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Three-Dimensional Transonic Potential Flow About Complex Three-Dimensional Configurations

Theodore A. Reyhner

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JULY 1984





Three-Dimensional Transonic Potential Flow About Complex Three-Dimensional Configurations

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Prepared under Cooperative Agreement with NASA Langley Research Center and Boeing Commercial Airplane Company



National Aeronautics and Space Administration

Scientific and Technical Information Branch

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1.0 SUMMARY

An analysis has been developed for the solution of the full three-dimensional potential flow equation for subsonic or transonic potential flowfields about arbitrary configurations. This analysis is an extension of an earlier analysis to more complex geometries and to lifting surfaces. Possible configurations include inlets, nacelles, nacelles with ground planes, S-ducts, turboprop nacelles, wings, and wing-pylon-nacelle configurations.

The solution procedure is to use an arbitrary mesh and difference quotients to create a system of nonlinear finite-difference equations. The grids used are Cartesian and cylindrical. The difference equations consist of a very large system of algebraic equations. They are solved iteratively by using the initial guess or the results of the previous iteration to linearize and partly decouple the equations. Successive line over-relaxation (SLOR) is used along horizontal, vertical, or alternating horizontal and vertical lines. A sequence of grids is used in combination with multigrid to improve convergence efficiency.

The analysis has been programed in FORTRAN for the CRAY-1 computer and in extended FORTRAN for the Control Data Corporation Cyber 203 computer. The computer code has been written to obtain maximum performance benefit from the vector operations capability of these computers. Included in this report are descriptions of the input and output files for the computer program.

Comparisons of the analysis results with experimental measurements are presented for several configurations.

2.0 INTRODUCTION

This report is a description of an analysis and computer code for the prediction of three-dimensional transonic potential flow about three-dimensional configurations. The program can handle a variety of geometries including lifting surfaces. Configurations include inlets, nacelles, nacelles with ground planes, S-ducts, turboprop nacelles, wings, and wing-pylon-nacelle combinations. The analysis and computer code are extensions of an earlier work (refs. 1 and 2). Extensions include multigrid for a more efficient solution process, greater geometric flexibility, and a Kutta boundary condition to handle wing or wing-like surfaces.

The basic approach is described in detail in Reference 1. In summary, the analysis solves the full compressible potential equation. Either cylindrical or Cartesian coordinates may be used. The analysis does not use a body-fitted grid, so there are no grid generation problems. The partial differential equations are replaced by finite-difference equations which are solved on a grid using successive line over-relaxation (SLOR). Horizontal lines, vertical lines, or alternating horizontal and vertical lines may be used. A sequence of grids is used to calculate the solution using what is commonly referred to as multigrid (ref. 3).

This document covers the use of multigrid, the logic for handling lifting surfaces, and a guide to using this version of the code. Also included are some examples computed using this version of the code.

3.0 NOMENCLATURE

a	speed of sound
C_1, C_2, C_3	difference quotient coefficients
C_p	coefficient of pressure, $(p/p_{\infty} - 1) / (\frac{1}{2} \gamma M_{\infty}^2)$
D	discrepancy
F, F	right-hand side of equation
I,Ī	interpolation/extrapolation operators
L	differential operator
М	Mach number
n	unit normal to surface oriented into flowfield
$n_x, n_y, n_z, or n_x, n_r, n_{\theta}$	components of n
р	static pressure
q	velocity, $\mathrm{u}^2 + \mathrm{v}^2 + \mathrm{w}^2$, or $\mathrm{u}^2 + \mathrm{u_r}^2 + \mathrm{u_{\dot{ heta}}}^2$
r	radius
r r _{max}	radius r at outer edge of computational cylinder
r r _{max} s	radius r at outer edge of computational cylinder arc length
r r _{max} s u	radius r at outer edge of computational cylinder arc length axial velocity component, $u = \phi_x$
r r _{max} s u u _r	radius r at outer edge of computational cylinder arc length axial velocity component, $u = \phi_x$ radial velocity component, $u_r = \phi_r$
r r _{max} s u u _r	radius r at outer edge of computational cylinder arc length axial velocity component, $u = \phi_x$ radial velocity component, $u_r = \phi_r$ component of velocity along cut of surface
r r_{max} s u u_{r} u_{s} u_{θ}	radiusr at outer edge of computational cylinderarc lengthaxial velocity component, $u = \phi_x$ radial velocity component, $u_r = \phi_r$ component of velocity along cut of surfacecircumferential velocity component, $u_{\theta} = \frac{1}{r} \phi_{\theta}$
r r_{max} s u u_{r} u_{g} u_{θ} v	radius r at outer edge of computational cylinder arc length axial velocity component, $u = \phi_x$ radial velocity component, $u_r = \phi_r$ component of velocity along cut of surface circumferential velocity component, $u_\theta = \frac{1}{r} \phi_\theta$ velocity component in y direction, $v = \phi_y$
r r_{max} s u u_{r} u_{θ} v w	radius r at outer edge of computational cylinder arc length axial velocity component, $u = \phi_x$ radial velocity component, $u_r = \phi_r$ component of velocity along cut of surface circumferential velocity component, $u_\theta = \frac{1}{r} \phi_\theta$ velocity component in y direction, $v = \phi_y$ velocity component in z direction, $w = \phi_z$
r r_{max} s u u_r u_g v_{θ} v w x	radius r at outer edge of computational cylinder arc length axial velocity component, $u = \phi_x$ radial velocity component, $u_r = \phi_r$ component of velocity along cut of surface circumferential velocity component, $u_\theta = \frac{1}{r} \phi_\theta$ velocity component in y direction, $v = \phi_y$ velocity component in z direction, $w = \phi_z$ axial coordinate
r r_{max} s u u_{r} u_{g} u_{θ} v w x y	radius r at outer edge of computational cylinder arc length axial velocity component, $u = \phi_x$ radial velocity component, $u_r = \phi_r$ component of velocity along cut of surface circumferential velocity component, $u_\theta = \frac{1}{r} \phi_\theta$ velocity component in y direction, $v = \phi_y$ velocity component in z direction, $w = \phi_z$ axial coordinate coordinate, y
r r_{max} s u u_r u_g u_{θ} v w x y z	radius r at outer edge of computational cylinder arc length axial velocity component, $u = \phi_x$ radial velocity component, $u_r = \phi_r$ component of velocity along cut of surface circumferential velocity component, $u_\theta = \frac{1}{r} \phi_\theta$ velocity component in y direction, $v = \phi_y$ velocity component in z direction, $w = \phi_z$ axial coordinate coordinate, y coordinate, z

NOMENCLATURE (Concluded)

α	flow angle
γ	ratio of specific heats
Г	$\Delta\phi$ for Kutta boundary condition jump
Δx, Δy,	step size in x, y, etc.
$\Delta\phi$	change in ϕ
θ	circumferential coordinate, arctan (y/z)
ϕ	potential function
$\phi^{ m k}_{ m (n)}$	ϕ at sweep n for grid k
ϕ_{n}	velocity normal to surface
$\phi_{ m s}$	$\partial \phi / \partial \mathbf{s}$, velocity in direction of \mathbf{s}
Φ	exact solution for ϕ
ω _x ,ω _y ,	over-relaxation parameters for x, y, etc.
Subscript	
i	index for x mesh values
j	index for y mesh values
k	index for z mesh values or grid number
A,A'	surface points
S,S_1,S_2	surface points
x,y,z,r,θ,s	partial derivatives
8	freestream
Superscripts	
k	grid number
Special	
$\phi_{\mathbf{z}_{ \mathbf{k} }}$	$\phi_{ m z}$ at point i, j, k
$\phi^{\mathbf{k-1}}_{\mathbf{F}-\mathbf{k}}$	ϕ on grid k-1 obtained by interpolation from grid k

4.0 ANALYSIS

The full partial differential equation for compressible transonic flow expressed in terms of a velocity potential, ϕ , is solved by replacing the partial derivatives with difference quotients. The difference quotients are formed using values of the potential at the discrete nodes formed by the intersection of a network of Cartesian or cylindrical grid lines with each other and the surface. The grid used is not body-fitted. When the partial derivatives are replaced with difference quotients, a nonlinear difference equation is obtained at each grid intersection in the flowfield. The difference equations form a large system of algebraic nonlinear equations which are not practical to solve directly. The solution of the difference equations is obtained by approximating the system of equations by a linear system and solving that system using successive line over-relaxation (SLOR). This process is iterated until convergence. The formulas for the difference quotients and the resulting difference equations are covered in Reference 1. This analysis differs from Reference 1 in that Cartesian coordinates can be used as well as cylindrical coordinates.

The primary difference between this analysis and that of Reference 1 is the way a sequence of grids is used to calculate the solution to the equations on the finest grid. The basic difficulty with using SLOR to solve a large system of difference equations such as these is that the convergence rate is very slow for fine meshes. An estimate or starting guess for the solution can be computed using coarse meshes, but relaxing the starting solution to a final solution on a fine mesh still takes a considerable number of sweeps. This code uses the multigrid technique which uses the coarse grids to correct the fine grid solution. This greatly reduces the number of cycles required on the fine mesh and the overall work required.

The logic for calculating lift about wing-like surfaces is also included in this version of the analysis.

4.1 EQUATION AND BOUNDARY CONDITIONS

The equation to be solved is the complete equation for inviscid, irrotational flow formulated in terms of a velocity potential, ϕ . The Cartesian coordinate system is shown in Figure 1.

4.1.1 POTENTIAL FLOW EQUATION

The equation for the velocity potential in Cartesian coordinates is

$$(a^{2}-\phi_{x}^{2})\phi_{xx} + (a^{2}-\phi_{y}^{2})\phi_{yy} + (a^{2}-\phi_{z}^{2})\phi_{zz} - 2\phi_{x}\phi_{y}\phi_{xy} - 2\phi_{y}\phi_{z}\phi_{yz} - 2\phi_{z}\phi_{x}\phi_{zx} = 0$$
(1)

where ϕ is the velocity potential and the local speed of sound, a, is given by

$$a^{2} = a_{\infty}^{2} - \frac{\gamma - 1}{2} \left(\phi_{x}^{2} + \phi_{y}^{2} + \phi_{z}^{2} - q_{\infty}^{2} \right).$$
⁽²⁾

The velocity components in the flowfield (u, v, w) are obtained from the potential function with the following relationships:

$$u = \phi_x,$$

$$v = \phi_y,$$

$$w = \phi_z,$$
(3)



Figure 1. Airplane Coordinate System

The potential equation in cylindrical coordinates (fig. 2) is

$$(a^{2}-\phi_{x}^{2})\phi_{xx} + (a^{2}-\phi_{r}^{2})\phi_{rr} + \left(a^{2}-\frac{\phi_{\theta}^{2}}{r^{2}}\right)\frac{\phi_{\theta\theta}}{r^{2}} - 2\phi_{x}\phi_{r}\phi_{xr} - 2\phi_{r}\phi_{\theta}\frac{\phi_{r\theta}}{r^{2}} - 2\phi_{\theta}\phi_{x}\frac{\phi_{\thetax}}{r^{2}} + \left(a^{2}+\frac{\phi_{\theta}^{2}}{r^{2}}\right)\frac{\phi_{r}}{r} = 0.$$

$$(4)$$

This is similar to the equation in Cartesian coordinates and the solution technique is the same. The axis points are special and the analysis for the axis is described in Reference 1. The coordinates are related by:

$$y = r \cos \theta$$

$$z = r \sin \theta$$

$$\phi_{r} = v \cos \theta + w \sin \theta$$
(5)

and

$$u_{\mathbf{r}} = \phi_{\mathbf{r}} = \mathbf{v} \cos \theta + \mathbf{w} \sin \theta$$
$$u_{\theta} = \frac{\phi_{\theta}}{\mathbf{r}} = -\mathbf{v} \sin \theta + \mathbf{w} \cos \theta.$$
(6)

4.1.2 BOUNDARY CONDITIONS

The boundary condition at solid surfaces is that the velocity normal to the surface, ϕ_n , equals zero. The boundary condition at the exit of a duct, or at the compressor face of an inlet, is that the axial velocity is fixed at the uniform value that gives a specified mass flow. At the left (inflow) boundary of the computational field, the potential function, ϕ , is specified. At the left boundary,

$$\phi = \mathbf{u}_{\infty}\mathbf{x} + \mathbf{v}_{\infty}\mathbf{y} + \mathbf{w}_{\infty}\mathbf{z} + \text{constant}.$$
 (7)

At the right boundary of the flowfield the outflow velocity, $u=\phi_n$, is specified. At the sides of the flowfield, for Cartesian coordinates, the outflow velocities, v or $w = \phi_n$, are specified. When cylindrical coordinates are used, the outflow velocity is specified on the top half of the cylinder ($\theta < 90 \deg$ and $\theta > 270 \deg$) and equation (7) is used to specify ϕ on the bottom half ($90 \deg \le \theta \le 270 \deg$) (fig. 2).

The choice of boundary conditions is not unique. Whether a velocity normal to the surface or ϕ is specified is a matter of choice except for the compressor face or duct exit. It is not possible to prescribe freestream velocity at the outer boundaries as that would be too many boundary conditions. Specifying ϕ at the boundary enforces the tangential velocity component and specifying ϕ_n enforces the normal component of the velocity. This is satisfactory as long as the boundaries are placed far enough from the object of interest. Five to ten diameters out appears to be satisfactory for inlet and nacelle computations.

4.2 KUTTA BOUNDARY CONDITION

The analysis of flows about lifting surfaces (wings) requires a means to compute a solution with circulation. The magnitude of the circulation is determined by applying the Kutta condition at the trailing edge of the lifting surface. A jump or discontinuity is required in the potential at some point in the flowfield in order to have circulation.



Figure 2. Geometry and Boundary Conditions for Inlet Flowfield Computation in Cylindrical Coordinates

The discontinuity in the potential for this analysis is taken as a cut in the flowfield extending straight back in x from the trailing edge of the lifting surface. At any trailing edge point on the surface a jump in the potential across the cut is computed and that jump value is applied whenever difference quotients are computed using points on both sides of the discontinuity surface. The jump value is held constant downstream of the trailing edge but can vary in the spanwise direction.

The jump in the potential, Γ_j , is determined by applying the Kutta condition, which requires that the trailing edge point be a stagnation point in the flowfield (two-dimensional flow). With the nonbody-fitted mesh, there is, in general, no mesh-surface intersect on the exact trailing edge and, thus, the Kutta condition must be applied in an approximate manner. The criterion used requires no flow around the trailing edge.

