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Transformation of Two and Three-Dimensional Regions by Elliptic Systems

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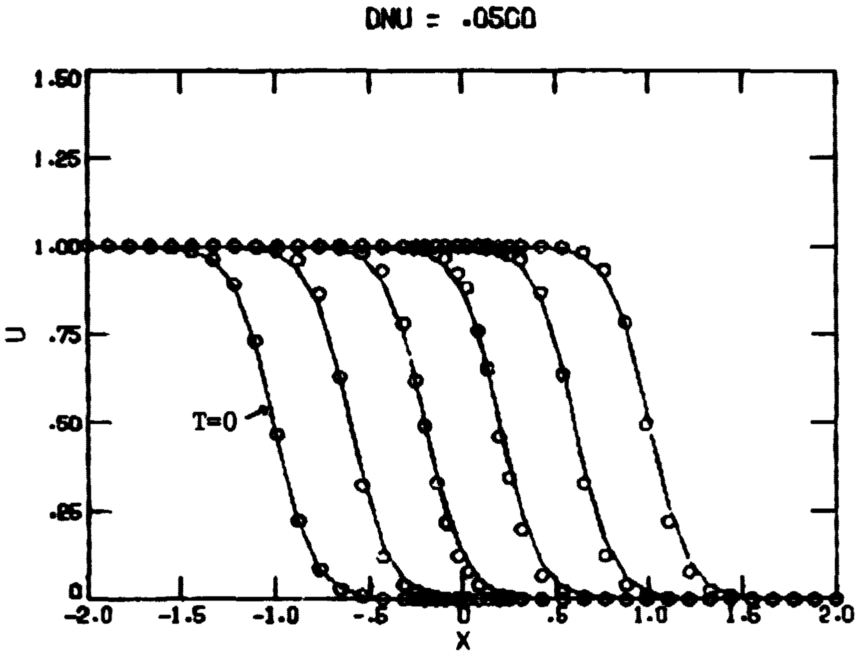
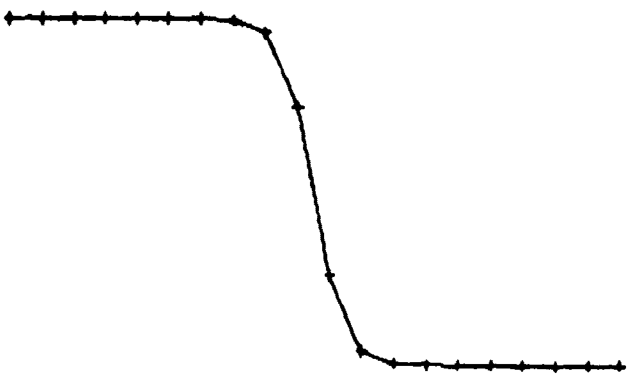
TRANSFORMATION OF TWO AND THREE-DIMENSIONAL REGIONS BY ELLIPTIC SYSTEMS

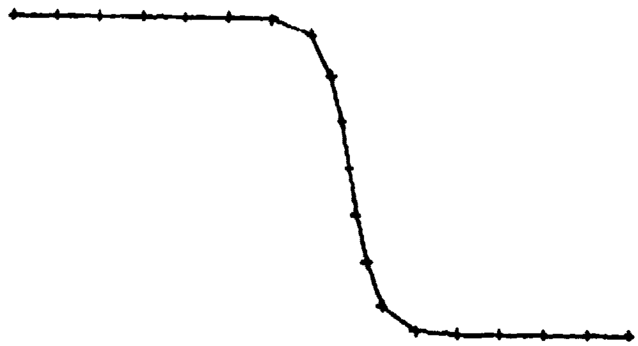
The research during this period continued to expand the class of numerical algorithms that can be accurately and efficiently implemented on overlapping grids. Whereas previous calculations have been used to solve elliptic equations and to find the steady-state solution of parabolic equations, the present work is aimed towards developing time-accurate solution techniques for parabolic and hyperbolic equations. The primary difficulty here is in the correct treatment of the interior boundary nodes that must be updated at each iteration. The implementation of explicit methods is straightforward. However, the common practice of lagging these values when using an implicit method leads to inconsistencies in the difference equation. One way to avoid this problem is to alternately calculate with an implicit and an explicit method on each subgrid. With this procedure, the explicit method generates boundary values at the next time level which are then used by the implicit step. It can be shown that when a backward implicit method is combined with a forward explicit method, the composite method is second order accurate and unconditionally stable for linear problems. Of course, one can view this method as a hopscotch algorithm applied to subgrids. A disadvantage of this method is that it requires the coding of both an explicit and an implicit algorithm. This was of little consequence for the one-dimensional model problems on which it was tested, but would be a consideration when solving a large multidimensional system of partial differential equations. The following graph indicates the type of problem that is being used for algorithm verification. The moving front problem was modeled by solving Burgers' equation on two uniform overlapping grids. The use of the above hopscotch algorithm resulted in a fifty percent error reduction when compared with a second order Crank-Nicolson scheme with lagged boundary values. A further improvement in the numerical solution was achieved by using a new interpolation procedure based on the approximation of the differential equation. We are presently investigating the feasibility of using such an interpolation procedure for multidimensional problems.



