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# SIMULTANEOUS OCEAN CROSS-SECTION AND RAINFALL MEASUREMENTS FROM SPACE WITH A NADIR-POINTING RADAR

Robert Meneghini and David Atlas

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National Aeronautics and  
Space Administration

Goddard Space Flight Center  
Greenbelt, Maryland 20771

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SIMULTANEOUS OCEAN CROSS-SECTION AND RAINFALL MEASUREMENTS  
FROM SPACE WITH A NADIR-POINTING RADAR

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GODDARD SPACE FLIGHT CENTER  
Greenbelt, Maryland 20770

## ABSTRACT

In the case of a nadir-looking space-borne or aircraft radar in the presence of rain the return power corresponding to secondary surface reflections may provide information on the scattering properties of the surface and the precipitation. The object of the present study is to evaluate a method for determining simultaneously the rainfall rate and the normalized backscattering cross section of the surface,  $\sigma^{\circ}$ . The method is based upon the mirror-reflected power,  $P_m$ , which corresponds to the portion of the incident power scattered from the surface to the precipitation, intercepted by the precipitation, and again returned to the surface where it is scattered a final time back to the antenna. Two approximations are obtained for  $P_m$  depending on whether the field of view at the surface is either much greater or much less than the height of the reflection layer. Since the dependence of  $P_m$  on the backscattering cross section of the surface differs in the two cases, two algorithms are given by which the path-averaged rain rate and  $\sigma^{\circ}$  can be deduced. We also discuss the detectability of  $P_m$ , the relative strength of other contributions to the return power arriving simultaneous with  $P_m$ , and the validity of the approximations used in deriving  $P_m$ .

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# SIMULTANEOUS OCEAN CROSS-SECTION AND RAINFALL MEASUREMENTS FROM SPACE WITH A NADIR-POINTING RADAR

## 1. INTRODUCTION

The importance of measuring precipitation globally has been discussed on many occasions (e.g., Atlas and Thiele, 1981). In the latter work, a broad spectrum of experts reviewed the state of the art and prospects for making precipitation measurements from space. Subsequently, Atlas et al. (1982) assessed a variety of spaceborne radar methods. This paper is a variation upon one of the methods discussed there, namely the surface reference method or SRT. The latter technique is treated in greater detail by Meneghini et al. (1983). One of its major shortcomings was the need to estimate the normalized backscattering cross section of the surface,  $\sigma^0$ , independently in order that the rainfall rate could be measured.

We shall show that for a nadir-looking radar, the present method offers the potential of measuring both the ocean radar cross-section  $\sigma^0$  and the path average rain rate simultaneously. This represents an improvement over the SRT method in that  $\sigma^0$  can be obtained directly from the measured powers.

This paper is a revision of an earlier, preliminary study on the same subject (Atlas and Meneghini, 1983). Here we derive expressions for the mirror-reflected power under fewer restrictions and discuss the error sources in more detail. Nevertheless, a rigorous error analysis is beyond the scope of this paper. Therefore, the study should be viewed as a first step in addressing the questions of the nature of the secondary surface reflections and in determining whether meteorological information can be extracted from such data.

## 2. BASIC CONCEPT

The basic concept of the method is an extension of the approach proposed by Eckerman et al. (1978). In essence, they proposed measurements of the surface radar cross-section in the absence of rain from either measurements outside of the rain in the immediate vicinity, or

from a look-up table generated from prior observations. The path average rain rate would then be deduced from the microwave attenuation calculated as a difference between the echo expected in the absence of rain and that actually measured, and using well established relations between attenuation and rain rate. Dual wavelength versions of this method also have been described (Moore, 1981; Atlas et al., 1982).

The primary limitation of the method focused around the accuracy with which one could expect to estimate the surface radar cross-section. Indeed, the minimum detectable rain rate was found to be dependent upon the RMS error of this estimate.

It is noteworthy that Inomata et al. (1981) conducted an aircraft experiment with two radars operating at 0.86 and 3.2 cm. By simply taking the ocean surface cross-section,  $10 \log \sigma^\circ$ , at nadir to be a constant 10 db at 0.86 cm, they were able to compute the attenuation at this wavelength and the average rain rate between the surface and the melting level. These values correlated well with those computed from measurements of the 3.2 cm reflectivity factor,  $Z$ , and resulted in a  $Z$ - $R$  relationship close to those generally accepted (Battan, 1973). More recently, Fujita et al. (1984) have found fair to good correlations between path-averaged attenuation as estimated from the surface reference method and a dual-wavelength algorithm (Fujita, 1983). They note that the major source of error is probably caused by the variability in  $\sigma^\circ$ . Indeed the nadir ocean surface cross-section is sensitive to roughness and thus decreases with windspeed (Moore and Fung, 1979). This sensitivity has been exploited in estimating ocean surface wind speeds from space (Fedor and Brown, 1982).