Referring to Figure 3 for notation ϕ_{upper} is defined as the potential function ϕ above the cut and ϕ_{lower} as the potential function ϕ below the cut. Then

$$\phi_{\text{upper}} = \phi_{\text{lower}} + \Gamma_{j} \tag{8}$$

where j is the index of the y mesh for the cross-section shown. The velocity of the flow about the trailing edge can be defined using either ϕ_{upper} or ϕ_{lower} . Using ϕ_{upper} , this velocity is defined by

$$\phi_{\rm s|trailing edge} = \frac{\phi_{\rm upper}|_{\rm A} - \phi_{\rm upper}|_{\rm A'}}{\Delta \rm s}.$$
 (9)

The condition that ϕ_s at the trailing edge equals zero gives

$$\phi_{\text{upper}|A} = \phi_{\text{upper}|A'}.$$
(10)

What is computed and stored is ϕ_{upper} above the cut and ϕ_{lower} below the cut. Thus, equation 10 becomes

$$\phi_{\text{upper}|A} = \phi_{\text{lower}|A'} + \Gamma_j. \tag{11}$$

 Γ_{i} is computed by setting

$$\Gamma_{j} = \phi_{|A} - \phi_{|A'} \tag{12}$$

where the subscripts upper and lower have been dropped. $\phi = \phi_{upper}$ above the cut and ϕ_{lower} below the cut.

Once the jump, Γ_j , is computed, it is used in the calculation of any difference quotients representing z derivatives that involve points on both sides of the flowfield discontinuity or cut. Since three-point derivatives are used, it is the difference quotients for k and k-1 that are affected. As an example, if the standard difference quotient for ϕ_z at k is given by

$$\phi_{\mathbf{z}_{|k}} = C_1 \phi_{k+1} + C_2 \phi_k + C_3 \phi_{k-1} , \qquad (13)$$



Figure 3. Flowfield and Mesh About a Trailing Edge, $y = y_j$

.

then when the discontinuity is present between k and k-1,

$$\phi_{z_{|k}} = C_1 \phi_{upper_{|k+1}} + C_2 \phi_{upper_{|k}} + C_3 \phi_{upper_{|k-1}}$$
(14)

is the correct formula except that $\phi_{\rm upper}$ is not what is saved at k-1. However, the formula can be rewritten as

$$\phi_{z|k} = C_1 \phi_{upper|k+1} + C_2 \phi_{upper|k} + C_3 \left(\phi_{lower|k-1} + \Gamma_j \right)$$
(15)

which can be again rewritten

$$\phi_{z_{|k}} = C_1 \phi_{k+1} + C_2 \phi_k + C_3 \phi_{k-1} + C_3 \Gamma_j.$$
(16)

Thus, the jump creates a correction $C_3\Gamma_j$ to the usual formula for the difference quotient. It can be shown that

$$\phi_{z_{|k-1}} = C_1 \phi_k + C_2 \phi_{k-1} + C_3 \phi_{k-2} - C_1 \Gamma_j$$
(17)

where for this equation C_1 , C_2 , and C_3 are the appropriate coefficients at k-1.

4.3 MULTIGRID

The multigrid procedure is an algorithm for solving the finite-difference equations. It is used in conjunction with the successive line over-relaxation (SLOR) technique to accelerate (in terms of number of sweeps and quantity of computational work) the convergence of the solution to the finite-difference equations. Theoretically, for some problems, very great reductions in computational work are possible.

In simplest terms, multigrid allows corrections to a solution being computed on a fine mesh to be computed using a coarser mesh. Computations on a coarser mesh are less expensive because there are fewer points and larger corrections to the solution per sweep are possible. The theory of multigrid (again in simpler terms, see References 3 and 4 for greater detail) is based on the error in a solution consisting of a variety of wavelengths. On a given mesh the error components with wavelengths of the order of the mesh spacing are reduced or eliminated quickly, that is, in a small number of sweeps. The error components with long wavelengths, those many times the mesh spacing, are reduced very slowly, and this is the primary difficulty with standard relaxation techniques. Multigrid uses a sequence of meshes and, in its pure form, every wavelength is eliminated on a mesh where it is of the order of the mesh spacings. Hence, very few sweeps are required on each mesh. For linear problems, the entire theory is relatively straightforward. For nonlinear problems, the theory is more complex. Success of multigrid depends on many things. One critical item is that sweeping on the fine mesh does indeed eliminate the high-frequency error terms, that is, smooth the error. The mesh aspect ratio does affect this item. Another factor is how the boundary points are processed. Both these items probably adversely affect the efficiency of the multigrid scheme in this code.

The finite-difference equation set can be written as

$$L^{k}(\phi_{(n)}^{k};\phi_{(n-1)}^{k}) = F^{k}$$
(18)

where L^k is an operator on grid k, $\phi_{(n)}^k$ is the solution matrix at sweep n on grid k, and F^k is independent of $\phi_{(n)}$ and $\phi_{(n-1)}$. L^k is a linear operator on $\phi_{(n)}^k$, but derived from a nonlinear equation which has been linearized by using values of ϕ^k from the previous sweep, n-1. An exact solution, Φ^k on grid k, is defined by

$$\mathbf{L}^{\mathbf{k}}(\Phi^{\mathbf{k}};\Phi^{\mathbf{k}}) = \mathbf{F}^{\mathbf{k}} \,. \tag{19}$$

An error or discrepancy term, D^k, can be defined by

$$\mathbf{D}_{(n)}^{k} = \mathbf{L}^{k} \left(\phi_{(n)}^{k}; \phi_{(n)}^{k} \right) - \mathbf{F}^{k} \,. \tag{20}$$

What is desired is a correction, $\Delta \phi^k$, to the solution, $\phi^k_{(n)}$, that reduces the error, D^k. That is,

$$L^{k}(\phi_{(n)}^{k} + \Delta \phi^{k}; \phi_{(n)}^{k} + \Delta \phi^{k}) - F^{k} \ll D_{(n)}^{k}.$$
(21)

Such a correction can be computed on a coarser mesh, grid k-1, and the error is reduced if the assumptions made previously are correct.

An injection operator, I_k^{k-1} , is required to generate an initial field on grid k-1 from that on grid k. If all the points of grid k-1 are in grid k, the simplest operator just takes those values from the grid k solution that corresponds to points in grid k-1. Then,

$$\phi_{\rm F-k}^{\rm k-1} = {\rm I}_{\rm k}^{\rm k-1} \, \phi_{\rm (n)}^{\rm k} \tag{22}$$

where $\phi_{\mathbf{F}}^{\mathbf{k}-1}$ is ϕ on grid k-1 obtained from grid k.

The error term $D_{(n)}^k$ can also be injected down to level k-1, giving us

$$\bar{l}_{k}^{k-1} D_{(n)}^{k}$$
 (23)

on level k-1. Note that \overline{I} need not be the same operator as I. What is desired is a $\Delta \phi$ calculated on level k-1 which, when added to ϕ_{F-k}^{k-1} , gives an error term on level k-1 equal to $-\overline{I}_{k}^{k-1} D_{(n)}^{k}$.

This is still not sufficient as even an exact solution Φ^k with resulting D^k equal to 0 will generally lead to an error term on a lower level. That is,

$$\mathbf{L}^{k-1} \left(\mathbf{I}_{k}^{k-1} \Phi^{k} ; \mathbf{I}_{k}^{k-1} \Phi^{k} \right) - \mathbf{F}^{k-1} \neq 0.$$
(24)

The problem to be solved on level k-1 is

$$L^{k}(\phi_{(m)}^{k-1};\phi_{(m-1)}^{k-1}) = \overline{F}^{k-1}$$
(25)

where

$$\overline{F}^{k-1} = L^{k-1} \left(\phi_{F-k}^{k-1}; \phi_{F-k}^{k-1} \right) - \overline{I}_{k}^{k-1} D_{(n)}^{k}.$$
(26)

Once a solution $\phi_{(m)}^{k-1}$ is obtained on level k-1, the difference between it and the initial solution on level k-1, ϕ_{F-k}^{k-1} , is expanded to level k using an expansion operator I_{k-1}^k and added to the previous solution on level k.

$$\phi_{\text{new}}^{k} = \phi_{\text{last}}^{k} + I_{k-1}^{k} (\phi_{(m)}^{k-1} - \phi_{F-k}^{k-1})$$
(27)

The solution on level k-1 can be obtained using levels k-2 and the same technique. Similarly, levels k-3, k-4, etc., can be used in the calculation.

This particular program uses a maximum of four levels. Grid k-1 is obtained from grid k by deleting every other mesh, except for θ mesh. With cylindrical coordinates, an option exists for leaving the θ mesh unchanged between grids. This is desirable, since for some θ meshes, smoothing of the error term in θ can be very poor and, hence, the multigrid process can fail if the θ mesh varies (ref. 4). Inadequate smoothing in the θ coordinate is caused by mesh aspect ratios, $r\Delta\theta/\Delta x$ and $r\Delta\theta/\Delta r$, that are much greater than one. These can occur because for near axisymmetric geometries it is possible to obtain good accuracy with relatively coarse θ meshes.

 I_k^{k-1} is a straight injection operator used with the potential function ϕ . This gives as the ϕ value in grid k-1 the ϕ value at the same point in grid k. The operator \overline{I}_k^{k-1} used with D^k is a volume weighted injection operator so that $\overline{I}_k^{k-1}D^k$ is a volume weighted average of the D^k in the immediate vicinity of the grid point. This provides additional smoothing of the error term beyond that of the relaxation scheme. The expansion operator I_{k-1}^k is a parameteric cubic interpolator which uses the function and first derivatives. The first derivatives are calculated on the coarse grid using three-point difference formulas.

Multigrid, as applied to this program, works best for fine meshes. As the mesh becomes coarser, more points involve boundary conditions and the multigrid procedure may not correctly handle the boundary and near boundary points. For very coarse grids there may be only one mesh line between surfaces, and all the field points in a region are adjacent to surfaces. The differencing for such field points is nonstandard, and the multigrid assumptions do not necessarily apply. It is not clear what happens in this situation except that it is certainly adverse.

4.4 RESULTS

This program and its predecessors have been used for several years to calculate inlet and nacelle flowfields. Typical results are shown in References 1 and 2. Good agreement with experiments for inlet calculations can be obtained by using cylindrical meshes with about 50 000 mesh nodes.

Other configurations that have been calculated with the code are flow in the lobe of a multilobe mixer, an inlet in the influence of a ground plane, and flow about a turboprop chin inlet. The turboprop chin inlet required the maximum number of grid points, and even this provided only an approximate flowfield. The difficulty was that the inlet was small relative to the nacelle and spinner and thus a mesh fine enough to resolve the inlet was too fine for the spinner. This problem would benefit greatly from a capability to locally embed a finer mesh or to use larger (and hence, denser) meshes. A wireframe of the geometry, a cross-section for y=0.0, and computed results are shown in Figure 4.

Another configuration that has been computed is an "S" duct for a turboprop installation such as the one described previously. This configuration included the shaft for the prop and a fairing around the shaft. This calculation was made with about 50 000 mesh nodes and gave good agreement with experimental data (fig. 5).



Figure 4. Turboprop Nacelle Computation



Figure 5. Comparison of Experiment With Analysis for Flow in Turboprop Inlet Diffuser

Several trial calculations were made with 2-D wing configurations to verify that the Kutta condition was properly applied. Figure 6 shows results for an NACA-0012 airfoil at Mach 0.75 and 1-deg angle of attack. Experimental results are from Reference 5. The computation used 35 600 nodes, nine mesh along the wing and 25 mesh chordwise across the wing. Results are excellent. Disagreement with the experiment shown is typical of potential flow results (no boundary-layer effects included). A finer grid would give a better defined (sharper) shock wave.

An attempt was made to compute a solution about the outer nacelle of a 747 including part of the wing and the pylon. Results were unsatisfactory. The primary problem was that the mesh along the wing was too coarse and, hence, the boundary conditions where the wing exited the computational volume could not be enforced properly. The problem was run using 250 000 grid points, the maximum available. Whether this problem could be solved successfully using a denser mesh is not known.

A solution was calculated for a straight wing, pylon, and flow-through nacelle model tested at the NASA-Langley Research Center (ref. 6). The straight two-dimensional wing made the side boundary condition simpler and allowed a coarser mesh to be used successfully. Good agreement with experiment was obtained by using the maximum number of mesh and mesh-surface intersects currently possible and by careful selection of mesh values. Top, front, and side views showing the location of the pressure tap rows used for data comparison are shown in Figure 7. Results are shown in Figures 8 through 10 for free-stream Mach numbers of 0.2, 0.6 and 0.8. The results for the wing upper surface Mach number at a freestream Mach number of 0.8 show an expansion approaching the trailing edge. This is appropriate for supersonic inviscid flow over a convex surface. The trailing edge shock is very strong, so it is reasonable to assume that the wing boundary layer separates and that the differences between experiment and analysis are viscous effects.

The calculations for the wing-pylon-nacelle configuration were made with a compressor face at x = -4.1, and an extension of the nacelle as a cylinder to the computational boundary. The Mach number at the compressor face was estimated from the data at the nacelle exit. Assuming that the flow at the duct exit was at M_{∞} would give approximately the same solution.

4.5 CONCLUSIONS

The analysis gives good results as long as two criteria are met. First, inviscid irrotational flow has to be the correct model for the flow; that is, boundary layers have to be thin and attached. Second, the geometry and, thus, the flowfield, cannot be too complicated. Flowfield features must be resolved by the mesh and the number of mesh is limited by the size of the core memory of the computer.

The required central processor times for this analysis are quite reasonable. Usually the times are less than five minutes even for 250 000 field points. This is primarily the consequence of using multigrid for the solution procedure, since with multigrid run times are approximately proportional to the number of points. If more memory, with the same computation speed, becomes available, solving for the flowfields about more complex configurations would be practical.



Figure 6. NACA-0012 Airfoil, $M_{\infty} = 0.75$, Angle of Attack = 1.0 deg



Figure 7. NASA-Langley Wing-Pylon-Nacelle Test Model – Location of Pressure Measurements Used for Analysis Comparison



(a) Wing Surface Mach Number, y/D = 0.5





Figure 8. NASA Wing, Pylon, and Nacelle Model, M_{∞} = 0.2, Angle of Attack = 5.0 deg, Configuration 5, D = 4.5 in



Figure 8. NASA Wing, Pylon, and Nacelle Model, $M_{\infty} = 0.2$, Angle of Attack = 5.0 deg, Configuration 5, D = 4.5 in



(a) Wing Surface Mach Number, y/D = 0.5







Figure 9. NASA Wing, Pylon, and Nacelle Model, M_{∞} = 0.6, Angle of Attack = 3.0 deg, Configuration 5, D = 4.5 in



Figure 9. NASA Wing, Pylon, and Nacelle Model, M_{∞} = 0.6, Angle of Attack = 3.0 deg, Configuration 5, D = 4.5 in



(a) Wing Surface Mach Number, y/D = 0.5

Figure 10. NASA Wing, Pylon, and Nacelle Model, $M_{\infty} = 0.8$, Angle of Attack = 2.0 deg, Configuration 5, D = 4.5 in











Figure 10. NASA Wing, Pylon, and Nacelle Model, $M_{\infty} = 0.8$, Angle of Attack = 2.0 deg, Configuration 5, D = 4.5 in

5.0 USE OF THE PROGRAM

This section describes part of the procedure for using the computer program based on the analysis described in the last section. Those subjects not discussed are control cards and preparation of the geometry. Mesh selection is discussed. Samples of input and output files are presented. Some guides to possible failure modes are provided.

