NASA TECHNICAL MEMORANDUM   NASA TM- 77767

EXPERIMENTAL DETERMINATION OF THE INERTIA
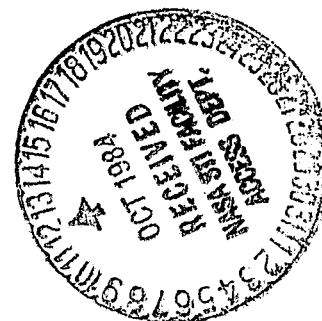CONSTANTS OF AN AIRPLANE OR OF A MISSILE

H. Loiseau

Translation of "Détermination expérimentale des
constantes d'inertie d'un avion ou d'un missile",
La Recherche Aerospatiale No. 120, 1967, pp. 53-62.

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16. Abstract

It is possible to measure the inertia constants of an airplane or of a missile by means of a unique experimental set-up without having to materialize axes of rotation. For this it is sufficient to suspend the structure appropriately, to obtain the six natural modes determined by the suspension and to introduce these results as data into a computer.

If the structure is very flexible and, therefore, if it is not possible to make it oscillate without deformation, it is then necessary to take into account the first natural modes of deformation.

Experiments on rigid and flexible models recently carried out have led to precise results and allow consideration of full-scale measurements.

Our final aim is to provide, by means of a standard ground vibration test completed by the measured characteristics of the suspension modes, the set of data necessary for flutter calculations and for the determination of all the inertia constants.

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It is possible to measure the inertia constants of an airplane or of a missile by means of a unique experimental set-up without having to materialize axes of rotation. For this it is sufficient to suspend the structure appropriately, to obtain the six natural modes determined by the suspension and to introduce these results as data into a computer.

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Our final aim is to provide, by means of a standard ground vibration test completed by the measured characteristics of the suspension modes, the set of data necessary for flutter calculations and for the determination of all the inertia constants.

I. Generalities

Calculations on the inertia constants of an airplane or missile are not very precise and the results difficult to verify. Direct measurement of these magnitudes is therefore desirable, but it would not be accepted by manufacturers unless it requires only a short period of im-

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*Numbers in the margin indicate pagination in the foreign text

mobilization of the prototype. It is in this spirit that we at O.N.E.
R.A. seek to use the same set-up for the standard ground vibration test
and for measurement of inertia constants.

The suspension usually used during records of modes of deformation
only serves to dynamically isolate the airplane from its environment.
It forms part of the sequence of measurement of inertia constants.
The precision of the results depends first on its accuracy and on its
linearity. It is furthermore necessary that it be simple enough so
that the frequencies of the six modes of rigid bodies are low in rela-
tion to the natural frequencies of the structure.

Theoretically, the six modes of rigid bodies can be anything. In
fact, in order to not amplify local errors of measurement, it is desir-
able that the six degrees of freedom be as uncoupled as possible: axes
of rotation very different from each other in direction and position,
frequencies very distinct. It is very rare that a sturcture does not
have at least a plane of symmetry without also having an axis of sym-
metry. The dimensions measured in a parallel direction and perpendic-
ularly to this plane or axis are often of a different order (flat bod-
ies, elongated bodies). A very brief examination of the structure,
associated with a rough knowledge of the center of gravity, generally
allows us to define the most suitable six modes of suspension. For ex-
ample, for an airplane, this would be rolling, pitching, exhaust, side-
slipping, swaying, and yawing motions. R. Kappus [1] prefixed these
words with a pseudo-prefix, to indicate that we have to deal with mo-
tions predominately of rolling, pitching, etc. and not of perfectly
pure modes.

Generally, before the test we arrange the order of magnitude of
calculated inertias on the plane, the calculated or measured mass, and
the position of the center of gravity (at least of its projection on
the plane parallel to its greatest dimensions). It is then very easy
to define the suspension, always foreseeing a margin of adjustment of
rigidity during the actual test (examine further in the case of an air-
plane).

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All of the test (obtained from parameters of the six modes of rigid bodies) must be carried out without any modification of the set-up. The amplitudes of vibration were measured in P points (P being large in relation of the number of modes considered [1]): these P points are compulsorily the same in all modes. The superabundance of P will be on the order of 4 to 5 for each factor, being 40 to 50 for 10 modes [1]).

The pick-ups are distributed in three groups such that they meas- ure the components of motion parallel to one or another of the three coordinate axes. The selection of location of the measurement points is very important, but difficult to make. It consists first of placing pick-ups as distant as possible from each other (in the three directions O_{xyz}).

To obtain the distribution of pick-ups which would provide the values of the amplitudes parallel to a given direction (O_y for example), we project the outline of the airplane on the plane perpendicular to that direction (figure 1-a). The projections of the points of measurement on this plane are, on one hand, located at the farthest points of the outline (1,2,3) in such a fashion to determine the projection of the mode line (on x-z) with the maximum of precision, and on the other hand, in intermediate positions 4, 5, 6 ... n such that in case of doubt we can easily verify the measurements by statistical smoothing.

If the airplane is indeformable in its motion, the extremes of the vector projections displaced on one of the planes of reference z-x remain aligned if the projections of the points on this same plane are themselves aligned. Figure 1-b demonstrates a useful distribution of points 1 to n.

