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ORBITAL STABILITY IN COMBINED UNIFORM AXIAL AND THREE-DIMENSIONAL WIGGLER MAGNETIC FIELDS FOR FREE-ELECTRON LASERS

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SUMMARY

Zachary [Phys. Rev. A 29 (6)m 3224 (1984)] has recently analyzed the instability of relativistic-electron helical trajectories in combined uniform axial and helical wiggler magnetic fields when the radial variation of the wiggler field is taken into account. It is shown here that the type II instability comprised of secular terms growing linearly in time, identified by Zachary and earlier by Diament [Phys. Rev. A 23 (5), 2537 (1981)], is an artifact of simple perturbation theory. A multiple-time-scale perturbation analysis reveals a nonsecular evolution on a slower time scale which accommodates an arbitrary initial perturbation. It is shown that, in the absence of exponential instability, the electron seeks a modified helical orbit more appropriate to its perturbed state and oscillates stably about it. Thus, the perturbed motion is oscillatory but nonsecular, and hence the helical orbits are stable.

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I. INTRODUCTION

A recent paper by W. W. Zachary[1] extends to arbitrary radius some earlier work by P. Diament[2] concerning the instability of relativistic-electron helical trajectories in combined uniform axial and helical wiggler magnetic fields when the radial variation and axial component of the wiggler field are taken into account. These authors find in particular that, in the absence of exponential instability, there remains a weak instability comprised of secular terms which grow linearly in time, casting doubt on the suitability of these helical orbits as a basis for free-electron laser calculations. Here, it is shown that a refined perturbation analysis employing a multiple-time-scale formalism reveals a nonsecular evolution of the orbit on a slower time scale which accommodates arbitrary perturbing initial conditions. In the absence of exponential instability, the electron seeks a modified helical orbit more appropriate to its perturbed state and oscillates stably about this modified orbit. Thus, the secular terms can be said to manifest physical stability of the orbits rather than instability.

The plan of the paper is as follows. In Sec. II, the formalism of the multiple-time-scale perturbation theory is introduced. In Sec. III, the first-order nonsecular solution is obtained by Laplace transformation methods and it is shown how the parameters of the zero-order helical orbit are modified by the perturbing initial conditions. In Sec. IV, the analogous parameter shifts in response to a small change in the guide magnetic field are derived. Finally in Sec. V, the weak instability of Diament[2] is interpreted in this context.

II. MULTIPLE-TIME-SCALE PERTURBATION THEORY

Zachary[1] considers the trajectory of a relativistic electron in the magnetic-field combination

$$\vec{B}(r,\phi,z) = \vec{\nabla}\left[B_o z + B_w r \frac{2I_1(kr)}{kr} \cos(kz - \phi)\right] \quad , \qquad (2\text{-}1)$$

where B_o and B_w denote the magnitudes of the axial guide and helical wiggler magnetic fields, respectively, k is the wave number of the axially periodic wiggler field, I_1 is the modified Bessel function of the first kind of order one, and (r,ϕ,z) denote cylindrical coordinates in space. We adopt Zachary's notation in the following but refine his perturbation analysis.

The equations governing the motion of a relativistic electron in the magnetostatic field (2-1) can be written schematically in the form

$$\dot{\vec{x}} = \vec{y}\,(x_1, \ldots, x_6) \quad , \qquad (2\text{-}2)$$

where $\vec{x}$ denotes the six-dimensional phase-space vector

$$\vec{x} = \left[r, \phi, z, V_r, V_\phi, V_z\right]^T \quad , \qquad (2\text{-}3)$$

and (V_r, V_ϕ, V_z) are the cylindrical components of the electron velocity multiplied by γ, the usual relativistic factor. Expressions for the six components of $\vec{y}$ are given in Ref. 1. The equations (2-2) admit the steady-state helical-orbit solution

$$\vec{x}_o(t) = \left[a, \gamma^{-1}kut + \theta \pm \pi/2, \gamma^{-1}ut + k^{-1}\theta, 0, kau, u \right]^T , \quad (2\text{-}4)$$

where θ is an arbitrary phase and the parameters a, u and γ are related by the conditions

$$\gamma^{-1}ku = \Omega_o \pm 2\Omega_w I_1(ka) \left[1 + (ka)^{-2} \right] , \quad (2\text{-}5)$$

$$\gamma^{-1}ku = kc\left\{ (1 - \gamma^{-2}) \left[1 + (ka)^2 \right]^{-1} \right\}^{\frac{1}{2}} , \quad (2\text{-}6)$$

with $\Omega_{o,w} = |e| B_{o,w} / (mc\gamma)$.

Consider now perturbations $\vec{w}(t)$ of the steady-state helical orbit $\vec{x}_o(t)$ by writing

$$\vec{x}(t) = \vec{x}_o(t) + \epsilon \vec{w}(t) + O(\epsilon^2). \quad (2\text{-}7)$$

In other words, we seek a perturbation solution of Eq. (2-2) correct to first order in ϵ where ϵ denotes the order of smallness of the perturbing initial conditions. However, to cope with the secular terms which arise in a conventional perturbation analysis[1], we introduce a multiple-time-scale formalism. The essential idea of this method is to extend the number of independent time variables to remove secularities order by order in the perturbation solution; the removal of time secularities on a fast time scale determines the time development of the motion on a slower time scale. The solution $\vec{x}(t)$ is expanded in the small parameter ϵ in the form

$$\vec{x}(t) = \vec{x}_o(t_o, t_1, t_2, \ldots) + \epsilon \vec{w}(t_o, t_1, t_2, \ldots) + \ldots , \quad (2\text{-}8)$$

where $t_o, t_1, t_2, \ldots$ denote a hierarchy of successively slower time scales:

$$\frac{dt_o}{dt} = 1 \, , \, \frac{dt_1}{dt} = \epsilon \, , \, \frac{dt_2}{dt} = \epsilon^2 , \; \ldots \qquad . \qquad (2\text{-}9)$$