The process of specifying a three-dimensional geometry and intersecting it with a mesh is a major task by itself. Therefore this procedure is documented separately (ref. 7). The computer program has been designed so that the geometry information can come from any source. The file of surface-mesh intersects, coordinates, and normals does not have to have any special ordering. The geometry documents previously referred to are guides to one possible way to generate the geometry portion of the input file, but certainly not the only way.

The control cards for executing the code on the computer are not described in this document. Operating systems, field lengths, file names, etc. change relatively frequently so it is not advisable to document procedures that will probably be different by the time the document is read.

The analysis currently exists in versions for both the CDC CYBER 203 computer and the CRAY-1 computer. This report describes the latest version of the program encompassing the capabilities of the earlier versions of the computer code.

5.1 INPUT FORMAT

The first two lines of the input file are title lines and are printed at the start of the output for identification purposes. All input except for the title lines is by means of *order independent* groups headed by key words. The purposes of this particular input format are to allow certain groups to be optional, make the input file more readable, facilitate checking of input data by the program, and make future additions to the program easier. Certain input groups are mandatory, while others are optional and may be omitted. All input, except for title lines and comment lines, consists of numbers or words (depending on group) in fields of 10 columns wide, *maximum of six fields per input line. All numbers are floating point* and require a decimal point. Only the first four characters of key words are checked.

Certain interrelations among various input groups have to be taken into account. If convergence is to be obtained on a sequence of meshes, the *number* of x, y, and z, or x, r, and θ mesh can only have certain values. This is because coarser meshes are formed by deleting every other mesh line. Also, a compressor face, if there is one, must lie on an x mesh belonging to the coarsest mesh. There are restrictions on the number of mesh and number of surface points relating to declared array lengths in the computer code. These limits as they currently exist are listed in Section 5.1.3.

The program has the capability to use up to four mesh-density levels to provide more efficient convergence. The number of levels is controlled by the SWEEPS option. The mesh and geometry for the finest mesh level must be input. Coarser meshes for x, y, z, and r are formed by deleting *exactly* every other mesh from the previous mesh. This places restrictions on the number of mesh allowed in the finest mesh, as the first and last mesh line have to remain when every other mesh is deleted. The θ mesh is a special case. There is an option to control the manner in which the θ mesh is varied between levels. The number of θ mesh can be held constant for two successive levels, or every other θ mesh value can be deleted for the coarser mesh.
The program allows the use of planes of symmetry for *cylindrical coordinates* to reduce the number of mesh needed to make a calculation. For a cylindrical mesh, if the largest θ mesh value input is 180.00 deg, the plane 0 = 0 deg, and 180 deg is taken to be a plane of symmetry. If the largest θ mesh input is less than 180 deg, the flow is assumed symmetrical about 0 deg and the largest θ value input. Zero deg must always be input as a theta mesh.

5.1.1 SUMMARY OF INPUT GROUPS

REQUIRED

Keyword		Description
FREEstream XMESh VMESh		Title lines Speed of sound, freestream velocity and flow angles Axial mesh values
RMESh }	or	y or radial mesh values
ZMESh TMESh	or	z or circumferential mesh values
GEOMetry		Surface-mesh intersections: coordinate and surface normal values
		OPTIONAL
Keyword		Description
COMMent		Allows comments describing the run to be printed on the first page of the output
WING		Indicates lifting surface calculation
CFACe		Indicates an inlet geometry and specifies inlet mass flow and location of the compressor face
SWEEps		Convergence control parameters
COPT ADI		Allows changing the criteria for the multigrid cycling between meshes Controls type of lines, y or z, r or θ , used for line relaxation. Allows
		requesting alternating y and z (r and θ) lines
THETa		Control of number of θ planes for each mesh-density level (cylindrical coordinates only)
SCDIff		Indicates special θ differencing to be used (cylindrical coordinates only)
PRINt op		Requests printout of various categories of geometric information
SFLOw		Requests surface flow variable printout at end of run
OUTT		Allows surface quantities to be printed in an alternate coordinate system
FLDT		Requests printout of flow variables at constant z or θ cuts of flowfield
IPRI		Requests printout at other than level 4 for multilevel calculation
NOMG		Suppresses multigrid convergence procedure
PLOT		Surface quantities are written to file FT03 in a format for IGDA GGP plots (BCS CRAY-1 version) only
RELX		Allows the specification of the over- or under-relaxation factors

OPTIONAL (Diagnostic)

Keyword	Description
DEBUg	Requests print of coefficients, velocities, and potential function for a specified axial cut and sweep number

5.1.2 DETAILED INPUT-GROUP DESCRIPTIONS

FREESTREAM

This group specifies the velocity and orientation for the freestream relative to the geometry.

The scaling of the velocities is essentially arbitrary except that they should be of order one to avoid difficulties with print formats. Note that $q_{\infty}/a_{\infty} = M_{\infty}$.

Required input group, no default values.

Card 1	Cols.	1-4	'FREE', Keyword
Card 2	Cols.	1-10 11-20 21-30 31-40	a_{∞} , freestream speed of sound q_{∞} , freestream velocity α_1 , degrees, $\arctan(v_{\infty}/u_{\infty})$ α_2 , degrees, $\arctan(w_{\infty}/u_{\infty})$

Special conventions:

If $\alpha_1 = 90 \text{ deg}$, $\alpha_2 = \arctan(w_{\infty}/v_{\infty}) (u_{\infty} = 0.0)$.

If $\alpha_1 = 90 \text{ deg and } \alpha_2 = 90 \text{ deg}$, $w_{\infty} = q_{\infty} (u_{\infty} = v_{\infty} = 0.0)$.

If airplane coordinates are used (z oriented up), α_2 is the angle of attack and α_1 is a yaw angle.

If cylindrical coordinates are used with $\theta = 0$ deg as the top or crown (inlet calculation), α_1 is the angle of attack and α_2 is a yaw angle.

Note: Input of "FREE STREAM" which is 11 characters instead of "FREE" or "FREESTREAM" will draw an error message.

XMESH YMESH or <u>RMES</u>H ZMESH or <u>TMES</u>H

These groups handle the input of the computational mesh, x, y, and z or x, r, and theta, theta in degrees. The values do not have to be in any order. Theta mesh must include 0, 90, 180, and 270 deg unless there is a plane of symmetry. Zero deg must always be included as a theta mesh.

The program creates lower mesh levels by deleting every other mesh value from the previous mesh. Also, the first and last mesh must remain and the compressor face, if there is one, must be in the coarsest mesh. If L is the number of mesh levels (see SWEEPS Group), then NX and NY (NR) can have the following values:

 $L = 4 \quad NX \text{ or } NY = 25, 33, 41, \dots .(8m+1) \qquad (m \text{ an integer}) \\ L = 3 \quad NX \text{ or } NY = 13, 17, 21, \dots .(4m+1)$

The number of z mesh behaves the same. For cylindrical coordinates, similar rules apply, except it is possible to keep the number of θ mesh constant for two adjacent levels and a periodic mesh (0 to 360 deg) is possible. See the THET option for the rules for such meshes.

See Section 5.2 for a discussion of the number of levels that can or should be used.

Required input groups, no default values.

Card 1	Cols.	1-4 11-20	'_MES', Keyword,_ can be X, Y, and Z or X, R, and T (theta) Number of mesh values to be read
Card 2	Cols.	$\left. \begin{array}{c} 1\text{-}10 \\ 11\text{-}20 \\ 21\text{-}30 \end{array} \right\}$	Axial, y or radial, or z or circumferential location of mesh, six values per card, as many cards as required. Theta must be in degrees.
•		·	

See Section 5.1.3 for the maximum number of mesh that can be used.

GEOMETRY

This group consists of the coordinates of the intersections of the mesh with the geometry and the direction cosines of the surface normal at each intersect.

Required input group, no default values.

Card 1	Cols.	1-4 11-20	'GEOM', Keyw number of inter	ord sects, NSURTOT
Card 2	Cols.	$ \begin{array}{r} 1-10\\ 11-20\\ 21-30\\ 31-40\\ 41-50\\ 51-60\\ \end{array} $	$\left.\begin{array}{c} x \\ y \text{ or } r \\ z \text{ or } \theta \\ (\text{degrees}) \\ n_x \\ n_y \text{ or } n_r \\ n_z \text{ or } n_{\theta} \end{array}\right\}$	one intersect per card
Card 3	Cols.	1-10		
•				
Card NSURT	OT+1			

Note:

- (1) See Section 5.1.3 for maximum value of NSURTOT.
- (2) Surface normal is unit surface normal $(n_x^2+n_y^2+n_z^2=1.0)$ and must be oriented into the flowfield.

COMMENT (Optional)

This group allows a multiline comment to be printed on the first page of a computation. This allows a longer description of a run than can be achieved using the required two title cards.

Card 1	Cols.	1-4 11-20	'COMM', Keyword Number of comment cards to be read
Card 2	Cols.	1-80	1st comment card
Card 3	Cols.	1-80	2nd comment card
•		•	•
•		•	•

WING (Optional)

This option allows lifting surface calculations, that is, wings with a Kutta boundary condition applied at the trailing edge. The program automatically locates wing-like surfaces.

Card 1 Cols. 1-4 'WING', Keyword

CFACE (Optional)

Signals that there is an inlet geometry and specifies the Mach number and station (x) of the compressor face.

Card 1	Cols.	1-4	'CFAC', keyword
Card 2	Cols.	1-10 11-20	x value at compressor face Mach number to be enforced at compressor face

The x station specified must be consistent with the input geometry and must be a mesh value for the coarsest mesh. The Mach number specified is the one-dimensional flow average Mach number for a cylinder with the cross-sectional area of the compressor face.

SWEEPS (Optional)

This group controls the sweeping process by allowing control of the number of mesh-density levels, the maximum number of sweeps on each mesh-density level, and a convergence criterion for each mesh-density level. The mesh numbering system is such that level 4 is the finest mesh. Levels 3, 2, and 1 may or may not exist. The number of levels to be used is set by the number of values entered for maximum number of sweeps on a level. There will be as many levels used as nonzero values entered.

In general, it is difficult to determine in advance how many mesh levels to use in a calculation. The multigrid is theoretically most efficient with many levels, but if the mesh becomes quite coarse relative to the geometry, the program can behave badly or fail on the coarser mesh levels. For standard calculations (not more than 56 000 grid points), three levels is probably the maximum that should be used. Less than three levels can be tried if the program fails while trying to use three levels.

Default is a three-level mesh with the following limits:

	Level Maximum n Convergenc	umber of s e paramete	weeps er	2 800.0 1.0	3 200.0 1.0	4 100.0 1.0
Card 1	Cols.	1-4	"SWEE', Ke	yword.		
Card 2	Colš.	1-10 11-20 21-30 31-40	Maximum n Coarsest me One to four y levels to be y	umber of s sh limit fi values. Th used in the	sweeps all rst and fin is card set calculation	owed at each level. est mesh limit last. s the number of on.
Card 3	Cols.	1-10 11-20 21-30 31-40	Convergence parameters for each level, coarsest level limit first, finest level limit last. Sweeping on a level is stopped and convergence is assumed when the average value of $ \Delta \phi $ (change in ϕ between sweep multiplied by 10 ⁶ and divided by ($\phi_{max} - \phi_{min}$) is less than t value of the convergence parameter for that level.			

Example:

Two-level calculation, maximum of 500 sweeps for the coarsest level, level 3, and 100 sweeps for the finest level. The calculation to be converged to a relative error of 0.5×10^{-6} on level 3 and 10^{-6} on level 4.

SWEE

500.0	100.0
0.5	1.0

<u>COPT</u> (Optional)

This option allows control of the convergence procedure used. The options are adaptive cycling, fixed cycling with default limits, or fixed cycling with user specified limits. The adaptive cycling does not move to a higher (denser) level until convergence is reached at the current level. This can cause a run failure if it happens that the calculation is unstable at one of the intermediate levels. The default for this option is fixed cycling with default limits. It is recommended that the adaptive cycling option not be used. The default method, in general, is the most reliable convergence option.

The default cycling pattern for a three-level calculation is diagramed below:



where(n)indicates sweeping on level n.

Level 2 is swept until convergence or 100 sweeps have been taken. Levels 3 and 4 are swept a minimum of six sweeps and a maximum of 20 sweeps. Sweeping on levels 3 and 4 is stopped if convergence or stalling occurs. Level 4 is visited at least twice and a maximum of 4 times.

Card 1 Cols. 1-4 11-20	 "COPT", Keyword 0.0 adaptive cycling 1.0 fixed cycling with default limits on the number of sweeps per visit to a level and the number of visits to any level (default) 2.0 fixed cycling with limits input
---------------------------	--

If the COPT option equals 2.0, the following cards are required:

Card 2	Cols.	1-10 11-20 21-30 31-40	Floating point values for the maximum number of sweeps per visit on each level starting with the lowest (coarsest) mesh level and ending with level 4. 1st value in cols. 1-10. One to four values depending on number of levels to be used (see "SWEE" option).
Card 3	Cols.	$ \begin{array}{c} 1-10\\ 11-20\\ 21-30\\ 31-40 \end{array} $	Floating point values for maximum number of visits to each level ordered as above.
Default	values (CO	OPT option (equals 1.0)
100	sweeps n	naximum pe	er visit on the lowest level (coarsest mesh)

20 sweeps maximum per visit otherwise

4 = maximum number of visits to finest level

Example:

Three level calculation, default limits input explicitly.

COPT	2.0	
100.0	20.0	20.0
20.0	20.0	4.0

ADI_(Optional)

This option permits user control over the direction of the lines for line relaxation. Either y (r) or z (θ) lines may be used or y and z lines can be used alternately.

Default is z lines for Cartesian meshes and r (radial) lines for cylindrical coordinates.

Card 1	Cols.	1-3	'ADI', Keyword
		4	Blank
		11-20	1.0 = y or r (radial) lines used
			$2.0 = z \text{ or } \theta$ lines used
			$3.0 = $ alternate y and z or r and θ lines starting with y (r) lines

THETA (Optional)

This group allows control of the number of θ grid used for each mesh-density level (cylindrical coordinate calculation only). The number of levels used is controlled by the SWEEPS option (default is four levels), and this option must be consistent with the SWEEPS option input. The

number of θ grid at level i, NT_i, may be held the same as level i+1 or every other grid of level i+1 can be deleted to form level i. The value specified for the number of θ grid at the finest level must be the same as used in the TMESH input group, and the values specified for coarser levels must be consistent with any flow symmetry that has been specified (TMESH input group).

Default for this option is no change in the number of θ mesh used for different levels.

Card 1	Cols.	1-4	'THET', Keyword
Card 2	Cols.	1-10 11-20 21-30 31-40	Number of θ grid for each level, coarsest level first and finest level last, one to four values starting in cols. 1-10

Notes:

(1) If there is a plane of symmetry (less than 0-deg to 360-deg geometry input)

$$NT_i = NT_{i+1} \text{ or } NT_i = (NT_{i+1}+1)/2.0,$$

otherwise,

$$NT_i = NT_{i+1}$$
 or $NT_i = NT_{i+1}/2.0$.

