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APPROXIMATION TECHNIQUES FOR PARAMETER
ESTIMATION AND FEEDBACK CONTROL FOR
DISTRIBUTED MODELS OF LARGE FLEXIBLE
STRUCTURES

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APPROXIMATION TECHNIQUES FOR PARAMETER

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Abstract

We discuss approximation ideas that can be used in parameter estimation and feedback control for Euler-Bernoulli models of elastic systems. Focusing on parameter estimation problems, we outline how one can obtain convergence results for cubic spline-based schemes for hybrid models involving an elastic cantilevered beam with tip mass and base acceleration. Sample numerical findings are also presented.

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I. Introduction

In this lecture we discuss some approximation techniques that may be used in algorithms for parameter estimation and/or feedback control in distributed models such as those arising in models typical of large flexible space structures. The focus of our recent efforts has been the development and analysis of computational algorithms, e.g., convergence analysis, numerical implementation (software development) and testing. While the ideas involved are also applicable to the computation of feedback controls, we restrict our discussions here to some of our efforts on techniques in the context of parameter estimation or "inverse" problems: given observations of a system, determine parameters in models which best describe structural/material properties manifested by the system in response to perturbations (loading, etc.).

The importance of such problems is twofold: (i) parameter estimation can be viewed as a primary tool in on-orbit model development and analysis where one seeks to understand elastic/viscoelastic material properties such as damping, stiffness, etc. and to detect changes in these due to aging, prolonged stress, etc.; (ii) parameter estimation is a precursor to and integral part of development of sophisticated feedback control laws (via feedback operators satisfying infinite dimensional Riccati equations involving functional parameters of the system).

Many of the structures of interest to aerospace engineers entail systems composed of composite materials in rather complex geometric/structural configurations. The need for methods to

investigate such variable structure distributed models has, in our opinion, been clearly established in a number of recent efforts including [1], [2], [3], [4]. A number of investigations of parameter estimation in models for elastic beams have involved approximation results (the Trotter-Kato theorem) from linear semigroup theory. In particular, problems for simple beams have been treated in this manner in [5], [6], [7], [8]. In [9] the Trotter-Kato ideas are employed to establish results for hybrid models similar to those introduced later in this presentation and which are important in the study of shuttle-deployed payloads. However, in some instances it is advantageous to use an alternate approach involving a variational (weak) formulation of the system equations along with estimates in the spirit of those found in numerous papers on finite element techniques in structural problems. In [10] such a treatment was given for damped cantilevered Euler-Bernoulli beams. In this presentation we outline this approach in the context of models for beams with tip masses and base acceleration. Full details of our results in this direction will be given in a more lengthy manuscript currently in preparation.

Fundamental to our discussions is a conceptual framework in which one has a dynamical model with "states" $u(t,x)$, $0 < t < T$, $x \in \Omega$, and "parameters" $q(t,x)$, $q \in Q$, where Q is an admissible class of parameter functions. The state system is an initial-boundary value problem involving a hybrid model (parameter dependent and coupled partial differential equations/ordinary differential equations). One is given observations (data) $\bar{u}_{ij}$ for $u(t_i, x_j)$ and seeks to solve the

optimization problem of finding parameters $\bar{q}$ in the feasible parameter set Q which give a best (in the least squares sense) fit of the model to the data.

We formulate this problem in an abstract setting with Hilbert state space V and parameter space Q. For computational purposes we then approximate V and Q by finite dimensional spaces V^N and Q^M respectively. We illustrate these ideas with a specific model and particular classes of approximations in our subsequent discussions here.

