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TABLE OF CONTENTS

TECHNICAL PAPER

ALGORITHM FOR ASTRONOMICAL, POINT SOURCE, SIGNAL TO NOISE RATIO CALCULATIONS

I. GENERAL DESCRIPTION

The starting point for this program is three other programs [1] (OTF, PSF and Camera) developed by Dr. D. J. Schroeder. References 2 through 4 provide the basis for the development work. The programs OTF and PSF compute the monochromatic point spread function for a star of a given visual magnitude as seen through a telescope with a circular obscuration ratio of 0.33, entrance pupil diameter D and system focal ratio F outside the Earth's atmosphere. In the absence of any degradations, the point spread function (PSF) is computed from the modulation transfer function (MTF) of a perfect lens with a central obscuration. Image degradations can be introduced by multiplying the perfect lens MTF by the MTF's associated with the different types of degradations. The above programs include a MTF for image jitter, and MTFs for high, mid and low frequency wave aberrations associated with the optical telescope assembly. The PSF is assumed to be radially symmetric to considerably simplify to the computations. No ray aberrations due to optical path differences, especially those associated with the off axis imaging of the telescope, are considered. Any off axis defects are assumed to be compensated for by the following instrument. The change in the PSF for a selected wavelength can be examined for different combinations and magnitudes of the degradations included in the program.

Program Camera, in conjunction with OTF and PSF, adds a detector pel array, to the focal plane, a uniform cosmic background and the option to add a second star in the vicinity of the target star. For a given wavelength and for each pel in the detector array (maximum array size is 20 x 8) two relative signals are computed. The first is a target star relative signal. The second is the target star plus cosmic background signal. If a second star has been included, three relative signals are computed: the target star relative signal, the second star relative signal, and the target star plus second star plus cosmic background relative signal.

The above program has been modified to also compute the encircled energy for the monochromatic PSFs, the polychromatic PSF, and signal to noise ratios (S/N) for a given wavelength interval as a function of observation time and observation times as a function of signal to noise ratios using the combined Space Telescope and Wide Field/Planetary Camera quantum efficiencies [5], filter functions [6], and a detector characterized by an rms readout noise, a mean dark current, and an optional dead space between the detector pels.

II. USER INSTRUCTIONS

The user is first asked to input the visual magnitude of the signal star and then to choose whether or not the spectral distribution of the star is uniform or blackbody. The cosmic background and background star (if requested) will also have the same type of distribution. If a blackbody distribution is chosen, the user is asked to input the effective temperature (degrees Kelvin) of the signal star.

The effective temperatures of the cosmic background and background star (if requested) will be asked for later.

Next the user is asked to input the number of wavelengths to consider and the lower and upper wavelengths (cm). For Space Telescope, the lower and upper wavelengths are 10^{-5} and 11×10^{-5} cm. The maximum number of wavelengths that the program can handle is 21, which makes the wavelength increment 0.5×10^{-5} cm. For this case, the filter functions ($0 \leq$ transmission ≤ 1) and quantum efficiencies (electrons/photon) must be specified and read in (all 21 of them) in increments of 0.5×10^{-5} cm. The user is asked if there is a table of quantum efficiencies and filter functions. If the answer is yes to both questions then two files of data must have been previously prepared, which are now read into the program. One file contains the quantum efficiencies and the other contains the filter transmission. Because the program integrates over wavelength, the first and last non-zero filter transmissions should be divided by 2 in order to use the trapezoidal rule for numerical integration. If the answer to both questions is no, the user is assumed to desire a system with 100 percent quantum efficiency and 100 percent transmission.

To obtain a monochromatic star and background, the lower wavelength will be chosen as the desired wavelength, if the number of wavelength calculations is set to 1. The user is then asked to input the quantum efficiency and filter transmission for that wavelength.

Next the user is asked to input the diameter (cm) of the telescope entrance pupil, the focal ratio of the optical system and the central obscuration ratio (ratio of obscured to clear entrance pupil diameter). For Space Telescope the obscuration ratio is 0.33.

The user is then asked to input the image degradation factors of jitter and high, mid and low frequency wave aberrations. For Space Telescope [1] the rms image jitter is anticipated to be 0.007 arc sec or less and the high, mid and low frequency aberrations are reported to be 0.121×10^{-5} cm, 0.1304×10^{-5} cm, and 0.2361×10^{-5} cm, respectively.

At this stage of the program the user may request a printout of the monochromatic point spread function and encircled energy. If this option is requested, the program outputs the Airy radius in arc seconds and cm using the shortest wavelength for which the product of the quantum efficiency and filter function is greater than 10^{-4}. The user is then asked to input the maximum radius (arc seconds) to be considered and the number of radius calculations desired. If the maximum radius to be considered is 0.2 and the number of radius calculations is 51, the point spread function and encircled energy values will be output in the range from 0 to 0.2 arc sec in increments of 0.004 arc sec. The wavelength is selected by entering a wavelength number from 1 to 21 for space telescope. For example, 9 is 5×10^{-5} cm, 10 is 5.5×10^{-5} cm, and 11 is 6×10^{-5} cm. The program then outputs the normalized point spread function [PSF(0)=1], the encircled energy, EE, [EE(∞)=1], the normalized point spread function scaled in units of energy cm^{-2} sec^{-1} as a function of radius along with the radius in units of cm and arc seconds. After the printout has been completed the user is asked if there is a desire to change any of the PSF parameters. A yes answer allows the user to re-input a maximum radius, change the number of radius calculations or select another wavelength. The procedure described in this paragraph is repeated until a no answer is received for changing the PSF parameters.

Next, the user is asked to enter the x,y widths of a pel (arc seconds) and the x,y pel center separation (arc seconds), the number of pels in the x,y directions and the x,y coordinates of the signal star chief ray in the detector plane. To visualize the effects of the inputs, see Figure 1, using the definitions listed below:

XPW = x direction width of the CCD pel in arc seconds

YPW = y direction width of the CCD pel in arc seconds

XPD = x distance between pel centers in arc seconds, XPD $\geq$ XPW

YPD = y distance between pel centers in arc seconds, YPD $\geq$ YPW

NPX = number of pels in the x direction

PSX = number of pels in the y direction

PSX = x coordinate (arc seconds) of signal star chief ray, 0 $\leq$ PSX $\leq$ NPX.PD/2

PSY = y coordinate (arc seconds) of signal star chief ray, 0 $\leq$ PSY $\leq$ NPY.YPD/2

Dead space between the pels is created whenever XPD > XPW and/or YPD > YPW, i.e., the pel center separation is greater than the pel width in the same direction. The chief ray coordinates, which are now considered to be the optic axis, should be as close or closer to the origin than to the farthest corner of the pel that is farthest from origin. Otherwise, redundant situations are created that serve no purpose other than to complicate the programming. Smaller array sizes produce more accurate results in the (S/N) calculations because the PSF is computed at 200 different radii that range from zero to a value in arc seconds that is equal to the distance from the signal star chief ray coordinates to the coordinates of the farthest corner of the farthest pel from the origin (i.e., the larger the array size, the larger radius increment). The normalized point spread function is integrated over a pel to determine the fraction of energy, which will be denoted as diffraction efficiency, that falls within the pel. This integration requires radial interpolation of the point spread function, which produces less error when the radial increments are made smaller. From these inputs just discussed in the paragraph, the program computes and outputs the diffraction efficiency as a function of wavelength for each pel in the array.

Next, the user is asked to enter the visual magnitude of the cosmic background, and if a blackbody distribution was previously selected, an effective background temperature (degrees Kelvin) will be requested. The user is also asked whether or not to include a background star. If the answer is yes the user is asked for the visual magnitude of the background star, and if a blackbody distribution was previously selected, the effective temperature (degrees Kelvin) of the star will be requested. The x,y coordinates (arc seconds) of the background star chief ray are now requested. The diffraction efficiency, as discussed before, is computed for this star as a function of wavelength for each pel. In this case the point spread is also computed at 200 different radii. If the chief ray coordinates lie within the detector array, the radius ranges from a value of zero to a value equal to the distance from the chief ray coordinates to the farthest corner of the pel that is farthest from the chief ray. If the chief ray coordinates lie outside the detector array, the radius ranges from a value that is equal to the minimum distance from the chief ray to the nearest pel to a value that is equal to the distance from the chief ray to the farthest corner of the farthest pel. From these just discussed inputs, the program computes and outputs the diffraction efficiency of the background star as a function of wavelength for each pel in the array.

To complete the detector characteristics, the user inputs the rms readout noise in electrons per pel and the detector temperature in degrees Kelvin. For the particular CCD array used with the Space Telescope and Wide Field/Planetary Camera, the readout noise has been suggested [5] to range from 13.9 to 17.8 electrons per pel rms. For the above CCD array, the maximum operating temperature [5] is expected to be around 178°K (-95°C), and the mean dark current in electrons per pel per second is computed from this output temperature. The user can input a mean dark current directly by first entering a temperature less than 4°K. A prompt will then appear requesting the desired dark current.

The next set of inputs are concerned with the output data that is desired. The user first specifies the observation start and end times in seconds and the number of time calculations. For example, to compute the (S/N) in hour intervals from 1 to 14 hours, the user would input a start time of 3600 seconds, and an end time of 50400 seconds and 14 is the number of time calculations. To reverse the situation and determine the observation times needed to achieve a desired range of (S/N), the user inputs a start and end (S/N) and the desired number of (S/N) calculations. The program will proceed to output the desired data.

After the output is complete, the user may repeat the calculations for a signal star with a different visual magnitude and/or a cosmic background with a different visual magnitude and/or a background star with a different visual magnitude, without having to re-input all the previous input data. An additional repeat calculation that can be accomplished with or without the magnitude changes, is to change the characteristics of the detector. This option, however, requires that the user re-input all data starting with the detector readout noise and ending with the number of (S/N) calculations. The program terminates when the user answers no to all the repeat calculation options.

II. EQUATIONS USED

The system of units used is the cgs system, except for the image jitter and detector focal plane coordinates and dimensions, which are in arc seconds. The total energy emitted in the eye responsive spectral region from a star of visual magnitude m and received per square meter per second outside the Earth's atmosphere is given by [7]

$$I_m = 2.54 \times 10^{-6} \times 10^{-0.4m} \text{ lux} \quad . \tag{1}$$

The above equation is converted from photometric to radiometric units by dividing by

$$0.68 \int_{3.8\times10^{-5}\ \mathrm{cm}}^{7.6\times10^{-5}\ \mathrm{cm}} K(\lambda)\, d\lambda \ \text{lux per (ergs cm}^{-2}\ \text{sec}^{-1}\ \Delta\lambda^{-1}) \quad , \tag{2}$$

where $\Delta\lambda$ is the wavelength interval (expressed in cm) and $K(\lambda)$ is the photopic eye response [7] given in Table 1. Dividing equation (1) by equation (2) is tantamount to assuming a uniform spectral distribution for the star and gives

$$I_m(\lambda) = \frac{3.7353\times10^{-6}\times10^{-0.4m}}{\int_{3.8\times10^{-5}\mathrm{cm}}^{7.6\times10^{-5}\mathrm{cm}} K(\lambda)\, d\lambda} \ \text{ergs cm}^{-2}\ \text{sec}^{-1}\ \Delta\lambda^{-1} \quad . \tag{3}$$

If $B(\lambda,T)$ is the blackbody distribution having an effective temperature T (degrees Kelvin), then equation (3) can be converted to a blackbody distribution in the following manner [8]:

$$I_m(\lambda,T) = \frac{3.7353\times10^{-6}\times10^{-0.4m}\ B(\lambda,T)}{\int_{3.8\times10^{-5}\mathrm{cm}}^{7.6\times10^{-5}\mathrm{cm}} K(\lambda)B(\lambda,T)d\lambda} \ \text{ergs cm}^{-2}\ \text{sec}^{-1}\ \Delta\lambda^{-1} \quad . \tag{4}$$

Any type distribution can be used for $B(\lambda,T)$ in equation (4), since multiplying equation (4) by $0.68K(\lambda)$ and integrating from 3.8×10^{-5} cm to 7.6×10^{-5} cm reproduces equation (1). If $B(\lambda,T)$ were a uniform distribution, then equation (3) would be obtained again. Assume that a uniform distribution has been selected and that the following quantities have been specified:

A = area of the telescope entrance pupil (cm^2)

hc/λ = ergs/photon

$Q(\lambda)$ = quantum efficiency in electrons per photon

$F(\lambda)$ = optical filter transmission

$DE_s(\lambda)$ = diffraction efficiency .