To the mass and inertias of the structure tested are added those of its environment (suspension rods, mobile portions of the dischargers and pick-ups, etc.). The calculation allows, after the event, plotting of the results relative to the structure alone. This reduction is still unwieldy in the general case. To simplify the operation, it is desirable that the attachments involved with the motion of the struc-
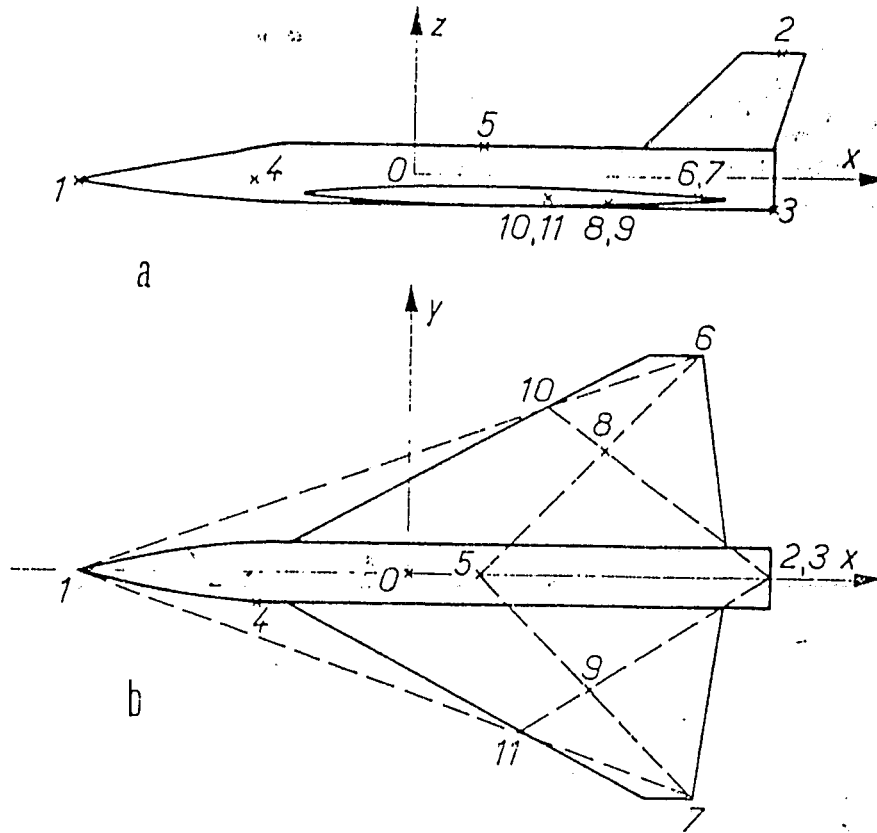
Figure 1. Distribution of measurement points

ture have a very low mass: a maximum acceptable order of magnitude for the ratio: $\dfrac{\text{extraneous mass}}{\text{structural mass}}$ is 5%. Their deformation should be negligible or known "a priori" (for example, "rigid" rods articulated at their two extremities, that is to say natural frequencies of the transversal mode of rods, clearly superior to the highest frequency of the modes considered). With these conditions, it is easy to replace the extraneous mass by the equivalent masses relating to the points of contact with the structure.

The experimental data necessary for the calculations are the following:

-coordinates of the P points of measurement

-coordinates of the pinpoint masses equivalent to the extraneous masses, as well as their values

-standardized amplitudes measured in P points;

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and for:

-modes of vibration

-generalized masses

-natural frequencies.

II. Principals of Measurement of Various Parameters
II.1 Measurement of Natural Forms

The instruments used are standard velocity pick-ups which have the advantage of simplicity, which do not require any feed mechanism, and which give an acceptable signal from the point of view of the reference level and linearity (figure 2: principle; figure 3: sensitivity as a function of mean position).
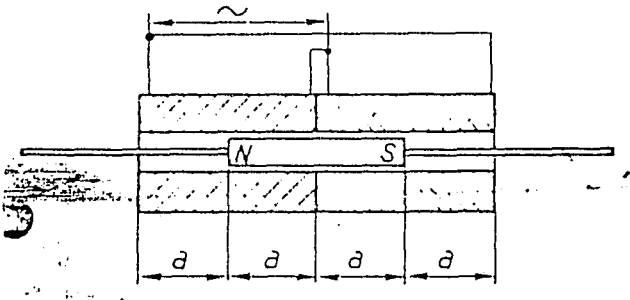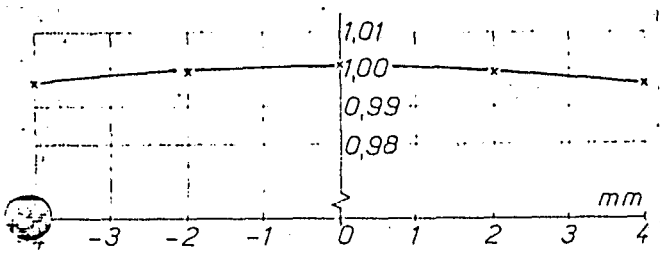
Figure 2. Principle of the instruments used.

Figure 3. Sensitivity of the instruments as a function of mean position.

$V = Be = \delta\omega$

where: e = voltage in volts

V = velocity in ms^{-1}

δ = amplitude in m

ω = frequency in s^{-1}

B is on the order of 0.3 with a 30 mm long stick of ferrite.

The mobile equipment is guided by bungee cords which ensure a pure translation without dry friction: it is linked to the test structure by the intermediary of long rods elastically articulated at their two extremities (figure 4) in such a manner to measure only the component of motion parallel to the axis of the pick-up.
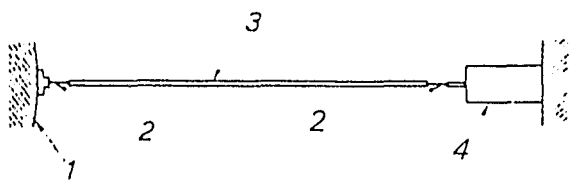

Figure 4. Set-up of mobile equipment.
1=Model 2=Elastic articulations
3=Rod 4=Pick-up

Knowledge of the amplitude of motion in each of the points of measurement is only necessary relative to that of a reference point R, selected from those which have a large displacement.

The precise measurement of magnitudes of low frequencies less than 5 Hz presents certain difficulties. In the particular case with the modes of vibration being well uncoupled and having a very low absorption, it is easy, with the assistance of some dischargers, to obtain pure vibrating responses containing only a single mode. In this case, all the points oscillate in phase or in opposition. We seek to calculate the ratio of the amplitudes of each of the sinusoidal voltages in relation to one of the others. Finally, we admit that these ratios do not depend on the absolute amplitude of the motion, which is verified experimentally. With these conditions, it suffices to measure simultaneously the instantaneous values of the voltages, in the vicinity of peaks by preference, and to figure the ratios from them (figure 5). To increase precision, it is desirable to figure a certain number of successive ratios $\frac{\delta_{i1}}{\delta_{r1}}, \frac{\delta_{i2}}{\delta_{r2}}$, etc., and to calculate the mean from them.