To reduce the errors caused by the variability in  $\sigma^\circ$  at least two approaches are possible. The first is to introduce a second radar wavelength and make use of the fact that the  $\sigma^\circ$  at the two wavelengths are well correlated, thus allowing measurements of differential path attenuation. A second approach is to attempt to measure both  $\sigma^\circ$  and rainfall attenuation simultaneously at a single wavelength. This paper is concerned exclusively with the latter approach for

the special case of a nadir-looking radar over the sea. Although the method is applicable to sensing over the land as well, higher transmit powers are needed to obtain detectable signal levels.

The scattering geometry is illustrated schematically in Fig. 1. Fig. 1a shows a nadir pointing radar beam viewing the ocean through a storm. In the absence of rain, the echo power is a measure of the ocean backscatter cross section per unit area  $\sigma^{\circ}$ . With a rainstorm in the path, the power is attenuated by A db, which is a measure of the path average rain rate,  $\bar{R}$ , (Atlas and Ulbrich, 1977). The latter authors show that, in the absence of vertical air motions, A(dB) is essentially linear with R regardless of the drop size distribution at 0.9 cm and approximately so in the band from about 0.7 to 2 cm. To get a true measure of A and thus of R (assuming the attenuating path length is known) it is necessary to know  $\sigma^{\circ}$ .

Figure 1b illustrates the path of a ray from the radar through the precipitation, scattered from the ocean surface back to a slab of precipitation at height H, scattered back to the ocean, and finally reflected once more from the ocean surface and passing through the precipitation back to the radar receiver. The paths of the radar transmitter and receiver are displaced from one another for ease of displaying the incoming and outgoing rays. Of course similar mirror reflected echoes of the precipitation are received from all heights from which they are detectable.

Figure 1c shows a schematic of the radar return power as a function of range, or equivalently, time. The first contribution received is the direct backscatter from the precipitation,  $P_d$ , followed by the echo from the surface at height zero,  $P_g$ . At still later times the sum of all scattering mechanisms corresponding to ray paths greater than  $2H_0$  (where  $H_0$  is the satellite altitude) will arrive at the receiver. As indicated in the figure, one set of ray paths corresponds to the mirror-reflected backscatter,  $P_m$ . In addition, other multi-path contributions will be present as well as the return from the surface received along the antenna side-lobes. If the mirror-reflection component dominates, then it can be seen that a single series of measurements made along the range direction will yield  $P_d$ ,  $P_g$  and  $P_m$ . In the next section we show

that under certain conditions the three unknown quantities,  $\sigma^\circ$ , the rain attenuation and rain reflectivity can be found from the above measurables.

### 3. DERIVATION

To simplify the equations, we introduce the quantities  $A_j$ ,  $j = 1, \dots, n$

$$A_j = \int_0^{H_j} k(s) ds \quad (1)$$

where  $H_j$  is the height of the  $j$ th range gate as measured from the surface. Letting the storm height  $H_s$  correspond to the  $n^{\text{th}}$  range gate, i.e.,  $H_s = H_n$ , then  $A_n$  represents the one way attenuation through the precipitation.

With this notation, the average echo power from the ocean surface under beam-limited conditions can be written (Appendix II)

$$P_g = (C_g \sigma^\circ / H_o^2) 10^{-0.2 A_n} \quad (2)$$

where the symbols are defined in Appendix I, and the constant  $C_g$  is approximately

$$C_g = 7 \times 10^{-3} \lambda^2 P_T / \theta_b^2 \quad (3)$$

Similarly, the echo power received directly from the precipitation at height  $H_j$  above the surface corresponding to a time of arrival of  $t_j$  prior to that of the ground echo, is given by

$$P_d(-t_j) = C_d \eta_j / (H_o - H_j)^2 10^{-0.2 (A_n - A_j)} \quad (4)$$

where

$$C_d = 3.5 \times 10^{-3} \lambda^2 P_T L / \theta_b^2 \quad (5)$$

and where  $\eta_j$  is the rain reflectivity in the  $j$ th range gate above the surface.

Under several simplifying assumptions, the expression for the mirror reflected power, measured at a time  $t_j$  subsequent to that of the ground return can be written as (Appendix II)



$$P_m(t_j) = 3.5 \times 10^{-3} \frac{\lambda^2 P_T \eta_j \Gamma^4 L \sigma^\circ 10^{-0.2 (A_n + A_j)}}{\sigma^\circ \rho_o^2 + 2.76 \Gamma^2 H_j^2} \quad (6)$$

where  $\rho_o$  is the radius of the field of view (FOV) given by

$$\rho_o = \theta_b H_o \quad (7)$$

For notational convenience we let  $q = (\sigma^\circ / 2.76 \Gamma^2)^{1/2}$ . Thus, when the height of the reflection layer,  $H_j$ , is either much greater or much less than  $q\rho_o$ , the mirror-reflected return power can be approximated by one of the following expressions

$$P_m(t_j) = \begin{cases} \frac{C_m' \eta_j \sigma^\circ \Gamma^2}{H_j^2} 10^{-0.2 (A_n + A_j)} ; H_j \gg q\rho_o & (8) \\ \frac{C_m \eta_j \Gamma^4}{H_o^2} 10^{-0.2 (A_n + A_j)} ; H_j \ll q\rho_o & (9) \end{cases}$$