The quantum efficiencies [5] used in the output example are shown in Table 2. This includes the combined effects of the optical telescope assembly and the Wide Field/Planetary Camera. The filter function [6] used in the output example is listed in Table 3. The diffraction efficiency is quite involved and will be discussed in detail later. However, it is the integral of the normalized point spread function over the area of a pel. For a given wavelength, this integral gives the fraction of energy intercepted by that pel, since the integral over all space of the normalized point spread function is one.

The rate at which electrons are liberated from a particular pel per wavelength interval at a chosen wavelength is given by

$$S_s(\lambda) = \frac{A}{hc} \lambda I_m(\lambda) Q(\lambda) F(\lambda) DE_s(\lambda) \text{ electrons sec}^{-1} \, \Delta\lambda^{-1} \quad . \tag{5}$$

Equation (5) is called the monochromatic signal current. Integrating equation (5) over the wavelength region influenced by the quantum efficiency and optical filter produces the polychromatic signal current. If there is a dead space between the detector pels, the diffraction efficiency is adjusted to account for the dead space and the location at the chief ray of the diffraction pattern relative to the pel in question. The polychromatic signal current is given by

$$S_s = \frac{A}{hc} \int_{\lambda_1}^{\lambda_2} \lambda I_m(\lambda) Q(\lambda) F(\lambda) DE_s(\lambda) d\lambda \text{ electrons sec}^{-1} \quad . \tag{6}$$

By the same analogy the signal current from a second star would be given by

$$S_u = \frac{A}{hc} \int_{\lambda_1}^{\lambda_2} \lambda I_{m}'(\lambda) Q(\lambda) F(\lambda) DE_u(\lambda) d\lambda \text{ electrons sec}^{-1} \quad , \tag{7}$$

where m' could be a different visual magnitude and $DE_u(\lambda)$ is the diffraction efficiency of the second star for the same pel. The signal current for the cosmic background is given by

$$S_b = \frac{A(d\Omega)}{hc} \int_{\lambda_1}^{\lambda_2} I_m''(\lambda) Q(\lambda) F(\lambda) \, d\lambda \text{ electrons sec}^{-1} \quad , \tag{8}$$

where $d\Omega$ is the area of a pel in arc seconds squared ($\square''$). The cosmic background is treated as an extended source of visual magnitude m" and $I_m''(\lambda)$ has units of per $\square''$. The $d\Omega$ for an extended source replaces $DE(\lambda)$ for a point source.

To compute a signal to noise ratio, the contributions to the signal term and the noise term must be defined. For the polychromatic case, for example, the signal is equation (6) multiplied by the time in seconds. If the incoming photon flux is assumed to be Poisson distributed, then the variance of the signal is also equal to the mean signal. If the signal to noise ratio is defined as the ratio of the mean to the standard deviation of the mean, then the signal star will contribute to the noise. Other contributions to the noise include the cosmic background signal, the unwanted star signal if requested, the rms readout noise in electrons per pel and the mean dark current in electrons per pel per second. If all of the noise contributors are assumed to be statistically independent, then the signal to noise ratio is given by

$$(S/N) = \frac{S_s . t}{(S_s . t + S_u . t + S_b . t + R^2 + D . t)^{1/2}} \quad , \tag{9}$$

where R is the rms readout noise, D is the mean dark current and t is the time in seconds. If the second star option was not requested, S_u would be set to zero. Equation (9) is computed for every pel and the only quantity that changes in the diffraction efficiency in the expressions for S_s and S_u. As previously mentioned, the user has the option of inputing the dark current directly or inputing the detector temperature and having the dark current computed for the CCD array on Space Telescope for the Wide Field/Planetary Camera. The equations (9) for the dark current are given by

$$D = 29.3 \times 10^9 \times T^{3/2} \exp(-5802.1Z) \quad , \tag{10}$$

where

$$Z = \frac{1.1557}{T} - \frac{7.021 \times 10^{-4}\ T}{1108 + T} \quad , \tag{11}$$

and where T is the detector temperature in degrees Kelvin.

Equation (9) can also be solved for the time required to achieve a particular (S/N). Solving for the time gives

$$t = \frac{(S/N)^2\ (S_s + S_u + S_b + D)}{2S_s^{\ 2}} + \left\{ \frac{(S/N)^4\ (S_s + S_u + S_b + D)^2}{4S_s^{\ 4}} + (S/N)^2\ (R/S_s)^2 \right\}^{1/2} \text{sec} . \tag{12}$$

How the diffraction efficiency is calculated will now be discussed. As previously mentioned, the diffraction efficiency is the integral of the normalized point spread function over the detector pel area. The analytic expression for the normalized rotationally symmetric point spread function is given by [2]

$$PSF(r,\lambda) = \frac{8}{\tau_p}\int_0^1 MTF(\nu_n,\lambda)\, J_0(2\pi r\, [Dp.\pi/\lambda.64800]\nu_n)\, \nu_n d\nu_n \quad , \tag{13}$$

where r is the radius of the PSF in arc seconds, D_p is the diameter of the telescope entrance pupil (cm), λ is the wavelength (cm), ν_n is the normalized spatial frequency, J_o is a zero order Bessel function of the first kind, τ_p is the pupil transmittance, 648000 is the number of arc seconds in pi radians, and MTF is the modulation transfer function. For a central obstruction, the pupil transmittance is given by $1-\varepsilon^2$, where ε is the ratio of the obstruction diameter to the clear diameter. Equation (13) is normalized such that $PSF(0,\lambda) = 1$, however it can be scaled [2] in units of energy $cm^{-2}\ sec^{-1}$ by multiplying by $\tau_p\ \pi^2\ D^2_p/16\lambda^2F^2$, where F is the system focal ratio. The MTF is composed of the product of the individual MTF's representing the perfect lens, the image jitter, and the high, mid and low frequency aberrations of the optical telescope assembly. The MTF equation for the perfect lens is given by [2]

$$TPL(\nu_n) = (A+B+C)/(1-\varepsilon^2) \quad , \tag{14}$$

where

$$A = \frac{2}{\pi}\left[\text{arc}\cos(\nu_n)-\nu_n(1-\nu_n{}^2)^{1/2}\right] \quad ; \quad 0 \le \nu_n \le 1.0$$
$$A = 0 \quad ; \quad \nu_n > 1.0 \tag{15}$$

$$B = \frac{2\varepsilon^2}{\pi}\left\{\text{arc}\cos(\nu_n/\varepsilon)-(\nu_n/\varepsilon)[1-(\nu_n/\varepsilon)^2]^{1/2}\right\} \quad ; \quad 0 \le \nu_n/\varepsilon \le 1.0$$
$$B = 0 \quad ; \quad \nu_n/\varepsilon > 1.0 \tag{16}$$

$$C = -2\varepsilon^2 \quad ; \quad 0 \le \nu_n \le (1-\varepsilon)/2$$
$$C = \frac{2}{\pi}\left\{\varepsilon \sin\phi + \frac{\phi}{2}(1+\varepsilon^2) - (1-\varepsilon^2)\ \text{arc}\tan\left(\frac{1+\varepsilon}{1-\varepsilon}\tan\frac{\phi}{2}\right)\right\} - 2\varepsilon^2 \quad ; \tag{17}$$
$$(1-\varepsilon/2) \le \nu_n \le (1+\varepsilon/2)$$
$$C = 0 \quad ; \quad \nu_n \ge (1+\varepsilon/2) \quad ,$$

and

$$\phi = \text{arc}\cos\left[\frac{1-\varepsilon^2-4\nu_n{}^2}{2\varepsilon}\right] \quad .$$

The MTF for image jitter is given by [2]

$$TJ(\nu_n,\lambda) = \exp\left[\frac{-2\pi^2 \cdot \sigma_j^2 \cdot D_p^2 \cdot \pi^2 \cdot \nu_n^2}{\lambda^2 \cdot 648000^2}\right] \quad , \tag{18}$$

where σ_j is the rms jitter in arc seconds, D_p is the telescope entrance pupil diameter (cm), 648000 is the number of arc seconds in pi radians and λ is the wavelength (cm). The MTF for the high frequency wave aberrations is given by [4]

$$TH(\lambda) = \exp\left[\frac{-4\pi^2 \cdot \sigma_h^2}{\lambda^2}\right] \quad , \tag{19}$$

where σ_h is the rms high frequency aberration (cm). The MTF for the mid frequency aberrations are given by [1]

$$TM(\nu_n,\lambda) = \exp\left[\left(\frac{-4\pi^2 \cdot \sigma_m^2}{\lambda^2}\right)\{1 - [1 - 18 \cdot \nu_n]^{3/2}\}\right] \quad ; \quad \nu_n < 1/18 \tag{20}$$

$$TM(\nu_n,\lambda) = \exp\left(\frac{-4\pi^2 \cdot \sigma_m^2}{\lambda^2}\right) \quad ; \quad \nu_n \geq 1/18$$

where σ_m is the rms mid frequency aberration (cm). The MTF for the low frequency aberrations is given by [1]

$$TL(\nu_n,\lambda) = 1 - \frac{2.815\ \sigma_L}{\lambda}\sin\left(\frac{5\pi\nu_n}{3}\right) \quad ; \quad \nu_n \leq 0.355 \tag{21}$$

$$TL(\nu_n,\lambda)\ 1 - \frac{\sigma_L}{0.0373\ \lambda}(-0.9 + \sec[3/4(\nu_n - 0.4)]) \quad ; \quad \nu_n > 0.355$$

where σ_L^2 is the rms low frequency aberration (cm). If $TL(\nu_n,\lambda)$ becomes negative, it is set to zero. Thus, the $MTF(\nu_n,\lambda)$ in equation (13) is given by

$$MTF(\nu_n,\lambda) = TPL(\nu_n)\ .TJ(\nu_n,\lambda)\ .TH(\lambda)\ .TM(\nu_n,\lambda)\ .TL(\nu_n,\lambda) \tag{22}$$

For a given wavelength, the encircled energy, $EE(r_o,\lambda)$, is given by [2]

$$EE(r_o,\lambda) = \frac{\pi\tau_p}{4}\left[\frac{D_p \cdot \pi}{\lambda \cdot 648000}\right]^2 \int_o^{2\pi}\int_o^{r_o} PSF(r,\lambda,\theta)\, rdrd\theta \quad . \tag{23}$$

