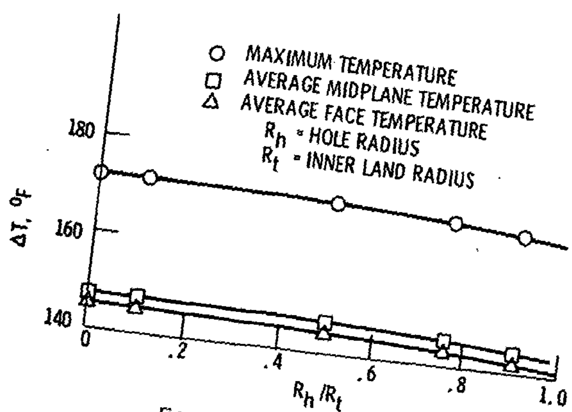


Figure 9. - Effect of center hole size.

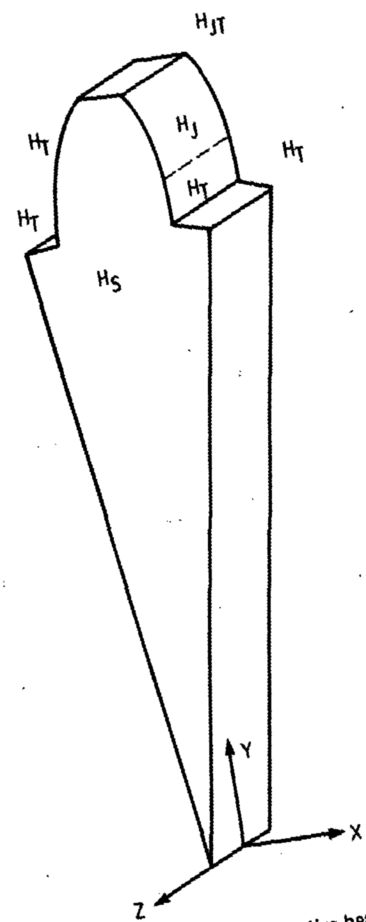


Figure 10. - Convective heat transfer coefficient zones.

1. Report No. NASA TM-83721	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Parameter Studies of Gear Cooling Using an Automatic Finite Element Mesh Generator		5. Report Date 6. Performing Organization Code 505-40-42
7. Author(s) Lotfi E. El-Bayoumy, Lee S. Akin, and Dennis P. Townsend		8. Performing Organization Report No. E-2187 10. Work Unit No.
9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135		11. Contract or Grant No. 13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Lofti E. El-Bayoumy and Lee S. Akin, California State University at Long Beach, Long Beach, California; Dennis P. Townsend, NASA Lewis Research Center. Prepared for the Fourth International Power Transmission and Gearing Conference sponsored by the American Society of Mechanical Engineers, Cambridge, Massachusetts, October 10-12, 1984.

16. Abstract

This work has investigated the range of accuracies achieved in the gear tooth temperature using an automatic finite element mesh generator. Gear web contribution to the gear cooling process is studied by introducing a varying size hole at the center of the gear because of the versatility of program TARG in allowing different heat transfer coefficients in different areas of the gear tooth. A study is carried out to evaluate the contribution of the loaded and unloaded faces as well as the top and bottom lands. A general purpose two-dimensional finite element preprocessor ATOGEN has been developed for automatic generation of a finite element mesh over a pie-shaped sector of a gear. The program is used for facilitating the input to an upgraded version of a previously developed program for the thermal analysis of running gears (TARG). The latter program determined the steady state temperature distribution throughout the specified gear. The automatic mesh generator program includes a band width minimization routine for reducing computer cost.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Gear Finite element Cooling Thermal analysis	Unclassified - unlimited STAR Category 37

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price*
Unclassified	Unclassified		

*For sale by the National Technical Information Service, Springfield, Virginia 22161

National Aeronautics and
Space Administration

Washington, D.C.
20546

Official Business
Penalty for Private Use, $300

SPECIAL FOURTH CLASS MAIL
BOOK

U.S. MAIL

Postage and Fees Paid
National Aeronautics and
Space Administration
NASA-451

NASA

POSTMASTER: If Undeliverable (Section 158
Postal Manual) Do Not Return