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REVIEW OF SOME VORTEX RELATIONS

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REVIEW OF SOME VORTEX RELATIONS

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Abstract

The evaluation of the circulation from numerical solutions of the momentum and energy equations is discussed for incompressible and compressible flows. It is shown how artificial damping directly influences the time rate of change of the circulation.

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Introduction

Most finite-difference approximations of the Euler equations and of the Navier-Stokes equations for large Reynolds numbers must resort to artificial damping in order to stabilize the solution of the difference equations. Since the discretization process itself and the introduction of the damping terms changes the differential balance of mass, momentum, and energy, it is of general interest to know to what extent the local flow behavior and the overall flow properties are affected. If, for example, unsteady vortex formation is to be predicted, the net spacing and the time steps must be appropriately chosen, or else the numerical solution may completely suppress the formation of vortices. Or, if solution branching occurs, the results predicted may not correspond to the correct branch at all [1]. Another example is the prediction of inviscid transonic flows. Severe falsification of the total pressure may result in the computation if either the truncation error is too large or the damping terms are not adequately modelled. In fact, for irrotational flows the total pressure is often taken as a measure of accuracy of the numerical solution.

For rotational flows, in particular for unsteady flows, such a measure is not always available. It is then advantageous, to trace back the influence of the artificial damping on the integral properties of the solution. As the circulation is a characteristic integral property for rotational flows, its dependence on the local flow properties is reviewed here in some detail. First the material derivative of the circulation is discussed in conjunction with the Thomson theorem. Thereby the overall influence of artificial damping terms on the circulation can be demonstrated. By making use of various forms of the energy equation, it is then shown how for compressible flows the material derivative of the circulation is related to the entropy, heat conduction, and dissipation.

2. The Material Derivative of the Circulation

As is well known (see, for example [2]), the material derivative of the circulation Γ is obtained by considering its change during an infinitesimal interval of time dt:

$$\frac{d\Gamma}{dt} = \oint_c \frac{d\overline{v}}{dt} \cdot d\overline{s} \ . \tag{2.1}$$

Equation (2.1) can be written in a different form by splitting up the material derivative of the velocity into its local and convective part and by eliminating the latter with the vector identity

$$(\overline{v}\cdot\nabla)\overline{v} = \nabla\left(\frac{v^2}{2}\right) - \overline{v}x(\nabla x\overline{v}). \tag{2.2}$$

There results

$$\frac{d\Gamma}{dt} = \oint_c \frac{\partial\overline{v}}{\partial t} \cdot d\overline{s} - \oint_c \left(\overline{v}x(\nabla x\overline{v})\right) \cdot d\overline{s}. \tag{2.3}$$

This relation is valid for incompressible and compressible flows, as is equation (2.1). Another way of eliminating the material derivative of the velocity in equation (2.1) is to replace it through the momentum equation

$$\frac{d\overline{v}}{dt} = -\frac{1}{\rho}\nabla p - \frac{1}{\rho}\nabla\cdot\sigma' \ . \tag{2.4}$$

In equation (2.4) the quantity σ' represents the Stokes stress tensor without the pressure. Equation (2.1) then yields

$$\frac{d\Gamma}{dt} = - \oint_c \frac{dp}{\rho} - \oint_c \frac{1}{\rho} (\nabla \cdot \sigma') \cdot d\overline{s}. \tag{2.5}$$

For incompressible flow the first line integral in equation (2.5), representing the change of the static pressure along the closed curve c, vanishes. Then $d\Gamma/dt$ is solely determined by viscous forces, i.e.

$$\frac{d\Gamma}{dt} = -\nu \oint_c (\nabla^2 \overline{v}) \cdot d\overline{s}. \tag{2.6}$$

Equation (2.6) contains Thomson's theorem: if the kinematic viscosity vanishes, the material derivative of the circulation is zero, i.e. $d\Gamma/dt = 0$, and the circulation remains constant, if it is initially constant. From equation (2.3) it follows for $d\Gamma/dt = 0$ that

$$\oint_c \frac{\partial \overline{v}}{\partial t} \cdot d\overline{s} = \oint_c \left(\overline{v} x (\nabla x \overline{v})\right) \cdot d\overline{s}. \tag{2.7}$$

Equation (2.6) can serve to evaluate the time development of the circulation in numerical solutions for incompressible flow.