SCDIFF (Optional)

This group allows use of special θ differencing (ref. 1) for improved accuracy with very coarse θ meshes (cylindrical coordinate calculations).

Default is regular differencing.

Card 1	Cols.	1-4 11-20	'SCDI', Keyword 0.0 regular differencing 1.0 special θ differencing (any other value than 0.0 1.0 will be treated as 0.0)
			1.0 will be treated as 0.0)

PRINT OP (Optional)

Inputs any or all of a group of keywords to obtain printed output of a variety of geometrical information.

Card 1	Cols.	1-4	'PRIN', Keyword
Card 2	Cols.	1-10 11-20	Up to five keywords as described below. Can be in any order (only first four characters of keyword are checked).
<u>SPIN</u> PUT I <u>SPOR</u> DER I <u>SPEC</u> PNTS I		List of th List of su List of sp	ne surface points in the order read. urface points in the internal ordering used in the analysis. pecial points.

TYPE2List of Type 2 points. These are points that are both surface and mesh
nodes.MAPLists of x, y, and z, or x, r, and θ constant cuts of the surface. Lists include

The default is none of the above output.

SFLOW (Optional)

This group controls printing of flow properties along the surface. Default is printing of every fourth cut for all surfaces. This default corresponds to coarse-mesh cuts for a three-level calculation.

Card 1	Cols.	1-4	'SFLO', Keyword
Card 2	Cols.	1-10 11-20 21-30	SKIPX value SKIPR value SKIPT value

For the fine mesh, every constant x cut will be printed if SKIPX = 1. If SKIPX = 0, no cuts will be printed. Otherwise, cuts will be printed for x = X(I), I = 1, 1 + SKIPX, 1 + 2*SKIPX, etc. SKIPR and SKIPT work the same for y or r, and z or θ constant cuts of the surfaces, respectively.

OUTT (Optional)

It is often desirable or necessary to use more than one coordinate system. As an example, an inlet may be described by either an engine centerline or an inlet centerline coordinate system, and the user may alternate between the two depending on the situation. This option allows output of flow properties along the surface in a coordinate system different from that of the computation.

Card 1	Cols.	1-4 11-20	'OUTT', Keyword (output transformation) 1.0 transformed coordinate output only 2.0 regular output <i>and</i> transformed coordinate output
Card 2	Cols.	1-10 11-20 21-30 31-40	0.0 transformed coordinates are Cartesian 1.0 transformed coordinates are cylindrical SKIPX SKIPR SKIPT SKIPT SKIPT
Card 3	Cols.	1-10 11-20 21-30 31-40 41-50 51-60	$\begin{array}{c} \Delta x_1 \\ \Delta y_1 \\ \Delta z_1 \\ \Delta x_3 \\ \Delta y_3 \\ \Delta z_3 \end{array}$
Card 4	Cols.	1-10 11-20 21-30	$t_{11} t_{12} t_{13}$

Card 5	Cols.	1-10 11-20 21-30	t ₂₁ t ₂₂ t ₂₃
Card 6	Cols.	1-10 11-20 21-30	t ₃₁ t ₃₂ t ₃₃

The transformation is defined as follows where the subscript 1 indicates the initial coordinates and the subscript 4 the final coordinates:

Note (\boldsymbol{t}_{ij}) must be such that :

$\begin{bmatrix} t_{11} \ t_{12} \ t_{13} \\ t_{21} \ t_{22} \ t_{23} \\ t_{31} \ t_{32} \ t_{33} \end{bmatrix}$	x	$\begin{bmatrix} t_{11} t_{21} t_{31} \\ t_{12} t_{22} t_{32} \\ t_{13} t_{23} t_{33} \end{bmatrix}$	=	$\begin{bmatrix} 1.0\ 0.0\ 0.0\\ 0.0\ 1.0\ 0.0\\ 0.0\ 0.0\ 1.0 \end{bmatrix}$
--	---	--	---	---

i.e.,

$$\begin{array}{ccc} 3 & & & & \delta_{ij} = 1 & \quad i = j \\ \sum\limits_{k=1}^{j} & t_{ik} t_{jk} = \delta_{ij} & & & \delta_{ij} = 0 & \quad i \neq j \end{array}$$

$y_1 = r_1 \cos\theta_1$)	if initial coordinates
$z_1 = r_1 \sin \theta_1$	are cylindrical,

$$\begin{array}{l} x_{2} = x_{1} + \Delta x_{1} \\ y_{2} = y_{1} + \Delta y_{1} \\ z_{2} = z_{1} + \Delta z_{1} \end{array} \\ \left[\begin{array}{c} x_{3} \\ y_{3} \\ z_{3} \end{array} \right] = \left[\begin{array}{c} t_{11} t_{12} t_{13} \\ t_{21} t_{22} t_{23} \\ t_{31} t_{32} t_{33} \end{array} \right] \left[\begin{array}{c} x_{2} \\ y_{2} \\ z_{2} \end{array} \right] \\ x_{4} = x_{3} + \Delta x_{3} \\ y_{4} = y_{3} + \Delta y_{3} \\ z_{4} = z_{3} + \Delta z_{3} \\ r_{4} = \sqrt{y_{4}^{2} + z_{4}^{2}} \\ \theta_{4} = \arctan\left(y_{4}/z_{4}\right) \end{array} \right]$$
 if final coordinates are cylindrical.

Example



Note that y is the vertical coordinate and z the side coordinate for this example.

If in engine centerline coordinates point P has coordinates:

 $\begin{aligned} \mathbf{x} &= \mathbf{a} \\ \mathbf{r} &= \mathbf{0.0} \\ \boldsymbol{\theta} &= --, \end{aligned}$

and in inlet centerline coordinates point P has coordinates:

then for calculation in engine centerline coordinates - print in inlet centerline coordinates,

 $\begin{array}{l} \Delta x_1 = -a \\ \Delta y_1 = 0.0 \\ \Delta z_1 = 0.0 \\ \Delta x_3 = b \\ \Delta y_3 = 0.0 \\ \Delta z_3 = 0.0 \\ t_{11} = \cos\beta \\ t_{12} = \sin\beta \\ t_{13} = 0.0 \\ t_{21} = -\sin\beta \\ t_{23} = 0.0 \\ t_{23} = 0.0 \\ t_{31} = 0.0 \\ t_{32} = 0.0 \\ t_{33} = 1.0. \end{array}$

For calculation in inlet centerline coordinates - print in engine centerline coordinates,

 $\begin{array}{l} \Delta x_1 = -b \\ \Delta y_1 = 0.0 \\ \Delta z_1 = 0.0 \\ \Delta x_3 = a \\ \Delta y_3 = 0.0 \\ \Delta z_3 = 0.0 \\ t_{11} = \cos\beta \\ t_{12} = -\sin\beta \\ t_{13} = 0.0 \\ t_{21} = \sin\beta \\ t_{22} = \cos\beta \\ t_{23} = 0.0 \\ t_{31} = 0.0 \\ t_{32} = 0.0 \\ t_{33} = 1.0. \end{array}$

FLDT (Optional)

This option determines z or θ mesh values for which the field properties are to be printed. Default is no field printout.

Card 1	Cols.	1-4	'FLDT', Keyword.
		11-20	NFFPR, number of z or θ values for which field
			properties are to be printed. NFFPR = 0.0 or NFFPR < 0.0 has
			special significance as described.

If NFFPR = 0.0, the entire flowfield is printed. Warning - this can cause an excessive quantity of printout.

If NFFPR > 0.0, the following cards are required:

Card 2	Cols.	1-10 11-20	Values of z or θ mesh for printing field properties, 6 values per card (θ in degrees)
		• }	
		•	
•		· · /	,
•	•		

If NFFPR < 0.0, the flowfield is printed between specified x values for each z or θ constant plane requested. The following cards are required:

Card 2	Cols.	1-10 11-20 21-30	T1 X11 X12	Field is printed for the z or θ mesh value(s), TI for XI1 $\leq x \leq$ XI2
Card 3	Cols.	1-10	T2 X21 X22	
Card NI +1	FFPR			

IPRI (Optional)

This option allows printing of solution properties for the coarse meshes including mass-flow conservation computation. Default is no solution printout for coarse meshes. Printout occurs only if convergence is obtained. The 0.0 option for the COPT group works best with this option.

Card 1 Cols. 1-4 'IPRI', Keyword

NOMG (Optional)

This keyword causes the program to use a multilevel procedure instead of the more efficient multigrid convergence procedure. The multilevel procedure is defined as a calculation on a sequence of successively finer grids. A multigrid calculation is a cycling between grids using coarse grid calculations to obtain corrections to the fine grid solution. Use of this option is not recommended.

Card 1 Cols. 1-4 'NOMG', Keyword

PLOT (Optional) - CRAY-1 version only

This option allows surface flow properties to be written to disk in a format suitable for plotting using the GGP plotting program on the Boeing IGDA graphics computer systems.

Card	.1	Cols.	1-4 11-20		PLOT', Keywor Number of const can be x constan of the surface. L	d ant coordinate curve families to be saved. These t cuts, y or r constant cuts, or z or θ constant cuts imit is 3.0.
Card	.2	Cols.	1-10 11-20 21-30		One to three val families request information, 2.0 2- or θ-constant	ues depending on number of curve ed. 1.0 to save x-constant cut to save y- or r-constant information, 3.0 to save information.
Exan	nples:					
	PLO] 1.0	Г <u>2</u> . З.	0 0	}		To save x- and z (θ)- constant cuts
	PLOI	Г <u>З</u> .	0	١	,	To save everything

Note: Plot information is written to disk (file FT03) and control cards are required to save the disk file.

RELX (Optional)

1.0

2.0

This option allows the user to specify the over- or under-relaxation factors used in the code. Use of this option is not recommended. If multigrid is being used, the over-relaxation factors are used only for the coarsest grid. Under- relaxation factors are used for all grids.

Default values are $\omega_{\mathbf{x}} = 1.85$, $\omega_{\mathbf{v}} (\omega_{\mathbf{r}}) = 0.90$, and $\omega_{\mathbf{z}} (\omega_{\theta}) = 0.90$.

3.0

Cols.	1-4	'RELX', Keyword
Cols.	1-10	ω
	11-20	$\omega_{\mathbf{v}}(\omega_{\mathbf{r}})$
	21-30	$\omega_{z}^{\prime}(\omega_{\theta})$
	Cols. Cols.	Cols. 1-4 Cols. 1-10 11-20 21-30

Note: $0 < \omega < 2.0$ required for stable solution.

DEBUG (Optional)

Diagnostic print option. Prints internal parameters for a given x mesh index and sweep number.

Card 1	Cols.	1-4	'DEBU', Keyword
Card 2	Cols.	1-10 11-20	Sweep number x-plane index

5.1.3 Limits and Timing

The limits on the number of mesh and the number of surface points are due to the declared array lengths in the code. They are subject to change. Current limits are given on Table 1. These limits are set with respect to the computer memory available, but could be increased to some extent.

Table 1. Program Internal Limitations

NX	161
NY (NR)	121
NZ (NT)	121 *
NX*NY	8 500
NX*NZ	8 500
NY*NZ	4 250
NX*NY*NZ	252 105
Number of surface points	8 500
Number of surface points on	500
or adjacent to any x constant plane	
Number of surface points on or	1 000
adjacent to any z (θ) constant plane	
Maximum number of planes of field	200
print out (NFFPR for FLDT input group)	

*120 for a periodic mesh (i.e., 0 to 360 deg)

The CRAY-1 version of the code presently requires approximately 1 600 000 words of memory. A maximum case on the NASA Langley Research Center CDC C-203 computer (a virtual memory machine with 1 million word real memory, but soon to be increased) will be very expensive (with the 1 million word real memory) because of the page faults. It is best to run smaller cases on this machine until the memory is increased.

Typical run times on the CRAY-1 are 1 to 2 min for 50 000 field points and 3 to 5 min for 250 000 field points. Run times on the CDC C-203 computer are approximately twice as long.

5.1.4 Sample Inputs

The first page of the listings of three different data cases are presented as examples (figs. 11, 12, 13). These files have been arranged so that the geometry group, by far the longest, is last. This is not required, but does make the files much easier to work with.

QFAN INLET Mach Inf. Freestream	REF. NA = 0.21 A	SA CR15192: LPHA = 60+	2 ; syberg ; D degrees	P465 DEMO	DNCSE Ky J. DNSTRATION	Dev JAN. 1977 Test case
1.0	0.21	60.0	0.0			
SHEEPS						
800.	200.	100.				
	1.	1.				
SFLUEFR 1.0	1 - 0	1.0				
TPRT	1.00					
THET						
9.0	9.0	9.0				
CFACE						
\$5.012	0.43					
XMESH	69.		`			
-180+0480	-168.7950	-157.5420	-146+2890	-135.0360	-123.7830	
-112.5300	-101-2770	-90.0240	-83.2722	-76.5204	-69•7686	
-63.0168	-56+2650	-49.5132	-42.7614	-36.0096	-31.5084	
-27.0072	-22.5060	-18-0048	-15-7542	-13.5036	-11.2530	
-9.0024		-5./518	-5+6265	-4.5012	-3+38/2	
-2+2/31	-101371 5 6 2 2 6	-•U43U	100713	2.2281	JeJ676	
4.5012	Jeb203	10 0040	100//1 20.255A	22.5060	24.7566	
27.0072	29,2578	31.5084	2002334	36.0096	39-2602	
40-5108	A2-761A	A5_0120	47-2626	49-5132	51-7638	
54-0144	58-5156	63-0168	67.5180	72.0192	76-5204	
81.0216	85.5228	90.0240	0100100		1000201	
RMESH	49.					
0.0000	1.4450	2.8900	4.3350	5.7800	7.2250	
8.6700	10.1150	11.5600	13.0050	14.4500	15.8950	
17.3400	18.7850	20+2300	21.6750	23.1200	23.8425	
24.5650	25.2875	26+0100	26.7325	27.4550	28.1775	
28.9000	50.3450	31.7900	33.2350	34.6800	37.5700	
40.4600	43.3500	46+2400	52.0200	57.8000	63.5800	
69.3600	80.9200	92.4800	104+0400	115.6000	137.2750	
158-9500	180.6250	202.3000	223.9750	245.6500	267.3250	
289-0000)					
1MC2H	7 • • • • • • • •	AE 0000	(7 E000		112 5000	
135-0000	1 157-5000	1.80.0000	6/03000	30.0000	11205000	
250NE.18A	991.	10010000				
-0000	30-3450	135-0000	- 999994	003517	•000180	
.0007	26.7325	0.0000	999799	020028	000012	
.0009	28.9000	90.0000	999471	-032450	002328	
•0029	27.4550	45.0000	994741	~102286	005338	
•0078	28+1775	67.5000	992455	•122305	008609	
•0099	26.7325	22.5000	994329	106305	•003078	
• 0532	30+3450	157.5000	980441	- 196739	•005447	
•0778	31.7900	180.0000	974464	+224543	000008	
•0863	27+4550	67-5000	952733	303124	•020380	
•0915	50.3450	180.0000	• 969981	24 3179	•000017	
+U998	50+3450	112+5000	- 0E1000			
-1171	2001113	9060000 45.0000			.01895A	
-1247	. 2001323 9 28-3000	112-5000			-020454	
1298	31.7900	157.5000	• 956120	#292836	008981	
•1550	27.4550	22.5000	871558	.490069	014798	
•1979	26+0100	0.0000	871352	490658	.000007	
• 2614	27.4550	0.0000	807477	√ 589899	000138	
•2695	i 28.9000	67.5000	346628	.530655	040326	