Since PSF is radially symmetric, equation (23) becomes

$$EE(r_o,\lambda) = \frac{\pi^2\tau_p}{2}\left[\frac{D_p \cdot \pi}{\lambda \cdot 648000}\right]^2 \int_o^{r_o} PSF(r,\lambda) rdr \quad . \tag{24}$$

The encircled energy is normalized such that $EE(oo,\lambda) = 1$. Substituting (13) into (24) gives

$$EE(r_o,\lambda) = 4\pi^2\left[\frac{D_p \cdot \pi}{\lambda \cdot 648000}\right]^2 \int_o^1 MTF(\nu_n,\lambda)\left[\int_o^{r_o} J_o\left(2\pi r\left(\frac{D_p \cdot \pi}{\lambda \cdot 648000}\right)\nu_n\right) rdr\right]\nu_n d\nu_n \tag{25}$$

or

$$EE(r_o,\lambda) = 2\pi r_o\left[\frac{D_p \cdot \pi}{\lambda \cdot 648000}\right] \int_o^1 MTF(\nu_n,\lambda)\, J_1\left(2\pi r_o\left[\frac{D_p \cdot \pi}{\lambda \cdot 648000}\right]\nu_n\right) d\nu_n$$

.

where J_1 is a first order Bessel function of the first kind, D_p is the diameter (cm) of the telescope entrance pupil, r_o the radius of the encircled energy in arc seconds, λ is the wavelength and 648000 is the number of arc seconds in pi radians. To achieve reasonable accuracy ν_n needs to be calculated for at least 400 points in the range $0 \leq \nu_n \leq 1$. Thus, ν_n is incremented in increments of 1/400. It is also recommended [1] that the following numerical integration scheme [10[be used:

$$\int_a^b f(x)\, dx = \frac{h}{4.5}\Big[1.4y_o+6.4y_1+2.4y_2+6.4y_3+2.8y_4 \\ +6.4y_5+2.4y_6+6.4y_7+2.8y_8 \\ \cdots\cdots\cdots\cdots \\ +6.4y_{4n-3}+2.4y_{4n-2}+6.4y_{4n-1}+1.4y_{4n}\Big] \tag{26}$$

where 4nh=b-a.

The computation of the diffraction efficiency is very similar to computing encircled energy except that the integration is performed over a square or rectangular area instead of a circular area. Thus, for a particular pel, equation (23) becomes

$$DE(\lambda) = \frac{\pi\tau_p}{4} \left[\frac{D_p \cdot \pi}{\lambda \cdot 648000} \right]^2 \int_{y_1}^{y_2} \int_{x_1}^{x_2} PSF(x,y,\lambda)\, dxdy \quad , \tag{27}$$

where x,y are expressed in arc seconds. Each pel is broken into a 20 by 20 subpixel array. The distance from each subpixel to the chief ray of the diffraction image is computed. Since the PSF is radially symmetric, the value of $PSF(x,y,\lambda)$ is found by interpolating the value of PS (r,λ) to a value of $r = (x^2+y^2)^{1/2}$. This procedure is used for the signal star as well as the second star.

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TABLE 1. PHOTOPIC EYE RESPONSE

λ (10^{-5}cm)	K(λ)	λ (10^{-5}cm)	K(λ)	λ (10^{-5}cm)	K(λ)
3.8	0.00004	5.1	0.503	6.4	0.175
3.9	0.00012	5.2	0.71	6.5	0.107
4.0	0.0004	5.3	0.862	6.6	0.061
4.1	0.0012	5.4	0.954	6.7	0.032
4.2	0.004	5.5	0.995	6.8	0.017
4.3	0.0116	5.6	0.995	6.9	0.0082
4.4	0.023	5.7	0.952	7.0	0.0041
4.5	0.038	5.8	0.87	7.1	0.0021
4.7	0.09	6.0	0.631	7.3	0.00051
4.8	0.139	6.1	0.503	7.4	0.00025
4.9	0.208	6.2	0.381	7.5	0.00012
5.0	0.323	6.3	0.265	7.6	0.00006

TABLE 2. QUANTUM EFFICIENCIES

λ (10^{-5}cm)	Q(λ)	λ (10^{-5}cm)	Q(λ)	λ (10^{-5}cm)	Q(λ)
1.0	0.0*	4.5	0.08*	8.0	0.12
1.5	0.028*	5.0	0.12	8.5	0.105*
2.0	0.05	5.5	0.15*	9.0	0.09
2.5	0.05	6.0	0.18	9.5	0.08*
3.0	0.05	6.5	0.18*	10.0	0.07
3.5	0.06	7.0	0.18	10.5	0.045*
4.0	0.04	7.5	0.15*	11.0	0.02

*Interpolated Values

TABLE 3. FILTER FUNCTION

λ (10^{-5} cm)	F(λ)
5.0	0.278*
5.5	0.899
6.0	0.578
6.5	0.0835*

*The values were divided by 2 for using trapezoidal rule in numerical integration over wavelength.

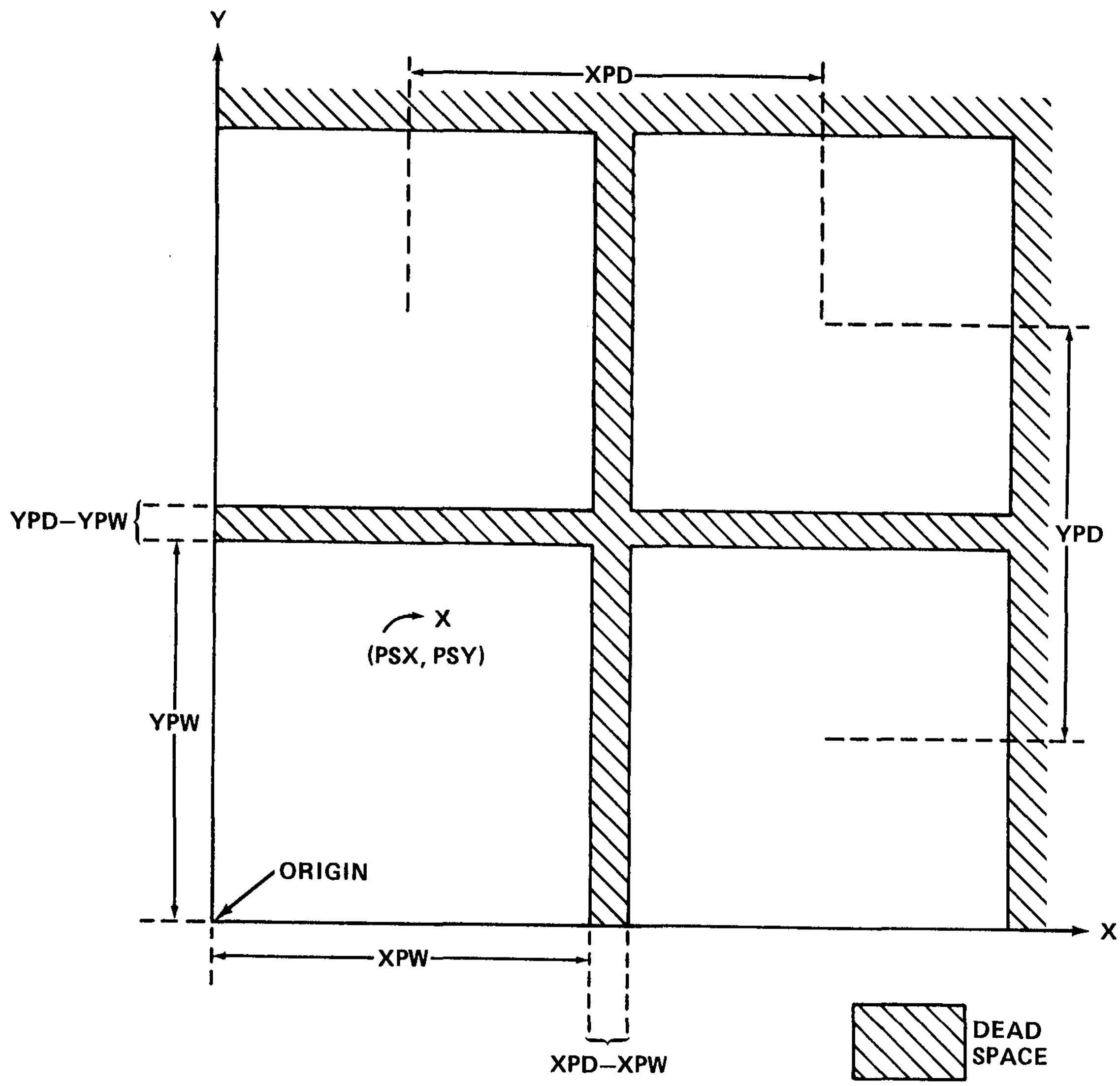

FIGURE 1. RELATIONSHIP OF POINT SOURCE DIFFRACTION IMAGE TO CCD CONFIGURATION

APPENDIX A

OUTPUT EXAMPLE

```
ENTER SIGNAL STAR(#1) VISUAL MAGNITUDE
?27.

ENTER CHOICE OF SPECTRAL DISTRIBUTION FOR SOURCE
FLAT DISTRIBUTION=0;BLACKBODY DISTRIBUTION=1
?0.

ENTER NUMBER OF WAVELENGTHS
?21.

ENTER LOWER AND UPPER WAVELENGTHS(CM)
?1.E-5,11.E-5

IS THERE A TABLE OF QUANTUM EFFICIENCIES?(YES=1;NO=0)
?1.

IS THERE A TABLE OF FILTER TRANSMISSIONS?(YES=1;NO=0)
?1.

ENTER TELESCOPE ENTRANCE PUPIL DIAMETER(CM)
?240.

ENTER SYSTEM FOCAL RATIO
?12.9

ENTER OBSCURATION RATIO
?0.33

ENTER RMS IMAGE JITTER(ARC SECONDS)
?0.007

ENTER HIGH FREQUENCY RMS ABERATIONS(CM)
?0.121E-5

ENTER MID FREQUENCY RMS ABERATIONS(CM)
?0.1304E-5

ENTER LOW FREQUENCY RMS ABERATIONS(CM)
?0.2361E-5

DO YOU WISH TO COMPUTE NORMALIZED POINT SPREAD
FUNCTION,PSF(0)=1,AND THE NORMALIZED ENCIRCLED ENERGY,
EE(INF)=1?(YES=1;NO=0)
?1.

MINIMUM AIRY RADIUS=  .78690000E-03 CM=  .52425639E-01 ARC SECONDS

ENTER MAXIMUM RADIUS TO CONSIDER(ARC SECONDS)
?0.5

ENTER NUMBER OF RADIUS CALCULATIONS
?51.

ENTER DESIRED WAVELENGTH NUMBER(1,2,...,21)
?9.

CLEAR SCREEN AND RETURN TO CONTINUE
?
```

MAXIMUM RADIUS= .50000000E 00 ARC SECONDS= .75049157E-02 CM
NUMBER OF RADIUS CALCULATIONS 51: WAVELENGTH= .50000000E-04 CM