II. The Identification Problem

We consider a flexible beam of length ℓ, spatially varying stiffness EI and linear mass density ρ which is clamped at one end and free at the other with an attached tip mass of magnitude m (see Figure 2.1).
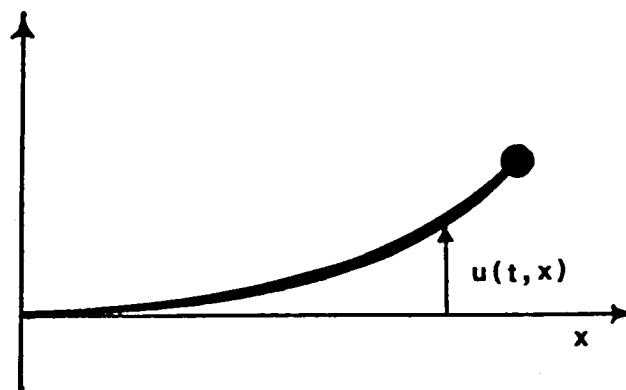

Figure 2.1

Using the Euler-Bernoulli theory to describe the transverse vibrations of the beam we obtain the partial differential equation (see [11], [12])

$$\rho(x)D_t^2 u(t,x) + D_x^2 EI(x)D_x^2 u(t,x) = D_x \sigma(t,x)D_x u(t,x) + f(t,x),$$

$$x \in (0,\ell), \quad t \in (0,T), \qquad (2.1)$$

for the transverse displacement u where σ denotes the internal tension, f is the net externally applied transverse or lateral load, and $D_t = \partial/\partial t, D_x = \partial/\partial x$. Use of principles of elementary Newtonian mechanics (i.e., force and moment balance equations) yields the boundary conditions at the free end. From translational equilibrium we obtain

$$mD_t^2 u(t,\ell) - D_x EI(\ell)D_x^2 u(t,\ell) = -\sigma(t,\ell)D_x u(t,\ell) + g(t), \quad t \in (0,T), \quad (2.2)$$

where g is the net external force on the tip mass. In a similar manner, requiring rotational equilibrium, we have

$$D_x^2 u(t,\ell) = 0, \qquad t \in (0,T). \qquad (2.3)$$

The geometric boundary conditions (zero displacement and zero slope) at the clamped end are given by

$$u(t,0) = 0, \qquad t \in (0,T), \qquad (2.4)$$

and

$$D_x u(t,0) = 0, \qquad t \in (0,T), \qquad\qquad (2.5)$$

respectively. The initial conditions are in the form of initial displacement

$$u(0,x) = \phi(x), \qquad x \in [0,\ell], \qquad\qquad (2.6)$$

and initial velocity

$$D_t u(0,x) = \psi(x), \qquad x \in [0,\ell]. \qquad\qquad (2.7)$$

In order to characterize solutions to the hybrid system (2.1) – (2.7) of ordinary and partial differential equations, boundary and initial conditions, we formally represent it as an abstract second-order system. Consider

$$M_0 D_t^2 \hat{u}(t) + A_0 \hat{u}(t) = B_0(t)\hat{u}(t) + \hat{F}(t), \quad t \in (0,T), \qquad (2.8)$$

$$\gamma_0 \hat{u}(t) = 0 \qquad\qquad \gamma_1 \hat{u}(t) = 0 \qquad\qquad \gamma_2 \hat{u}(t) = 0$$
$$\text{at } x = 0, \qquad\qquad \text{at } x = 0, \qquad\qquad \text{at } x = \ell, \qquad\qquad (2.9)$$

$$\hat{u}(0) = \hat{\phi}, \qquad D_t \hat{u}(0) = \hat{\psi}, \qquad\qquad (2.10)$$

where $\hat{u}(t) = \big(u(t,\ell), u(t,\cdot)\big)$, and the operators M_0, A_0, $B_0(t)$ and γ_i, $i = 0,1,2$ are defined by (for $\hat{v} = \big(v(\ell),v\big) \in R \times H^0(0,\ell)$)

$$M_0 \hat{v} = (mv(\ell),\ \rho v),$$

$$A_0 \hat{v} = (-D_x EI(\ell) D_x^2 v(\ell),\ D_x^2 EID_x^2 v)$$

$$B_0(t)\hat{v} = (-\sigma(t,\ell) D_x v(\ell),\ D_x \sigma D_x v)$$

and

$$\gamma_i \hat{v} = D_x^i v, \qquad\qquad i = 0,1,2,$$

$$\hat{F}(t) = (g(t), f(t,\cdot)), \quad \hat{\phi} = (\phi(\ell),\phi), \quad \text{and} \quad \hat{\psi} = (\psi(\ell),\psi).$$