Operationally, we treat $t_o, t_1, t_2, \ldots$ as independent variables, expanding the time derivative in Eq. (2-2) according to

$$\frac{d}{dt} = \frac{\partial}{\partial t_o} + \epsilon \frac{\partial}{\partial t_1} + \epsilon^2 \frac{\partial}{\partial t_2} + \ldots \qquad . \qquad (2\text{-}10)$$

Since the number of time variables has been increased, there is additional freedom in the perturbation analysis which we make use of to remove, order by order, any time secularities which occur in the solution. This approach ensures that the perturbation solution represented by (2-8) is uniformly valid, order by order. It should be emphasized that there is not sufficient freedom in a conventional perturbation analysis of Eq. (2-2) to remove secularities order by order; only if the conventional expansion procedure is carried out to all orders in ϵ and the secular terms summed, can the slow evolution of the motion be determined. However, in a multiple-time-scale analysis of Eq. (2-2), the condition that the first-order solution $\vec{w}(t_o, t_1, \ldots)$ be nonsecular as $t_o \to \infty$ determines the slow evolution of $\vec{x}_o(t_o, t_1, \ldots)$ on the t_1 time scale. When the multiple-time-scale perturbation solution has been obtained to the desired accuracy, one returns to the physical variable t by making the replacements $t_o = t$, $t_1 = \epsilon t$, $t_2 = \epsilon^2 t$, ... in the final expressions for $\vec{x}_o(t_o, t_1, \ldots)$, $\vec{w}(t_o, t_1, \ldots)$, etc.

Accordingly, our procedure now is to substitute (2-8) and (2-10) into Eq. (2-2), and then to equate to zero the coefficients of successive powers of ϵ. To lowest order (ϵ^o), we have

$$\frac{\partial \vec{x}_o}{\partial t_o} = \vec{y}(x_{o1}, \ldots, x_{o6}) \quad , \tag{2-11}$$

which admits the axisymmetric helical motion (2-4) for a certain class of initial conditions. We assume here that the given initial conditions fall into this class with perturbations to these initial conditions to be considered in next order. Note that the time variable t in (2-4) is now to be replaced by t_o and, furthermore, that the parameters a, u and θ, which characterize the helical solution (2-4), are constants on the fast time scale t_o but can vary on the slower time scale t_1.

Proceeding to first order in ϵ, we obtain the equation

$$\frac{\partial \vec{w}}{\partial t_o} + \frac{\partial \vec{x}_o}{\partial t_1} = \underline{A} \cdot \vec{w} \quad , \tag{2-12}$$

where $\underline{A}$ is a matrix whose entries are constant on the fast time scale t_o and which has the explicit form

$$\underline{A} = \begin{bmatrix} 0 & 0 & 0 & \gamma^{-1} & 0 & 0 \\ -\frac{ku}{\gamma a} & 0 & 0 & 0 & (\gamma a)^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma^{-1} \\ \alpha & 0 & 0 & 0 & \zeta & -\psi \\ 0 & -\frac{\kappa}{k} & \kappa & -\frac{\psi}{ka} & 0 & 0 \\ 0 & \kappa a & -ka\kappa & \psi & 0 & 0 \end{bmatrix} , \tag{2-13}$$

where we have followed Zachary[1] in defining the parameters

$$\alpha = -ku\left\{\Omega_o \pm 2\Omega_w \left[I_1(ka) + I_1'(ka)(ka + (ka)^{-1})\right]\right\} \quad , \tag{2-14}$$

$$\zeta = \Omega_o \pm 2\Omega_w I_1(ka)\left[1 + 2\ (ka)^{-2}\right] \quad , \qquad (2\text{-}14)$$

$$\kappa = \mp 2\Omega_w ku\ I_1'\ (ka) \quad ,$$

$$\psi = \pm 2\Omega_w\ (ka)^{-1}\ I_1\ (ka) \quad .$$

The term $\partial\vec{x}_o/\partial t_1$ in (2-12) is absent in a conventional perturbation analysis. It is to be obtained from the lowest-order solution (2-4) by differentiating the parameters a, u and θ with respect to t_1, and it has the form

$$\frac{\partial\vec{x}_o}{\partial t_1} = \vec{p} + t_o\vec{q} \quad , \qquad (2\text{-}15)$$

where the vectors $\vec{p}$, $\vec{q}$ are constants on the fast time scale t_o. Noting that the time dependences of $u(t_1)$ and $a(t_1)$ are related by the equilibrium constraint (2-5) which implies

$$\frac{ku}{\gamma}\ \frac{\partial u}{\partial t_1} = -\ \frac{(\alpha + ku\,\zeta)}{ka}\ \frac{\partial a}{\partial t_1} \quad , \qquad (2\text{-}16)$$

we obtain for $\vec{p}$ and $\vec{q}$ the expressions

$$\begin{aligned}\vec{p} = {} & da/dt_1\ \left[1,\ 0,\ 0,\ 0,\ (ku + ka\gamma\eta),\ \gamma\eta\right]^T \\ & + k^{-1}d\theta/dt_1\ \left[0,\ k,\ 1,\ 0,\ 0,\ 0\right]^T \quad ,\end{aligned} \qquad (2\text{-}17)$$

$$\vec{q} = \eta\ da/dt_1\ \left[0,\ k,\ 1,\ 0,\ 0,\ 0\right]^T \quad , \qquad (2\text{-}18)$$

where we have defined $\eta = -(kuka)^{-1}(\alpha + ku\,\zeta)$. The combination of parameters $(\alpha + ku\,\zeta)$ appearing in (2-16) - (2-18) has been analyzed by Zachary[1], who notes that it can only vanish if (ka) takes on one and only one critical value, viz., $ka \cong 0.850$.