where  $\Gamma^2$  is the Fresnel reflectivity of the surface and

$$C_m' = 1.26 \times 10^{-3} \lambda^2 P_T L \quad (10)$$

$$C_m = C_d = 3.5 \times 10^{-3} \lambda^2 P_T L / \theta_b^2 \quad (11)$$

The approximation to  $P_m$  given by (8), apart from a constant factor that depends only on the particular antenna pattern function assumed, is the same as previously derived (Atlas and Meneghini, 1983). On the other hand, when  $H_j \ll q\rho_o$  and  $H_o \gg H_j$ ,  $P_m$  is the same as would be derived for the mirror reflected power from a perfectly smooth surface. In fact, in this latter case, a ratio of (9) to (4) yields  $\Gamma^4 10^{**(-0.4 A)}$  which implies that the ratio of the mirror-reflected power to the direct power near the surface, where  $A_j \approx 0$ , is approximately equal to the square of the Fresnel reflectivity of the surface.

There are several features of equations (8) and (9) that are of interest: (1) the precipitation parameters (reflectivity and attenuation) enter in the same way in both approximations:

(2) For  $H_j \gg q\rho_0$ ,  $P_m$  is independent of the antenna parameters and satellite altitude but dependent on the height of the reflection layer,  $H_j$ , while for  $H_j \ll q\rho_0$ ,  $P_m$  is inversely proportional to the area of the FOV but independent of  $H_j$ ; (3) for  $H_j \gg q\rho_0$ ,  $P_m$  is directly proportional to  $\sigma^\circ$  while for  $H_j \ll q\rho_0$ ,  $P_m$  is independent of  $\sigma^\circ$ .

Before discussing equations (8) and (9) it is worthwhile first to comment on why the form of  $P_m$  should change according to whether  $H_j \gg q\rho_0$  or  $H_j \ll q\rho_0$ . Consider a small volume element containing precipitation located along the vertical at a height  $H_j$  above the surface and upon which is incident a ray scattered from the surface. If  $2\rho_0 \gg H_j$  then the majority of the energy scattered from the precipitation volume will be returned to the illuminated portion of the surface. However, if  $H_j \gg 2\rho_0$ , then of the energy scattered from the precipitation volume, only those rays nearly anti-parallel to the incident ray will be returned to the illuminated portion of the surface. In the former case, the precipitation is analogous to a mirror with a 'reflection coefficient' of  $\eta_j$ ; in other words, apart from the energy transmitted through the precipitation, all the energy incident on the precipitation is returned to the illuminated portion of the surface. On the basis of this difference we can expect the form of  $P_m$  to change according to whether  $H_j$  is much greater or much less than the diameter of the FOV, i.e.,  $2\rho_0$ . Of course, this argument does not account for the fact that the actual quantity to be compared to  $H_j$  is not  $2\rho_0$  but  $q\rho_0$  and therefore is a function of  $\sigma^\circ$  and  $\Gamma^2$  as well as  $\rho_0$ .

An understanding of the dependence of  $P_m$  on  $\sigma^\circ$  may be gained by considering radiation incident on a surface area element which is scattered bistatically out to the precipitation, a portion of which is returned to a second surface element. The final process is the scattering from the secondary area element back to the antenna. The total mirror reflected power, therefore, is given by the double sum of all such primary and secondary scattering elements within the illuminated surface region.

In the notation of Appendix II the total mirror-reflected power is proportional to  $J$  where

$$J = \int_{\underline{x}_1} \int_{\underline{x}_2} I(\underline{x}_1, \underline{x}_2) e^{-\beta^2 (|\underline{x}_1|^2 + |\underline{x}_2|^2)} d\underline{x}_1 d\underline{x}_2 \quad (12)$$

and where  $\underline{x}_1, \underline{x}_2$ , are the locations of the primary and secondary surface scattering elements. The exponential term of the integrand represents the antenna pattern function at  $\underline{x}_1$  and  $\underline{x}_2$ , with  $\beta = \gamma^2 / \rho_0^2$ ,  $\gamma = 0.86$ . It may be shown numerically that a good approximation to  $I(\underline{x}_1, \underline{x}_2)$  is

$$I(\underline{x}_1, \underline{x}_2) = I_s e^{-\alpha^2 |\underline{x}_1 - \underline{x}_2|^2} \quad (13)$$

where, for  $\sigma^0 \gg \Gamma^2$ ,  $H_j \gg L$

$$I_s = \pi L \Gamma^2 \sigma^0 / H_j^2 \quad (14)$$

$$\alpha^{-2} = a_e^2 = 8\Gamma^2 H_j^2 / \sigma^0 \quad (15)$$

A heuristic argument for the form of  $I(\underline{x}_1, \underline{x}_2)$  given by (13) is presented in Appendix II. In this approximation,  $I(\underline{x}_1, \underline{x}_2)$  is a function only of the separation between the scattering centers. As the separation distance goes to zero,  $I(\underline{x}_1, \underline{x}_2)$  reduces to  $I_s$  which is directly proportional to  $\sigma^0$ . On the other hand, the distance between the scattering centers for which