3. Compressible Flows

The material derivative of the circulation can also be obtained for compressible flow. Since the density is then not constant along the curve c, the first integral in equation (2.5) does not vanish. The differential of the pressure in equation (2.4) can be expressed by the differentials of enthalpy and entropy according to the second law of thermodynamics,

$$\frac{1}{\rho}\, dp = dh - Tds, \tag{3.1}$$

and equation (2.5) becomes

$$\frac{d\Gamma}{dt} = \oint_c Tds - \oint_c \left(\frac{1}{\rho}\, \nabla\cdot\sigma'\right)\cdot d\overline{s}. \tag{3.2}$$

The line integral over dh vanishes for the same reasons given in conjunction with equation (2.6). For inviscid flows equation (3.2) reduces to [2]

$$\frac{d\Gamma}{dt} = \oint_c Tds. \tag{3.3}$$

In contrast to incompressible flow, circulation can be generated in compressible inviscid flow, as long as the entropy and temperature vary along the path of integration in such a way that the line integral in equation (3.3) does not vanish.

Equation (2.3) can also be derived from equation (3.3) by using Crocco's theorem. It is obtained by expressing the static enthalpy h in equation (3.1) through the total enthalpy and the kinetic energy

$$dh = dh_0 - d\,\frac{v^2}{2}\,, \tag{3.4}$$

and by using the vector identity, equation (2.2), and the momentum equation, (equation (2.4))

$$T\nabla s = \nabla h_0 + \frac{\partial \bar{v}}{\partial t} - \bar{v}x(\nabla x\bar{v}) + \frac{1}{\rho}\nabla\cdot\sigma' . \tag{3.5}$$

This relation is identical with Crocco's theorem if the term describing the viscous forces is set equal to zero. If equation (3.5) is inserted in equation (3.2), it is seen that equation (2.8) is recovered.

In numerical computations the material derivative of the circulation can be evaluated either from equation (2.5), equation (3.2) or, for inviscid flows, from equation (3.3). The influence of heat conduction and energy dissipation can be demonstrated by differentiating equation (3.4) with respect to time, and by inserting the energy equation into the resulting relation. With

$$\frac{dh_0}{dt} = \frac{1}{\rho}\frac{\partial p}{\partial t} + \frac{1}{\rho}\nabla\cdot(\lambda\nabla T - \sigma'\cdot\bar{v}), \tag{3.6}$$

there is then obtained

$$T\frac{ds}{dt} = \frac{1}{\rho}\nabla\cdot(\lambda\nabla T) - \frac{1}{\rho}(\sigma':\nabla\bar{v}) . \tag{3.7}$$

A direct substitution of this relation into equation (3.2) is, however, not possible.

4. Applications

Some of the relations discussed can be used for numerical evaluation. If artificial damping is introduced, the momentum equation, (equation (2.4)), may contain two additional terms of second and fourth order:

$$\frac{d\bar{v}}{dt} = -\frac{1}{\rho}\nabla p - \frac{1}{\rho}\nabla\cdot\sigma' + f_1 \nabla^2 \bar{v} + f_2 \nabla^4 \bar{v}, \tag{4.1}$$

where f_1 and f_2 designate the damping coefficients. The material derivative of the circulation is then

$$\frac{d\Gamma}{dt} = -\oint_c \frac{dp}{\rho} - \oint_c \frac{1}{\rho}(\nabla\cdot\sigma')\cdot d\bar{s} \tag{4.2}$$

$$+ \oint_c f_1(\nabla^2 \bar{v}) \cdot d\bar{s} + \oint_c f_2(\nabla^4 \bar{v})\cdot d\bar{s}.$$

The last two integrals can be evaluated separately so that their contribution to the circulation can be checked as a function of time. The damping of the energy equation can be estimated from equation (3.7), if corresponding damping terms are added.

5. Conclusions

The evaluation of the circulation in numerical solutions was discussed for incompressible and compressible flows. It was shown that the material derivative of the circulation is directly influenced through artificial damping terms in the momentum equation, and indirectly through damping terms in the energy equation.

References

[1] Krause, E.: Remarks on computations of viscous flows, Comm. Pure Appl. Math., Vol. XXXII, 1979, pp. 749-781.

[2] Oswatitsch, K.: Gas Dynamics, Academic Press, New York, p. 200, 1956.

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