For this operation, it is not necessary that the vibration be stabilized in amplitude, which allows a gain of considerable time if we have to deal with poorly damped systems of low natural frequency. In particular, the measurement can be made at freely dampled load.
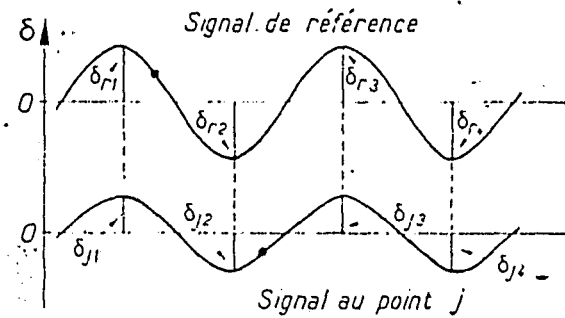
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Figure 5. Measurement of instantaneous values of voltages.

The operation is repeated for each of the pick-ups, that is to say N successive times.

The diagram of this portion of the sequence of measurement is given in figure 6.
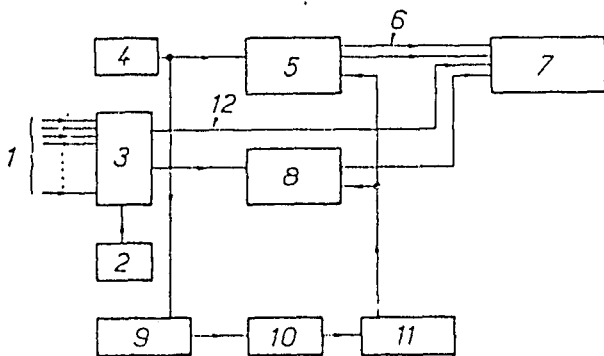

Figure 6. Diagram of the sequence of measurement.
1=N pick-ups 2=Commutator control 3=Commutator 4=Reference pick-up 5=Analog-digital converter 6=Printer control 7=Printer or keypunch 8= Analog-digital converter 9=Integrator 10=Amplifier 11=Master oscillator: made in the form of a signal of simultaneous controls of both converters 12=Pick-up number

The reference pick-up is permanently connected on a numerical ana- /56 log converter, the other pick-ups are successively plugged in on a second converter through the intermediary of a commutator with manual or automatic control. The reference signal is set out of phase by $\frac{\pi}{2}$ (through integration by preference), amplified, then put into such a form that passage through the maximum or minimum of the "reference sinusoid" provides a short time signal which simultaneously controls both converters.

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The precision of the measurement is only limited by the error of synchronization of the converters (this error increases when the frequency decreases). Another limitation in frequency is imposed by the rhythm of the printer or keypunch. If we use a printer with a sufficient number of columns, the results would appear in a line: pick-up number, reference voltage, and reading with indication of the sign and decimal point. In this case, we can then figure the ratios, without risk of registration errors, with a hand calculator. Use of a keypunch allows complete automization of attainment of the modal matrix. An idea of the precision is given by the measurements on successive maxi- and mini-peaks when the commutator is on the "reference pick-up" position. The value found should be equal to unity.

II.2. Measurement of Generalized Masses

It is preferable to avoid all manual intervention on the structure during the measurements. The precision is tied, on the other hand, to errors of measurement of frequency. We presently have sufficiently precise instruments at our disposal, but it is necessary that the structure be constant during the period of determination of generalized mass. We therefore have the advantage of operating rapidly. A useful method which responds to these objectives is that of frequencies displaced by reinjection, with the current dischargers in phase or in opposition with displacement (called electric strength method) [2]. The principle is the following (figure 7) (system with one degree of freedom):
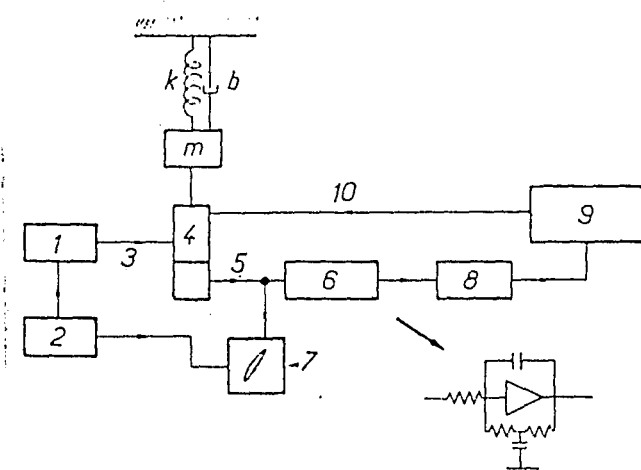

Figure 7. Principle of measurement of generalized masses. 1=Amplifier 2=Generator 3=Toward coil number 1 of the discharger 4= Discharger 5=Signal velocity 6=Integration 7=Lissajou oscillograph 8=Amplifier 9=Power amplifier-gain adjustable -G to +G 10=Toward coil number 2 of the discharger.

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The left portion of the design diagram comprises the generator power amplifier, velocity pick-up, and oscillograph necessary for investigation of phase resonance. The signal velocity is integrated, amplified in voltage and power, and sent into the discharger; the power gain is adjustable from $-G$ to $+G$. In a fashion to separate the excitation and reinjection funcitons, the discharger includes two distinct coils wound on the same frame. The force, in phase with the motion, is[1]: $F = Ai$

where i=current in the second coil of the discharger

A=discharger constant.

The amplitude at the point of excitation is: $\delta = \frac{V}{\omega} - \frac{Bv}{\omega}$

Where i is proportional to V, and therefore to δ, the force acting on the system is therefore a force of rigidity k_E: $k_E = \frac{F}{\delta} = \frac{F}{V\omega} \cdot \frac{A}{B} \cdot \frac{i}{v} \cdot \omega$

If ω and ω_0 are the natural frequencies with and without reinjection, the generalized mass is given by: $m \sim \frac{\omega}{\omega^2 - \omega_0^2} \cdot \frac{A}{B} \cdot \frac{i}{v} \cdot \frac{1}{2\pi} \cdot \frac{TT_0^2}{T_0^2 - T^2} \cdot \frac{A}{B} \cdot \frac{i}{v}$

T, T_0 : period

The value of $\frac{A}{B}$ can be obtained by separate calibration of the discharger and pick-ups, but it is preferable to verify these calibrations by carrying out beforehand measurement of the generalized mass of a perfectly understood system with one degree of freedom.