RADIUS(CM)	PNT.SPREAD FN.	ENCIRCLED ENERGY	(ARC SECONDS)	ENERGY/SEC/CM**2
.00000000E 00	.71783407E 00	.00000000E 00	.00000000E 00	.54630266E 11
.15009831E-03	.62818690E 00	.80034691E-01	.10000000E-01	.47807730E 11
.30019663E-03	.41356896E 00	.26385750E 00	.20000000E-01	.31474380E 11
.45029494E-03	.19260445E 00	.43750811E 00	.30000000E-01	.14658029E 11
.60039325E-03	.57680580E-01	.53230129E 00	.40000000E-01	.43897407E 10
.75049157E-03	.20511074E-01	.56671843E 00	.50000000E-01	.15609811E 10
.90058988E-03	.32076381E-01	.59954483E 00	.60000000E-01	.24411508E 10
.10506882E-02	.41359947E-01	.65898910E 00	.70000000E-01	.31476702E 10
.12007865E-02	.32244479E-01	.72702312E 00	.80000000E-01	.24539438E 10
.13508848E-02	.16216223E-01	.77586572E 00	.90000000E-01	.12341244E 10
.15009831E-02	.61343854E-02	.79957848E 00	.10000000E 00	.46685316E 09
.16510814E-02	.32642920E-02	.81042938E 00	.11000000E 00	.24842668E 09
.18011798E-02	.27686108E-02	.81841905E 00	.12000000E 00	.21070321E 09
.19512781E-02	.21322428E-02	.82599344E 00	.13000000E 00	.16227286E 09
.21013764E-02	.18672712E-02	.83234957E 00	.14000000E 00	.14210739E 09
.22514747E-02	.21197206E-02	.83921188E 00	.15000000E 00	.16131987E 09
.24015730E-02	.19664822E-02	.84735678E 00	.16000000E 00	.14965777E 09
.25516713E-02	.12263025E-02	.85378890E 00	.17000000E 00	.93326904E 08
.27017696E-02	.83088452E-03	.85760040E 00	.18000000E 00	.63233892E 08
.28518679E-02	.12570514E-02	.86245988E 00	.19000000E 00	.95667029E 08
.30019663E-02	.17948766E-02	.86935760E 00	.20000000E 00	.13659784E 09
.31520646E-02	.16565774E-02	.87816518E 00	.21000000E 00	.12607268E 09
.33021629E-02	.10036829E-02	.88541796E 00	.22000000E 00	.76384591E 08
.34522612E-02	.49217134E-03	.88942075E 00	.23000000E 00	.37456360E 08
.36023595E-02	.34663041E-03	.89171255E 00	.24000000E 00	.26380068E 08
.37524578E-02	.32668767E-03	.89350920E 00	.25000000E 00	.24862339E 08
.39025561E-02	.28592982E-03	.89511280E 00	.26000000E 00	.21760491E 08
.40526545E-02	.30576034E-03	.89710784E 00	.27000000E 00	.23269679E 08
.42027528E-02	.38363752E-03	.89876181E 00	.28000000E 00	.29196469E 08
.43528511E-02	.37126165E-03	.90232322E 00	.29000000E 00	.28254611E 08
.45029494E-02	.25106076E-03	.90436156E 00	.30000000E 00	.19106805E 08
.46530477E-02	.20103978E-03	.90672520E 00	.31000000E 00	.15299993E 08
.48031460E-02	.31268894E-03	.90812325E 00	.32000000E 00	.23796975E 08
.49532443E-02	.43781817E-03	.91195925E 00	.33000000E 00	.33319849E 08
.51033426E-02	.40859452E-03	.91475336E 00	.34000000E 00	.31095804E 08
.52534410E-02	.26334991E-03	.91809605E 00	.35000000E 00	.20042063E 08
.54035393E-02	.15212166E-03	.91884714E 00	.36000000E 00	.11577114E 08
.55536376E-02	.12271794E-03	.92050378E 00	.37000000E 00	.93393643E 07
.57037359E-02	.11636423E-03	.92055190E 00	.38000000E 00	.88558192E 07
.58538342E-02	.10572196E-03	.92173590E 00	.39000000E 00	.80458970E 07
.60039325E-02	.11850355E-03	.92409051E 00	.40000000E 00	.90186305E 07
.61540308E-02	.14663707E-03	.92394983E 00	.41000000E 00	.11159713E 08
.63041292E-02	.14006352E-03	.92556743E 00	.42000000E 00	.10659437E 08
.64542275E-02	.99118686E-04	.92801915E 00	.43000000E 00	.75433591E 07
.66043258E-02	.87950077E-04	.92731732E 00	.44000000E 00	.66933798E 07
.67544241E-02	.13343053E-03	.92950553E 00	.45000000E 00	.10154638E 08
.69045224E-02	.17811796E-03	.93103175E 00	.46000000E 00	.13555545E 08
.70546207E-02	.16339423E-03	.93423958E 00	.47000000E 00	.12435005E 08
.72047190E-02	.10848708E-03	.93366166E 00	.48000000E 00	.82563339E 07
.73548173E-02	.68956311E-04	.93559926E 00	.49000000E 00	.52478724E 07
.75049157E-02	.58774463E-04	.93612036E 00	.50000000E 00	.44729899E 07

DO YOU WISH TO CHANGE ANY PSF PARAMETERS?(YES=1;NO=0)
?0.

CLEAR SCREEN AND RETURN TO CONTINUE
?

```
ENTER X,Y PEL WIDTHS(ARC SECONDS)
?0.1,0.1

ENTER X,Y PEL CENTER SEPARATIONS(ARC SECONDS)
?0.1,0.1

ENTER NUMBER OF PELS IN X,Y DIRECTIONS
?2.,2.

ENTER X,Y COORDINATES OF SIGNAL STAR(ARC SECONDS)
?0.05,0.05
WAV=  .50000000E-04 IX=1 IY=1 DIFE=  .57745045E 00
WAV=  .50000000E-04 IX=1 IY=2 DIFE=  .47940138E-01
WAV=  .50000000E-04 IX=2 IY=1 DIFE=  .47940138E-01
WAV=  .50000000E-04 IX=2 IY=2 DIFE=  .10263014E-01
WAV=  .55000000E-04 IX=1 IY=1 DIFE=  .57273850E 00
WAV=  .55000000E-04 IX=1 IY=2 DIFE=  .47984166E-01
WAV=  .55000000E-04 IX=2 IY=1 DIFE=  .47984166E-01
WAV=  .55000000E-04 IX=2 IY=2 DIFE=  .12232071E-01
WAV=  .60000000E-04 IX=1 IY=1 DIFE=  .56934979E 00
WAV=  .60000000E-04 IX=1 IY=2 DIFE=  .46483964E-01
WAV=  .60000000E-04 IX=2 IY=1 DIFE=  .46483964E-01
WAV=  .60000000E-04 IX=2 IY=2 DIFE=  .14751427E-01
WAV=  .65000000E-04 IX=1 IY=1 DIFE=  .56424891E 00
WAV=  .65000000E-04 IX=1 IY=2 DIFE=  .44573175E-01
WAV=  .65000000E-04 IX=2 IY=1 DIFE=  .44573175E-01
WAV=  .65000000E-04 IX=2 IY=2 DIFE=  .17411530E-01

ENTER COSMIC BACKGROUND VISUAL MAGNITUDE
?23.

DO YOU WISH TO INCLUDE A BACKGROUND STAR?(YES=1;NO=0)
?0.

ENTER RMS READOUT NOISE(ELECTRONS/PEL)
?18.

ENTER DETECTOR TEMPERATURE(KELVIN)
?178.

ENTER OBSERVATION START AND END TIMES(SECONDS)
?3600.,50400.

ENTER NUMBER(>1) OF TIME CALCULATIONS
?14.

ENTER START AND END SIGNAL TO NOISE RATIOS(S/N)
?1.,10.

ENTER NUMBER(>1) OF (S/N) CALCULATIONS
?10.

CLEAR SCREEN AND RETURN TO CONTINUE
?
```

TELESCOPE ENTRANCE PUPIL DIAMETER 240.000 CM
OPTICAL SYSTEM FOCAL RATIO 12.90

SOURCE STAR CHARACTERISTICS
SOURCE MAGNITUDE 27.00

COSMIC BACKGROUND CHARACTERISTICS
COSMIC BACKGROUND MAGNITUDE 23.00

SYSTEM SPECTRAL CHARACTERISTICS

WAVELENGTH(CM)	QUAN.EFFIC.	FILTER FUNCTION
.5000E-04	.1200E 00	.2780E 00
.5500E-04	.1500E 00	.8990E 00
.6000E-04	.1800E 00	.5780E 00
.6500E-04	.1800E 00	.8350E-01

DETECTOR CHARACTERISTICS
PEL DIMENSIONS(ARC SECONDS):X= .1000 Y= .1000
PEL CENTER SEPARATION(ARC SECONDS):X= .1000 Y= .1000
SOURCE STAR COORDINATES(ARC SECONDS):X= .0500 Y= .0500
ARRAY SIZE:X= 2 BY Y= 2
RMS READOUT NOISE(ELECTRONS/PEL):18.00
DETECTOR TEMPERATURE(KELVIN):178.0000
DARK CURRENT(ELECTRONS/PEL/SECOND): .53327224E-02

SIGNAL TO NOISE RATIO AS A FUNCTION OF OBSERVATION TIME
AND PEL NUMBER

TIME(SECONDS)	(S/N)	PEL NUMBER	
.36000E 04	.7986E 01	1	1
.72000E 04	.1288E 02	1	1
.10800E 05	.1662E 02	1	1
.14400E 05	.1975E 02	1	1
.18000E 05	.2248E 02	1	1
.21600E 05	.2493E 02	1	1
.25200E 05	.2718E 02	1	1
.28800E 05	.2925E 02	1	1
.32400E 05	.3119E 02	1	1
.36000E 05	.3303E 02	1	1
.39600E 05	.3476E 02	1	1
.43200E 05	.3642E 02	1	1
.46800E 05	.3800E 02	1	1
.50400E 05	.3952E 02	1	1
.36000E 04	.7756E 00	1	2
.72000E 04	.1329E 01	1	2
.10800E 05	.1772E 01	1	2
.14400E 05	.2147E 01	1	2
.18000E 05	.2478E 01	1	2
.21600E 05	.2775E 01	1	2
.25200E 05	.3047E 01	1	2
.28800E 05	.3299E 01	1	2
.32400E 05	.3535E 01	1	2
.36000E 05	.3757E 01	1	2
.39600E 05	.3968E 01	1	2
.43200E 05	.4168E 01	1	2
.46800E 05	.4360E 01	1	2
.50400E 05	.4544E 01	1	2
.36000E 04	.7756E 00	2	1
.72000E 04	.1329E 01	2	1
.10800E 05	.1772E 01	2	1
.14400E 05	.2147E 01	2	1
.18000E 05	.2478E 01	2	1
.21600E 05	.2775E 01	2	1
.25200E 05	.3047E 01	2	1
.28800E 05	.3299E 01	2	1
.32400E 05	.3535E 01	2	1
.36000E 05	.3757E 01	2	1
.39600E 05	.3968E 01	2	1
.43200E 05	.4168E 01	2	1
.46800E 05	.4360E 01	2	1
.50400E 05	.4544E 01	2	1
.36000E 04	.2214E 00	2	2
.72000E 04	.3816E 00	2	2
.10800E 05	.5105E 00	2	2
.14400E 05	.6202E 00	2	2
.18000E 05	.7168E 00	2	2
.21600E 05	.8038E 00	2	2
.25200E 05	.8834E 00	2	2
.28800E 05	.9572E 00	2	2
.32400E 05	.1026E 01	2	2
.36000E 05	.1091E 01	2	2
.39600E 05	.1153E 01	2	2
.43200E 05	.1211E 01	2	2
.46800E 05	.1268E 01	2	2
.50400E 05	.1321E 01	2	2