III. FIRST-ORDER NONSECULAR SOLUTION

Combining (2-12) and (2-15), we seek a nonsecular solution of the first-order equation

$$\frac{\partial \vec{w}}{\partial t_o} = \underline{A} \cdot \vec{w} - \vec{p} - t_o \vec{q} \quad . \tag{3-1}$$

Equation (3-1) is conveniently solved by Laplace transformation in the variable t_o in terms of specified perturbing initial conditions $\vec{w}_o$ at $t_o = 0$. Upon introduction of the Laplace transform of $\vec{w}(t_o)$,

$$\vec{\omega}(s) = \int_0^\infty dt_o \; e^{-st_o} \; \vec{w}\,(t_o) \quad , \tag{3-2}$$

there results from (3-1) the algebraic equation

$$(\underline{A} - s\underline{I}) \cdot \vec{\omega}(s) = -\vec{w}_o + \vec{p}/s + \vec{q}/s^2 \quad , \tag{3-3}$$

where $\underline{I}$ denotes the 6 x 6 identity matrix. Solving (3-3) for $\vec{\omega}(s)$, we find

$$\vec{\omega}(s) = \frac{\underline{R}(s) \cdot (-\vec{w}_o + \vec{p}/s + \vec{q}/s^2)}{\det\,(\underline{A} - s\underline{I})} \quad , \tag{3-4}$$

where $\underline{R}(s)$ denotes the transpose of the matrix of cofactors for $(\underline{A} - s\underline{I})$. Explicit expressions for the thirty-six entries in $\underline{R}(s)$ are given in the Appendix.

To evaluate the Laplace inversion integral corresponding to (3-4) by means of the Residue Theorem, we require the zeroes of the denominator, i.e., the eigenvalues of the matrix $\underline{A}$. Direct evaluation of det $(\underline{A} - s\underline{I})$ from (2-13) shows that the six eigenvalues are the six roots of the equation

$$\det\,(\underline{A} - s\underline{I}) = s^2\,(s^4 + bs^2 + d) \;=\; 0 \quad , \tag{3-5}$$

where, in agreement with Zachary[1],

$$b \;=\; \psi\left[\psi + (ka)^{-1}\zeta\right] + \gamma^{-1}\kappa\left[ka + (ka)^{-1}\right] - \gamma^{-1}\alpha \;, \tag{3-6}$$

$$d \;=\; -\gamma^{-2}ka\kappa\left\{\left[1 + (ka)^{-2}\right]\;(\alpha + ku\zeta) - \gamma^{-1}k^2u^2\right\} . \tag{3-7}$$

As an independent check on the algebra, we have related the results (3-6) and (3-7) to those obtained by Freund and co-workers[3,4] without Laplace transformation. We have, by judicious use of the equilibrium condition (2-5), carefully verified the relations

$$b = \Omega_1^2 + \Omega_2^2 \quad , \qquad d = \Omega_1^2\,\Omega_2^2 \quad , \tag{3-8}$$

where Ω_1 and Ω_2 are identical to the (possibly complex) characteristic response frequencies of the equilibrium helical orbit derived by Freund, Johnston and Sprangle[3] and reported again by Freund and Ganguly[4], viz.,

$$\Omega_{1,2}^2 \;=\; \frac{1}{2}\,(\omega_1^2 + \omega_2^2) \pm \frac{1}{2}\;\sqrt{(\omega_1^2 - \omega_2^2)^2 + 4AB} \quad , \tag{3-9}$$

where

$$\omega_1^2 = \gamma^{-2}k^2u^2 \;\mp\; \gamma^{-1}ku\;2\Omega_w\;I_2(ka)\;\left[\,ka + (ka)^{-1}\right] \quad , \tag{3-10}$$

$$\begin{aligned}\omega_2^2 = &\left[\Omega_o - \gamma^{-1}ku \;\pm\; 2\Omega_w\;I_1(ka)\,\right]\;(\Omega_o - \gamma^{-1}ku)\\ &\pm \gamma^{-1}ku\;2\,\Omega_w I_2\;(ka)\;\left[\,ka + (ka)^{-1}\right] \quad ,\end{aligned} \tag{3-11}$$

$$A = \pm\;ca^{-1}\;(\Omega_o - 2\,\gamma^{-1}ku) \quad , \tag{3-12}$$

$$B = -2\Omega_w u\,(c\gamma)^{-1}\Big\{\Big[\Omega_o - \gamma^{-1}ku\,(1-k^2a^2)\Big]\ I_2(ka)$$
$$+\ \gamma^{-1}ku\ \Big[(1+k^2a^2)\ I_1'\ (ka) - (ka)^{-1}\ I_1\ (ka)\Big]\Big\}\ . \qquad (3\text{-}13)$$

The conditions for exponential instability of the orbit are, as shown by Zachary[1], either $d < 0$ with $(b^2 - 4d) > 0$, or $d > 0$ with $(b^2 - 4d) < 0$. The marginal stability condition $d = 0$ agrees exactly with that found by Freund and Ganguly[4]. In the domain $d > 0$ with $(b^2 - 4d) > 0$, the quantities Ω_1^2 and Ω_2^2 are real and positive and so there is no exponential instability of the orbit. However, Zachary[1] finds a non-exponential instability instead in this case, associated with secular terms which grow linearly in time. The proper interpretation of these secular terms in terms of orbital <u>stability</u> is the subject of this paper.

Insertion of (3-5) and (3-8) into (3-4) yields the relation

$$\vec{\omega}\,(s) = \frac{\underline{R}(s)\ \cdot\ (-\ \vec{w}_o + \vec{p}/s + \vec{q}/s^2)}{s^2(s^2 + \Omega_1^2)\ \ (s^2 + \Omega_2^2)}\ , \qquad (3\text{-}14)$$

and it remains simply to invert the Laplace transformation to obtain $\vec{w}(t_o)$. The singularities of $\vec{\omega}(s)$ occur at the simple poles $s = \pm\, i\Omega_1$, $s = \pm\, i\Omega_2$, and also at $s = 0$. It appears from the form of the $\vec{q}$ term in (3-14) that the pole at $s = 0$ may be as high as fourth order. However, explicit evaluation of the $\vec{q}$ term reveals that $\underline{R}(s)\ \cdot\ \vec{q}$ is proportional to s and so the pole at $s = 0$ is at most of third order. Accordingly, we rewrite (3-14) in the form

$$\vec{\omega}(s) = \frac{\vec{N}\ (s)}{s^3(s^2 + \Omega_1^2)\ \ (s^2 + \Omega_2^2)}\ , \qquad (3\text{-}15)$$

where the vector numerator $\vec{N}(s)$ is analytic at $s = 0$ (as well as at the other zeroes of the denominator) and is given by the expression

$$\vec{N}(s) = \underline{R}(s) \cdot (-s\vec{w}_o + \vec{p} + \vec{q}/s) \quad . \tag{3-16}$$