We determine n by several successive tests corresponding to 2 or 3 positive and negative rigidities, and through interpolation (figure 8).

Precision is on the order of 0.001 with a system with one degree of freedom.

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[1] The rigidity and damping forces are of different orders of magnitude (ratio 2 x. x being damping coefficient). It is necessary to displace the velocity $\pi/2$ as precisely as possible, otherwise we reintroduce significant damping (positive for a given sign of $\pi/2$, negative for the opposite sign) and there is risk of instability. It is very easy to displace the phase of $\pi/2$ with very little error with the assistance of an operational feedback amplifier through resistance and capacitance with suppression of very low frequency components.
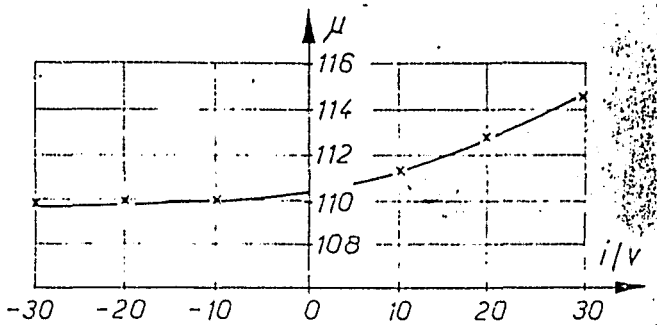
Figure 8. Calibration of the instrument.

We easily establish the expression giving the generalized mass in the case where we distribute rigidity on a system with n degrees of freedom (generalized mass $\mu = \int dm\, \delta^2$ extended for the whole structure):

$$\mu = \frac{1}{2\pi} \frac{T T_0^2}{T^2 - T_0^2} \cdot \sum_{j=1}^{N} \left(\frac{\lambda}{l} \cdot \frac{i}{v} \right)_j \cdot \delta_{jN}^2$$

where: N=number of reinjection points

δ_{jN}=amplitude normalized at point of reinjection.

For $\frac{T - T_0}{T_0} = 0.02$ it is necessary to measure the periods with an error less than 0.0001 to get the mass with a precision greater than 1%. In the case of weakly damped low frequency systems, investigation of phase resonance is very long. It is faster to use excitation in order to better approximate resonance, then let the system dampen itself freely. The period is measured during free oscillation conditions at regular intervals, at the same time as the amplitude of oscillations. We can thus graph curves giving the period as a function of amplitude to reveal very low non-linear rigidity of the suspension. In then taking T and T_0 at the same amplitudes, we eliminate through difference the errors due to these variaitons of rigidity. The design diagram for this portion of the apparatus is given in figure 9 [4].

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[1]This method is the same as that of frequencies displaced by additional masses [3], these being replaced by pure rigidities.
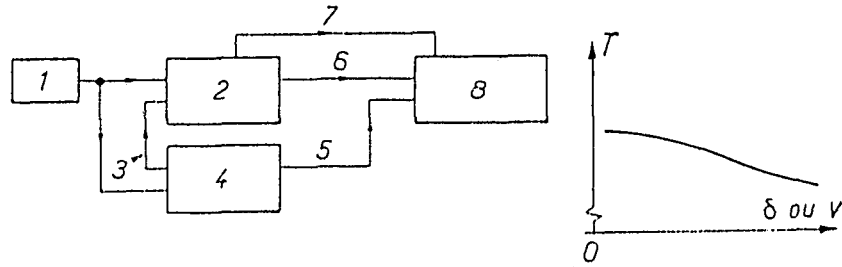
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Figure 9. Design diagram for measurement of amplitude
1=Reference pick-up 2=Analog-digital converter 3=Converter
control by period meter 4=Period meter 5=Period 6=Amplitude
7=Printer control 8=Printer.

Approximation of Electric Rigidity

The arbitrary application of additional rigidity at various points
of the structure modifies the vibration mode (this is seen immediately
from the fact that the theoretical law of figure 9 appeared as whatever
curve). Although the interpolated value of μ at the abscissa point
$\frac{i}{e} = 0$ remains correct, there is nevertheless an advantage in not moving
further away from the horizontal theoretical law. This is attained
through distribution of additional rigidity proportional to the rigidity
of the suspension, taking care to place the reinjection dischargers at
the points of suspension or in their immediate vicinity.

The electric rigidity at point j is: $K_j = \left(\frac{\lambda}{b} \cdot \frac{i}{c}\right) \cdot \omega$

Since the modes remain unchanged it is necessary that $K_{ej}/K_{sj} = c$,
a constant independent of j (K_s=rigidity of the suspension at j).

It therefore suffices to adjust the currents i in each discharger
so that: $i_j = \frac{c}{\omega} \cdot K_{sj} \cdot c \cdot \frac{b}{\lambda}$

Knowing the order of magnitude of K_{sj}, and on the other hand, by
the values c, provided by the modal plots, it is then very easy to cor-
rectly distribute the electric rigidity.

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Given that the frequencies are low, the currents can be adjusted in each discharger by shunting the latter through a variable resistance (the $L\omega/R$ values of the circuit are insignificant, the phase displacement practically nil).

II.3. Measurement of Natural Frequencies

They are obtained during the measurements of generalized mass. They correspond to the frequencies measured with zero reinjection (remark: determination of the dampings fortunately is unnecessary, a circumstance which is favorable to fine measurements).

III. Suspension and Excitation

It is necessary to distinguish the suspension modes and deformation modes.

III.1. Deformation Modes

Their intervention in the calculation series has the objective of correcting the six suspension modes which include a slight deformation of the structure, that is to say reducing the frequency and generalized mass values to those which we would have obtained with an absolutely rigid model. They interfere in the form of corrective terms which do not need to be known with such exactness as the principal term.