Figure 11. Part of Input File for Inlet Geometry Data Case

NACA 0012	AIRFOIL W	ING ALONE F	INE HESH SE	PT 21, 1982	2
FREESTREAM	175 ALFA,	-100 DEBK	LEJ		
1.0	0.75	0.0	1.0		
SWEEPS					
800.0	200.0	60+0			
0.1	0.1	0.1			
ADI	3.0				
IPRI					
SELON					
1.0	1.0	1.0			
KMESH	65.	,			
-2+5000	-2.2500	-2.0000	-1.7500	-1.5000	-1.2500
-1.0000	8500	7000	5500	4000	3500
3000	2500	02000	1750	1500	1250
1000	0750	-•0500	0250	0.0000	•0250
•0500	•0750	•1000	•1375	•1750	•2125
•25UU 5500	· • 3000	J •3500	+4000	• 4500	•5000
• 3300		UUCCe U	+/UUU 0511	+ / J V U 1907 5	+8000
1.0375	1.0625	•7100 5 1-0875	• 7551	• 707 3	1.1344
1.1500	1.1875	1.2250	1.2625	1.3006	1.1750
1.4500	1.5250	1.6000	-3-0000	-2.75000	100100
YMESH	91	•			
2000	1500	1000	0500	0.0000	•0500
-1000	●1500	•2000			
Z ME SH	61.	•			
-5.0000	-4.5001	0 -4.0000	-3.5000	-3.0000	-2.5000
-2.0000	-1.7500	0 -1.5000	-1.2500	-1.0000	9000
8000	7000	6000	5250	4500	3750
~ ●3000	- 262		1875	1500	-+1219
			-00125	•0125	•U3/5
. 2250		5 .3000	+1201	- 1500	-5250
•6003	.7000	.8000	•9000	1.0000	1.2500
1.5000	1.750	2.0000	2.5000	3.0000	3.5000
4.0000	4.5000	5.0000	5.5000	6.0000	-6.0000
-5.5000)				
GEOMETRY	495 •				
0.0000	-•2000	0.000.0	-L.000000	0.00000	0.000000
0.0000	1500	0.0000	-1.000000	0-000000	0.000000
0.0000		0 0.0000	-1.000000	0-000000	0.000000
		0.0000	-1.000000	0.000000	0.000000
0.0000	0.0000	0.0000	-1.000000	00.00000	0.000000
0.0000			-1-000000	0-0000000	0.000000
0.0000	•1500		-1.000000	0_0000000	0.000000
0.0000	.2000	0.0000	-1.000000	0.000000	0.000000
•0053	2000	0125	760089	0.000000	-+649819
.0053	51500	••0125	760089	0-000000	649819
•0053	5 -+1000	0125	760089	0.00000	649819
•0053	0500	-+0125	760089	0.000000	-•649819
•0053	0.0000	••0125	•.760089	0.000000	649819
•0053	•0500		+.760089	U-UU0000	**649819
●UU53 0041	•1000	J =+0125	- 760089	04000000	
-00-1		-+U125		0.0000000	-+077017 +.K44914
•0053	2000	•0125	760089	0-000000	a649819
.0053	1500	.0125	• • 760089	0-000000	•649819

Figure 12. Part of Input File for NACA-0012 Airfoil Analysis

CONE. 5	<pre>K WING; PY M INE =0.</pre>	LON AND NA 20 Alpha	CELLE 11/ = 5-0 DEGI	/09/83 7 Rees no	$7 \times 41 \times 65$	MESH
FREESTREAN	1					
I OU CFACE	0.2	0.0	5.0			
-4-1	•1					
SWEEPS	204-0	100-0				
1.0	1.0	1.0				
WING						
SFLUW 1-0	1.0	1.0				
XMESH	77.	100				
-20.0000	-18.000	0 -16+000	0 -12.000	0 -11.000	0 -10.0000	
-9.5000	-9-000	0 -8.700	0 -8.400	0 -8.100	0 -7.8000	
-/.5000		0 -6.500	0 -6+000	0 -5.500		
-2.5000		0 -1.750	0 -1.500	0 -1.250		
7500	500	0250	.010	.250	•5000	
•7500	1.000	0 1.250	0 1.500	0 2.000	0 2.5000	
3.0000	J 3.500	0 4-000			0 5.2500	
2000	U 30/3U	0 8.250	0 8-500	0 6.300 1 8. 7 00		
9.1000	9.300	0 9.500	0 9.700	D 9.90 0	0 10.1000	
10.3000	10.500	0 11.000	0 11.500	0 12.000	12.5000	
13.0000	14.000	0 16.000	0 18+000	0 20.000	0	
YMESH	41.	-17 5	-11 0	-0.0	-/ 75	
-2000	=16+3	-13+5	= 11 + 0	- 7. 0	-6./J -2.0	
-1.75	-1.5	-1.25	-1+0	86		
***2	-+<1	0.0	• 21	• 42	•64	
• 86	-•21 1•0	1.25	•21 1•5	•42 ·· 1•75	•64 2•0	
• • • 2 • 86 2 • 25	-•21 1•0 2•5	0.0 1.25 3.0	•21 1•5 ••0	•42 ··· 1•75 5•3	•64 2•0 6•75	
- + + 2 + 86 2 + 25 9 + 0 2 MESH	-•21 1•0 2•5 11•0 65•	0.0 1.25 3.0 13.5	•21 1•5 4•0 16•5	•42 1•75 5•3 20•0	•64 2•0 6•75	
•••2 •86 2•25 9•0 2#ESH 72•0000	21 1.0 2.5 11.0 65. -56.000	0.0 1.25 3.0 13.5 0 -45.000	•21 1•5 4•0 16•5 0 -40•000	•42 1•75 5•5 20•0 -32•000	•64 2•0 6•75	
+2 .86 2.25 9.0 2HESH 72.0000 -24.0000	21 1.0 2.5 11.0 65. 0 -56.000 0 -20.000	0.0 1.25 3.0 13.5 0 -45.000 0 -16.000	•21 1•5 4•0 16•5 0 -40•0000 0 -14•0000	•42 1•75 5•5 20•0 0 -32•000 0 -12•000	•64 2•0 6•75 0 -28•0000 0 -10•0000	
+2 -86 2-25 9-0 2HESH -24-0000 -8-0000	$ \begin{array}{r} -21\\ 1.0\\ 2.5\\ 11.0\\ 65.\\ -56.000\\ -20.000\\ -7.000 \end{array} $	0.0 1.25 3.0 13.5 0 -45.000 0 -16.000 0 -6.500	•21 1.5 4.0 16.5 0 -40.0000 0 -14.0000 0 -6.050	• 42 1.75 5.5 20.0 0 -32.000 0 -12.000 0 -5.700	•64 2•0 6•75 0 -28•0000 0 -10•0000 0 -5•3500	
	$\begin{array}{c} -21 \\ 1 \cdot 0 \\ 2 \cdot 5 \\ 11 \cdot 0 \\ 65 \cdot \\ 0 & -56 \cdot 000 \\ 0 & -20 \cdot 000 \\ 0 & -7 \cdot 000 \\ 0 & -4 \cdot 650 \\ 0 & -2 \cdot 56 \end{array}$	$\begin{array}{c} 0 & 0 \\ 1 & 25 \\ 3 & 0 \\ 13 & 5 \\ 0 & -45 & 000 \\ 0 & -16 & 000 \\ 0 & -6 & 500 \\ 0 & -4 & 300 \\ 0 & -2 & 350 \end{array}$	$\begin{array}{r} \bullet 21 \\ 1 \bullet 5 \\ 4 \bullet 0 \\ 16 \bullet 5 \\ 0 & -40 \bullet 0000 \\ 0 & -14 \bullet 0000 \\ 0 & -6 \bullet 0500 \\ 0 & -3 \bullet 9750 \\ 0 & -3 \bullet 9750 \\ 0 & -3 \bullet 9750 \\ \end{array}$	$\begin{array}{c} .42 \\ 1.75 \\ 5.5 \\ 20.0 \\ 0 \\ -32.000 \\ 0 \\ -12.000 \\ 0 \\ -5.700 \\ 0 \\ -3.650 \\ 0 \\ -1.700 \\ 0 \\ -3.650 \\ 0 \\ -1.700 \\ 0 \\ -1.700 \\ 0 \\ -1.700 \\ 0 \\ -1.700 \\ 0 \\ -1.700 \\ 0 \\ -1.700 \\ 0 \\ -1.700 \\ 0 \\ -1.700 \\ 0 \\ -1.700 \\ 0 \\ -1.700 \\ 0 \\ -1.700 \\ 0 \\ -1.700 \\ 0 \\ -1.700 \\ 0 \\ -1.700 \\ 0 \\ -1.700 \\ 0 \\ -1.700 \\ 0 \\ 0 \\ -1.700 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 $	•64 2•0 6•75 0 -28•0000 0 -10•0000 0 -5•3500 0 -3•3250 0 -1 5000	
	$\begin{array}{c} -21 \\ 1 \cdot 0 \\ 2 \cdot 5 \\ 11 \cdot 0 \\ 65 \cdot \\ 0 & -56 \cdot 000 \\ 0 & -20 \cdot 000 \\ 0 & -7 \cdot 000 \\ 0 & -4 \cdot 650 \\ 0 & -2 \cdot 675 \\ 0 & -1 \cdot 100 \end{array}$	$\begin{array}{c} 0 & 0 \\ 1 & 25 \\ 3 & 0 \\ 1 & 5 \\ 0 & -45 & 000 \\ 0 & -16 & 000 \\ 0 & -6 & 500 \\ 0 & -4 & 300 \\ 0 & -2 & 350 \\ 0 & -2 & 900 \\ \end{array}$	$\begin{array}{r} \bullet 21 \\ 1 \bullet 5 \\ 4 \bullet 0 \\ 16 \bullet 5 \\ 0 & -40 \bullet 0000 \\ 0 & -14 \bullet 0000 \\ 0 & -6 \bullet 0500 \\ 0 & -3 \bullet 9750 \\ 0 & -2 \bullet 0250 \\ 0 & -2 \bullet 026750 \\ 0 & -6750 \\ 0 & -6750 \\ \end{array}$	$\begin{array}{c} .42 \\ \\ 1.75 \\ 5.5 \\ 20.0 \\ 0 \\ -32.000 \\ 0 \\ -12.000 \\ 0 \\ -5.700 \\ 0 \\ -3.650 \\ 0 \\ 450 \\ 0 \\ 450 \\ \end{array}$	•64 2•0 6•75 0 -28•0000 0 -10•0000 0 -5•3500 0 -3•3250 0 -1•5000 0 -•2250	
+2 	$\begin{array}{c} -21 \\ 1 \cdot 0 \\ 2 \cdot 5 \\ 11 \cdot 0 \\ 65 \cdot \\ 0 \\ -56 \cdot 000 \\ -20 \cdot 000 \\ 0 \\ -7 \cdot 000 \\ 0 \\ -4 \cdot 650 \\ 0 \\ -2 \cdot 675 \\ 0 \\ -1 \cdot 100 \\ 0 \\ -225 \end{array}$	$\begin{array}{c} 0 & 0 \\ 1 & 25 \\ 3 & 0 \\ 13 & 5 \\ 0 & -45 & 000 \\ 0 & -16 & 000 \\ 0 & -6 & 500 \\ 0 & -6 & 500 \\ 0 & -4 & 300 \\ 0 & -2 & 350 \\ 0 & -900 \\ 0 & -450 \end{array}$	$\begin{array}{r} \bullet 21 \\ 1 \bullet 5 \\ 4 \bullet 0 \\ 16 \bullet 5 \\ 0 & -14 \bullet 0 0 0 \\ 0 & -14 \bullet 0 0 0 \\ 0 & -6 \bullet 0 5 0 \\ 0 & -3 \bullet 975 \\ 0 & -2 \bullet 0 25 \\ 0 & -675 \\ 0 & -675 \\ \end{array}$	$\begin{array}{c} .42 \\ $	•64 2•0 6•75 0 -28•0000 0 -10•0000 0 -5•3500 0 -3•3250 0 -1•5000 0 -•2250 0 1•2000	
+2 	$\begin{array}{c} -21 \\ 1 \cdot 0 \\ 2 \cdot 5 \\ 11 \cdot 0 \\ 65 \cdot \\ 0 \\ -56 \cdot 000 \\ -20 \cdot 000 \\ 0 \\ -7 \cdot 000 \\ 0 \\ -4 \cdot 650 \\ 0 \\ -2 \cdot 675 \\ 0 \\ -1 \cdot 100 \\ 0 \\ -225 \\ 0 \\ 1 \cdot 800 \end{array}$	$\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 0 \\ 1 \\ 3 \\ 5 \\ 0 \\ -4 \\ 3 \\ 0 \\ -6 \\ 5 \\ 0 \\ -6 \\ 5 \\ 0 \\ -6 \\ 5 \\ 0 \\ -2 \\ 3 \\ 0 \\ 0 \\ -9 \\ 0 \\ 0 \\ -9 \\ 0 \\ 0 \\ -9 \\ 0 \\ 0 \\ 2 \\ -100 \end{array}$	$\begin{array}{c} \bullet 21 \\ 1 \bullet 5 \\ 4 \bullet 0 \\ 16 \bullet 5 \\ 0 & -14 \bullet 0 0 0 \\ 0 & -14 \bullet 0 0 0 \\ 0 & -6 \bullet 0 5 0 \\ 0 & -3 \bullet 975 \\ 0 & -2 \bullet 0 575 \\ 0 & -675 \\ 0 & -675 \\ 0 & -675 \\ 0 & 2 \bullet 500 \\ \end{array}$	$\begin{array}{c} .42 \\ $	•64 2•0 6•75 0 -28•0000 0 -10•0000 0 -5•3500 0 -3•3250 0 -1•5000 0 -•2250 0 -2250 0 3•5000	
62 .86 2.25 9.0 2MESH 72.0000 -24.0000 -3.0000 -3.0000 0.0000 1.5000 4.00000	$\begin{array}{c} -21 \\ 1 \cdot 0 \\ 2 \cdot 5 \\ 11 \cdot 0 \\ 65 \cdot \\ 0 -56 \cdot 000 \\ 0 -20 \cdot 000 \\ 0 -4 \cdot 650 \\ 0 -2 \cdot 675 \\ 0 -1 \cdot 100 \\ 0 -225 \\ 1 \cdot 800 \\ 0 -5 \cdot 000 \end{array}$	$\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 0 \\ 1 \\ 3 \\ 0 \\ 1 \\ 3 \\ 5 \\ 0 \\ -4 \\ 3 \\ 0 \\ 0 \\ -6 \\ 5 \\ 0 \\ 0 \\ -2 \\ 3 \\ 0 \\ 0 \\ -9 \\ 0 \\ 0 \\ -9 \\ 0 \\ 0 \\ -9 \\ 0 \\ 0 \\ -9 \\ 0 \\ 0 \\ -9 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ $	$\begin{array}{c} \bullet 21 \\ 1 \bullet 5 \\ 4 \bullet 0 \\ 16 \bullet 5 \\ 0 & -14 \bullet 0000 \\ 0 & -14 \bullet 0000 \\ 0 & -6 \bullet 0500 \\ 0 & -3 \bullet 9750 \\ 0 & -2 \bullet 02750 \\ 0 & -2 \bullet 02750 \\ 0 & -6750 \\ 0 & -6750 \\ 0 & 2 \bullet 5000 \\ 0 & 7 \bullet 0000 \\ 0 & 0 & 0000 \end{array}$	$\begin{array}{c} .42 \\ $	•64 2•0 6•75 0 -28•0000 0 -10•0000 0 -5•3500 0 -3•3250 0 -1•5000 0 -2250 0 1•2000 0 3•5000 0 10•0000	
	$\begin{array}{c} -21 \\ 1 \cdot 0 \\ 2 \cdot 5 \\ 11 \cdot 0 \\ 65 \cdot \\ 0 -56 \cdot 000 \\ 0 -20 \cdot 000 \\ 0 -7 \cdot 000 \\ 0 -4 \cdot 650 \\ 0 -2 \cdot 675 \\ 0 -1 \cdot 100 \\ 0 -225 \\ 0 -1 \cdot 800 \\ 0 -5 \cdot 000 \\ 1 + 600 \\ 0 + 0 \cdot 600 \end{array}$	$\begin{array}{c} 0 & 0 \\ 1 & 25 \\ 3 & 0 \\ 13 & 5 \\ 0 & -16 & 0 & 0 \\ 0 & -16 & 0 & 0 \\ 0 & -6 & 5 & 0 \\ 0 & -2 & 35 & 0 \\ 0 & -2 & 35 & 0 \\ 0 & -2 & 35 & 0 \\ 0 & -4 & -5 & 0 \\ 0 & -4 & -5 & 0 \\ 0 & -$	$\begin{array}{c} \bullet 21 \\ 1 \bullet 5 \\ 4 \bullet 0 \\ 16 \bullet 5 \\ 0 & -14 \bullet 000 \\ 0 & -14 \bullet 000 \\ 0 & -3 \bullet 975 \\ 0 & -2 \bullet 025 \\ 0 & -2 \bullet 025 \\ 0 & -675 \\ 0 & -675 \\ 0 & -675 \\ 0 & -675 \\ 0 & -675 \\ 0 & -675 \\ 0 & -7 \bullet 000 \\ 0 & 20 \bullet 000 \\ 0 & 56 \bullet 000 \\ \end{array}$	$\begin{array}{c} .42 \\ $	•64 2.0 6.75 0 -28.0000 0 -10.0000 0 -5.3500 0 -3.3250 0 -1.5000 02250 0 1.2000 0 3.5000 0 3.5000 0 28.0000	
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Figure 13. Part of Input File for Wing-Pylon-Nacelle Geometry