Define the Hilbert space H by

$$H = R \times H^0(0,\ell)$$

with innerproduct

$$\langle (\eta,\phi),(\xi,\psi)\rangle_H = \eta\xi + \langle \phi,\psi\rangle_0$$

where $\{H^1,\ \langle \cdot,\cdot\rangle_i\}$ denote the usual Sobolev spaces together with the usual Sobolev inner products. Let V be the Hilbert space defined by

$$V = \{(\eta,\phi) \in H : \phi \in H^2(0,\ell),\ \phi(0) = D\phi(0) = 0,\ \eta = \phi(\ell)\}$$

together with the innerproduct

$$\langle \hat{\phi}, \hat{\psi} \rangle_V = \langle D^2\phi, D^2\psi \rangle_0,$$

where $\hat{\phi} = (\phi(\ell),\phi)$, $\hat{\psi} = (\psi(\ell),\psi) \in V$. It is easily shown that V is dense in H and choosing H as our pivot space we have the continuous embeddings $V \subset H \subset V^\sim$ where $V^\sim$ is the space of continuous linear functionals on V.

Of particular interest to us here will be the notion of a weak solution to (2.8) - (2.10). Interpreting the derivatives in the definitions of the operators A_0, $B_0(t)$ and γ_1 in the distributional sense, we rewrite (2.8) - (2.10) in variational form as

$$\langle M_0 \, D_t^2 \hat{u}(t), \hat{\theta} \rangle_H + a(\hat{u}(t),\hat{\theta}) = b(t)(\hat{u}(t),\hat{\theta}) + \langle \hat{F}(t),\hat{\theta} \rangle_H, \quad (2.11)$$

$$\hat{\theta} \in V, \; t \in (0,T)$$

$$\hat{u}(0) = \hat{\phi}, \qquad D_t\hat{u}(0) = \hat{\psi} \qquad\qquad (2.12)$$

where the sesquilinear forms a and $b(t)$ on $V \times V$ are defined by

$$a(\hat{\phi},\hat{\psi}) = \langle EID_x^2\phi, D_x^2\psi \rangle_0$$

and

$$b(t)(\hat{\phi},\hat{\psi}) = -\langle \sigma D_x\phi, D_x\psi \rangle_0$$

respectively, and the H innerproduct is interpreted as the duality pairing between $V^\sim$ and V (see [13], [14])) wherever appropriate. Under the assumption that EI, $\rho \in L_\infty(0,\ell)$, $\sigma \in L_2([0,T],H^1(0,\ell))$,

$f \in L_2([0,T], H^0(0,\ell))$ and $g \in L_2(0,T)$, it is not difficult to show (see [15]) that the system (2.11), (2.12) admits a unique solution $\hat{u}$ with values in V and which satisfies $\hat{u} \in C([0,T],V)$, $D_t\hat{u} \in C([0,T],H)$ and $D_t^2\hat{u} \in L_2([0,T],V^-)$. The existence of strong solutions can be demonstrated by rewriting (2.8) − (2.10) as an equivalent first-order system and using linear semigroup theory [16], [17] and evolution operators. The details involve standard ideas for evolution systems such as those found in [17] and under additional regularity assumptions (e.g., $EI \in H^2(0,\ell)$, $\sigma \in C^1([0,T], H^1(0,\ell))$, $\hat{\phi} \in V \cap \{(v(\ell),v)|v \in H^4(0,\ell), D^2v(\ell) = 0\}$, $\hat{\psi} \in V)$, one can argue existence of strong solutions (i.e., $\hat{u} \in C([0,T],V)$, $D_t\hat{u} \in C([0,T],H)$, $D_t^2\hat{u} \in L_2([0,T],H)$ with $\hat{u}$ satisfying (2.8), (2.9) almost everywhere on $[0,T])$ with sufficient smoothness to carry out the convergence arguments underlying the results presented in the next section.