We can therefore consider that the measurements carried out at the time of the ground test with standard procedures are sufficiently pre- cise that any modification for measurement of the structure-apparatus set-up should in practice be limited to the six suspension modes.

The plot of deformation modes is therefore considered here as an independent problem, and we will only concern ourselves with the case of rigid-body models (suspension modes), without losing sight of the fact that an appreciable gain of time will be acquired if the two types of tests have the maximum number of points in common.

III.2. Suspension

For a precise determination of 6 modes, it is necessary to have 6 independent rigidities at our disposal. For an airplane of the shape given by figure 10 (pitch, roll, and yaw axes G_{xyz}), the following is true.
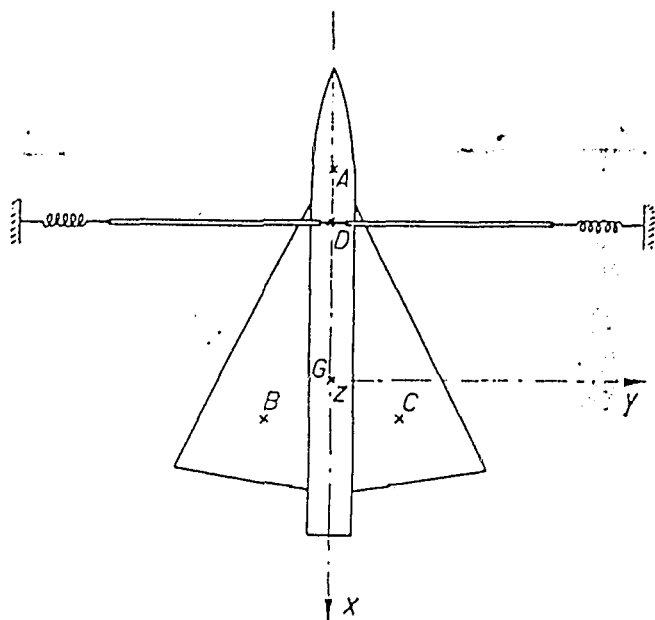

Figure 10. Airplane tested.

Three vertical suspension points suffice. Their coordinates are:

$$x_A, x_B, x_C \qquad x_B = x_C$$
$$y_A, y_B, y_C \qquad y_A = 0 \qquad y_B = -y_C$$
$$z_A, z_B, z_C \qquad z_A, z_B, z_C = 0$$

They determine the modes of rolling, pitching, and exhaust, completely uncoupled if: $(k_B + k_C) \cdot x_B, \quad k_A \cdot x_A$
$$k_B = k_C$$

The frequency of exhaust is selected to be as low as possible in relation to that of the first symmetric mode of the airplane. For ω_n, an acceptable value: $k_A = k_B = k_C = M \cdot \omega_n^2$.

For the modes of yawing and side-slipping, two horizontal rigidities applied at 2 points D and E of coordinates x_D, $x_E \neq 0$ and of dif-

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ferent signs, $z_0, z_1, y_0, y_1 \neq 0$ with $x_0 \cdot y_1 \cdot z_0 \cdot z_1 y$ suffice for a correct decoupling.

Finally, a sixth rigidity acting horizontally along G_x gives an independent mode of examination.

For the six rigidities to be independent from each other, it is necessary that the extension of one of them be possible without extension of any of the others. For this, it is necessary to place them at the ends of long enough braces which must be of the same quality as that of the braces used for the pick-ups. The vertical braces stretched by the weight of the structure have high frequencies. So that they will be the same as the horizontal braces, we divide the horizontal rigidity into two equal portions which we divide symmetrically in relation to the airplane, and which we prestress with voltage (figure 10).

The elasticities give support on one or several gantries: the vertical suspension gantry to be independent of those used for the horizontal rigidity. For these last, it is relatively easy to render them very rigid. Thus there is not always a principal gantry. Nevertheless it is easy to demonstrate that if the framework participates slightly in the motion of the structure, the error committed on the generalized mass determinations are low. For example, suppose that the framework was propelled by a translation motion equal to one tenth of the amplitude of the exhaust structure, with M the mass of the model, m that of the framework. The relatve error on the generalized mass is: $\frac{m}{M} \cdot \left(\frac{1}{10}\right)^2$ where m is much lower than M; for example, one tenth leads to a very acceptable error of 1/1000.

On the other hand, the velocity pick-ups must not give support to deformable suspension frameworks, but to very rigid elements (the ground when that is possible-it is nevertheless necessary to make the ground apparently very rigid) but that really are composed of such thin plates above the cavities.

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III.3.   Excitation

Six discharges suffice: the rigidity being distributed in such a fashion to uncouple the modes at best, the best location for the dischargers is confused with the suspension points. But it is necessary to distinguish two cases:

-dischargers for investigation of resonances: they can be placed at the suspension points, even in the case of a flexible framework

-reinjection dischargers: if they are coupled with a velocity pick-up, they should be placed on a very rigid element. If they are separated from the pick-up, the state of rigidity of the support is nevertheless still severe enough, since the forces of rigidity are large enough, and the support can have a significant amplitude even far from a resonance frequency. In each case, this support should be distinct from the suspension frameworks if these are too flexible.

IV.   Experimental Accomplishments and Results
IV.1.   Symmetrical Rigid Model (Figure 11)

This first model was composed of elements of simple geometric shapes assembled by bonding or screwing the same materials. It had a vertical plan of symmetry. The set was very rigid and rigorously inde- formable at the test frequencies (less than 3 Hz). As well as for the other models, it was easy to calculate with precision the inertia constants then to compare them to the measured results.
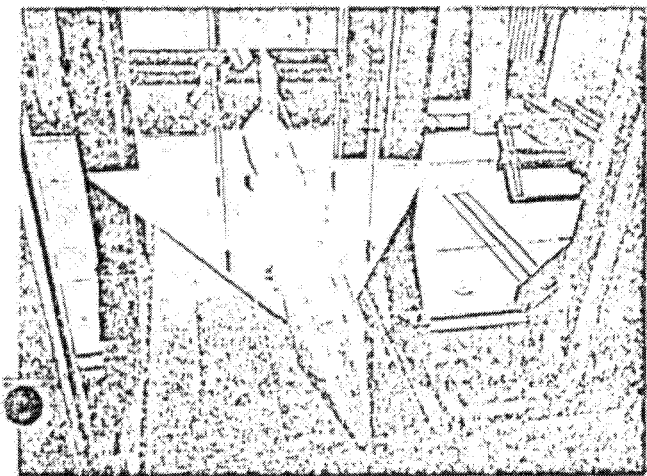

Figure 11.   Symmetrical rigid model.