5.2 INPUT RECOMMENDATIONS

Mesh selection is probably the most important step in obtaining accurate flowfield predictions for a given analysis problem. There are two primary reasons for this situation. The first reason is the limitation on the maximum number of grid lines and surface-grid intersects due to computer memory size and the way the analysis was coded (see Section 5.1.3). In theory, as the mesh spacing becomes very small, the solution of the difference equations should approach the solution of the partial differential equations. Thus, in the limit, as the mesh spacing becomes very small, the solution is independent of mesh. Unfortunately, for most problems of interest this limit is of no consequence because the meshes that can be used are very coarse, in fact barely adequate in many cases.

A second consideration in mesh selection is convergence rate. Solutions using uniform meshes converge fastest with least possibility of instability. However, for most problems, given the limitation of maximum number of grid lines, uniform meshes give unacceptable accuracy. Hence, grid lines must be packed in areas of interest (i.e., rapid flow changes) in order to resolve the flowfield. The mesh can be sparser far from the body where the flow properties vary slowly with position. This packing has to be accomplished with discretion or the solution can fail to converge.

The following discussion can be used as a guide for mesh placement.

Ground rules for mesh selection are:

- Sufficient mesh are required to resolve the geometry and flowfield in critical regions (fig. 14).
- o Mesh spacing should vary gradually from one region to another (fig. 15). Adjacent grid spacings should differ by no more than a factor of two within any mesh level.
- o Mesh aspect ratios, $\Delta x/\Delta y$, $\Delta x/\Delta z$, etc., should be approximately one in important flow regions. This becomes even more important when the flow is transonic.
- o The edges of the computational volume must be located far enough from the bodies that they do not greatly affect the flowfield. That is, the flow in the vicinity of the edges of the computational volume can be reasonably approximated by freestream conditions.

As an example, a nondimensional mesh for calculation of the flowfield around a typical turbofan engine inlet is shown in Figure 16 and tabulated in Table 2. What is shown is the coarsest (level 2) of a sequence of three meshes. The fine mesh (level 4) is generated by adding 3 mesh equally spaced between each of the given mesh.

This mesh is a compromise between cost of a computation and accuracy. It is just adequate for making the calculation. It is safest to make the calculations with the number of theta mesh held constant for all three levels. Otherwise, for sample inlet calculations, there have been problems with the multigrid convergence.

It has been found during limited numerical experiments that for wing calculations it takes a minimum of 20 to 30 grid in the chordwise direction across a wing to obtain any kind of reasonable (though not extremely accurate) results.

Selection of mesh for other configurations is a difficult problem. The best advice would be to try several meshes and/or compare with experiments for similar configurations.



Figure 14. Mesh Density Required to Resolve Geometry Effects on Flowfield



(a) Poor Spacing



(b) Recommended Mesh Spacing Variation

Figure 15. Mesh Spacing Control

-



Figure 16. Standard Inlet Mesh (Coarse)

	x/L		r/R		θ (degrees)
1	-4.0	1	0.0	1	0
2	-3.0	2	0.2	2	22.5
3	-2.0	3	0.4	3	45.0
4	-1.4	4	0.6	4	67.5
5	-0.8	5	0.8	5	90.0
6	-0.4	6	0.9	6	112.5
7	-0.2	7	1.0	7	135.0
8	-0.1	8	1.2	8	157.5
9	0.0	9	1.6	9	180.0
10	0.1	10	2.4	10	202.5
11	0.2	11	4.0	11	225.0
12	0.4	12	7.0	12	247.5
13	0.6	13	10.0	13	270.0
14	0.8	l l		14	292.5
15	1.0			15	315.0
16	1.2			16	337.5
17	1.6	1			
18	2.0		•		

Table 2. Standard Inlet Mesh (Level 2)

The program can make calculations using up to four levels. Every lower level is generated from the next higher level by deleting every other mesh line. The exception is for cylindrical coordinate computations where the theta (circumferential) grid can be held constant between levels. This option exists because of multigrid convergence problems (ref. 4) that have been noted with some geometries for cylindrical coordinates.

Multiple levels are used with multigrid to give orders of magnitude decreases in run costs by faster and less expensive convergence. In theory it is best to use many levels. In practice, problems occur at the lower (coarser mesh) levels because the mesh can become very sparse near the body(s) and fail to resolve geometry features. For problems that use only about 50 000 to 70 000 grid intersects, three levels are all that should be used. If problems with convergence seem to be occurring between levels 2 and 3, then two levels may be used.

The code is most stable when the flow is subsonic. More problems have been noticed for transonic flow and the possibility of problems increases with the maximum Mach numbers. The code usually works for transonic flows, but occasionally it will fail. If it fails, recommendations include:

- o Check that a physically realistic problem is being run.
- o Lower the freestream or compressor face Mach number if possible.
- o Try an alternate grid with a squarer mesh in the region of difficulty.
- o Experiment by moving mesh around.

The surface geometry must be complete (closed) or end on a computational boundary. That is, a surface cannot just end in the middle of the flowfield. A closed surface such as a wing or sphere is acceptable. Computational boundaries are defined by the compressor face and the smallest and largest value for each mesh. Geometry cannot extend beyond or through the computational boundaries.

An inlet diffuser geometry must end at the compressor face. (This does not have to be the physical compressor face; the diffuser can be shortened or extended for computational purposes.)

5.3 PROGRAM OUTPUT

The computer code calculates the potential function and, hence, all flow properties at all nodes. These include mesh nodes as well as all intersections of the mesh with the surface. The quantity of possible output is very large, especially for a dense mesh. What is printed is controlled by the user with the input print options.

The default output includes headings, listings of many of the program input parameters, and lists of the options that have been selected. The actual mesh and number of surface points are listed for each level. If the geometry has a duct, an area table for the duct is printed. If a lifting surface calculation is being made, tables are printed of the geometry information for the trailing edge of the wing as determined by the program. The default output includes the complete iteration history of the run including timing information. After convergence is obtained or an iteration limit is reached, a mass flow table is printed if there is a duct in the flowfield. This table is a check on the solution. The computed mass flow at a cross section is compared to the value being enforced as the boundary condition at the duct exit. Next is the surface properties printed for every fourth cut of the surface, x constant cuts first followed by y (r) constant and $z(\theta)$ constant cuts. Last is a summary of the convergence history.

A variety of additional output can be requested. A listing of the input surface coordinate and normal values can be obtained. This list can be either ordered as input or in the internal ordering the computer code uses. Lists can be obtained of surface points that are special cases. These include surface nodes that are also mesh nodes and pairs of surface points adjacent to the same field point in the same coordinate. The internal program maps can also be obtained. These are lists of surface points in order along constant coordinate cuts of the surface and include the program calculated arc length. The user has some control of the density of surface properties output. Printout of the flowfield solution can be requested. Surface properties can be requested to be printed in terms of an alternate coordinate system.

5.3.1 DESCRIPTION OF PRINTED OUTPUT

Much of the printed output from the code is self explanatory. Tables 3 through 7 define some of the quantities in the printout.

5.3.2 SAMPLE OUTPUT

The sample output presented in Figure 17 is for a two-dimensional airfoil, NACA-0012, run as a test case for checking the Kutta condition implementation in the code. This is a very simple geometry, but it makes a good example for the program output. Only a small repesentative selection is presented. This run was made on the Cyber 203 at the NASA-Langley Research Center. The answers are the same using the CRAY-1. However, the timing and cost information on the Sweeping History table would be different.

5.4 DIAGNOSTICS AND TROUBLESHOOTING

Many factors can cause difficulties with a computation. If no results are obtained, it is obvious there is a problem. In many ways, a more serious problem occurs when results are obtained that, to casual observation, appear reasonable but are, in fact, incorrect. Errors include bad inputs, inputs inconsistent with code limitations, logic errors in the code, poor choice of mesh, and wrong flow model, to name some possibilities. Some possible problems will be discussed and some of the code diagnostics are listed and explained. Their order is approximately that in which items are processed in the code.

The best way to be sure that the program is providing acceptable answers is to use the code on similar problems for which experimental results are available for comparison. If this is not possible, then the next best alternatives are as follows. Prepare sample plots of cross sections of the geometry with the mesh to be used. They should look reasonable. (See the discussion in Section 5.2 and Figures 14 and 15.) If they do not look reasonable, computed results will probably be poor. A table of computed mass flow is printed for ducts. This table is generated from the solution and is a check of the solution. Typical errors (leakage) are of the order of 1 to 2%. Much greater errors indicate an inadequate mesh and/or poor convergence. Another check is to see how quickly the solution changes relative to the mesh. Very large changes in velocity or flow direction between two mesh lines are an indication that a denser mesh is required.

5.4.1 INCORRECT INPUT OTHER THAN GEOMETRY

The first thing done by the program is to read the input cards. These are processed group by group. If the input is not formatted correctly the program will get "out of step" and expect a keyword where there is none. The message

Table 3. Headings for Surface Point and Surface Geometry Printout

INDEX X Y or R Z or THETA S	Surface point index x y or r z or θ s, arc length along cut
NX NY or NR NZ or NT I J K	n_x , component of unit surface normal n_y or n_r , component of unit surface normal n_z or n_{θ} , component of unit surface normal indices of the mesh lines defining the field point adjacent to the surface point
SÚRFARM	Δx , Δy , Δr , Δz or $\Delta \theta$ between surface point and adjacent field point $\begin{cases} -1 = x \text{ intersect} \\ 0 = z \text{ or theta intersect} \end{cases}$
IYPE	1 = y or r intersect 2 = both mesh and surface node

Table 4. Convergence History Headings

SWEEP **FIELD POINTS**		Relaxation sweep number, (n)
AVE RESIDUE		Sum of $ \phi_{i,j,k}^{(n)} - \phi_{i,j,k}^{(n-1)} $ over all points i, j,k in the flowfield divided by $\phi_{\max}^{(n)} - \phi_{\min}^{(n)}$ and the number of such field points
I J K MAX RESIDUE "CONVERGING/ DIVERGING"	{	The indices i, j,k of the field point having the maximum change in ϕ and the maximum value of $ \phi_{i,j,k}^{(n)} - \phi_{i,j,k}^{(n-1)} $ divided by $\phi_{max}^{(n)} - \phi_{min}^{(n)}$ ***, MAX RESIDUE decreasing, or ****, MAX RESIDUE increasing
SURFACE POINTS		
AVE RESIDUE		Sum of $ \phi_S^{(n)} - \phi_S^{(n-1)} $ over all surface points S divided by $\phi_{max}^{(n)} - \phi_{min}^{(n)}$ and the number of surface points
I J K INDEX MAX RESIDUE M > 1.0		The mesh indices i, j,k of the field point next to the surface point with the maximum change in ϕ , the index of that surface point, and the maximum value of $ \phi_{\rm S}^{(n)} - \phi_{\rm S}^{(n-1)} $ divided by $\phi_{\rm max}^{(n)} - \phi_{\rm min}^{(n)}$ number of field points for which Mach number > 1.0
EIGEN1 EIGEN2 "EXTRAPOLATION FLAG"		$1/(1 - \lambda_1)$, see ref. 1 $1/(1 - \lambda_2)$, see ref. 1 * indicates flowfield extrapolation was made after this sweep

INDEX	Surface point number
x	x
Y or R	y or r
Z or THETA	z or θ
S	s, arc length along the cut of the surface
масн	Mach Number
СР	C_p , coefficient of pressure, $(p/p_{\infty}-1)/(\frac{1}{2}\gamma M_{\infty}^2)$ (if $M_{\infty} = 0.0$, then $C_p = (p/p^*-1)/(\frac{1}{2}\gamma)$, where the * indicates sonic conditions)
РНІ	ϕ , potential function
Q	$q = (u^{2} + v^{2} + w^{2})^{\frac{1}{2}} = (u^{2} + u_{r}^{2} + u_{\theta}^{2})^{\frac{1}{2}}$
PHI,S	u _s , component of velocity along the cut
U	u, axial velocity component
V or U-RADIAL	v or u _r velocity component
W or U-THETA	w or u_{θ} , velocity component