$$I(\underline{x}_1, \underline{x}_2) = I_s e^{-1}$$

is equal to  $a_e$  which in turn is inversely proportional to  $\sqrt{\sigma^0}$ . We can interpret this last result in the following way: about each primary scattering element on the surface,  $\underline{x}_1$ , there corresponds a secondary scattering region of dimension  $a_e$ , centered at  $\underline{x}_1$ , which contributes the majority of the power returned to the antenna. This last statement must be qualified, however, if  $a_e \gg \rho_0$  since in this case  $\rho_0$ , the radius of the FOV, determines the effective

dimension of the secondary scattering region. In other words, the secondary scattering area can be defined approximately by

$$A_e = \min(\pi a_e^2, \pi \rho_0^2)$$

To understand this in a more quantitative manner we assume for simplicity that  $\underline{x}_1$ , centered at the surface area element  $\Delta A_1$ , is located at the center of the FOV, i.e.  $|\underline{x}_1| = \rho_1 = 0$ . The contribution to the mirror reflected power is then

$$J_1 = 2\pi \Delta A_1 \int_0^\infty I_s e^{-\rho^2(\alpha^2 + \beta^2)} \rho d\rho$$

or

$$J_1 = \frac{\pi \Delta A_1 I_s \rho_0^2 a_e^2}{\rho_0^2 + \gamma^2 a_e^2}$$

Thus

$$J_1 = \pi \Delta A_1 I_s \begin{cases} \rho_0^2/\gamma^2 & ; \gamma a_e \gg \rho_0 \\ a_e^2 & ; \gamma a_e \ll \rho_0 \end{cases}$$

Since  $I_s$  and  $a_e^{-2}$  are directly proportional to  $\sigma^\circ$ , the above equations show that  $J_1$  is independent of  $\sigma^\circ$  for  $\gamma a_e \ll \rho_0$  while  $J_1$  is directly proportional to  $\sigma^\circ$  for  $\gamma a_e \gg \rho_0$ . This accounts for the  $\sigma^\circ$  dependence in equations (8) and (9). The dependencies of  $P_{in}$  on the antenna parameters and satellite and reflection layer altitudes can be found by summing the contributions from all primary surface scattering elements (Appendix II).

#### 4. ALGORITHMS

Case I:  $H_j \gg \rho_0$ ; Height of reflection layer much larger than diameter of FOV

To obtain an algorithm that yields path attenuation and thus mean rainfall rate, we note that equations (4) and (8) are functions of  $\eta_j$  thus allowing its elimination.

Explicitly,

$$\frac{P_m(t_j)}{P_d(-t_j)} = \frac{C_m' \sigma^\circ \Gamma^2 (H_o - H_j)^2}{C_d H_j^2} 10^{-0.4 A_j} \quad (16)$$

To eliminate  $\sigma^\circ$ , (2) is used in (16). Taking the logarithm of the result yields

$$2A_j - A_n = 5 \left\{ -\text{Log} [P_m(t_j)/P_d(-t_j) P_g] + S_j + S_o \right\} \quad (17)$$

where

$$S_j = \text{Log} [H_o^2 (H_o^2 - H_j^2) \Gamma^2 / H_j^2] \quad (18)$$

$$S_o = \text{Log} (C_m' / C_d C_g) \quad (19)$$

When the range gate at the storm top is chosen,  $H_n = H_s$ , (17) reduces to an estimate for the total one-way attenuation,  $A_n$ :

$$A_n = 5 \left\{ -\text{Log} [P_m(t_n)/P_d(-t_n) P_g] + S_n + S_o \right\} \quad (20)$$

From (20), a measurement of  $H_s$ , and the k-R relationship, the path-averaged rain rate can be deduced (Atlas and Ulbrich; 1977).

The estimate of attenuation given by (20) depends upon the calibration constants  $C_m'$ ,  $C_d$ ,  $C_g$  through the term  $S_o$ . If we assume that detectable levels of  $P_m$  can be obtained in light rain (where the attenuation is negligible), then the left hand side of (20) is approximately zero. This implies that a best-fit value of  $S_o$  can be obtained from measurements of  $P_m$  and  $P_d$  near the storm top under conditions of light rain. Returning to (20) and comparing it with (2) shows that  $\sigma^\circ$  can be found from the equation

$$\sigma^\circ = (P_g H_o^2 / C_g) 10^{0.2 A_n} \quad (21)$$

The vertical profiles of reflectivity factors,  $\eta_j$ , and the associated rain rates  $R_j$  can be ob-

tained from  $A_n$ , equation (20), and measurements of  $P_d(-t_j)$  via the Hitschfeld-Bordan algorithm (see Lu and Hai, 1980; Meneghini et al., 1983).