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The test primarily had the aim of selection and perfection of a method of measurement. The suspension had been adjusted in such a way that the axes of oscillation of three modes of rotation coincided with the principal axes of inertia, in such a way that verification of the experimental results was immediate. Despite the simplicity of the apparatus, the results obtained are acceptable. These experiments have allowed preparation of the subsequent tests and definition of better adapted apparatuses (the clearance in particular was revealed to be far too much).

We present below a summary of the results obtained:

Roll Inertia:

    Calculated value = 7.038

    Mean of measured value = 7.076

    Mean error = + 0.54%

    Deviation of results = ± 2%

Pitching Inertia

    Calculated value = 55.83

    Mean of measured values = 55.96

    Mean error = + 0.3%

    Deviation of results = ± 1.7%

Yaw Inertia

    Calculated value = 60.93

    Mean of measured values = 61.70

    Mean error = 1.27%

    Deviation of results = ± 1/2%

Mass

    Value weighed = 215.31 kg

    Mean of measured values = 215.96 kg

    Mean error = + 0.3%

    Deviation of results = + 1.8%
                           − 1.2%

IV.2. Unsymmetrical Rigid Model (Figure 12)

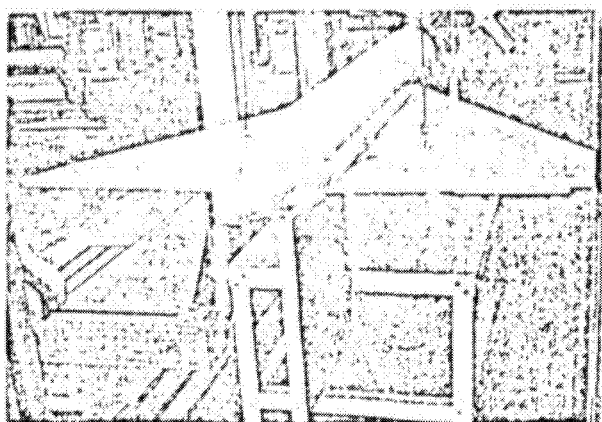We again find the first model modified: very heavy keel placed

Figure 12. Unsymmetrical rigid model.

toward the bottom, mass below the right wing in a manner to make the set-up unsymmetrical and the principal axis of roll inertia slanted. The suspension did not have to be modified in relation to the preceding test, in the way that the axes of rotation for the six modes were what- ever. The apparatus used was the same. The objective investigated was to inspect the calculation program on a concrete example. The re- sults were satisfactory:

Error on Mass = − 2.8%

Position of the Center of Gravity

Error along G_x = + 10.2 mm

Error along G_y = − 4.8 mm

Error along G_z = − 12.0 mm

Error on the Inertias

$O_{\overline{x}}$ = + 0.66%

$O_{\overline{y}}$ = − 0.73%

$O_{\overline{z}}$ = − 3.95%

Product of Inertia

$O_{\overline{x}\overline{z}}$ = − 8.31%

For the product of inertia, of very small value in relation to $O_{\overline{z}}$, /60
O_x, $O_{\overline{z}}$, we still observe a low enough error of − 8.31%.

IV.3. Symmetrical Flexible Model (Figure 13)

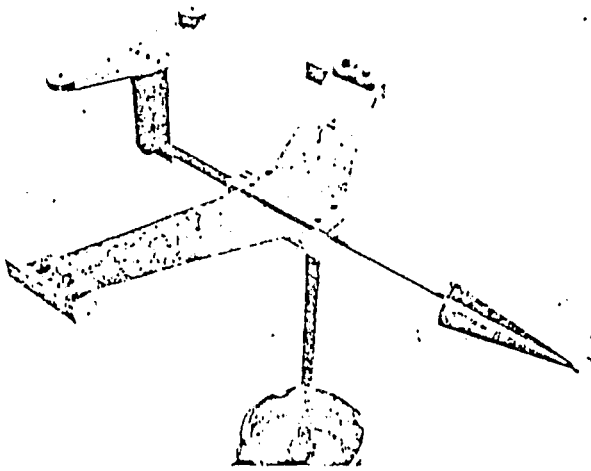We can see here a reduction (scale ½) of the test model, always

Figure 13. Symmetrical flex-
ible model.

composed of single geometric elements leading to precise calculated re-
sults. The difficulties have been voluntarily increased in bringing
together as much as possible the frequencies of the deformation and
suspension modes, and arranging a very elevated T-rudder.

The measurement apparatus used conformed to the diagrams previous-
ly given. The aggregate of the experimental plots has been conducted
in its term of 20 hours by 3 persons: 6 modes of suspension and 4
modes of deformation. This test, being absolutely similar to that of
an airplane in actual magnitude, gives a precise idea of the additional
immobilization time anticipated (taking into account the fact that the
deformation modes are still plotted):

Suspension Frequencies

    Exhaust = 3.05 Hz
    Pitch = 1.99 Hz
    Roll = 2.58 Hz
    Sway = 2.92 Hz
    Side-slip = 2.06 Hz
    Yaw = 1.75 Hz

Deformation Frequencies

    Symmetrical wing flexion = 9.6 Hz
    Vertical flexion of fuselage = 5.8 Hz
    Horizontal flexion of fuselage = 5.5 Hz
    Fuselage torsion = 7.4 Hz

18

Very significant deformations have been noted in exhaust and pitch. The results obtianed are very satisfactory:

Errors

     Mass = 1% (actual mass: 73 kg)

     Coordinates of center of gravity

          Along: G_x = + 6.4 mm

                    G_y = + 2.0 mm

                    G_z = – 0.8 mm

        Inertias:

                  O_z = – 1.98%

                  O_{Λ} = – 2.27%

                  O_z = – 4.87%

        Product of inertia:

                  O_{zz} (very small) = – 17.6%

The number of points of measurement of amplitude was 34 (8 along G_x, 9 parallel to G_y, 17 vertically).