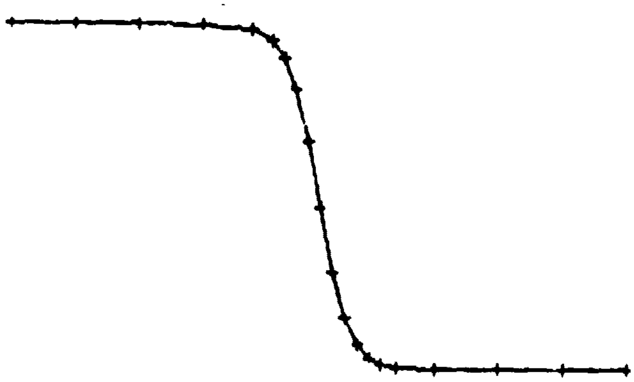
A second area in which progress can be reported is in the distribution of grid points on curves and surfaces. Here the problem is to select a set of parameter values so that the corresponding points on the curve are correctly distributed. The desired distribution would be influenced to some extent by the physical problem one wishes to solve. Typically, one may want the grid points along a boundary curve equidistributed relative to arclength, or it may be desirable to have more points where the curvature is greatest. Either choice is possible with the general reparameterization algorithm which is presently under development. The problem of finding grid points at equal spacing along a curve is nothing new. Previous solutions have required a numerical integration method to define the arclength parameter. The present method allows the specification of an arbitrary grid spacing function and the numerical values for the parameter are computed by solving an ordinary differential equation. The decision to formulate the problem as a differential equation was primarily motivated by the multitude of highly accurate and stable solution algorithms. The current calculations use a fourth order Runge-Kutta method with variable steplength. The following examples indicate the natural distribution of grid points, obtained with equal parameter values, and a redistribution determined by a specified grid spacing function. In the plots, the grid points are connected by straight lines and not the actual curve.