Case II:  $H_j \ll q\rho_0$ ; Height of reflection layer much less than diameter of FOV

As in the first case, we begin by taking the ratio of  $P_m$  to  $P_d$  where  $P_m$  is now given by (9):

$$\frac{P_m(t_j)}{P_d(-t_j)} = \frac{C_m \Gamma^4 (H_0 - H_j)^2 \cdot 10^{-0.4 A_j}}{C_d H_0^2} \quad (22)$$

Noting that  $C_m = C_d$ , then

$$A_j = \frac{5}{2} \left\{ \text{Log} \left( \frac{P_m(t_j)}{P_d(-t_j)} \right) + 2 \text{Log} \left( \frac{H_0}{\Gamma^2 (H_0 - H_j)} \right) \right\} \quad (23)$$

Setting  $j = n$  in (23) again provides an estimate for the total path integrated attenuation,  $A_n$ . Estimates of  $\sigma^\circ$  and the range-profiled rain rates are found by a procedure identical to that previously described. It is not necessary to assume that  $C_m$  and  $C_d$  are equal, however. Since these quantities are range independent, it is clear from (22) that the interval attenuation between range gates  $k$  and  $j$  can be found. Solving for  $A_{jk}$  where  $A_{jk} = \int_{r_k}^{r_j} k(s) ds$ ,  $r_j > r_k$  we obtain

$$A_{jk} = -\frac{5}{2} \left\{ \text{Log} \left[ \frac{P_m(t_j) P_d(-t_k)}{P_m(t_k) P_d(-t_j)} \right] + 2 \text{Log} \left( \frac{H_0 - H_k}{H_0 - H_j} \right) \right\} \quad (24)$$

For  $H_0 \gg H_n$ , the second term is negligible and equation (24) is essentially the same as would be obtained for a dual-wavelength algorithm (Eccles and Mueller, 1971) if we interpret  $P_m$  and  $P_d$  respectively as the attenuating and nonattenuating wavelength returns from the  $j$ th and  $k$ th range gates. In the dual-wavelength algorithm, the reflectivity factors are assumed to be equal at each range gate, i.e., independent of wavelength. The analogous assumption here is that the reflectivity of the directly viewed rain volume is identical to that of the rain volume viewed via the first reflection from the surface (see section 6.3).

## 5. PRELIMINARY RESULTS

The first objective is to determine how the mirror reflected precipitation echo behaves with respect to that directly scattered and that received from the surface. We also want to know the dynamic range of rain rates over which we can expect detectable mirror echoes, the smallest of the three measurable. For the purpose of these calculations we have assumed the radar, storm, and ocean parameters shown in Table 1. Initially we assume a very large antenna diameter of 7.5 m to ensure that  $H_s \gg q\rho_0$  and therefore that equation (8) is applicable. The effects of reducing the antenna size on  $P_m$  are discussed below.

Table 1: Model Radar, Storm, and Ocean Parameters

Parameter	Value
Wavelength - $\lambda$	1.87 cm
Peak Power - $P_T$	10 kw
Pulse Width - $\tau$	1.33 sec
Antenna Diameter - D	7.5 m
Radar Noise Figure	7.0 dB
Orbit Height - $H_0$	500 km
Top of rain - $H_s$	3 km
Ocean Cross-Section - $\sigma^0$	10 dB
Range Resolution - $L/2$	0.2 km

We also use  $\sigma^0 = 10$  which corresponds to a wind speed of about  $10 \text{ ms}^{-1}$ . Table 2 shows values of  $10 \text{ Log } \sigma^0$  versus wind speed for the SEASAT 13.5 GHz radar (Fedor and Brown, 1982)

Table 2: Ocean Normalized Backscatter Cross-Sections ( $10 \text{ Log } \sigma^0$ ) at Nadir Versus Windspeed (Fedor and Brown, 1982)

Windspeed ( $\text{ms}^{-1}$ )	1	3	5	10	20	50	100
Cross-Section (dB)	19.5	13.5	11.8	9.7	7.2	5.1	4.0

The results are shown in Fig. 2. All curves are signal to noise versus rain rate on a single pulse basis. To use a uniform scale for all three quantities the plotted values of  $10 \text{ Log}(P_g/N)$ ,

corresponding to the sea echo, have been reduced by 60 dB while the values of the direct rain return to noise.  $10 \text{ Log}(P_d/N)$  have been reduced by 11 dB. Clearly, with such a powerful radar there is an abundance of surface signal with the signal to noise curve remaining approximately constant at about 73 dB up to moderate rain rates beyond which the two-way attenuation through the 3 km depth of rain increases sharply.

The curve of  $10 \text{ Log}(P_d/N)$ , corresponding to the echo received directly from the precipitation in the range gate immediately above the surface, exhibits adequate dynamic range with values exceeding 7 db over the full range from 0.1 to 100 mm/hr. At  $R < 10 \text{ mm/hr}$ ,  $10 \text{ Log } P_d$  is proportional to  $15.2 \text{ Log } R$  corresponding to the assumed  $Z - R$  relation,  $Z = 424 R^{1.52}$ . At  $R > 40 \text{ mm/hr}$ , the curve is dominated by attenuation and the slope is proportional to  $- 22 \text{ Log } R$  corresponding to a two way path through rain with an attenuation coefficient  $k = 0.042 R^{1.1} \text{ db/km}$ . That is to say that a 10-fold increase in  $R$  produces a 22 db increase in attenuation and corresponding decrease in received power.