Table 5. Surface Point Printout Headings

Table 6. Field Point Printout Headings

Y or R	y or r
МАСН	Mach Number
PHI	ϕ , potential function
СР	C_p , coefficient of pressure, $(p/p_{\infty}-1)/(1/2\gamma M_{\infty}^2)$ (if $M_{\infty} = 0.0$, then $C_p = (p/p^*-1)/(1/2\gamma)$, where the * indicates sonic conditions)
U	u, axial velocity component
v	v, y component of velocity
w	w, z component of velocity
U-RADIAL	u _r radial velocity component
U-THETA	u_{θ} , circumferential velocity component

Table 7. Headings for Kutta Boundary Condition Printout

Ν	Jump number
SURFACE	Surface number
SEQ. NO.	Sequence number of jump in order along cut
I	Index of last $x = constant plane to cut trailing edge$
J	Index of $y = constant plane that cut is in$
JJUMP	Index of first y mesh above trailing edge
к	Index of $z = constant plane that cut is in$
KJUMP	Index of first z mesh above trailing edge
NDEXL	Index of last surface point below the body on this cut which is not an x intersect
NDEXU	Index of last surface point above the body on this cut which is not an x intersect
DELPHI	$\Delta \phi = \Gamma$ at trailing edge
DPHICL	Calculated value of Γ at trailing edge (used in multigrid cycling)
ІТҮРСЈ	= 0 Γ calculated directly = n Γ interpolated from parameters on line n of interpolation table
JUMP INTERPOLATIO	N INFORMATION
N	Interpolation index
ITPIJP1 ITPIJP2 NDEXJP1 NDEXJP2 ETAJMP1 ETAJMP2	$\Delta \phi = \text{ETAJ MP1}^* \Gamma_1 + \text{ETAJ MP2}^* \Gamma_2$ where Γ_i is Γ of jump NDEXJPi where NDEXJPi is jump number. NDEXJPi refers to y constant cut table if ITPIJPi = 2, z constant table if ITPIJPi = 1.

********* P 4 6 5 C - THREE-DIMENSIONAL TRANSCRIC POTENTIAL FLOW PROGRAM

VERSION COD - OCTOBER 11, 1983

RUN DATE - 10/25/83

ABSTRACT -

THIS IS A COMPUTER PROGRAM WRITTEN FOR THE CONTROL DATA CORPORATION CYBER 203 COMPUTER. ITS PURPOSE IS THE COMPUTATION OF TRANSONIC POTENTIAL FLOW ABOUT THREE DIMENSIONAL INLETS, DUCTS AND BODIES. IT IS PROGRAMMED IN CYBER 203 EXTENDED FORTRAN IV.

REFERENCES -

REYHNER. T. A... "TRANSUNIC POTENTIAL FLOW COMPUTATION ABOUT THREE-DIMENSIONAL INLETS. DUCTS AND BODIES." ATAA JOURNAL, VOL. 19, SEPTEMBER 1981, PP. 1112-1121.

REYHNER, T. A., COMPUTATION OF TRANSCNIC POTENTIAL FLOW ABOUT THREE-DIMENSIONAL INLETS, DUCTS, AND BODIES, NASA CR-3514, HARCH 1982

PROPRIETARY NOTICE -

and a second of the content of the c 🔺 THE COMPUTER PROGRAM, P465 - VERSION C, IS THE SOLE PROPERTY OF THE BOEING COMMERCIAL AIRPLANE * COMPANY UNTIL JANUARY 1, 1986 DURING WHICH TIME NASA (THE NATIONAL AERONAUTICS AND SPACE AUMINISTRATION) HAS RIGHTS OF USE *****-

VERSION C -

and a second VERY FINE MESH KUTTA CONDITION BOUNDARY CONDITION Y OR Z LINE RELAXATION 4 LEVEL CALCULATIONS VERY COARSE GRID CAPABILITY

CONSULTATION -

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R. G. JORSTAD	BOEING COMPUTER SERVICES, INC.	(206) 656-5745
D. E. REUBUSH	NASA LANGLEY RESEARCH-CENTER	(804) 865- 2675

RUN TITLE -

NACA 0012 AIRFOIL WING ALONE FINE MESH SEPT 21, 1982 M INF = 0.75 ALPHA-=1.0-DEGREES-

(a) Sample Output

.....

THE FLOWFIELD PARAMETERS ARE -

A INF	=	1.00000	
Q INF	=	•75000	
ALPHAL - ARCTAN(V	INF/U INF) =	•000	DEGREES
ALPHA2 - ARCTANCH	INF/U INF) =	1.000	DEGREES
M INF	=	•750	
U INF	=	• 74989	
V INF	2	-00000	
W INF	=	•01309	

WING OR WING LIKE GEOMETRY HAS BEEN SPECIFIED

MESH AND CONVERGENCE PARAMETERS -

LEVEL NUMBER	2	3	4
NX	17	33	65
NY	3	5	9
NZ	16	31	61
MAXIMUM NUMBER OF SWEEPS	800	200	60
CONVERGENCE TEST VALUES +(10++6)	.1000	.1000	.1000
MAX. NUMBER OF SWEEPS PER VISIT	100	20	20
MAXIMUM NUMBER OF VISITS	10	20	•

ALTERNATING DIRECTION LINE RELAXATION (ADI) USED - ALTERNATING BETWEEN Y AND Z

LINES.

MULTIGRID PROCEDURE WILL BE USED.

CARTESIAN COORDINATES WILL BE USED.

SURFACE FLOW PROPERTIES PRINT REQUESTED -

X CONSTANT	CUTS	x=x(I)	(I-1)	DIVISIBLE	BY	1
Y CONSTANT	CUTS	¥=¥(J)	(J-1)	OIVISIBLE	8Y	1
Z CONSTANT	cuts	2=Z(K)	(K-1)	DIVISIBLE	BY	1

(b) Sample Output (Continued)

PRINTOUT REQUESTED ON INTERMEDIATE LEVELS.

THERE ARE 495 SURFACE POINTS. THE LIMIT IS 8500

MESH -

iπ •

NX+NY+NZ =	35685	THIS	DATA	CASE	USES	35685	OF THE	AVAILABLE	252105.	FIELD	POINTS
NY+NZ -= ···	549	THE (LIMIT	-15	+250						
NX+NY =	585	THE I	LIMIT	IS	8500						
NX+NZ =	3965	THE I	LIMIT	IS	8500				•••••		

11	-3.0000	221	a-0750		7600
11	-310000	<i>c21</i>	-40/30	737	• / 50 0
2)	-2.7500	23)	-+0500	44)	-8000
3)	-2.5000	24)	0250	45)	•8500
43	-2.2500	25)	•0000	46)	•8844
5)	-2.0000	26)	.0250	47)	•9188
6)	-1.7500	27)	.0500	48)	•9531
7)	-1.5000	28)	e0750	49)	♦9875
8)	-1.2500	29)	-1000	50)	1.0125
9)	-1.0000	30)	•1375	51)	1.0375
10)	8500	31)	. 1750	52)	1.0625
11)	7000	32)	•2125	53)	1.0875
12)	5500	33)	•2500	54)	1.1031
13)	4000	34)	-3000	55)	1.1188
14)	3500	35)	•3500	56)	1.1344
15)	3000	36)	-4000	57)	1.1500
16)	2500	37)	.4500	58)	1.1075
17)	2000	38)	-5000	59)	1.2250
18)	1750	39)	.5500	60)	1.2625
19)	1500	40)	-6000	61)	1.3000
20)	1250	41)		62)	1.3750
21)	1000	42)	.7000	63)	1.4500
				64)	1.5250
				651	1.6000
					¥ = 0 0 0 0

(c) Sample Output (Continued)

********* ********* ¥ MESH ******** *********

1)	2000	4)	0500	7)	.1000
2)	1500	5)	-0000	8)	•1500
3)	1000	6)	•0500	9)	•2000

********	*
******* Z MESH	******
	▲

INPUT PROCESSING COMPLETED

THE FOLLOWING SURFACE POINTS ARE NOT ADJACENT TO FIELD POINTS

. -----

.

INDEX	· ··· X ·· · ····	ү	· ···Z · · · ·	NX	- · · · NY	NZ	TYPE
19	.000000	200000	.000000	-1.00000000	.00000000	.00000000	0
20	-000000	150000	.000000	-1.00000000	.00000000	.00000000	0
21	.000000	100000	.000000	-1.00000000	.00000000	.00000000	0
22	.000000	050000	.000000	-1.00000000	•00000000	.00000000	0
23	- 000000		.000000-	1-000000000	.00000000	.00000000	0
24	- 000000	.050000	.000000	-1.00000000	.00000000	.00000000	0
25	-000000	-100000	.000000	-1-00000000	.00000000	.00000000	0
26	- 000000	-150000	-000000	-1.00000000	.000000000	.000000000	0
27	• 000000	.200000	.000000	-1.00000000	.00000000	+00000000	0

(d) Sample Output (Continued)

KUTTA CONDITION JUMP INFORMATION -

. JUMPS LYING IN Y CONSTANT CUTS . . 2 . . ---- 0 -- Ð-49-----2 479 ---Ð 0 49 0 49 4 31 0 49 0 49 0 49 . 5 6 7 Ð 5 31 6 31 7 31 475 0 49

(e) Kutta Boundary Condition Geometry Table

Figure 17. Sample Program Output

CONVE	RGENCE HISTO	R¥		LEVE	L NUMBER 4	VIS	SIT NUMBER 3	•			-				
SWEEP	AVE RESIDUE	FIELI I) J	OINTS. K	MAX RESIDUE		AVE RESIDUE	∎∎ SUR I	FACI J	E P(K	DINTS Index	MAX RESIDUE	H>1	EIGEN1	EIGEN2
532	• 20 65 7E-06	34	1	+1	67916E-05		+++0275-06	-34 -	1	33	190	59000E-05	378	•10000E+01	•10000E+01
533	•73068E-07	34	5	33	•61222E-05	**	•45603E-06	34	5	33	194	•51122E-05	378	.10000E+01	•10000E+01
534	•12337E-06	34	1	35	•61124E-05	**	•27221E-06	33	1	33	172	•42597E-05	378	•10000E+01	•10000E+01
535	•67958E-07	34	5	33	•62691E-05	***	•36198E-06	34	5	33	194	•54313E-05	378	•31765E+01	•51287E-01
536	•11635E-06	34	1	34	•63610E-05	***	•22088E -06	34	1	33	190	•57858E-05	378	•14594E+02	30942E+02
537	•63708E-07	34	5	33	•6064 3E=05	**	•30031E-06	34	5	33	194	•52171E-05	378	•15583E+02	-•28264E+03
CONVER	GENCE CRITE	RION	SAT	ISFIED	ON LEVEL 4	VIS	LT 3								

(f) Sample of Convergence History

KUTTA CONDITION JUMP INFORMATION -

JUMPS LYING IN Y CONSTANT CUTS

N	SURFACE	SE Q.	NO.	I	j.	KJUMP	NDEXL	NDEXU	ITYPCJ	DELPHI	DPHICL	DPHICR
1	1		1	49	1	31	469	478	0	•064593	•064593	-000000
2	1		2	49	2	31	470	479	0	•064593	.064593	.000000
3	1		3	49	3	31	471	480	0	•064593	.064593	-000000
4	1		4	49	4	31	472	481	0	•064592	+064592	.000000
5	1		5	49	5	31	473	482	0	.064592	.064592	.000000
6	1		6	49	6	31	474	483	0	.064592	.064592	.000000
7	1		7	49	1	31	475	484	0	+064593	.064593	.000000
8	1		8	49	8	31	476	485	0	•064593	•064593	.000000
9	1		9	49	9	31	477	486	0	•064593	.064593	.000000