(S/N)	SECONDS	PEL NUMBER	
1.00	.88949E 02	1	1
2.00	.21742E 03	1	1
3.00	.39291E 03	1	1
4.00	.62068E 03	1	1
5.00	.90407E 03	1	1
6.00	.12451E 04	1	1
7.00	.16449E 04	1	1
8.00	.21042E 04	1	1
9.00	.26234E 04	1	1
10.00	.32028E 04	1	1
1.00	.24769E 04	1	2
2.00	.90138E 04	1	2
3.00	.19865E 05	1	2
4.00	.35049E 05	1	2
5.00	.54569E 05	1	2
6.00	.78426E 05	1	2
7.00	.10662E 06	1	2
8.00	.13915E 06	1	2
9.00	.17602E 06	1	2
10.00	.21723E 06	1	2
1.00	.24769E 04	2	1
2.00	.90138E 04	2	1
3.00	.19865E 05	2	1
4.00	.35049E 05	2	1
5.00	.54569E 05	2	1
6.00	.78426E 05	2	1
7.00	.10662E 06	2	1
8.00	.13915E 06	2	1
9.00	.17602E 06	2	1
10.00	.21723E 06	2	1
1.00	.25804E 05	2	2
2.00	.10210E 06	2	2
3.00	.22926E 06	2	2
4.00	.40729E 06	2	2
5.00	.63618E 06	2	2
6.00	.91593E 06	2	2

```
      7.00        .12465E 07     2  2
      8.00        .16280E 07     2  2
      9.00        .20604E 07     2  2
     10.00        .25436E 07     2  2

DO YOU WISH TO CHANGE SOURCE MAGNITUDE?(YES=1;NO=0)
?0.

DO YOU WISH TO CHANGE COSMIC BACKGROUND MAGNITUDE?(YES=1;NO=0)
?0.

DO YOU WISH TO CHANGE BACKGROUND STAR MAGNITUDE?(YES=1;NO=0)
?0.

DO YOU WISH TO CHANGE DETECTOR EFFECTS?(YES=1,NO=0)
?0.

DID YOU MAKE ANY CHANGES?(YES=1;NO=0)
?0.
 *STOP* 0
```