The Laplace inversion integral can now be evaluated using the Residue Theorem. In the domain $d > 0$ with $(b^2 - 4d) > 0$, the contributions from the simple poles at $s = \pm i\Omega_1$ and $s = \pm i\Omega_2$ lead to stable sinusoidal oscillations at the characteristic frequencies Ω_1 and Ω_2. Thus, the solution $\vec{w}(t_o)$ can be decomposed into an oscillatory part, $\vec{w}_{osc}(t_o)$, and the contribution from the pole at $s = 0$, $\vec{w}_*(t_o)$, which includes any time secularities:

$$\vec{w}(t_o) = \vec{w}_{osc}(t_o) + \vec{w}_*(t_o) \quad . \tag{3-17}$$

Use of the Residue Theorem leads directly to the oscillatory terms

$$\vec{w}_{osc}(t_o) = - \frac{1}{2(\Omega_1^2 - \Omega_2^2)} \sum_{+,-} \left(\frac{1}{\Omega_1^4} \vec{N}(\pm i\Omega_1) e^{\pm i\Omega_1 t_o} - \frac{1}{\Omega_2^4} \vec{N}(\pm i\Omega_2) e^{\pm i\Omega_2 t_o} \right) \quad . \tag{3-18}$$

It remains, then, to investigate the contribution $\vec{w}_*(t_o)$ from the pole at $s = 0$.

Since it is of third order, the pole at $s = 0$ leads to a contribution of the form

$$\vec{w}_*(t_o) = \vec{\zeta} + \vec{\mu}\, t_o + \vec{\xi}\, t_o^2/2 \quad , \tag{3-19}$$

where $\vec{\delta}$, $\vec{\mu}$ and $\vec{\xi}$ are vectors, constant on the fast time scale t_o. Following the philosophy of the multiple-time-scale approach, we seek to eliminate the secular time-growing terms in (3-19), i.e., to arrange that $\vec{\mu} = 0$ and $\vec{\xi} = 0$ by an appropriate choice of the slow time dependence of $a(t_1)$ and $\theta(t_1)$. The term $\vec{\delta}$ represents a small net phase-space displacement of the orbit which need not vanish.

Upon use of the Residue Theorem, there result the following expressions for the vectors $\vec{\delta}$, $\vec{\mu}$, $\vec{\xi}$:

$$\vec{\delta} = \frac{1}{2\Omega_1^4\,\Omega_2^4} \lim_{s\to 0} \left(\Omega_1^2\Omega_2^2 \frac{d^2\,\vec{N}(s)}{ds^2} - 2\,(\Omega_1^2 + \Omega_2^2)\vec{N}(s) \right), \tag{3-20}$$

$$\vec{\mu} = \frac{1}{\Omega_1^2\,\Omega_2^2} \lim_{s\to 0} \frac{d\vec{N}(s)}{ds}, \tag{3-21}$$

$$\vec{\xi} = \frac{1}{\Omega_1^2\,\Omega_2^2} \lim_{s\to 0} \vec{N}(s) . \tag{3-22}$$

Examining (3-20) - (3-22), we see from (3-16) that we must determine the matrices $\underline{R}(o)$, $(d\underline{R}/ds)|s = 0$ and $(d^2\underline{R}/ds^2)|s = 0$. The explicit formulas for the entries in the cofactor matrix $\underline{R}(s)$ given in the Appendix make these determinations straightforward.

Consider first the vector $\vec{\xi}$ given by (3-22). From (3-16), we see that

$$\lim_{s\to 0} \vec{N}(s) = \underline{R}(o) \cdot \vec{p} + \left.\frac{dR}{ds}\right|_{s\,=\,0} \cdot \vec{q} . \tag{3-23}$$

From expressions (2-17), (2-18) for $\vec{p}$, $\vec{q}$, there results then the formula

$$\vec{\xi} = \frac{da}{dt_1} \frac{2(\alpha + ku\zeta)}{(ku)(ka)} [0, k, 1, 0, 0, 0]^T \quad . \tag{3-24}$$

Since $(\alpha + ku\zeta) \neq 0$ except for the one critical value of ka, we must therefore set

$$\frac{da}{dt_1} = 0 \tag{3-25}$$

in order to eliminate the quadratic time secularity in (3-19). It follows from the constraint (2-16) that we must also have

$$\frac{du}{dt_1} = 0 \quad , \tag{3-26}$$

and from (2-18) that $\vec{q} = 0$.

Proceeding to the evaluation of $\vec{\mu}$ and noting that

$$\lim_{s \to 0} \frac{d\vec{N}(s)}{ds} = - \underline{R}(o) \cdot \vec{w}_o + \left. \frac{d\underline{R}}{ds} \right|_{s=0} \cdot \vec{p} \quad , \tag{3-27}$$

we find, making use of (3-25), the explicit result

$$\vec{\mu} = \left(- \frac{\kappa}{\gamma^3} \frac{(\alpha + ku\zeta)}{ka} \frac{(kaw_{o5} + w_{o6})}{\Omega_1^2 \Omega_2^2} - \frac{1}{k} \frac{d\theta}{dt_1} \right)$$

$$\times [0, k, 1, 0, 0, 0]^T \quad . \tag{3-28}$$

Note that the condition that $\vec{\mu}$ vanish is again a single <u>scalar</u> equation rather than a vector equation, this time leading to a determination of the slow precession of the phase $\theta(t_1)$. From (3-28), the condition for a nonsecular solution with $\vec{\mu} = 0$ is

$$\frac{d\theta}{dt_1} = -\frac{K}{\gamma^3 a} \frac{(\alpha + ku\zeta)}{\Omega_1^2 \Omega_2^2} (ka\, w_{o5} + w_{o6}) \quad . \tag{3-29}$$