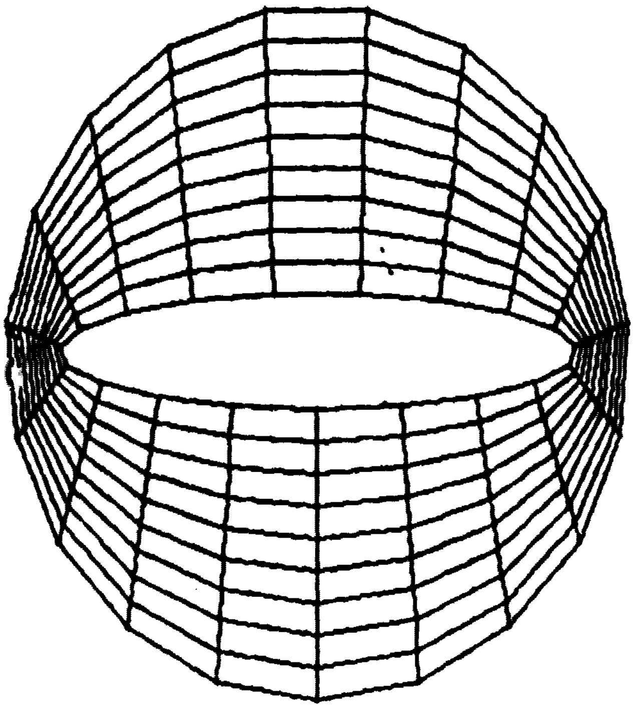
Natural Distribution



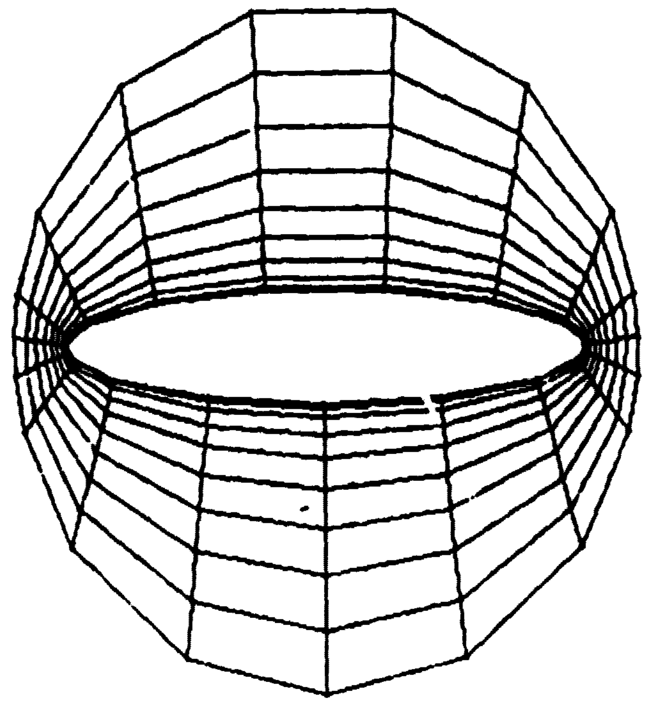
Arclength Distribution



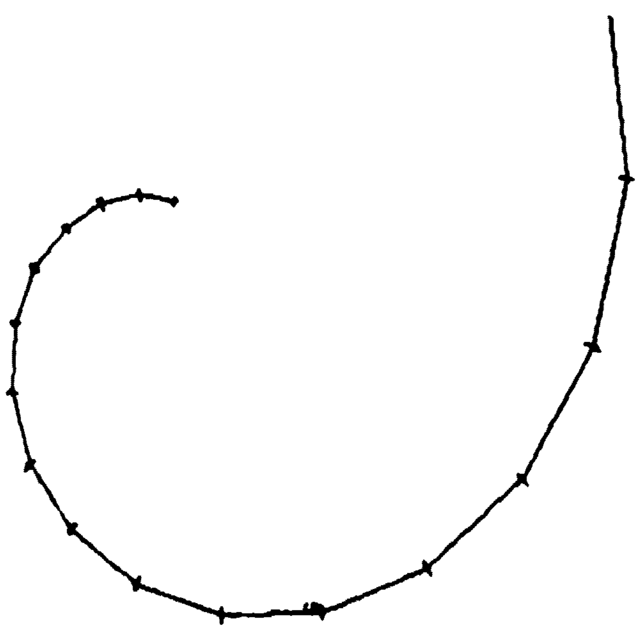
Curvature Distribution



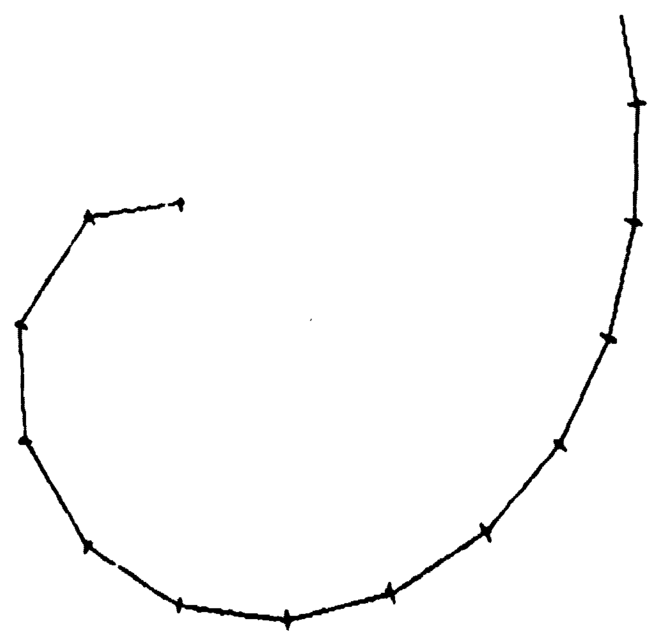
Natural Distribution



Reparameterization



Natural Distribution



Arclength Distribution

This report contains only a brief discussion of our two most recent research projects. Both projects are under continuing investigation. A detailed report on the distribution of grid points on parametric curves and surfaces will be prepared for presentation at a conference this summer.