APPENDIX B

PROGRAM LISTING

```
07:46 AUG 10,'84 DC/PNTSRC.JAYROE

      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION TPL(400),TJ(400),TH(21)
      DIMENSION TM(400,21),TL(400,21),FX1(400),DFE(10,10)
      DIMENSION SIGNL(200),PELS(10,10),PELB(10,10),DIFE(10,10)
      COMMON /BESF/FX0(400),PR(200,21),TTOT(400,21),QUEF(21),FF(21)
      COMMON /PHOF/XK(40)
      WRITE(102,6000)
 6000 FORMAT('ENTER SIGNAL STAR(#1) VISUAL MAGNITUDE')
      READ(101,4000)XS1
 4000 FORMAT(20G)
      XIS1=0.37353*10.0**(-0.4*XS1-5.)
      HC=1.9865E-16
      HCOK=1.4388
      DLA=1.E-6
      KRESP=0
      WRITE(102,6001)
 6001 FORMAT(/,'ENTER CHOICE OF SPECTRAL DISTRIBUTION FOR SOURCE')
      WRITE(102,6002)
 6002 FORMAT('FLAT DISTRIBUTION=0;BLACKBODY DISTRIBUTION=1')
      READ(101,4000)KRESP
      IF(KRESP.EQ.1)GO TO 90
      XISL1=(XIS1*10.0**5)/(1.0682*HC)
      GO TO 120
   90 WRITE(102,6004)
 6004 FORMAT(/,'ENTER SIGNAL STAR EFFECTIVE TEMPERATURE(KELVIN)')
      READ(101,4000)ST1
      DO 110 IK=1,39
      READ(20,5000)XK(IK)
 5000 FORMAT(E15.8)
  110 CONTINUE
      CALL PHOI(ST1,HCOK,DLA,HC,XINT)
      XISL1=XIS1*XINT
  120 WRITE(102,6008)
 6008 FORMAT(/,'ENTER NUMBER OF WAVELENGTHS')
      READ(101,4000)NWAV
      DL=0.
      DO 125 I=1,21
      QUEF(I)=1.
      FF(I)=1.
  125 CONTINUE
      FF(1)=0.5
      FF(21)=0.5
      IF(NWAV.EQ.1)GO TO 130
      WRITE(102,6012)
 6012 FORMAT(/,'ENTER LOWER AND UPPER WAVELENGTHS(CM)')
      READ(101,4000)WAVL,WAVH
      X=NWAV-1
      DL=(WAVH-WAVL)/X
      GO TO 140
  130 WRITE(102,6016)
```

```
6016 FORMAT(/,'ENTER WAVELENGTH(CM)')
     READ(101,4000)WAVL
     WRITE(102,6020)
6020 FORMAT(/,'ENTER QUANTUM EFFICIENCY AND FILTER TRANSMISSION')
     READ(101,4000)QUEF(1),FF(1)
     GO TO 180
 140 WRITE(102,6024)
6024 FORMAT(/,'IS THERE A TABLE OF QUANTUM EFFICIENCIES?(YES=1;NO=0)')
     READ(101,4000)IRESP
     IF(IRESP.NE.1)GO TO 160
     DO 150 I=1,NWAV
 150 READ(22,5000)QUEF(I)
 160 WRITE(102,6028)
6028 FORMAT(/,'IS THERE A TABLE OF FILTER TRANSMISSIONS?(YES=1;NO=0)')
     READ(101,4000)IRESP
     IF(IRESP.NE.1)GO TO 180
     DO 170 I=1,NWAV
 170 READ(24,5000)FF(I)
 180 WRITE(102,6032)
6032 FORMAT(/,'ENTER TELESCOPE ENTRANCE PUPIL DIAMETER(CM)')
     READ(101,4000)DP
     WRITE(102,6036)
6036 FORMAT(/,'ENTER SYSTEM FOCAL RATIO')
     READ(101,4000)FR
     CUTF=1./(FR*WAVL)
     WRITE(102,6040)
6040 FORMAT(/,'ENTER OBSCURATION RATIO')
     READ(101,4000)EN
     EN2=EN*EN
     ENPLS=1.+EN2
     ENMNS=1.-EN2
     ENRAT=(1.+EN)/(1.-EN)
     PI=3.1415926
     TOPI=2./PI
     DM=0.0025063
     XARG=-DM
     DO 210 I=1,400
     A=0.
     B=0.
     C=0.
     XARG=XARG+DM
     IF(XARG.GT.1.)GO TO 210
     A=TOPI*(ACOS(XARG)-XARG*SQRT(1.-XARG*XARG))
     IF(EN.LT.0.001)GO TO 210
     YARG=XARG/EN
     IF(YARG.GT.1.)GO TO 195
     B=TOPI*EN2*(ACOS(YARG)-YARG*SQRT(1.-YARG*YARG))
 195 ZARG=XARG*2./(1.-EN)
     IF(ZARG.GT.1.)GO TO 200
     C=-2.*EN2
     GO TO 210
 200 WARG=XARG*2./(1.+EN)
```

```
      IF(WARG.GT.1.)GO TO 210
      PHI=ACOS((1.+EN2-4.*XARG*XARG)/(2.*EN))
      C=EN*SIN(PHI)+PHI*ENPLS/2.-ENMNS*ATAN(ENRAT*TAN(PHI/2.))
      C=C*TOPI-2.*EN2
  210 TPL(I)=(A+B+C)/ENMNS
      DN=0.0025063*WAVL
      WRITE(102,6044)
 6044 FORMAT(/,'ENTER RMS IMAGE JITTER(ARC SECONDS)')
      READ(101,4000)SIGJ
      WRITE(102,6048)
 6048 FORMAT(/,'ENTER HIGH FREQUENCY RMS ABERATIONS(CM)')
      READ(101,4000)SIGH
      WRITE(102,6052)
 6052 FORMAT(/,'ENTER MID FREQUENCY RMS ABERATIONS(CM)')
      READ(101,4000)SIGM
      WRITE(102,6056)
 6056 FORMAT(/,'ENTER LOW FREQUENCY RMS ABERATIONS(CM)')
      READ(101,4000)SIGL
      WAV=WAVL-DL
      FACJ=PI*SIGJ*DP*PI/648000.
      FACJ=-2.*FACJ*FACJ
      FACH=-4.*PI*PI*SIGH*SIGH
      FACM=-4.*PI*PI*SIGM*SIGM
      DNU=DM
      DO 250 NW=1,NWAV
      PROD=QUEF(NW)*FF(NW)
      WAV=WAV+DL
      XNU=-DNU
      TH(NW)=EXP(FACH/WAV**2)
      DO 250 I=1,400
      XNU=XNU+DNU
      TJ(I)=EXP(FACJ*XNU*XNU/WAV**2)
      TTOT(I,NW)=0.
      IF(PROD.LT.1.E-4)GO TO 250
      XARG=XNU
      TM(I,NW)=EXP(FACM/WAV**2)
      YARG=1.-18.*XARG
      IF(YARG.LT.0.)GO TO 220
      ZARG=-FACM*YARG**1.5/WAV**2
      TM(I,NW)=TM(I,NW)*EXP(ZARG)
  220 IF(XARG.GT.0.355)GO TO 230
      YARG=PI*XARG/0.6
      TL(I,NW)=1.-2.815*SIGL*SIN(YARG)/WAV
      GO TO 240
  230 YARG=0.75*(XARG-0.4)
      TL(I,NW)=1.-SIGL*(-0.9+1./COS(YARG))/(0.0373*WAV)
  240 IF(TL(I,NW).LT.0.)TL(I,NW)=0.
      TTOT(I,NW)=TPL(I)*TJ(I)*TM(I,NW)*TL(I,NW)*TH(NW)
  250 CONTINUE
      WRITE(102,6060)
 6060 FORMAT(/,'DO YOU WISH TO COMPUTE NORMALIZED POINT SPREAD')
      WRITE(102,6064)
```

```
6064 FORMAT('FUNCTION,PSF(0)=1,AND THE NORMALIZED ENCIRCLED ENERGY,')
     WRITE(102,6065)
6065 FORMAT('EE(INF)=1?(YES=1;NO=0)')
     READ(101,4000)IRESP
     IF(IRESP.NE.1)GO TO 350
     WAV=WAVL-DL
     DO 255 I=1,NWAV
     WAV=WAV+DL
     PROD=QUEF(I)*FF(I)
     IF(PROD.LT.1.E-4)GO TO 255
     WAVM=WAV
     GO TO 256
 255 CONTINUE
 256 RAD=1.22*WAVM*FR
     ANG=1.22*WAVM*648000./(DP*PI)
     WRITE(102,6066)RAD,ANG
6066 FORMAT(/,'MINIMUM AIRY RADIUS=',E15.8,' CM=',E15.8,
    &' ARC SECONDS')
 260 WRITE(102,6068)
6068 FORMAT(/,'ENTER MAXIMUM RADIUS TO CONSIDER(ARC SECONDS)')
     READ(101,4000)RMAX
     WRITE(102,6072)
6072 FORMAT(/,'ENTER NUMBER OF RADIUS CALCULATIONS')
     READ(101,4000)NR
     WRITE(102,6076)
6076 FORMAT(/,'ENTER DESIRED WAVELENGTH NUMBER(1,2,...,21)')
     READ(101,4000)JW
     WRITE(102,6077)
6077 FORMAT(/,'CLEAR SCREEN AND RETURN TO CONTINUE')
     READ(101,4000)DUMMY
     X=NR-1
     RCM=RMAX*DP*FR*PI/648000.
     DR=RCM/X
     DRY=RMAX/X
     DWAV=JW-1
     WAVX=WAVL+DWAV*DL
     SCAL=ENMNS*(PI*DP/(4.*FR*WAVX))**2
     DRZ=RMAX*DP*PI/(X*WAVX*648000.)
     ZR=-DRZ
     YR=-DRY
     XR=-DR
     WRITE(102,7000)RMAX,RCM
7000 FORMAT('MAXIMUM RADIUS=',E15.8,' ARC SECONDS=',E15.8,' CM')
     WRITE(102,7004)NR,WAVX
7004 FORMAT('NUMBER OF RADIUS CALCULATIONS',I4,': WAVELENGTH=',E15.8,
    &' CM')
     WRITE(102,7008)
7008 FORMAT(/,'    RADIUS(CM)   PNT.SPREAD FN. ENCIRCLED ENERGY',
    &'  (ARC SECONDS) ENERGY/SEC/CM**2')
     DO 340 IR=1,NR
     XR=XR+DR
     YR=YR+DRY
```

```
      ZR=ZR+DRZ
      XNU=-DNU
      DO 310 I=1,400
      XNU=XNU+DNU
      X=2.*PI*ZR*XNU
      IF(X.GT.0.)GO TO 285
      BES0=1.
      BES1=0.
      GO TO 300
  285 IF(X.GT.4.)GO TO 290
      X2=X/2.
      X22=X2*X2
      X24=X22*X22
      X26=X24*X22
      X28=X24*X24
      BES0=1.-X22+X24/4.-X26/36.+X28/576.-X28*X22/14400.
      BES0=BES0+X28*X24/518400.-X28*X26/25401600.
      X3=X22*X2
      X5=X24*X2
      X7=X26*X2
      X9=X28*X2
      BES1=X2-X3/2.+X5/12.-X7/144.+X9/2880.-X7*X22/86400.
      BES1=BES1+X9*X22/3628800.-X7*X28/203212800.
      GO TO 300
  290 X2=X*X
      X3=X2*X
      X4=X3*X
      X5=X4*X
      X6=X5*X
      P0=1.-0.0703125/X2+0.112152/X4-0.5725/X6
      Q0=-0.125/X+0.0732422/X3-0.227108/X5
      BES0=0.7978846/SQRT(X)
      AQ=X-0.7853982
      BES0=BES0*(P0*COS(AQ)-Q0*SIN(AQ))
      P1=1.+0.1171875/X2-0.1441956/X4+0.6765926/X6
      Q1=0.375/X-0.1025391/X3+0.2775764/X5
      BES1=0.7978846/SQRT(X)
      BQ=X-2.35619449
      BES1=BES1*(P1*COS(BQ)-Q1*SIN(BQ))
  300 FX0(I)=TTOT(I,JW)*BES0*XNU
      FX1(I)=TTOT(I,JW)*BES1
  310 CONTINUE
      A2=0.
      A3=0.
      A4=0.
      B2=0.
      B3=0.
      B4=0.
      DO 320 J=2,399,2
      A2=A2+FX0(J)
      B2=B2+FX1(J)
  320 CONTINUE
```

```
      DO 330 J=3,398,4
      A3=A3+FX0(J)
      A4=A4+FX0(J-2)+FX0(J+2)
      B3=B3+FX1(J)
      B4=B4+FX1(J-2)+FX1(J+2)
  330 CONTINUE
      SAS=DNU*(1.4*A4+2.4*A3+6.4*A2)/4.5
      SBS=DNU*(1.4*B4+2.4*B3+6.4*B2)/4.5
      PSF=8.*SAS/ENMNS
      SPSF=SCAL*PSF
      EE=2.*PI*ZR*SBS
      WRITE(102,7012)XR,PSF,EE,YR,SPSF
 7012 FORMAT(5(E15.8,1X))
  340 CONTINUE
      WRITE(102,6080)
 6080 FORMAT(/,'DO YOU WISH TO CHANGE ANY PSF PARAMETERS?(YES=1;NO=0)')
      READ(101,4000)IRESP
      IF(IRESP.EQ.1)GO TO 260
  350 WRITE(102,6077)
      READ(101,4000)DUMMY
      WRITE(102,6084)
 6084 FORMAT(/,'ENTER X,Y PEL WIDTHS(ARC SECONDS)')
      READ(101,4000)XPW,YPW
      WRITE(102,6088)
 6088 FORMAT(/,'ENTER X,Y PEL CENTER SEPARATIONS(ARC SECONDS)')
      READ(101,4000)XPD,YPD
      WRITE(102,6092)
 6092 FORMAT(/,'ENTER NUMBER OF PELS IN X,Y DIRECTIONS')
      READ(101,4000)NX,NY
      DO 360 IX=1,NX
      DO 360 IY=1,NY
  360 PELB(IX,IY)=0.
      WRITE(102,6096)
 6096 FORMAT(/,'ENTER X,Y COORDINATES OF SIGNAL STAR(ARC SECONDS)')
      READ(101,4000)PSX,PSY
      XN=NX
      YN=NY
      XMAX=XN*XPD
      YMAX=YN*YPD