It remains, finally, to evaluate and interpret the constant shift vector $\vec{\delta}$ in (3-19). Using the relation

$$\lim_{s\to 0} \frac{1}{2} \frac{d^2\vec{N}(s)}{ds^2} = \frac{1}{2} \frac{d^2\underline{R}}{ds^2}\Bigg|_{s=0} \cdot \vec{p} - \frac{d\underline{R}}{ds}\Bigg|_{s=0} \cdot \vec{w}_o \quad , \tag{3-30}$$

we obtain for the six components of $\vec{\delta}$ the results

$$\delta_1 = \frac{K(\zeta - \psi/ka)}{\gamma^2 \Omega_1^2 \Omega_2^2} (ka\, w_{o5} + w_{o6}) \quad ,$$

$$\delta_2 = k\,\delta_3 = \frac{-Kk}{\gamma^2 \Omega_1^2 \Omega_2^2} \Big\{ \alpha a\, w_{o2} + \frac{ku}{\gamma} w_{o4} + \frac{1}{ka}\big[\alpha + ku\,(\zeta + ka\psi)\big] w_{o3} \Big\} \quad ,$$

$$\delta_4 = 0 \quad , \tag{3-31}$$

$$\delta_5 = \frac{-K(u\psi + \alpha a)}{\gamma^2 a\, \Omega_1^2 \Omega_2^2} (ka\, w_{o5} + w_{o6}) \quad ,$$

$$\delta_6 = \frac{-K(\alpha + ku\zeta)}{\gamma^2 ka\, \Omega_1^2 \Omega_2^2} (ka\, w_{o5} + w_{o6}) \quad .$$

These results can be understood in terms of shifts δa, δu, $\delta\theta$ in the parameters a, u, θ which characterize the equilibrium helical orbit (2-4). They admit the interpretation

$$\begin{aligned} &\delta_1 = \delta a \quad , \quad \delta_2 = k\,\delta_3 = \delta\theta \quad , \quad \delta_4 = 0 \quad , \\ &\delta_5 = (ku)\,\delta a + (ka)\,\delta u \quad , \quad \delta_6 = \delta u \quad , \end{aligned} \tag{3-32}$$

where

$$\delta a = \frac{\kappa k u}{\gamma^3} \frac{(ka\, w_{o5} + w_{o6})}{\Omega_1^2 \; \Omega_2^2} \quad ,$$

$$\delta u = \frac{\gamma}{k} \frac{d\theta}{dt_1} \quad , \tag{3-33}$$

$$\delta\theta = \frac{k}{\Omega_1^2 \Omega_2^2} \left\{ \Omega_1^2 \; \Omega_2^2 \; w_{o3} - \frac{\kappa}{\gamma^2} \left[\alpha a (w_{o2} - k\, w_{o3}) + \frac{ku}{\gamma} w_{o4} \right] \right\} ,$$

with $d\theta/dt_1$ given by formula (3-29). It follows that the perturbed motion (3-17) comprises stable oscillations at frequencies Ω_1, Ω_2 about a modified helical orbit whose parameters are shifted slightly by the perturbing initial conditions.

To conclude this section, it should be noted that the quantity $(ka\, w_{o5} + w_{o6})$ appearing in (3-33) vanishes for an initial perturbation which conserves energy [see the equilibrium solution (2-4)]. Since the basic equations (2-2) conserve energy, such an initial energy perturbation $\delta\gamma$ remains constant and could be incorporated into the zero order energy γ. For energy-conserving initial perturbations with $\delta\gamma = 0$, the secular terms vanish identically and the only non-vanishing shift in (3-33) is $\delta\theta$. However, this is not the case for the response to perturbations in the magnetic field structure, as the next two sections will demonstrate.

IV. ORBITAL RESPONSE TO A SMALL CHANGE IN THE GUIDE MAGNETIC FIELD

It has recently been proposed[5] that the efficiency of free-electron lasers can be enhanced by an appropriate taper in the axial guide magnetic field strength. In such a scheme, it is important to understand the effects of the changes in the guide field on the motion of the electrons. In particular, one can ask whether the helical orbits persist or are destroyed (by secular terms) as such changes occur. In this section, we show by means of a calculation completly analogous to that in Sec. III that the effects of a small change in the guide magnetic field are two-fold: the parameters of the helical orbit are shifted slightly and oscillations about this modified orbit occur. In addition to the reason stated above, these results are also important to the extent that they reflect a generic resiliency of the helical orbits in response to more general magnetic perturbations.

Consider a small change δB_o in the guide magnetic field where $|\delta B_o| \ll B_o$, and define

$$\delta\Omega_o = \frac{|e|\,\delta B_o}{mc\,\gamma} \quad . \tag{4-1}$$

The linearized equations of motion [cf. (3-1)] then take the form

$$\dot{\vec{w}} = \underline{A} \cdot \vec{w} - \vec{p} - t_o\vec{q} + \delta\Omega_o\,[0, 0, 0, -kau, 0, 0]^T \quad , \tag{4-2}$$

where we treat δB_o as a first-order perturbation and where the quantities $\underline{A}$, $\vec{p}$ and $\vec{q}$ are given as before by (2-13), (2-17) and (2-18), respectively. It is clear from (4-2) that the net effect of the perturbation δB_o on our calculations in Sec. III will be to change the vector $\vec{p}$ by an amount

$$\delta\vec{p} = kau\, \delta\Omega_o\, [\,0,\ 0,\ 0,\ 1,\ 0,\ 0\,]^T \quad . \tag{4-3}$$

For simplicity, we set the initial perturbation $\vec{w}_o = 0$ and take (4-3) instead as the source of the perturbed motion.