IV.4.   Elements Common to All These Experiments
Framework (Figure 14)

The framework was very stiff and absolutely immobile during all of the modal plots.

Elements of Suspension–Excitation Pick-Up (Figure 15)

When the vertical and horizontal suspension frameworks are very rigid, it is most desirable to center at the same points the springs, and the excitation and reinjection tanks associated with a velocity pick-up. The number of braces surrounding the structure is thus much lower and the installation simpler.

It is thus that the element represented by figure 15 includes first a central cylindrical portion containing the double coiling dis-charger and the pick-ups: 8 bungee cords connected between them by two rigid struts, serve to guide the moving contact in a way which can only
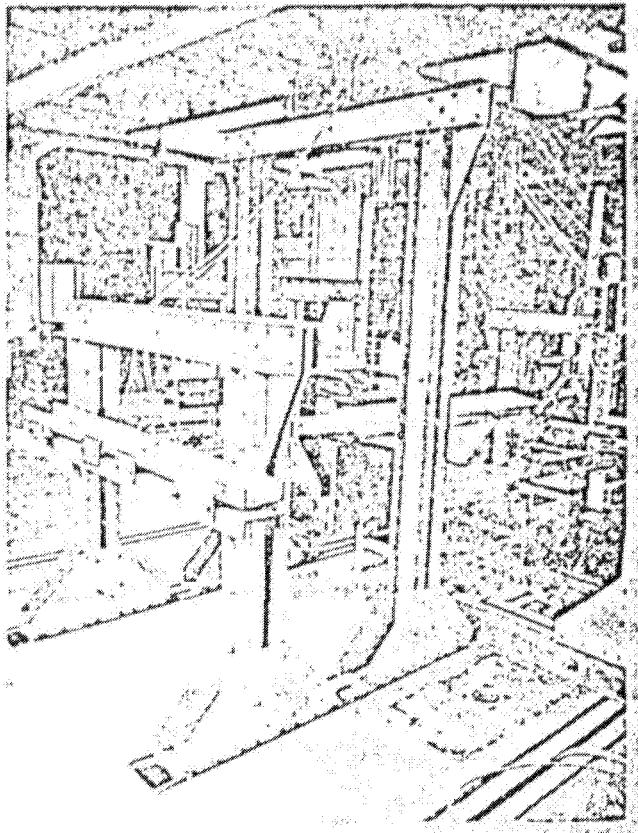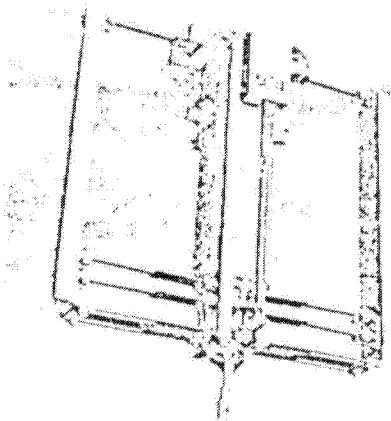
Figure 14. Test set-up.

Figure 15. Elements of suspension-
excitation pick-up

displace them along the discharger-pick-up axis without flexion and
without lateral motion. Finally we see a helicoidal spring (whose sym- /61
metry is concealed by the cylindrical portion). These springs are in-
terchangeable, which allows us to adjust the rigidity n by discrete
values. The lateral struts can slide on the bungee cords in a way in

which the model apparatus oscillates about its mean position. Security
stops protect the delicate portions. These set-ups have given complete
satisfaction. They allow us to suspend weights from 200 to 1000 N:
their extension to much higher values is possible.

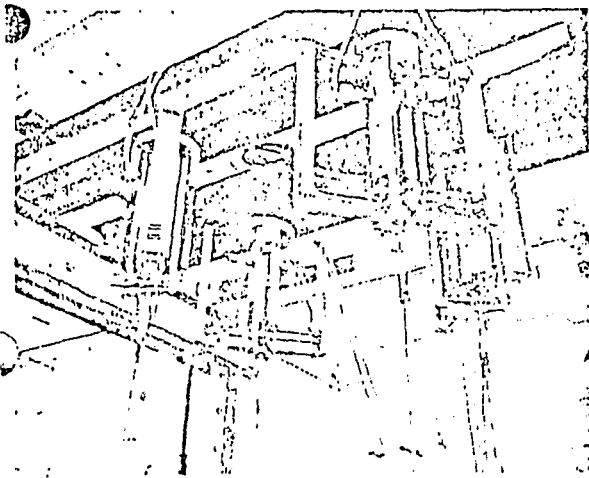Figure 16 demonstrates a set-up of 4 elements used in the case of
rigid models.

Figure 16. Set-up of 4 ele-
ments used with rigid models.

The tests on rigid models have been carried out with a reduced
number of fixed pick-ups (only the pick-ups incorporated in the pre-
viously described elements). The superabundant measurements were car-
ried out with the assistance of a mobile pick-up. The precision of the
results did not seem to be affected, but the measurements have been
much longer and more delicate. It is most desirable that all of the
pick-ups be fixed.

Figure 17 represents a velocity pick-up.