(g) Kutta Boundary Condition Γ Table

INDEX	x	Y	z	S	HACH	CP	PHI	Q	PHI,S	U	۷	u
469	.9875	2000	0030	.0000	.6222	.3014	3.0156	•6322	6322	• 6263	.0002	•0866
451	•9531	2000	0077	.0347	• 6680	•1950	2.9929	•6751	6751	• 6692	0001	.0888
433	• 9188	2000	0125	• 0694	•7049	.1078	2.9688	•7091	7091	•7034	0000	•0895
415	.8844	2000	0164	1040	•7285	•0515	2.9438	•7306	7306	• 7252	.0000	•0896
397	.8500	2000	0205	•1386	•7464	•0086	2.9182	•7468	7468	•7417	•0000	.0871
379	.8000	2000	0262	•1889	• 7684	-+0440	2.8801	.7664	-•7664	•7618	0000	•0844
361	•7500	2000	0316	• 2392	.7879	0910	2.8411	• 7838	7838	• 7796	0000	.0812
343	•7000	2000	0366	• 2895	• 8061	1347	2.8013	•7998	7998	• 7961	0000	.0774
325	.6911	2000	0375	•2984	.8092	1422	2.7941	•8026	8026	• 7989	0000	•0767
307	•6500	2000	0413	• 3397	.8235	1766	2.7607	•8151	8151	•8118	0000	•0731
289	•6000	2000	0456	• 3899	.8411	2187	2.7194	+8304	8304	8276	0000	• 0680
271	.5500	2000	0495	• 4400	• 8593	2621	2.6774	• 8460	8460	•8437	0000	•0619
253	.5000	2000	0529	• 4902	• 8782	3072	2.6346	•8622	8622	•8604	0000	•0543
235	•4500	2000	0558	• 5402	.8979	3540	2.5910	• 8789	8789	.8777	0000	•0450
217	- 4000	2000	0580	• 5903	•9186	-+4028	2.5466	8962	-+8962	.8956	0000	.0334
199	.3500	2000	0595	•6403	•9400	-+4529	2.5013	•9140	9140	•9138	0000	•0188
181	• 3000	2000	0600	.6903	.9604	5003	2.4552	.9307	9307	•9307	0000	0000
163	•2500	2000	0594	• 7403	• 9766	-• 5376	2.4082	• 94 39	9439	• 9436	0000	0239
145	•2125	2000	0580	• 7778	• 9829	5522	2.3727	•9491	9491	• 9479	0000	0463
127	1750	2000	0557	.8154	•9812	-•5482	2.3370	•9477	9477	•9448	0000	0735
109	.1375	2000	0521	•8531	•9672	5160	2.3014	• 9363	9363	•9301	0000	1072
91	.1000	2000	- • C46 8	.8910	. 5370	4460	2.2662	•9115	9115	• 8988	0000	1519
73	.0750	2000	0420	.9164	•9039	-•3680	2+2433	• 8839	8839	•8631	0000	-+1903
37	• 0567	2000	0375	•9353	.8602	2644	2.2269	• 8468	8468	•8163	.0001	2255
55	.0500	2000	0356	• 9422	.8314	1955	2.2210	• 8220	8220	• 7867	.0001	2382
19	•0250	2000	0262	•9690	•6543	•2269	2.2003	•6624	-+6624	• 5983	.0003	2842
1	.0053	2000	0125	• 9932	.2353	1.0091	2.1867	•2469	2469	•1604	0001	-•1976
10	.0053	2000	.0125	1.0200	•4023	•7595	2.1895	• 4176	•4176	•2714	0003	• 3174
28	•0250	2000	.0262	1.0441	.8046	1310	2.2064	• 7985	• 7985	• 7213	.0003	• 3426
64	•0500	2000	.0356	1.0709	• 9908	5702	2.2307	• 9554	• 955 4	• 9145	.0002	• 2768
46	•0567	2000	.0375	1.0778	1.0220	6409	2.2375	•9804	•980 +	• • 9450	•0002	•2611
82	.0750	2000	.0+20	1.0967	1.0691	-•7455	2.2564	1.0174	1.0174	• 9935	.0001	•2191
100	.1000	2000	•0468	1.1221	1.0991	8102	2.2827	1.0404	1.0404	1.0258	•0001	•1733
118	•1375	-•2000	•0521	1.1600	1.1273	-•8701	2.3226	1.0617	1.0617	1.0547	.0002	• 1215
136	•1750	2000	•0557	1.1977	1+1495	-•9164	2.3629	1.0783	1.0783	1.0751	•0001	• 0836
154	•2125	2000	.0580	1.2353	1.1658	9498	2.4037	1.0903	1.0903	1.0890	0002	•0531
172	.2500	2000	.0594	1.2728	1.1739	-•9664	2.4448	1.0963	1.0963	1.0959	.0002	•0278
190	•3000	2000	•0600	1.3228	1.1566	-• 9351	2.4997	1.0850	1.0850	1.0850	.0007	• 0000
208	•3500	2000	•0595	1.3728	1.0574	7196	2.5533	1.0082	1.0082	1.0080	0001	0207
226	• 4000	2000	•0580	1.4228	•9663	֥5139	2.6006	•9356	•9356	•9349	0000	0349
244	+4500	2000	•0558	1.4729	.9414	4562	2.6469	•9152	•9152	•9140	0000	0468
262	•5000	2000	•0529	1.5230	•9148	-•3939	2.6922	• 8931	• 8931·	•8913	• 0000	0563
280	• 5500	2000	•0495	1.5731	.8900	 3353	2.7364	•8722	•8722	•8699	•0000	0638
298	•6000	2000	• 04 56	1.6232	.8670	2804	2.7797	.8526	•8526	. 8497	.0000	0698
316	•6500	-•2000	.0413	1+6734	8452	2285	2.8220	• 83 39	• 83 3 9	•8305	•0000	0747
334	.6911	2000	• 03/5	1.7147	.8278	-•1869	2.8561	• 81 89	•8189	•8151	-0000	-+0783
352	• 7000	2000	•0366	1.7236	•8241	1779	2.8634	•8156	.8156	•8118	0000	0790
370	•7500	2000	.0316	1.7739	.8027	1266	2.9039	• 7969	• 7969	• 7926	0000	0825
388	.8000	2000	.0262	1.8242	•7802	0726	2.9435	•7770	• 7770	•7723	0000	0356
406	.8500	2000	•0205	1+8745	• 7556	0133	2.9821	•7550	•7550	•7498	0000	0881
424	.8844	2000	• 0164	1.9091	• 7358	•0340	3.0080	• 7372	•7372	•7318	0000	0894
442	•9188	-+2000	.0125	1.9438	•7103	•0949	3.0332	•7140	.7140	•7083	0000	0901
460	•9531	2000	•0077	1.9784	•6713	•1871	3+0574	•6782	•6782	•6723	0001	0892
478	•9875	2000	.0030	2.0131	.6234	.2987	3.0802	•6333	•6333	•6274	•0002	0867

2.0131 .6234 .2987 3.0802 .63 (h) Sample of Surface Properties Printout

Figure 17. Sample Program Output

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SURFACE FLOW PROPERTIES -

Y = CONSTANT CUTS

	• •						
	· ***** SWEEPING HISTORY - ******						
	• •						

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LEVEL NO•	VISIT NO.	INITIAL SWEEP NO.	FINAL SWEEP NO.	TOTAL Sweeps	++ CHANGE II + AVER INITIAL	N PHI BETWEEN Age + Final	SWEEPS X (+ Maxi Initial	10++6) ++ Mum + Final	M>1	EXTRAP.	SECONDS PER Sweep	OVERHEAD Seconds	WORK Units
2	1	1	100	100	43.1879	113.3141	-1096.8747	4703.2465	5	2	•071	1.881	8.659
3	1	101	120	20	- 100.1466	12.0684	1839•3208	-212.3870	40	0	•249	•130	4.899
2	2	121	220	100	76.2857	1.5660	408.4872	-16-1846	6	3	.074	• 328	7.413
3	2	221	236	16	13.5406	2.3926	-233.3536	-68.2228	55	0	•250	•143	3.981
4	- 1	237	2 56	20	2+9514	♦9189		-26.0873	378	. 0	1.037	• 455	20.348
3	3	257	267	11	2.8198	3,2064	30.8287	22.2553	55	0	•255	1.182	3.823
2	3	268	367	100	10.6248	.8034	-57.2768	-7.9568	6	2	.074	• 337	7.423
3	4		383	16	2.2223	. 3438	21.6036	7.3227	55	0	•254	-143	4.034
4	2	384	403	20	• 6295	•1873	-32.1730	-7.3376	387	0	1.038	• 476	20.395
3	5	404	411		.6364	.4359	-3.7587	5.7691	55	0	•255	1.188	3.097
2	•	412	511	100	2.5285	. 4190-	-9.7500	5.5537	6	3	.074	• 329	7•416
3	6	512	531	20	1.0744	.1037	-9.0064	-1.5826	55	0	•253	•143	5.005
4	3	532	537	6	•2066	•0637	-6.7916	6.0643	378	0	1.042	• 476	6.457

***** TOTAL WORK UNITS (EQUIVALENT FINE MESH SWEEPS) = 102.951 ******

(i) Convergence History

Figure 17. Sample Program Output

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THE FOLLOWING WAS NOT RECOGNIZED AS A KEYWORD THE PROGRAM WAS EXPECTING A KEYWORD THE NEXT CARDS ON THE INPUT FILE ARE -

is printed followed by the four characters not recognized and then the next twenty cards of the input file. The difficulty is usually in the previous input group. A common problem is the wrong number of intersects on the GEOM card (Card 1 for the GEOM input group).

The next step in the code is checking of many of the input quantities for reasonableness or values that are consistent with declared array lengths in the code. Most of these messages are self-explanatory. The following should help in interpreting some of the messages.

- NTMESH See THET input group. Values must be consistent with the number of theta mesh specified with the TMESH group.
- NFFPR See FLDT input group. NFFPR larger than declared array storage.

5.4.2 GEOMETRY ERRORS

The next step in the code is the processing of the input geometry points. Bad points or missing points will cause fatal errors. The process of finding missing points can be quite difficult. The diagnostics and explanations are listed approximately in the order processed.

EITHER THE FOLLOWING SURFACE POINTS ARE BAD OR THE INPUT MESH IS INCORRECT

At least two of the three coordinate values for a surface point must be mesh values. That is because a surface point is defined as the intersection of a grid line with the surface, and a grid line is defined by two mesh values (for example, x=X(6), y=Y(8)). Thus, either the point is not a surface intersect or a mistake has been made in input of the mesh values. Either remove the bad point(s) from the file or correct the input mesh.

THE FOLLOWING SURFACE POINTS ARE NOT ADJACENT TO FIELD POINTS

This is only an informative message, *not an error*. The situation is shown in Figure 18a. These points are not used in the calculations.

THE FOLLOWING SURFACE POINTS ARE MULTIPLY ADJACENT IN THE SAME DIRECTION

This is fatal geometry error and indicates a physically impossible situation. Examples are shown in Figures 18b and 18c. Figure 18b shows two overlapped surfaces. Figure 18c shows a physically correct geometry, but a surface intersect (the question mark) missing from the input file. Duplicate points would also draw this diagnostic. Another possible problem is a point that is correctly located, but the coordinates of the normal are bad, reversed in sign for example.

THE FOLLOWING SURFACE POINTS ARE ADJACENT TO A ISURFTP()=2 POINT AND/OR THERE ARE DUPLICATE POINTS

An ISURFTP()=2 point is created (fig. 19a) when a mesh node and the surface coincide. There are several possibilities when this diagnostic occurs. If there are duplicate points, the correction is to eliminate the duplicate points. Another possibility is shown in Figure 19b where the surface is very near a mesh node. Normally, three points will be generated as shown, but one or more may be rounded to a ISURFTP()=2 point. The correction is to delete the other one or two points, thus



(a) Surface Points Not Adjacent to Mesh Nodes (Nonpoints)



(b) Two Points Adjacent to the Same Mesh Node



(c) Two Points Adjacent to the Same Mesh Node

Figure 18. Possible Geometry Configurations and Errors


(a) ISURFTP () = 2 Point, Mesh Node and Surface Coincide



Figure 19. ISURFTP () = 2 Points and Possible Problems

moving the intersect to the mesh node. This is basically a tolerance problem in the geometry code. An additional possibility is shown in Figure 19c. For this situation, point S_2 must be deleted from the input file. Other possibilities include configurations similar to Figures 19b and 19c. Once the situation is understood, the means of correction should be clear.

 $\begin{array}{cccc} FATAL \mbox{ ERROR - } & \mbox{ BITSET - } & \mbox{ INCONSISTENT GEOMETRY } \\ X = & Y = & Z = \\ or & & \\ X = & R = & THETA = \\ FATAL \mbox{ ERROR - } & \mbox{ BITSET - } & \mbox{ GEOMETRY PROBLEMS } \\ X = & \dots & \end{array}$

BITSET, BITSET1 and BITSET2 are subroutines that sweep through the field and attempt to determine if mesh nodes are inside the flow or outside the flow (internal to a surface). Typical problems are missing points, extra points, and reversed normals. BITSET1 sweeps along y or radial lines, BITSET2 sweeps along z or circumferential lines. Normally, BITSET1 executes first and sets as many field nodes as possible. Then BITSET2 sweeps the other direction, checks for consistency with the BITSET1 results, and sets the remaining nodes. An extra pass is made with BITSET1 to ensure everything is consistent. BITSET1 and BITSET2 work on an x constant plane. BITSET compares adjacent x constant planes to insure they are consistent with each other and the surface intersects between them.

As an example, referring to Figure 20a, if point S_2 is missing from the input file, an error will be detected by BITSET1 when point S_3 is detected, and the coordinates printed will be that of mesh node P_3 . If S_2 is present, but the normal has the wrong sign, an error will be detected when S_2 is encountered, and the coordinates printed will be those of P_2 . If S_1 is missing, the error may not be detected until BITSET2 executes.

Referring to Figure 20b, if everything else is correct and point S_x is missing or the normals are bad, then BITSET will detect an error and the coordinates printed will be those of S_x , except that the x value printed will be x_2 .

A problem that will cause error messages from BITSET, BITSET1 or BITSET2 is a configuration such as Figure 19b where a surface point is very near a mesh node. This configuration generates three surface points that are not all calculated simultaneously by the geometry code (ref. 7). It is possible that tolerances are such that one of these points can be found on the wrong side of the mesh node relative to the others. If this occurs, the simplest solution is to replace all three points by a single point located at the mesh node.

A FATAL ERROR IN THE GEOMETRY DEFINITION HAS BEEN DETECTED.

ERROR DISCOVERED WHILE TRYING TO FOLLOW THE FOLLOWING CURVE (followed by a list of surface points)

The computer program attempts to link the surface points in sequence along x, y, and $z(x, r, and \theta)$ cuts of the surface. This is done by starting at a point and finding the next ones in the sequence until a boundary is reached or the sequence has returned to the starting point forming a closed loop. If a boundary is reached, the search starts in the reverse direction from the initial point until another boundary is reached. A boundary is the edge of the computational field or the compressor face. If the search does not find a next point in the sequence and is not at a boundary, the preceding message is printed and the table of points found is printed. The points printed are in sequence starting at the initial point, except that if a boundary is reached and the search started in the reverse direction from the initial point, those points follow the boundary point



(b) Sample Geometry for BITSET Error Explanation

Figure 20. Geometry for BITSET, BITSET1, and BITSET2 Explanations

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without a break. The area of difficulty is at the end of the table. Typical problems are identical to those associated with the BITSET routines. They are missing points, bad or reversed normals, and points slightly out of place.

SPECIAL SURFACE POINTS ON LEVEL _____ MESH IS VERY COARSE

This diagnostic occurs when two different surfaces are separated by only one grid line. The calculations in this region of the flowfield will be very inaccurate and also code failures can result. If the message occurs on levels 1 and 2, and the code appears to converge properly, the computation is probably all right. This is a warning message, not a fatal error.

5.4.3 CODE FAILURES

The hardest part of getting the code to successfully run a data case has been getting geometry points that are complete and self-consistent. Code failures beyond that point are less frequent and often due to errors in the computer code. The other problems causing code failures once the geometry is correct include geometries the code cannot handle properly and supersonic flow. The program will compute transonic and potential flow most of the time, but large regions of supersonic flow can lead to convergence problems and code failure. Suggestions are to check inputs to make sure that the Mach numbers requested are physically reasonable and then see an analyst to determine if the code is at fault. The message

FATAL ERROR - AA.LT.0.0

is associated with code failures. It means that the speed of sound has been calculated to be negative, indicating that physically impossible velocities have been computed.

WARNING - AA.LT.O.O_PLACES. THE PROGRAM WILL TRY TO RECOVER

or

WARNING - HIGH VELOCITY OUT OF SURFACE - RECOVERY WILL BE ATTEMPTED

followed by values for the mesh indices and then coordinate values may be printed. The most common problem is a bad value for ϕ obtained when interpolating up to a denser mesh for the first visit to the denser mesh. If the program recovers, that is, continues computing without further diagnostics, the run is probably okay.

Failure of the code to converge on lower levels is not very significant. Failure to converge on level 4 indicates results are suspect, but not necessarily incorrect.

5.4.4 OTHER PROBLEMS

A problem with the multilevel calculations is the coarseness of the lowest grid level. There is code in the computer program to attempt to handle the very coarse meshes, but it is not always successful. A possible situation is shown in Figure 21. If this is the coarsest of four levels, there should be no problem with the final results if no problems develop on the coarsest mesh. If there are problems on the coarsest mesh, possible solutions are to use fewer levels or to add or move mesh.

Another source of difficulty is the logic for detecting the wing trailing edges and setting up the internal tables for calculations with a Kutta boundary condition. At present, this code does not work for all possible configurations, but it is usually possible to hand-correct these internal tables for a given flow problem. Consultation should be sought before this is attempted.



Figure 21. Very Coarse Mesh

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