      RMAX=SQRT((XMAX-PSX)**2+(YMAX-PSY)**2)
      DRZ=RMAX*DP*PI/(199.*648000.)
      DRR=RMAX/199.
      ZR=-DRZ
C     ***** COMPUTE POINT SPREAD FUNCTION **************************
      CALL BES(NWAV,DRZ,ZR,DNU,ENMNS,WAVL,DL)
C     ***** INTEGRATE PSF OVER WAVELENGTH **************************
      IF(NWAV.GT.1)GO TO 440
      PROD=QUEF(1)*FF(1)*XISL1*PI*DP*DP/4.
      DO 430 IR=1,200
      IF(KRESP.EQ.1)GO TO 425
      SIGNL(IR)=PROD*PR(IR,1)/WAVL
      GO TO 430
```

```
  425 Y=HCOK/(WAVL*ST1)
      SIGNL(IR)=PROD*PR(IR,1)/((EXP(Y)-1.)*WAVL**6)
  430 CONTINUE
      GO TO 480
  440 MWAV=NWAV-1
      FAC=XISL1*PI*DP*DP*DL/4.
      DO 470 IR=1,200
      WAV=WAVL
      PROD=QUEF(1)*FF(1)*PR(IR,1)
      IF(KRESP.EQ.1)GO TO 441
      SUM=PROD/WAV
      GO TO 442
  441 Y=HCOK/(WAV*ST1)
      SUM=PROD/((EXP(Y)-1.)*WAV**6)
  442 IF(NWAV.EQ.2)GO TO 460
      DO 450 JW=2,MWAV
      WAV=WAV+DL
      PROD=QUEF(JW)*FF(JW)
      IF(PROD.LT.1.E-4)GO TO 450
      IF(KRESP.EQ.1)GO TO 446
      SUM=SUM+PROD*PR(IR,JW)/WAV
      GO TO 450
  446 Y=HCOK/(WAV*ST1)
      SUM=SUM+PROD*PR(IR,JW)/((EXP(Y)-1.)*WAV**6)
  450 CONTINUE
  460 WAV=WAV+DL
      PROD=QUEF(NWAV)*FF(NWAV)*PR(IR,NWAV)
      IF(KRESP.EQ.1)GO TO 465
      SUM=SUM+PROD/WAV
      GO TO 466
  465 Y=HCOK/(WAV*ST1)
      SUM=SUM+PROD/((EXP(Y)-1.)*WAV**6)
  466 SIGNL(IR)=SUM*FAC
  470 CONTINUE
C     ***** INTEGRATE SIGNAL OVER PELS ******************************
  480 DPX=XPW/20.
      DPY=YPW/20.
      DO 500 IX=1,NX
      DO 500 IY=1,NY
      PELS(IX,IY)=0.
      XI=IX-1
      YI=IY-1
      XX=-DPX/2.+XI*XPD
      SUM=0.
      DO 490 IDX=1,20
      YY=-DPY/2.+YI*YPD
      XX=XX+DPX
      DO 490 IDY=1,20
      YY=YY+DPY
      DIF=(PSX-XX)**2+(PSY-YY)**2
      DST=SQRT(DIF)
      FRAC=DST/DRR
```

```
      NUM=FRAC
      IF(NUM.GE.199)GO TO 500
      XNUM=NUM
      FRAC=FRAC-XNUM
      FAC=1.
      IF((IDY.EQ.1).OR.(IDY.EQ.20))FAC=0.5
      IF((IDX.EQ.1).OR.(IDX.EQ.20))FAC=FAC*0.5
      SUM=SUM+FAC*(SIGNL(NUM+1)+FRAC*(SIGNL(NUM+2)-SIGNL(NUM+1)))
  490 CONTINUE
      PELS(IX,IY)=SUM*ENMNS*PI*DPX*DPY*(PI*DP/1296000.)**2
  500 CONTINUE
C     ***** CALCULATE DIFFRACTION EFFICIENCY ************************
      WAV=WAVL-DL
      DO 920 NW=1,NWAV
      WAV=WAV+DL
      PROD=QUEF(NW)*FF(NW)
      IF(PROD.LT.1.E-4)GO TO 920
      DO 920 IX=1,NX
      DO 920 IY=1,NY
      XI=IX-1
      YI=IY-1
      XX=-DPX/2.+XI*XPD
      SUM=0.
      DIFE(IX,IY)=0.
      DO 900 IDX=1,20
      YY=-DPY/2.+YI*YPD
      XX=XX+DPX
      DO 900 IDY=1,20
      YY=YY+DPY
      DST=SQRT((PSX-XX)**2+(PSY-YY)**2)
      FRAC=DST/DRR
      NUM=FRAC
      IF(NUM.GE.199)GO TO 910
      XNUM=NUM
      FRAC=FRAC-XNUM
      FAC=1.
      IF((IDY.EQ.1).OR.(IDY.EQ.20))FAC=0.5
      IF((IDX.EQ.1).OR.(IDX.EQ.20))FAC=FAC*0.5
      SUM=SUM+FAC*(PR(NUM+1,NW)+FRAC*(PR(NUM+2,NW)-PR(NUM+1,NW)))
  900 CONTINUE
      DIFE(IX,IY)=SUM*ENMNS*PI*DPX*DPY*(PI*DP/(WAV*1296000.))**2
  910 WRITE(102,7017)WAV,IX,IY,DIFE(IX,IY)
 7017 FORMAT('WAV=',E15.8,' IX=',I1,' IY=',I1,' DIFE=',E15.8)
  920 CONTINUE
C     ***** CALCULATE BACKGROUND SIGNAL ****************************
      WRITE(102,6100)
 6100 FORMAT(/,'ENTER COSMIC BACKGROUND VISUAL MAGNITUDE')
      READ(101,4000)XMB
      XIMB=0.37353*10.0**(-0.4*XMB-5.)
      IF(KRESP.EQ.1)GO TO 510
      XIMLB=(XIMB*10.0**5)/(1.0682*HC)
      GO TO 515
```

```
  510 WRITE(102,6104)
 6104 FORMAT(/,'ENTER BACKGROUND EFFECTIVE TEMPERATURE(KELVIN)')
      READ(101,4000)BEFT
      CALL PHOI(BEFT,HCOK,DLA,HC,XINT)
      XIMLB=XIMB*XINT
C     ***** INTEGRATE OVER WAVELENGTH ******************************
  515 IF(NWAV.GT.1)GO TO 520
      IF(KRESP.EQ.1)GO TO 517
      SUM=XIMLB*QUEF(1)*FF(1)*WAVL
      GO TO 550
  517 Y=HCOK/(WAVL*BEFT)
      SUM=XIMLB*QUEF(1)*FF(1)/((EXP(Y)-1.)*WAVL**4)
      GO TO 550
  520 WAV=WAVL
      IF(KRESP.EQ.1)GO TO 521
      SUM=QUEF(1)*FF(1)*WAVL
      GO TO 522
  521 Y=HCOK/(WAV*BEFT)
      SUM=QUEF(1)*FF(1)/((EXP(Y)-1.)*WAV**4)
  522 IF(NWAV.EQ.2)GO TO 540
      MWAV=NWAV-1
      DO 530 I=2,MWAV
      WAV=WAV+DL
      PROD=QUEF(I)*FF(I)
      IF(PROD.LT.1.E-4)GO TO 530
      IF(KRESP.EQ.1)GO TO 528
      SUM=SUM+PROD*WAV
      GO TO 530
  528 Y=HCOK/(WAV*BEFT)
      SUM=SUM+PROD/((EXP(Y)-1.)*WAV**4)
  530 CONTINUE
  540 WAV=WAV+DL
      IF(KRESP.EQ.1)GO TO 545
      SUM=SUM+QUEF(NWAV)*FF(NWAV)*WAV
      GO TO 546
  545 Y=HCOK/(WAV*BEFT)
      SUM=SUM+QUEF(NWAV)*FF(NWAV)/((EXP(Y)-1.)*WAV**4)
  546 SUM=SUM*DL*XIMLB
  550 BGRND=0.7854*DP*DP*SUM*XPW*YPW
      IRESP=0
      WRITE(102,6108)
 6108 FORMAT(/,'DO YOU WISH TO INCLUDE A BACKGROUND STAR?(YES=1;NO=0)')
      READ(101,4000)IRESP
      IF(IRESP.NE.1)GO TO 700
      WRITE(102,6112)
 6112 FORMAT(/,'ENTER VISUAL MAGNITUDE OF BACKGROUND STAR')
      READ(101,4000)XVM
      XIMV=0.37353*10.0**(-0.4*XVM-5.)
      IF(KRESP.EQ.1)GO TO 560
      XIMLV=(XIMV*10.0**5)/(1.0682*HC)
      GO TO 570
  560 WRITE(102,6116)
```

```
 6116 FORMAT(/,'ENTER BACKGROUND STAR EFFECTIVE TEMPERATURE(KELVIN)')
      READ(101,4000)ST2
      CALL PHOI(ST2,HCOK,DLA,HC,XINT)
      XIMLV=XIMV*XINT
  570 WRITE(102,6120)
 6120 FORMAT(/,'ENTER X,Y COORDINATES OF BACKGROUND STAR(ARC SECONDS)')
      READ(101,4000)BSX,BSY
      DX=BSX-XMAX
      DY=BSY-YMAX
      D00=BSX*BSX+BSY*BSY
      DX0=DX*DX+BSY*BSY
      DXY=DX*DX+DY*DY
      D0Y=BSX*BSX+DY*DY
      DMAX=0.
      DMIN=10000.
      IF(D00.GT.DMAX)DMAX=D00
      IF(DX0.GT.DMAX)DMAX=DX0
      IF(DXY.GT.DMAX)DMAX=DXY
      IF(D0Y.GT.DMAX)DMAX=D0Y
      IF(D00.LT.DMIN)DMIN=D00
      IF(DX0.LT.DMIN)DMIN=DX0
      IF(DXY.LT.DMIN)DMIN=DXY
      IF(D0Y.LT.DMIN)DMIN=D0Y
      DMAX=SQRT(DMAX)
      DMIN=SQRT(DMIN)
      IF(DX.GT.0.)GO TO 580
      IF(DY.GT.0.)GO TO 600
      DMIN=0.
      GO TO 610
  580 IF(DY.GT.0.)GO TO 610
      DMIN=DX
      GO TO 610
  600 IF(DX.GT.0.)GO TO 610
      DMIN=DY
  610 RMAX=DMAX-DMIN
      DRZ=RMAX*DP*PI/(199.*648000.)
      DRR=RMAX/199.
      ZR=DMIN*DP*PI/648000.-DRZ
      CALL BES(NWAV,DRZ,ZR,DNU,ENMNS,WAVL,DL)
C     ***** INTEGRATE OVER WAVELENGTH ********************************
      IF(NWAV.GT.1)GO TO 630
      PROD=QUEF(1)*FF(1)*XIMLV*PI*DP*DP/4.
      DO 620 IR=1,200
      IF(KRESP.EQ.1)GO TO 615
      SIGNL(IR)=PROD*PR(IR,1)/WAVL
      GO TO 620
  615 Y=HCOK/(WAVL*ST2)
      SIGNL(IR)=PROD*PR(IR,1)/((EXP(Y)-1.)*WAVL**6)
  620 CONTINUE
      GO TO 670
  630 MWAV=NWAV-1
      FAC=XIMLV*PI*DP*DP*DL/4.
```

```
      DO 660 IR=1,200
      WAV=WAVL
      PROD=QUEF(1)*FF(1)*PR(IR,1)
      IF(KRESP.EQ.1)GO TO 631
      SUM=PROD/WAV
      GO TO 632
  631 Y=HCOK/(WAV*ST2)
      SUM=PROD/((EXP(Y)-1.)*WAV**6)
  632 IF(NWAV.EQ.2)GO TO 650
      DO 640 JW=2,MWAV
      WAV=WAV+DL
      PROD=QUEF(JW)*FF(JW)
      IF(PROD.LT.1.E-4)GO TO 640
      IF(KRESP.EQ.1)GO TO 636
      SUM=SUM+PROD*PR(IR,JW)/WAV
      GO TO 640
  636 Y=HCOK/(WAV*ST2)
      SUM=SUM+PROD*PR(IR,JW)/((EXP(Y)-1.)*WAV**6)
  640 CONTINUE
  650 WAV=WAV+DL
      PROD=QUEF(NWAV)*FF(NWAV)*PR(IR,NWAV)
      IF(KRESP.EQ.1)GO TO 655
      SUM=SUM+PROD/WAV
      GO TO 656
  655 Y=HCOK/(WAV*ST2)
      SUM=SUM+PROD/((EXP(Y)-1.)*WAV**6)
  656 SIGNL(IR)=SUM*FAC
  660 CONTINUE
C     ***** INTEGRATE SIGNAL OVER PEL ********************************
  670 DPX=XPW/20.
      DPY=YPW/20.
      DO 690 IX=1,NX
      DO 690 IY=1,NY
      PELB(IX,IY)=0.
      XI=IX-1
      YI=IY-1
      XX=-DPX/2.+XI*XPD
      SUM=0.
      DO 680 IDX=1,20
      YY=-DPY/2.+YI*YPD
      XX=XX+DPX
      DO 680 IDY=1,20
      YY=YY+DPY
      DIF=(BSX-XX)**2+(BSY-YY)**2
      DST=SQRT(DIF)-DMIN
      FRAC=DST/DRR
      NUM=FRAC
      IF((NUM.GE.199).OR.(NUM.LT.0))GO TO 690
      XNUM=NUM
      FRAC=FRAC-XNUM
      FAC=1.
      IF((IDY.EQ.1).OR.(IDY.EQ.20))FAC=0.5
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```
      IF((IDX.EQ.1).OR.(IDX.EQ.20))FAC=FAC*0.5
      SUM=SUM+FAC*SIGNL(NUM+1)+FAC*FRAC*(SIGNL(NUM+2)-SIGNL(NUM+1))
  680 CONTINUE
      PELB(IX,IY)=SUM*ENMNS*PI*DPX*DPY*(PI*DP*FR/1296000.)**2
  690 CONTINUE
C     ***** CALCULATE DIFFRACTION EFFICIENCY ***********************
      WAV=WAVL-DL
      DO 950 NW=1,NWAV
      WAV=WAV+DL
      PROD=QUEF(NW)*FF(NW)
      IF(PROD.LT.1.E-4)GO TO 950
      DO 950 IX=1,NX
      DO 950 IY=1,NY
      XI=IX-1
      YI=IY-1
      XX=-DPX/2.+XI*XPD
      SUM=0.
      DFE(IX,IY)=0.
      DO 930 IDX=1,20
      YY=-DPY/2.+YI*YPD
      XX=XX+DPX
      DO 930 IDY=1,20
      YY=YY+DPY
      DST=SQRT((BSX-XX)**2+(BSY-YY)**2)-DMIN
      FRAC=DST/DRR
      NUM=FRAC
      IF((NUM.GE.199).OR.(NUM.LT.0))GO TO 940