For present purposes the most interesting curve is  $10 \text{ Log } (P_m/N)$  corresponding to the range gate at the top of the rainfall at 3 km height. At low rain rates where attenuation is negligible  $P_d$  is approximately 11 dB greater than  $P_m$  and the curves are parallel to one another. As the rain rate increases, the behavior of the ratio  $P_m/P_d$  is dominated by the attenuation term for a two way path between the surface and height  $H_j$ . For the case shown, the gate for the mirror-reflected power is positioned at the top of the rain area,  $H_j = 3 \text{ km}$ , so that the attenuation terms for  $P_g$  and  $P_d$  are half that for  $P_m$ . When the range gates of the mirror reflection and the direct precipitation returns are near the ocean surface, the attenuation terms in  $P_m$ ,  $P_d$  and  $P_g$  are approximately equal.

As mentioned earlier, the size of the antenna was chosen to insure the validity of equation (8); i.e., using the approximations  $\rho_0 = \theta_b H_0$ ,  $\theta_b = \lambda/2D$  and  $D = 7.5 \text{ m}$  yield  $\rho_0 = 0.62 \text{ km}$  which is much less than the maximum height of the reflection layer. For  $D = 1 \text{ m}$ ,  $H_s \ll \rho_0$  so that  $P_m$  is given by equation (9). As shown in Fig. 3,  $P_m/N$  is reduced by about 7 dB from



the previous case and the range of rain rates for which  $P_m/N$  exceeds unity is between about  $1.0 \text{ mm hr}^{-1}$  and  $30 \text{ mm hr}^{-1}$ . It is also worth noting that at light rains (before the effects of attenuation are evident)  $P_d$  exceeds  $P_m$  by about 4.4 db. This difference, which is present as long as  $H_j \ll q\rho_0$  results entirely from the fact that a ratio of equation (9) to equation (4) is equal to  $\Gamma^4 10^{**(-0.4A_j)}$ , which for light rain rates and  $\Gamma^2 = 0.6$ , is equal to - 4.4 dB.

In Table 3 we present the range of rain rates for which  $P_m/N$  exceeds unity for selected values of antenna diameter and radar wavelength, for values of the peak transmit power of 1 kw and 10 kw. We see that the 1 kw system is inadequate with a 1 m diameter antenna. At 3.2 cm, a combination of 1 kw and 3 m antenna diameter would serve well except at the light rain rates. On the other hand, with 10 kw, a 1 m antenna diameter provides adequate signal to noise over the lighter rain rates up to about 8 mm/hr at 0.86 cm, but quickly loses signal at the larger rain rates where attenuation is excessive. Thus, in order to achieve sufficient dynamic range in rain rate, we require a dual wavelength system. This is the case for any method, radar or radiometric, which relies on rainfall attenuation (Atlas et al., 1984). We should note, nevertheless, that the condition required for the present method,  $P_m \gg N$ , is significantly more stringent than the condition  $P_g \gg N$ , needed in the surface reference methods.

Table 3: Range of Rain Rates ( $\text{mm hr}^{-1}$ ) for which  $P_m/N$  exceeds unity. The results in brackets correspond to  $P_T = 1 \text{ kw}$ ; otherwise  $P_T = 10 \text{ kw}$ .

		Wavelength (cm)		
		0.86	1.87	3.2
Antenna Diameter (m)	1	0.1 $\text{mm hr}^{-1}$ $\rightarrow$ 8 $\text{mm hr}^{-1}$ [1.25 $\text{mm hr}^{-1}$ $\rightarrow$ 3.2 $\text{mm hr}^{-1}$ ]	1 $\rightarrow$ 30 [none]	4 $\rightarrow$ 70 [none]
	3	0.1 $\rightarrow$ 10 [0.25 $\rightarrow$ 5]	0.3 $\rightarrow$ 40 [1.5 $\rightarrow$ 2.5]	1.3 $\rightarrow$ 100 [6.0 $\rightarrow$ 64]
	7.5	0.1 $\rightarrow$ 10 [0.25 $\rightarrow$ 6.5]	0.2 $\rightarrow$ 40 [1 $\rightarrow$ 25]	0.5 $\rightarrow$ 100 [2.5 $\rightarrow$ 80]

## 6. SOURCES OF ERROR

There are a number of difficulties associated with the method that must be resolved before establishing its feasibility. Some of the difficulties are common to all meteorological radar methods; e.g., sampling errors and errors caused by fluctuations in the drop size distribution. Other errors are, for the most part, characteristic of the method itself. Unfortunately, a rigorous error analysis requires a knowledge of the bistatic cross sections of the rain drops, a specification of the antenna pattern and the evaluation of a seven-fold integral. In the following sections, we discuss in a qualitative manner the error sources and indicate the circumstances under which they may be significant.