V. Common Suspension

V.1. Tests of Vibration and Measurements of Inertia Constants

It is advisable not to use bungee cords. These are not very well
adapted to the case of normal ground vibration tests, but are too inac-
curate to be integrated within a sequence of measurements. Elastic

Figure 17. Velocity pick-up.

mechanical components are preferable: leaf springs, etc. Low damping offers a considerable advantage for measurements of inertia constants, as we have previously seen. It is nevertheless awkward to attain complete arrest of oscillations in response to a measurement or an accidental bump on the structure. To avoid such losses of time, it generally suffices to short-circuit the dischargers (which induces a significant enough damping). But it is easy to provide for a looping of each discharger-pick-up according to the diagram of figure 18, which we can establish occasionally or leave in service continuously (for example, during the plots of deformation modes). The loopings must be distinct from one discharger to the next, or else auto-oscillation can appear on one or several modes.
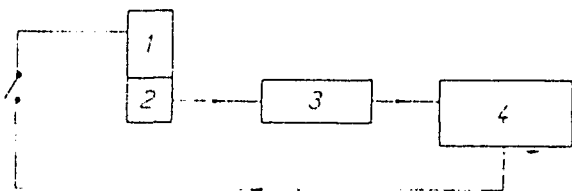

Figure 18. Looping of discharger-pick-up.
1=Discharger 2=Velocity pick-up
3=Voltage amplifier 4=Power amplifier (with springs or bungee cords)

Case of Airplanes of Large Dimensions

In the case of big airplanes, it is impossible to accomplish a suspension with braces or bungee cords whose frequencies are low in relation to those of the initial natural modes. We indeed know that elongation of the braces under the effect of weight is equal to $\frac{g}{\omega^2}$ (g=ac-

celeration of gravity). For $\int 0.5 \frac{g}{2} = 4.00$ n, which leads to elasticities of very large sizes, very heavy and unacceptable.

A solution for this study consists of using a pneumatic suspension. As a guide (tests are in preparation) we indicate below the principle (figure 19).
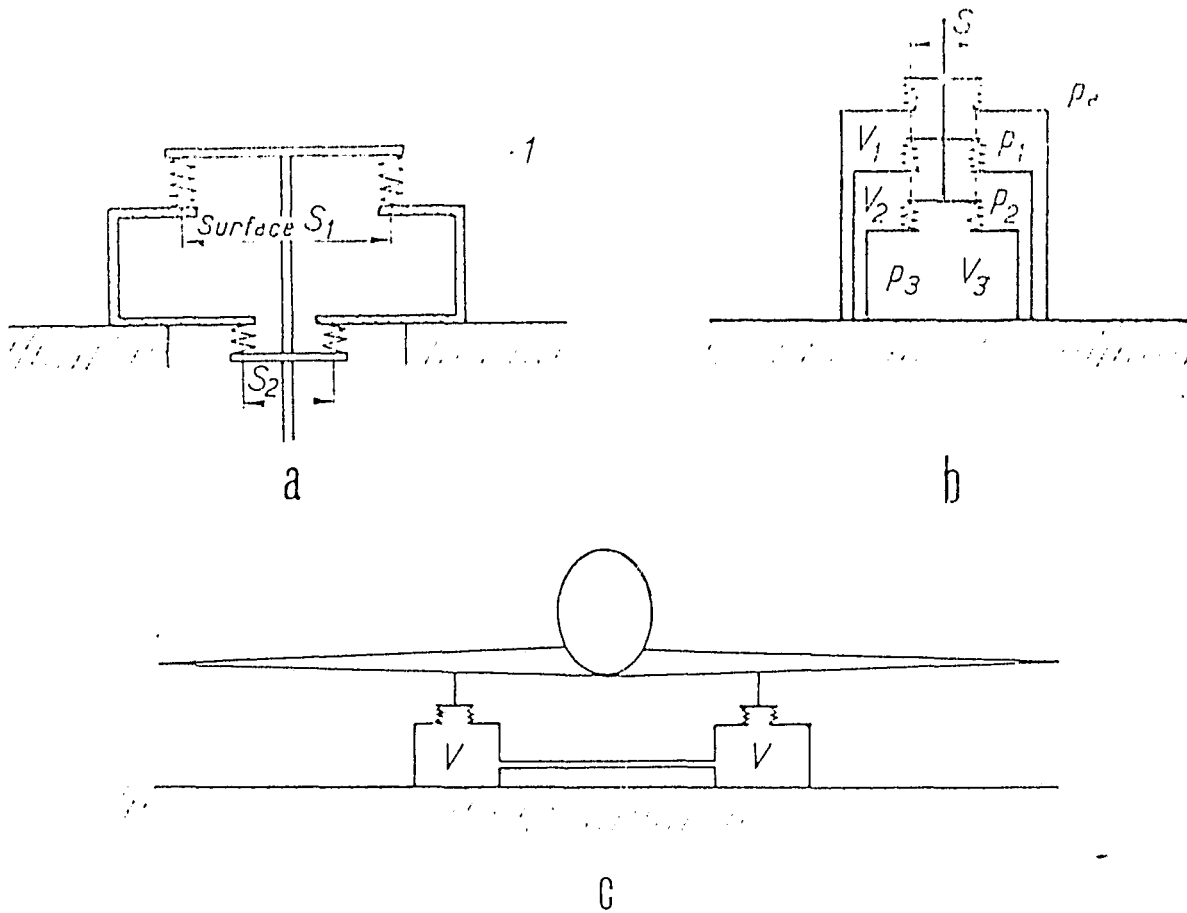
Figure 19. Elastic moistureproof joints.

In the case of figure 19a the pneumatic rigidity is: k_n p_i

The weight of the mobile portion is low, construction easy, and bulkiness reduced.

We can devise various arrangements, for example those of figures

19b and c. In 19b the rigidity is: k_r $\mu_r \frac{c_1^2}{c_1}$ where c_1 and c_2 are constants.

In 19c, the lateral pneumatic rigidity is nil, and reduced at the elastic spings due to the moistureproof joints.

For an airplane of 1500 KN with 2 principal points of suspension, with c_1 $0.11 \times^{-1} k_r$ $750 \wedge$ m we have c_3 (figure 19b) approximately 2 m^3.

The airplane can be suspended or supported on the suspension elements (it is then necessary to provide for lateral stabilization by braces with security stops).

VI. Conclusions

The results obtained on rigid models and flexible models are very satisfactory. The measurement apparatus used is directly transposable to any scale. The increase of the time of the standard vibration tests involved is on the order of 15 hours (therefore acceptable in all cases). It is now possible to take preliminary steps for measurement on airplanes of average tonnage (10 to 20 tons). For sturctures of great sizes the study of the problem of the suspension is underway for the scale model and will be examined for large models as a function of the initial results obtained.

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