The solution of (4-2) again takes the form

$$\vec{w}(t_o) = \vec{w}_{osc}(t_o) + \vec{\delta} + \vec{\mu}t_o + \vec{\zeta}t_o^2/2 \quad , \tag{4-4}$$

where $\vec{w}_{osc}(t_o)$ is given by (3-18) and $\vec{\delta}$, $\vec{\mu}$ and $\vec{\zeta}$ by (3-20) - (3-22), provided we now take

$$\vec{N}(s) = \underline{R}(s) \cdot (\vec{p} + \delta\vec{p} + \vec{q}/s) \quad . \tag{4-5}$$

As in Sec. III, the condition $\vec{\zeta} = 0$ implies $da/dt_1 = 0$ and hence $du/dt_1 = 0$ and $\vec{q} = 0$. The condition $\vec{\mu} = 0$ again leads to an expression for the slow evolution of the phase $\theta(t_1)$, viz.,

$$\frac{d\theta}{dt_1} = \frac{k^2u^2}{\gamma^2} \quad \frac{k\ a\ \kappa}{\gamma\,\Omega_1^2\Omega_2^2}\,\delta\Omega_o \quad . \tag{4-6}$$

Finally, the vector $\vec{\zeta}$ can be calculated and interpreted according to (3-32), i.e., in terms of shifts δa, δu and $\delta\theta$ in the parameters a, u and θ which characterize the unperturbed helical orbit. Our results for these shifts are

$$\frac{\delta a}{\delta\Omega_o} = - \frac{\kappa\ u}{\gamma^2\,\Omega_1^2\,\Omega_2^2}\,(\,1 + k^2a^2) \quad , \tag{4-7}$$

$$\frac{\delta\theta}{\delta\Omega_0} = 0 \quad , \tag{4-8}$$

$$\frac{\delta u}{\delta\Omega_0} = \frac{K q (ku)^2}{\gamma^2 \Omega_1^2 \Omega_2^2} \quad , \tag{4-9}$$

where the ratio form displays the rates of change of these quantities with respect to a smoothly varying guide field. Our results imply that the helical orbits persist in the presence of such a smooth variation.

V. DIAMENT'S WEAK INSTABILITY

The weak instability of Diament[2] differs somewhat from the type II instability of Zachary[1] which was discussed in Sec. III. Diament takes for the unperturbed motion the helical orbit associated with the idealized wiggler field in which all radial dependence is ignored. He then treats the radially-dependent corrections to the wiggler field as a perturbation and asks what will be the first-order effect of these realizability corrections on the idealized helical motion. As Zachary points out[1], this approach is clearly limited to the case of small ka. Diament finds[2] that when the idealized orbit is exponentially stable, there remains a weak instability associated with secular terms growing linearly in time except for special choices of perturbing initial conditions. He concludes from these results that realizable wigglers tend to induce outwardly spiralling motion in general. The purpose of this section is to properly interpret the secular terms and to show this conclusion to be erroneous.

Following Diament[2], we take for the unperturbed magnetic field the common approximation

$$B_r = B_w \cos(kz - \phi) \ , \ B_\phi = B_w \sin\ (kz - \phi) \ , \ B_z = B_0 \ , \tag{5-1}$$

which is the limit of (2-1) as $r \to 0$. The radially-dependent corrections to (5-1), i.e., the terms which must be added to yield (2-1), are thus

$$\delta B_r = B_w \left[2\ I_1'\ (kr) - 1\right] \cos\ (kz - \phi) \quad , \tag{5-2}$$

$$\delta B_\phi = B_w \left[2\ (kr)^{-1}\ I_1\ (kr) - 1\right] \sin\ (kz - \phi) \quad , \tag{5-3}$$

$$\delta B_z = -\ 2\ B_w I_1\ (kr)\ \ \sin\ (kz - \phi) \quad , \tag{5-4}$$

and are to be treated as a small perturbation. The limiting field (5-1) admits a steady-state helical orbit as a solution of the equations of motion (2-2) which again takes the form

$$\vec{x}_o\ (t) = \left[a\ ,\ \gamma^{-1}kut + \theta \pm \pi/2\ ,\ \gamma^{-1}ut + k^{-1}\theta\ ,\ 0\ ,\ kau\ ,\ u\right]^T . \tag{5-5}$$

However, relations (2-5) between the parameters a and u now simplifies to

$$\gamma^{-1}ku = \Omega_o \pm (ka)^{-1}\Omega_w \quad . \tag{5-6}$$

We now proceed with a multiple-time-scale perturbation calculation of the first-order correction to the helical motion (5-5), following closely our work in Secs. III and IV.

The linearized equations of motion take the form [cf. (4-2)]

$$\dot{\vec{w}} = \underline{A}' \cdot \vec{w} - \vec{p}' - t_0\vec{q}' + \vec{\Delta} \quad , \tag{5-7}$$

where henceforth a prime will denote association with the approximation (5-1). The matrix $\underline{A}'$ has exactly the same structure as the matrix $\underline{A}$ given by (2-13), but the definitions of the parameters $\alpha, \zeta, \kappa, \psi$ must be modified as follows:

$$\alpha' = -\gamma^{-1} k^2 u^2 \quad , \qquad \zeta' = -\Omega_o + 2\gamma^{-1} ku \quad ,$$

$$\kappa' = \mp ku \quad , \qquad \psi' = \pm\Omega_w \quad . \tag{5-8}$$

In fact, most of our earlier calculations can be adopted without change provided that the replacements (5-8) are made; in particular, the formulas given in the Appendix for the entries in the cofactor matrix $\underline{R}(s)$ remain valid. The characteristic response frequencies Ω_1, Ω_2 of the equilibrium helical orbit now simplify to

$$\Omega_1' = \gamma^{-1}ku \quad , \quad (\Omega_2')^2 = (\gamma^{-1}ku - \Omega_o)\left[\gamma^{-1}ku - \Omega_o\,(1 + k^2a^2)\right] , \tag{5-9}$$

and the condition for exponential instability reduces to $(\Omega_2')^2$ in (5-9) being negative.