### 6.1 *Multi-path and side-lobe contributions*

There are a number of other scattering mechanisms that correspond to ray paths whose distances are equal to the mirror reflected path. Since these contributions arrive at the receiver at the same time as  $P_m$ , they constitute a source of error. Two kinds of paths are the dominant contributors to this error. The first is the direct backscattered power from the surface along the antenna side lobes.

To find the direct side lobe contribution that arrives at the receiver simultaneously with  $P_m$ , we set the path length of the mirror reflection equal to the path length of the side lobe contribution. This condition along with a knowledge of  $D/\lambda$  and the antenna pattern function specify the side lobe(s) of interest. As an example, we assume that the primary reflector has a parabolic illumination function which produces an electric field in the far zone of the form (Silver, 1949)

$$E = 8 J_2(x)/x^2$$

where  $x = \pi D \sin \theta/\lambda$  and  $J_2$  is the Bessel function of the first kind of second order. For a mirror-reflection height of 3 km and for the parameters of Table 1, the side lobe that arrives at the same time as  $P_m$  is well beyond the 10th side lobe and therefore negligible in compar-

ison to  $P_m$ . If the antenna diameter is decreased to 1 m, the nearest side lobe of interest is the fifth. To obtain an upper bound on this contribution we take the maximum value of the gain within the fifth side lobe and the region of scattering to be equal to the area of the annulus  $\pi H_0 L$ . This procedure gives a side-lobe to noise ratio,  $P_{SL}/N$ , approximately 83 dB down from the  $P_g/N$  curve shown in Fig. 3 which is well below the corresponding  $P_m/N$  curve for this case. In the algorithm for the interval attenuation (equation [24]),  $P_m$  is needed at two distinct range gates. When the height of the reflection layer,  $H_j$ , is near the surface the side-lobe contribution arriving simultaneously with  $P_m$  will be larger. For example, if  $H_j = 0.5$  km and  $D = 1$  m then  $P_{SL}/N$  is down by about 35 dB from the plotted  $P_g/N$  curve in Fig. 3. As compared to  $P_m(0.5 \text{ km})/N$  this is an unacceptably high level. To reduce  $P_{SL}$  an additional 25 db the antenna diameter would need to be increased to approximately 1.5 m.

The second contribution that arrives at the receiver simultaneous with  $P_m$  can be understood by visualizing a ray path incident along the main beam (nadir direction) which scatters out of the vertical direction. This ray is then scattered by the precipitation and received along one of the antenna side lobes. The volume of interest is approximately that portion of the "shell" of two confocal prolate spheroids which contains precipitation. The foci of the ellipsoids are the antenna and the center of the surface illuminated by the main beam (Fig. 4).

To investigate this multi-path contribution to the return power we begin by equating this path length to the mirror reflection path which gives

$$\rho = \left\{ \left( \frac{H_0 (2H_j + z) + 2H_j^2}{H_0 + H_j} \right)^2 - z^2 \right\}^{1/2} : H_0 \gg z$$

where  $\rho$  is the radial distance measured from the axis of the main antenna beam (vertical) to the multi-path rain volume  $\Delta V$ . We also define  $\psi$  as the angle between the ray directed from the surface to  $\Delta V$  and the scattered ray from  $\Delta V$  to the antenna (Fig. 4). Note that for  $H_0 \gg H_j$ ,  $\psi$  is nearly equal to the bistatic scattering angle  $\theta$  shown in the figure.

As before  $H_j$  is the height of the mirror-reflection layer while  $z$  is the height of  $\Delta V$  which ranges from ground level to the storm top. Table 4 gives values of  $\rho$ ,  $\psi$  and  $n$  for three values of  $H_j$ ;  $n$  is the side lobe nearest the main beam into which the scattered energy from  $\Delta V$  is received (an  $n = 0$  implies that the energy is received along the main beam of the antenna). The results have been tabulated for an altitude of  $H_0 = 500$  km, where we have assumed  $\lambda = 1.87$  cm,  $z = H_s = 3$  km and parabolic illumination of the reflector.

Table 4

$H_0 = 500$  km

<u><math>H_j</math>(km)</u>	<u><math>\rho</math>(km)</u>	<u><math>\psi</math></u>	<u><math>n</math></u>		
			$D = 1$ m	$D = 5$ m	$D = 7.5$ m
0.5	2.7	41.8°	0	0	1
1	4	53.3°	0	1	2
3	8.4	71.3°	0	3	6

The large values of  $\psi$  in Table 4 indicate that the rain volume of interest is outside the cone into which most of the incident power is bistatically scattered from the surface (see Fig. 5). Moreover, since  $\psi$  also represents the bistatic scattering angle of the rain drops, only a fraction of the energy incident on  $\Delta V$  will be scattered towards the antenna. However, in four of the cases shown in the table, a portion of the multi-path contribution will be received along the main beam of the antenna ( $n = 0$ ) which produces only a small reduction in the one-way antenna gain. Unfortunately, it is difficult to estimate the magnitude of the multi-path contribution without evaluating the bistatic cross section of the droplets and without accounting for the variations of  $\psi$  and  $\rho$  throughout the full range of  $z$ . Although crude estimates indicate that this contribution is smaller than  $P_m$ , more detailed calculations are needed. It should also be noted that a Doppler radar may provide a means of discriminating  $P_m$  from side lobe and multi-path contributions (Atlas and Matejka, 1984).