In formulating the identification problem, for ease of exposition we assume that we wish to identify the parameters m, EI, ρ and σ only. We do note, however, that our general approach is in fact applicable to a wider class of problems involving the estimation of the forcing terms and initial conditions (see [18], [19]). Let Q be a compact subset of $\mathcal{Q} = R \times L_\infty(0,\ell) \times L_\infty(0,\ell) \times L_2([0,T],H^0(0,\ell))$. We assume that we have been provided with displacement observations $\{\bar{u}(t_i,x_j) : i=1,\cdots,\mu, j=1,\cdots,\nu\}$ at times $t_i \in [0,T]$ and positions $x_j \in [0,\ell]$ and formulate the identification problem as a least squares fit to data:

<u>(ID)</u>:   Find $q = (m, EI, \rho, \sigma) \in Q$ which minimizes

$$J(q; \hat{u}) = \sum_{i=1}^{\mu} \sum_{j=1}^{\nu} |u(t_i, x_j) - \overline{u}(t_i, x_j)|^2,$$

subject to $\hat{u}(t) = \big(u(t, \ell), u(t, \cdot)\big)$ being the solution to (2.11), (2.12)

corresponding to $q \in Q$.

III. The Approximation Scheme

Our approximation scheme is based upon the use of a standard
(finite element) Galerkin approach to construct a sequence of finite
dimensional approximating identification problems. For each
$N = 1, 2, \cdots,$ let $V^N \subset V$ be a finite dimensional subspace of H.
Let P^N denote the orthogonal projection of H onto V^N with respect
to the H innerproduct. The Galerkin equations for the system (2.11),
(2.12) are

$$\langle M_0 D_t^2 \hat{u}^N(t), \theta^N \rangle_H + a(\hat{u}^N(t), \theta^N) = b(t)(\hat{u}^N(t), \theta^N) + \langle \hat{F}(t), \theta^N \rangle_H, \quad (3.1)$$

$$\theta^N \in V^N, \ t \in (0, T),$$

$$\hat{u}^N(0) = P^N \hat{\phi}, \qquad D_t \hat{u}^N(0) = P^N \hat{\psi}, \qquad (3.2)$$

where $\hat{u}^N(t) = \big(u^N(t, \ell), \ u^N(t, \cdot)\big) \in V^N.$ The approximating
identification problems then take the form

(IDN):   Find $q^N = \left(m^N, EI^N, \rho^N, \sigma^N\right) \in Q$ which   minimizes $J(q; \hat{u}^N)$ subject to $\hat{u}^N$ being the solution to (3.1), (3.2) corresponding to $q \in Q$.

Of particular interest to us here is a scheme involving the use of cubic spline functions. Let $\left\{B_j^N\right\}_{j=1}^{N+1}$ denote the (modified) cubic B-splines on the interval $[0, \ell]$ corresponding to the uniform partition $\left\{0, \dfrac{\ell}{N}, \dfrac{2\ell}{N}, \cdots, \ell\right\}$ which satisfy $B_j^N(0) = DB_j^N(0) = 0$, $j = 1, 2, \cdots, N+1$. Let $V^N = \text{span}\left\{(B_j^N(\ell), B_j^N)\right\}_{j=1}^{N+1}$. Then $V^N \subset V$ and (3.1), (3.2) take the form

$$M^N \ddot{w}^N(t) + A^N w^N(t) = B^N(t) w^N(t) + F^N(t), \quad t \in (0, T) \qquad (3.3)$$

$$w^N(0) = [W^N]^{-1} w_0^N, \qquad \dot{w}^N(0) = [W^N]^{-1} w_1^N \qquad (3.4)$$