The vectors $\vec{p}'$ and $\vec{q}'$ in (5-7) again arise from the slow-time-scale term $\partial\vec{x}_o/\partial t_1$, with (2-17) and (2-18) simplifying to the expressions

$$\vec{p}' = \gamma da/dt_1 \left[1, 0, 0, 0, \Omega_o, (-ka)^{-1}(\gamma^{-1}ku - \Omega_o)\right]^T$$

$$+ k^{-1} d\theta/dt_1 \left[0, k, 1, 0, 0, 0\right]^T \quad , \tag{5-10}$$

$$\vec{q}\,' = -\,(ka)^{-1}\,(\gamma^{-1}ku - \Omega_o)\;da/dt_1\,[0,\,k,\,1,\,0,\,0,\,0]^T\;. \quad (5\text{-}11)$$

The source term $\vec{\Delta}$ in (5-7) is due to the realizability corrections (5-2) - (5-4) and takes the form

$$\vec{\Delta} = \mp\, u\,\Omega_w\left[2\,ka\,(1 + k^{-2}a^{-2})\;I_1\,(ka) - 1\right]\,[0,\,0,\,0,\,1,\,0,\,0]^T\;. \quad (5\text{-}12)$$

From the form of (5-7), we see that the net effect of $\vec{\Delta}$ is again to modify the vector $\vec{p}\,'$, just as in Sec. IV. For simplicity, we again set $\vec{w}_o = 0$. Since the rest of the calculation proceeds exactly as in Sec. IV, we shall simply state and interpret the results.

The slow phase precession is found to be

$$\frac{d\theta}{dt_1} = -\frac{ku\,\Omega_w^2}{\gamma(\Omega_2')^2}\left[2\,ka\,(1 + k^{-2}a^{-2})\;I_1\,(ka) - 1\right]\,, \quad (5\text{-}13)$$

with da/dt_1 and du/dt_1 both zero. The shifts δa, $\delta\theta$, δu in the idealized helical orbit parameters (5-5) are as follows:

$$\delta a = \frac{\Omega_w^2}{(\Omega_2')^2}\;\frac{(1 + k^2a^2)}{k^2a}\left[2\,ka\,(1 + k^{-2}a^{-2})\;I_1\,(ka) - 1\right], \quad (5\text{-}14)$$

$$\delta\theta = 0\,, \quad (5\text{-}15)$$

$$\delta u = -\,u\,\frac{\Omega_w^2}{(\Omega_2')^2}\left[2\,ka\,(1 + k^{-2}a^{-2})\;I_1\,(ka) - 1\right]. \quad (5\text{-}16)$$

As in the previous sections, the corresponding perturbed motion is now nonsecular.

These results admit the following interpretation when exponential instability is absent. An electron assumed to be executing an idealized helical orbit satisfies relation (5-6),

$$\gamma^{-1} ku' = \Omega_o \pm (ka')^{-1} \Omega_w \quad , \tag{5-17}$$

whereas, in the presence of the realizability corrections (5-2) - (5-4), the proper relation between a and u is (2-5), i.e.,

$$\gamma^{-1} ku = \Omega_o \pm 2\Omega_w I_1 (ka) (1 + k^{-2}a^{-2}) \quad . \tag{5-18}$$

The electron in question is therefore not on the helical orbit appropriate to its perturbed state and so, in accordance qualitatively with our rsults in Secs. III and IV, it seeks out this proper helical orbit by shifting its orbit parameters and by oscillating stably about this modified helix. To support this interpretation, set a = $(a' + \delta a)$ and u = $(u' + \delta u)$ in (5-17) and (5-18), with δa and δu given by (5-14) and (5-16), respectively. Upon subtraction of (5-17) from (5-18), the consistency condition is seen to be

$$\gamma^{-1} k \delta u \pm (ka^2)^{-1} \Omega_w \delta a = \pm (ka)^{-1} \Omega_w \left[2 ka (1 + k^{-2}a^{-2}) I_1 (ka) - 1 \right] . \tag{5-19}$$

To leading order in (ka), i.e., in the small expansion parameter, the dominant contribution to the left-hand side of (5-19) comes from δa and condition (5-19) is found to be verified.

In conclusion, it should be stressed that realizable wigglers do <u>not</u> induce outwardly spiralling motion in general. We have shown here that the secular terms found by Diament[2] are just an artifact of a direct perturbation calculation and that they disappear in a more refined analysis. Diament's assertion[2] that, even if the secular growth were bounded, the electrons would typically strike the wall of the drift tube before turning around, is not correct. The radial excursion associated with the shift (5-14) and the amplitude of the stable oscillations is of the same order of smallness as the perturbation parameter (ka), and the smallness of (ka) is a necessary prerequisite for Diament's analysis to be valid.

APPENDIX: ENTRIES IN THE COFACTOR MATRIX

In this Appendix, we summarize the entries in $\underline{R}(s)$, the transpose of the matrix of cofactors for $(\underline{A} - s\underline{I})$. Each of the thirty-six entries below results from the evaluation of an appropriate five-by-five determinant.

$$R_{11} = -s^3\left\{s^2 + \psi\left[\psi + (ka)^{-1}\zeta\right] + \gamma^{-1}\kappa\left[ka + (ka)^{-1}\right]\right\} .$$

$$R_{12} = s^2\gamma^{-1}\kappa a\left[\psi + (ka)^{-1}\zeta\right] .$$

$$R_{13} = -s^2\gamma^{-1}\kappa ka\left[\psi + (ka)^{-1}\zeta\right] .$$

$$R_{14} = -s^2\gamma^{-1}\left\{s^2 + \gamma^{-1}\kappa\left[ka + (ka)^{-1}\right]\right\} .$$

$$R_{15} = -s\gamma^{-1}\left[\zeta s^2 - \gamma^{-1}\kappa(\psi - ka\zeta)\right] .$$

$$R_{16} = s\gamma^{-1}\left[\psi s^2 + (\gamma ka)^{-1}\kappa(\psi - ka\zeta)\right] .$$

$$R_{21} = s^2(\gamma a)^{-1}\left[kus^2 + ku(\psi^2 + \gamma^{-1}\kappa ka) + (ka)^{-1}\psi(\alpha + ku\zeta)\right] .$$