      XNUM=NUM
      FRAC=FRAC-XNUM
      FAC=1.
      IF((IDY.EQ.1).OR.(IDY.EQ.20))FAC=0.5
      IF((IDX.EQ.1).OR.(IDX.EQ.20))FAC=FAC*0.5
      SUM=SUM+FAC*(PR(NUM+1,NW)+FRAC*(PR(NUM+2,NW)-PR(NUM+1,NW)))
  930 CONTINUE
      DFE(IX,IY)= SUM*ENMNS*PI*DPX*DPY*(PI*DP/(WAV*1296000.))**2
  940 WRITE(102,7027)WAV,IX,IY,DFE(IX,IY)
 7027 FORMAT('WAV=',E15.8,' IX=',I1,' IY=',I1,' DIFE=',E15.8)
  950 CONTINUE
  700 WRITE(102,6124)
 6124 FORMAT(/,'ENTER RMS READOUT NOISE(ELECTRONS/PEL)')
      READ(101,4000)RON
      RN=RON*RON
      WRITE(102,6128)
 6128 FORMAT(/,'ENTER DETECTOR TEMPERATURE(KELVIN)')
      READ(101,4000)DTEM
      IF(DTEM.LT.4.)GO TO 710
      FAC=-6705.487/DTEM+4.0737*DTEM/(1108.+DTEM)
      FAC=EXP(FAC)*(29.3E+9)
      DC=FAC*DTEM**1.5
      GO TO 720
  710 WRITE(102,6132)
 6132 FORMAT(/,'ENTER MEAN DARK CURRENT(ELECTRONS/PEL/SEC)')
```

```
      READ(101,4000)DC
  720 WRITE(102,6136)
 6136 FORMAT(/,'ENTER OBSERVATION START AND END TIMES(SECONDS)')
      READ(101,4000)STRT,ENDT
      WRITE(102,6140)
 6140 FORMAT(/,'ENTER NUMBER(>1) OF TIME CALCULATIONS')
      READ(101,4000)NT
      X=NT-1
      DT=(ENDT-STRT)/X
      WRITE(102,6144)
 6144 FORMAT(/,'ENTER START AND END SIGNAL TO NOISE RATIOS(S/N)')
      READ(101,4000)SNST,SNND
      WRITE(102,6148)
 6148 FORMAT(/,'ENTER NUMBER(>1) OF (S/N) CALCULATIONS')
      READ(101,4000)NS
      Y=NS-1
      DNS=(SNND-SNST)/Y
      WRITE(102,6077)
      READ(101,4000)DUMMY
  725 WRITE(102,6152)DP
 6152 FORMAT(/,'TELESCOPE ENTRANCE PUPIL DIAMETER ',F7.3,' CM')
      WRITE(102,6156)FR
 6156 FORMAT('OPTICAL SYSTEM FOCAL RATIO ',F5.2)
      WRITE(102,6160)
 6160 FORMAT(/,'SOURCE STAR CHARACTERISTICS')
      WRITE(102,6164)XS1
 6164 FORMAT(5X,'SOURCE MAGNITUDE ',F5.2)
      IF(KRESP.EQ.0)GO TO 726
      WRITE(102,6168)ST1
 6168 FORMAT(5X,'SOURCE TEMPERATURE(KELVIN) ',F9.3)
  726 WRITE(102,6172)
 6172 FORMAT(/,'COSMIC BACKGROUND CHARACTERISTICS')
      WRITE(102,6176)XMB
 6176 FORMAT(5X,'COSMIC BACKGROUND MAGNITUDE ',F5.2)
      IF(KRESP.EQ.0)GO TO 727
      WRITE(102,6180)BEFT
 6180 FORMAT(5X,'COSMIC BACKGROUND TEMPERATURE(KELVIN) ',F9.3)
  727 IF(IRESP.NE.1)GO TO 730
      WRITE(102,6184)
 6184 FORMAT(/,'BACKGROUND STAR CHARACTERISTICS')
      WRITE(102,6188)XVM
 6188 FORMAT(5X,'BACKGROUND STAR MAGNITUDE ',F5.2)
      WRITE(102,6192)ST2
 6192 FORMAT(5X,'BACKGROUND STAR TEMPERATURE(KELVIN) ',F9.3)
  730 WRITE(102,6196)
 6196 FORMAT(/,'SYSTEM SPECTRAL CHARACTERISTICS')
      WRITE(102,6200)
 6200 FORMAT(4X,'WAVELENGTH(CM)',5X,'QUAN.EFFIC.',3X,'FILTER FUNCTION')
      WAV=WAVL
      DO 740 I=1,NWAV
      PROD=QUEF(I)*FF(I)
      IF(PROD.LT.1.E-4)GO TO 739
```

```
      WRITE(102,6204)WAV,QUEF(I),FF(I)
 6204 FORMAT(3(6X,E11.4))
  739 WAV=WAV+DL
  740 CONTINUE
      WRITE(102,6208)
 6208 FORMAT(/,'DETECTOR CHARACTERISTICS')
      WRITE(102,6212)XPW,YPW
 6212 FORMAT(5X,'PEL DIMENSIONS(ARC SECONDS):X=',F8.4,'  Y=',F8.4)
      WRITE(102,6216)XPD,YPD
 6216 FORMAT(5X,'PEL CENTER SEPARATION(ARC SECONDS):X=',F8.4,'  Y=',
     &F8.4)
      WRITE(102,6220)PSX,PSY
 6220 FORMAT(5X,'SOURCE STAR COORDINATES(ARC SECONDS):X=',F8.4,'  Y=',
     &F8.4)
      IF(IRESP.NE.1)GO TO 750
      WRITE(102,6224)BSX,BSY
 6224 FORMAT(5X,'BACKGROUND STAR COORDINATES(ARC SECONDS):X=',F8.4,
     &'  Y=',F8.4)
  750 WRITE(102,6228)NX,NY
 6228 FORMAT(5X,'ARRAY SIZE:X=',I2,' BY Y=',I2)
      WRITE(102,6232)RON
 6232 FORMAT(5X,'RMS READOUT NOISE(ELECTRONS/PEL):',F5.2)
      IF(DTEM.LT.4.)GO TO 760
      WRITE(102,6236)DTEM
 6236 FORMAT(5X,'DETECTOR TEMPERATURE(KELVIN):',F8.4)
  760 WRITE(102,6240)DC
 6240 FORMAT(5X,'DARK CURRENT(ELECTRONS/PEL/SECOND):',E15.8)
      WRITE(102,6246)
 6246 FORMAT(/,'SIGNAL TO NOISE RATIO AS A FUNCTION OF OBSERVATION',
     &' TIME')
      WRITE(102,6250)
 6250 FORMAT('AND PEL NUMBER')
      WRITE(102,6254)
 6254 FORMAT(4X,'TIME(SECONDS) ',2X,'     (S/N)      ',2X,'PEL NUMBER')
      DO 770 IX=1,NX
      DO 770 IY=1,NY
      T=STRT-DT
      DO 770 IT=1,NT
      T=T+DT
      XNUM=PELS(IX,IY)*T
      XDNOM=XNUM+BGRND*T+PELB(IX,IY)*T+RN+DC*T
      SNP=XNUM/SQRT(XDNOM)
      WRITE(102,6256)T,SNP,IX,IY
 6256 FORMAT(5X,E12.5,6X,E11.4,2X,2I3)
  770 CONTINUE
      WRITE(102,6260)
 6260 FORMAT(/,5X,'(S/N)',4X,'  SECONDS       ',2X,'PEL NUMBER')
      DO 780 IX=1,NX
      DO 780 IY=1,NY
      SN=SNST-DNS
      DO 780 IS=1,NS
      SN=SN+DNS
```

```
      A=SN*SN/(PELS(IX,IY)**2)
      B=A*(PELS(IX,IY)+BGRND+PELB(IX,IY)+DC)*0.5
      C=A*RON
      T1=B+SQRT(B*B+C)
      WRITE(102,6264)SN,T1,IX,IY
 6264 FORMAT(5X,F5.2,5X,E12.5,2X,2I3)
  780 CONTINUE
      WRITE(102,6268)
 6268 FORMAT(/,'DO YOU WISH TO CHANGE SOURCE MAGNITUDE?(YES=1;NO=0)')
      READ(101,4000)JRESP
      IF(JRESP.EQ.0)GO TO 800
      WRITE(102,6272)
 6272 FORMAT(/,'ENTER NEW SOURCE MAGNITUDE')
      READ(101,4000)XSM
      XNEW=0.37353*10.0**(-0.4*XSM)
      XNEW=XNEW/XIMS
      DO 790 IX=1,NX
      DO 790 IY=1,NY
  790 PELS(IX,IY)=PELS(IX,IY)*XNEW
  800 WRITE(102,6276)
 6276 FORMAT(/,'DO YOU WISH TO CHANGE COSMIC BACKGROUND MAGNITUDE?',
     &'(YES=1;NO=0)')
      READ(101,4000)JRESP
      IF(JRESP.EQ.0)GO TO 810
      WRITE(102,6280)
 6280 FORMAT(/,'ENTER NEW COSMIC BACKGROUND MAGNITUDE')
      READ(101,4000)XMB
      XNEW=0.37353*10.0**(-0.4*XMB)
      XNEW=XNEW/XIMB
      BGRND=BGRND*XNEW
  810 WRITE(102,6284)
 6284 FORMAT(/,'DO YOU WISH TO CHANGE BACKGROUND STAR MAGNITUDE?',
     &'(YES=1;NO=0)')
      READ(101,4000)JRESP
      IF(JRESP.EQ.0)GO TO 830
      WRITE(102,6288)
 6288 FORMAT(/,'ENTER NEW BACKGROUND STAR MAGNITUDE')
      READ(101,4000)XMV
      XNEW=0.37353*10.0**(-0.4*XMV)
      XNEW=XNEW/XIMV
      DO 820 IX=1,NX
      DO 820 IY=1,NY
  820 PELB(IX,IY)=PELB(IX,IY)*XNEW
  830 WRITE(102,6292)
 6292 FORMAT(/,'DO YOU WISH TO CHANGE DETECTOR EFFECTS?(YES=1,NO=0)')
      READ(101,4000)JRESP
      IF(JRESP.EQ.1)GO TO 700
      WRITE(102,6296)
 6296 FORMAT(/,'DID YOU MAKE ANY CHANGES?(YES=1;NO=0)')
      READ(101,4000)JRESP
      IF(JRESP.EQ.1)GO TO 725
      STOP
```

```
      END
      SUBROUTINE PHOI(TEMP,HCOK,DLA,HC,XINT)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      COMMON /PHOF/XK(40)
      X=3.8E-5
      Y=HCOK/(X*TEMP)
      SUM=XK(1)/((EXP(Y)-1.)*X**5)
      X=X+DLA
      DO 10 IK=2,38
      ICHEK=2*(IK/2)
      FAC=2.
      IF(IK.EQ.ICHEK)FAC=4.
      Y=HCOK/(X*TEMP)
      SUM=SUM+FAC*XK(IK)/((EXP(Y)-1.)*X**5)
      X=X+DLA
   10 CONTINUE
      Y=HCOK/(X*TEMP)
      SUM=SUM+XK(39)/((EXP(Y)-1.)*X**5)
      XINT=3./(SUM*DLA*HC)
      RETURN
      SUBROUTINE BES(NWAV,DRZ,ZR,DNU,ENMNS,WAVL,DL)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      COMMON /BESF/FX0(400),PR(200,21),TTOT(400,21),QUEF(21),FF(21)
      TWOPI=6.283185307
      WAV=WAVL-DL
      DO 60 NW=1,NWAV
      WAV=WAV+DL
      DR=DRZ/WAV
      Z=ZR/WAV
      PROD=QUEF(NW)*FF(NW)
      DO 60 IR=1,200
      PR(IR,NW)=0.
      Z=Z+DR
      IF(PROD.GT.1.E-4)GO TO 5
      GO TO 60
    5 XNU=-DNU
      DO 31 I=1,400
      XNU=XNU+DNU
      X=TWOPI*Z*XNU
      IF(X.GT.0.)GO TO 10
      BS=1.
      GO TO 30
   10 IF(X.GT.4.)GO TO 20
      X2=X/2.
      X22=X2*X2
      X24=X22*X22
      X26=X24*X22
      X28=X24*X24
      BS=1.-X22+X24/4.-X26/36.+X28/576.-X28*X22/14400.
      BS=BS+X28*X24/518400.-X28*X26/25401600.
      GO TO 30
   20 X2=X*X
```

```
      X3=X2*X
      X4=X3*X
      X5=X4*X
      X6=X5*X
      P=1.-0.0703125/X2+0.112152/X4-0.5725/X6
      Q=-0.125/X+0.0732422/X3-0.227108/X5
      BS=0.7978846/SQRT(X)
      AQ=X-0.7853982
      BS=BS*(P*COS(AQ)-Q*SIN(AQ))
   30 FX0(I)=TTOT(I,NW)*BS*XNU
   31 CONTINUE
      A2=0.
      A3=0.
      A4=0.
      DO 40 I=2,399,2
      A2=A2+FX0(I)
   40 CONTINUE
      DO 50 I=3,398,4
      A3=A3+FX0(I)
      A4=A4+FX0(I-2)+FX0(I+2)
   50 CONTINUE
      SAS=DNU*(1.4*A4+2.4*A3+6.4*A2)/4.5
      PR(IR,NW)=8.*SAS/ENMNS
   60 CONTINUE
      RETURN
      END
```

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16. ABSTRACT

An algorithm was developed to simulate the expected signal to noise ratios as a function of observation time in the charge coupled device detector plane of an optical telescope located outside the Earth's atmosphere for a signal star, and an optional secondary star, embedded in a uniform cosmic background. By choosing the appropriate input values, the expected point source signal to noise ratio can be computed for the Hubble Space Telescope using the Wide Field/Planetary Camera science instrument.

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