where

$$\hat{u}^N(t) = \sum_{j=1}^{N+1} w_j^N(t) \left(B_j^N(\ell), B_j^N\right),$$

$$[M^N]_{ij} = m B_i^N(\ell) B_j^N(\ell) + \int_0^\ell \rho B_i^N B_j^N,$$

$$[A^N]_{ij} = \int_0^\ell EI D^2 B_i^N D^2 B_j^N,$$

$$[B^N(t)]_{ij} = \int_0^\ell \sigma(t, \cdot) DB_i^N DB_j^N,$$

$$[F^N(t)]_i = g(t) B_i^N(\ell) + \int_0^\ell f(t, \cdot) B_i^N,$$

$$[w_0^N]_i = \phi(\ell)B_i^N(\ell) + \int_0^\ell \phi B_i^N,$$

$$[w_1^N]_i = \psi(\ell)B_i^N(\ell) + \int_0^\ell \psi B_i^N,$$

and

$$[W^N]_{ij} = B_i^N(\ell)B_j^N(\ell) + \int_0^\ell B_i^N B_j^N, \quad i,j = 1,2,\cdots,N+1.$$

Our convergence results for the cubic spline approximation schemes are summarized in the following two theorems.

Theorem 1. Suppose $\{q^N\} \subset Q$ with $q^N \to q$ as $N \to \infty$. Suppose further that $\hat{u}(q)$, the solution to (2.11), (2.12) corresponding to $q \in Q$ is a strong solution. Then if $\hat{u}^N(q^N)$ is the solution to (3.1), (3.2) corresponding to q^N we have

$$|\hat{u}^N(q^N) - \hat{u}(q)|_V \to 0 \quad \text{and} \quad |D_t\hat{u}^N(q^N) - D_t\hat{u}(q)|_H \to 0,$$

as $N \to \infty$ for each $t \in [0,T]$.

Theorem 2. Let $\bar{q}^N$ be a solution to problem (IDN). Then the sequence $\{\bar{q}^N\}$ admits a convergent subsequence $\{\bar{q}^{N_k}\}$ with $\bar{q}^{N_k} \to \bar{q}$ as $k \to \infty$. Moreover, $\bar{q}$ is a solution to problem (ID).

Theorem 1 can be established using approximation properties of cubic splines (see [20]) and variational arguments which are similar in spirit to those found in [21] for second-order hyperbolic systems and in

[10] for damped cantilevered beams without tip mass. Continuous dependence of J and $\hat{u}^N$ and compactness of Q allow us to conclude that problem (IDN) admits a solution. The existence of a convergent subsequence also follows from the compactness of Q. Finally an application of Theorem 1 yields

$$J(\bar{q},\hat{u}(\bar{q})) = \lim_{k \to \infty} J(\bar{q}^{N_k},\hat{u}^{N_k}(\bar{q}^{N_k})) \leqslant \lim_{k \to \infty} J(q,\hat{u}^{N_k}(q)) = J(q,\hat{u}(q))$$

for all $q \in Q$ and Theorem 2 is thus proven.

Although the state equation in problem (IDN) is finite dimensional, the admissible parameter space Q is a function space and hence the minimization of J is over an infinite dimensional space. We briefly indicate a means of overcoming this difficulty which involves the introduction of a second level of approximation into our scheme. A detailed discussion of these ideas along with several numerical examples for problems with parabolic, hyperbolic, and simple Euler-Bernoulli equations can be found in [22], [23], and [7] respectively.

For each $M = 1,2,\cdots$, define the set $Q^M \subset \mathcal{Q}$ by $Q^M = I^M(Q)$ where I^M is a mapping which satisfies

Q1. $I^M : Q \to \mathcal{Q}$ is continuous

Q2. $I^M(q) \to q$ as $M \to \infty$ uniformly in q for all $q \in Q$.