$$R_{22} = -s\left\{s^4 + s^2\left[\psi(\psi + k^{-1}a^{-1}\zeta) + \gamma^{-1}(ka\kappa - \alpha)\right] - \gamma^{-2}ka\kappa\alpha\right\} .$$

$$R_{23} = -s\kappa(\gamma a)^{-1}\left[s^2 - \gamma^{-1}\alpha - \gamma^{-1}ku(\zeta + ka\psi)\right] .$$

$$R_{24} = s(\gamma a)^{-1}\left\{s^2\left[\gamma^{-1}ku + (ka)^{-1}\psi\right] + \gamma^{-2}kuka\kappa\right\} .$$

$$R_{25} = -(\gamma a)^{-1}(s^2 + \gamma^{-1}ka\kappa)\left[s^2 - \gamma^{-1}(\alpha + ku\zeta)\right] - (\gamma a)^{-1}s^2\psi^2 .$$

$$R_{26} = s^2(\gamma a)^{-1}\left[(ka)^{-1}\psi^2 - \gamma^{-1}ku\psi - \gamma^{-1}\kappa\right] + (\gamma^3 a)^{-1}\kappa(\alpha + ku\zeta) .$$

$$R_{31} = s^2\gamma^{-1}(\gamma^{-1}ku\kappa - \alpha\psi) .$$

$$R_{32} = -s\gamma^{-1}\kappa a\,(s^2 - \gamma^{-1}\alpha) \quad .$$

$$R_{33} = -s\Big\{s^4 + s^2\big[\psi(\psi + k^{-1}a^{-1}\zeta) + \gamma^{-1}(k^{-1}a^{-1}\kappa - \alpha)\big] - (\gamma^2 ka)^{-1}\big[\alpha + ku(\zeta + ka\psi)\big]\Big\} \quad .$$

$$R_{34} = -s\gamma^{-1}(\psi s^2 - \gamma^{-2}ku\kappa) \quad .$$

$$R_{35} = s^2\gamma^{-1}(\gamma^{-1}\kappa - \zeta\psi) + \gamma^{-3}\kappa(\alpha + ku\zeta) \quad .$$

$$R_{36} = -\gamma^{-1}s^4 - (ka\gamma)^{-1}s^2(\gamma^{-1}\kappa + \zeta\psi - \gamma^{-1}ka\,\alpha) + (\gamma^3 ka)^{-1}\kappa(\alpha + ku\zeta) \quad .$$

$$R_{41} = -s^2\Big\{\alpha s^2 + \gamma^{-1}\alpha\kappa\big[ka + (ka)^{-1}\big] + \gamma^{-1}ku\kappa\big[\psi + (ka)^{-1}\zeta\big]\Big\} \quad .$$

$$R_{42} = s^3 k^{-1}\kappa(\zeta + ka\psi) \quad .$$

$$R_{43} = -s^3\kappa(\zeta + ka\psi) \quad .$$

$$R_{44} = -s^3\Big\{s^2 + \gamma^{-1}\kappa\big[ka + (ka)^{-1}\big]\Big\} \quad .$$

$$R_{45} = -s^2\big[\zeta s^2 - \gamma^{-1}\kappa(\psi - ka\zeta)\big] \quad .$$

$$R_{46} = s^2\big[\psi s^2 + (\gamma ka)^{-1}\kappa(\psi - ka\zeta)\big] \quad .$$

$$R_{51} = s^3(ka)^{-1}(\alpha\psi - \gamma^{-1}ku\kappa) \quad .$$

$$R_{52} = s^2k^{-1}\kappa(s^2 - \gamma^{-1}\alpha) \quad .$$

$$R_{53} = -s^2\kappa(s^2 - \gamma^{-1}\alpha) \quad .$$

$$R_{54} = s^2(ka)^{-1}(\psi s^2 - \gamma^{-2}ku\kappa) \quad .$$

$$R_{55} = -s\big[s^4 + s^2(\psi^2 + \gamma^{-1}ka\kappa - \gamma^{-1}\alpha) - \gamma^{-2}k\,\kappa(\alpha a + u\psi)\big] \quad .$$

$$R_{56} = -s\Big\{s^2\big[(ka)^{-1}\psi^2 + \gamma^{-1}\kappa\big] - (\gamma^2 a)^{-1}\kappa(\alpha a + u\psi)\Big\} \quad .$$

$$R_{61} = - s^3 \left(\alpha\psi - \gamma^{-1} ku\kappa \right) .$$

$$R_{62} = - s^2 \kappa a \left(s^2 - \gamma^{-1}\alpha \right) .$$

$$R_{63} = s^2 ka\kappa \left(s^2 - \gamma^{-1}\alpha \right) .$$

$$R_{64} = - s^2 \left(\psi s^2 - \gamma^{-2} ku\kappa \right) .$$

$$R_{65} = - s \left[s^2 \left(\psi\zeta + \gamma^{-1}\kappa \right) - \gamma^{-2}\kappa \left(\alpha + ku\zeta \right) \right] .$$

$$R_{66} = - s \left\{ s^4 + s^2 \left[(ka)^{-1} \left(\psi\zeta + \gamma^{-1}\kappa \right) - \gamma^{-1}\alpha \right] - \left(\gamma^2 ka \right)^{-1} \kappa \left(\alpha + ku\zeta \right) \right\} .$$

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16. Abstract

Zachary [Phys. Rev. A 29 (6), 3224 (1984)] has recently analyzed the instability of relativistic-electron helical trajectories in combined uniform axial and helical wiggler magnetic fields when the radial variation of the wiggler field is taken into account. It is shown here that the type II instability comprised of secular terms growing linearly in time, identified by Zachary and earlier by Diament [Phys. Rev. A 23 (5), 2537 (1981)], is an artifact of simple perturbation theory. A multiple-time-scale perturbation analysis reveals a non-secular evolution on a slower time scale which accommodates an arbitrary initial perturbation. It is shown that, in the absence of exponential instability, the electron seeks a modified helical orbit more appropriate to its perturbed state and oscillates stably about it. Thus, the perturbed motion is oscillatory but nonsecular, and hence the helical orbits are stable.

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