The approximating identification problems now take the form

(IDNM): Find $q_M^N = (m_M^N, EI_M^N, \rho_M^N, \sigma_M^N) \in Q^M$ which minimizes $J(q; \hat{u}^N)$

subject to $\hat{u}^N$ being the solution to (3.1), (3.2) corresponding to q.

Typically, the spaces Q^M and the mappings I^M are realized using finite dimensional spaces of interpolating linear or cubic spline functions. In this case under sufficient regularity assumptions on Q, it can be shown that conditions Q1 and Q2 above are satisfied.

Using conditions Q1, Q2, and the compactness of Q, one can readily establish a convergence result analogous to that given in Theorem 2. Specifically, if $\{\overline{q}_M^N\}$ is any sequence of solutions to the problems (IDNM), there exists a convergent subsequence $\{\overline{q}_{M_j}^{N_k}\}$ with $\overline{q}_{M_j}^{N_k} \to \overline{q}$ as $N_k \to \infty$, $M_j \to \infty$, where $\overline{q}$ is a solution to (ID).

IV. A Numerical Example

We present a representative example to illustrate some of the numerical results we have obtained using the methods outlined above. Further details and other numerical findings will be presented elsewhere.

We consider the problem of estimating the spatially invariant stiffness EI and linear mass density ρ of a cantilevered beam of length $\ell = 1$ with an attached tip mass at the free end of unknown magnitude m which is also to be identified. We also assume that the entire system is subjected to a time varying base acceleration $a_0(t)$. The internal tension σ is then given by (see [9], [12])

$$\sigma(t,x) = - a_0(t)\big(\rho(\ell-x) + m\big).$$

The system was assumed to be initially at rest $(\phi = \psi = 0)$ and then acted upon by the distributed transverse load

$$f(t,x) = e^x \sin 2\pi t \quad .$$

and point load at the tip

$$g(t) = 2e^{-t}.$$

The base acceleration a_0 was taken to be

$$a_0(t) = \begin{cases} 1 & 0 < t < 1.5 \\ 0 & t > 1.5 \end{cases} .$$

"Observations" (i.e., displacement values to be used as data in the inverse algorithm) at positions $x_j = .75, .875, 1.0$ at times $t_i = .5,$ $1.0, \cdots, 5.0$ were generated using the "true" values of the parameters $m = 1.5$, $EI = 1.0$, and $\rho = 3.0$, the first two natural modes of the unforced, unaccelerated system and a standard Galerkin scheme. The approximating optimization problems were solved using a Levenberg-Marquardt iterative steepest descent method. "Start up" values for the parameters to be estimated were chosen as $m_0 = 1.7$, $EI_0 = .7$ and $\rho_0 = 2.7$. The initial value problem (3.3), (3.4) was solved at each iteration using a variable step size Adams predictor corrector method. The system did not appear to be stiff. Our results are summarized in Table 4.1 below.

Table 4.1

N	$\overline{EI}^N$	$\overline{\rho}^N$	$\overline{m}^N$	$\overline{J}^N$	CPU (min/sec)
2	1.0016	3.0997	1.4793	$.17 \times 10^{-4}$	0/18.01
3	1.0012	3.0636	1.4873	$.18 \times 10^{-4}$	0/35.93
4	1.0009	3.0414	1.4921	$.19 \times 10^{-4}$	1/26.19
5	1.0006	3.0306	1.4944	$.19 \times 10^{-4}$	4/15.87
6	1.0012	3.0319	1.4944	$.35 \times 10^{-4}$	5/21.34

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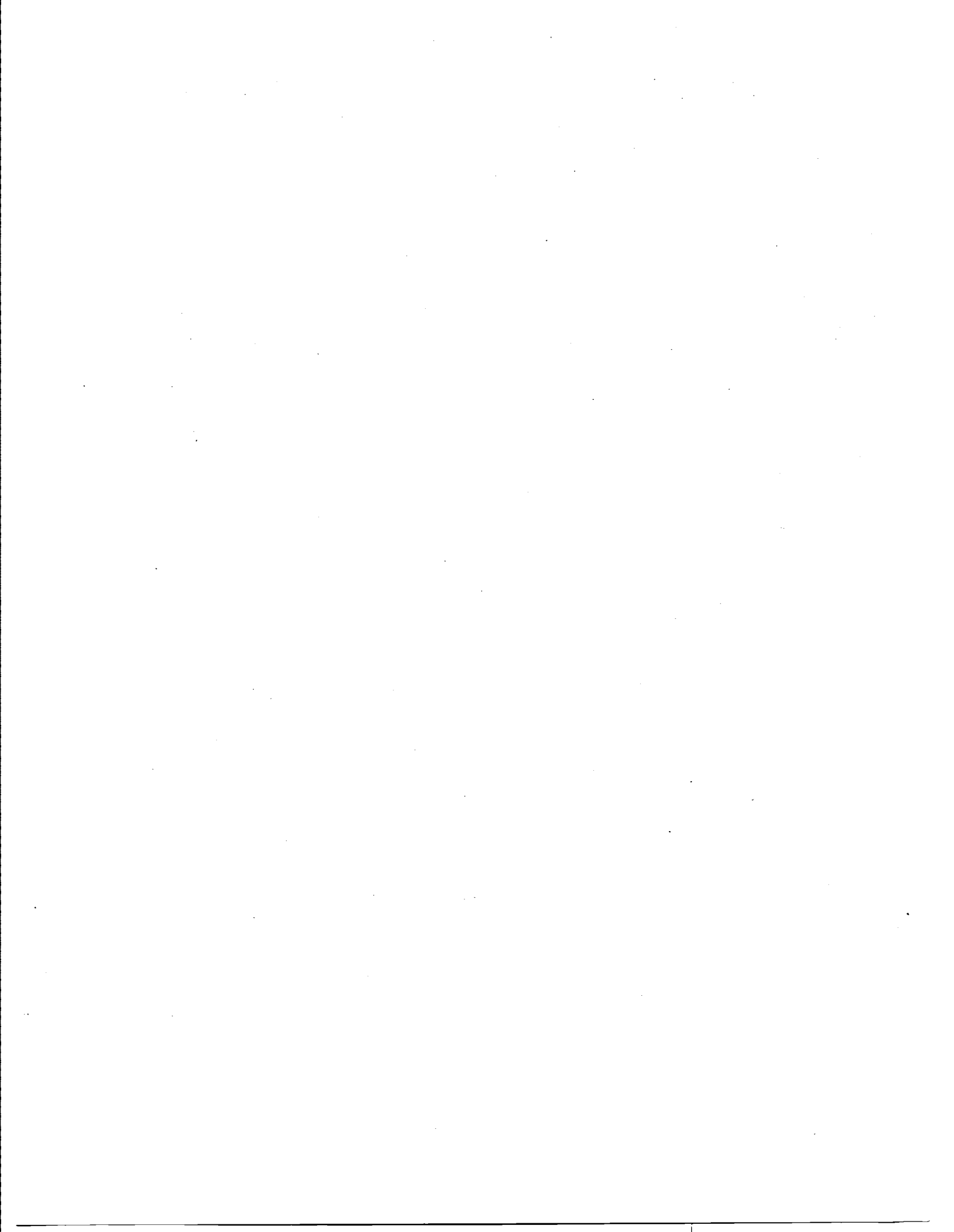[11] Clough, R. W. and Penzien, J., Dynamics of Structures, McGraw Hill, New York, 1975.

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15. Supplementary Notes
Langley Technical Monitor: Robert H. Tolson Final Report

16. Abstract

We discuss approximation ideas that can be used in parameter estimation and feedback control for Euler-Bernoulli models of elastic systems. Focusing on parameter estimation problems, we outline how one can obtain convergence results for cubic spline-based schemes for hybrid models involving an elastic cantilevered beam with tip mass and base acceleration. Sample numerical findings are also presented.

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