## 6.2 Bistatic Rain Reflectivity

In the derivation of  $P_m$  the bistatic rain reflectivity was replaced by the backscatter reflectivity. The conditions under which the approximation is valid are not as stringent as might be thought since the pairs of surface elements that contribute most to the return power,  $P_m$ , are separated by a distance less than the minimum of  $a_c$  and  $2\rho_0$  (Appendix II). Since  $a_c$  is given by  $H_j \Gamma \sqrt{8/\sigma^\circ}$  the range of bistatic scattering angles is given approximately by

$$\pi \geq \psi \geq \pi - \phi$$

where

$$\phi = \text{Tan}^{-1} \left[ \min \left( \left( \frac{8\Gamma^2}{\sigma^\circ} \right)^{1/2}, 2\rho_0/H_j \right) \right]$$

For example, if  $\phi = \text{Tan}^{-1} \left( \left( \frac{8\Gamma^2}{\sigma^\circ} \right)^{1/2} \right)$ , then for  $\sigma^\circ = 10$  dB, and  $\Gamma^2 = 0.6$  the minimum bistatic scattering angle is approximately  $180^\circ - 35^\circ = 145^\circ$ .

## 6.3 Mismatches Between Rain Scattering Volumes

Because the ocean surface is roughened by the wind and waves, energy incident on the surface is scattered in all directions. This is why the mirror precipitation echo must be integrated over all scattering angles. The bistatic scatter cross-section corresponding to Eq. (A9) after Ishimaru (1978) is shown in Fig. 5 with windspeed as a parameter. In order to assign a wind speed to each curve, we matched the value of  $10 \text{ Log } \sigma^\circ$  for direct backscatter (i.e., 0 deg) to the corresponding values read from the nadir values of  $\sigma^\circ$  versus windspeed given by Fedor and Brown (1982) and reproduced in Table 2. Since the latter data were based on observations taken at vertical incidence, while the curves are theoretical, one should not regard them as rigorous. Nevertheless they indicate reasonable behavior. In essence they show that as the wind increases the nadir surface cross-section decreases and that increasing energy is scattered to angles farther off nadir. The inset table in Fig. 5 lists the full angle around nadir for which the cumulative power which is scattered from the rain is equal to half the total power radiated back toward the radar. For example, at a windspeed of  $10 \text{ ms}^{-1}$  (correspond-

ing to  $10 \text{ Log } \sigma^{\circ} = 10 \text{ dB}$ ), we must include the energy scattered from within a cone of  $34.4$  deg to account for half the total mirror reflected power at the radar.

As indicated in Fig. 5, even light winds will roughen the ocean sufficiently to spread the reflected beams over fairly sizable angles. This has the effect of increasing the horizontal dimensions of the volume of precipitation which is viewed in mirror reflection relative to that which is viewed directly by the incident beam, the more so the higher the volume element. The effects of beam divergence call into question two assumptions made previously. In the derivation of  $P_m$  it was assumed that most of the scattering takes place along the vertical, implying that the general path attenuation could be replaced by the attenuation along the vertical. However, as the beam divergence increases, the attenuation must be considered as a weighted average along the various directions of the surface scattered power. The errors resulting from the vertical scattering assumption can be studied quantitatively by including the general path attenuation into the definition of the integral  $I(\underline{x}_1, \underline{x}_2)$  (equation (A13)). A study of this type, however, is beyond the scope of this paper. Secondly, in the algorithms for path attenuation, we implicitly assumed that the precipitation within the pulse volume seen in mirror reflection be identical to or of the same reflectivity to that viewed directly. This can be a demanding requirement at a windspeed of  $10 \text{ ms}^{-1}$ , with the half angle of  $17.2$  deg, and a pulse volume at  $3 \text{ km}$  height. The horizontal dimension of the mirror scattering volume is then increased by approximately  $0.93 \text{ km}$  on either side of the incident beam. For  $D = 7.5 \text{ m}$ ,  $\lambda = 1.87$ ,  $H_0 = 500 \text{ km}$  the diameter of the incident beam near the surface is approximately  $1.2 \text{ km}$ . In this case the mirror reflected scattering volume, at a height  $3 \text{ km}$  above the surface is over a factor of  $6$  greater than the direct rain scattering volume. On the other hand, for an antenna diameter of  $1 \text{ m}$  the ratio of the mirror reflected volume to the direct volume decreases to about  $1.44$ .

## 7. SUMMARY

The method proposed here for the remote measurement of rainfall rate either from space

or aircraft is motivated by the need to circumvent one of the limitations of the single wavelength surface reference technique; namely, that estimates of  $\sigma^{\circ}$  are required to extract meteorological information from the surface return power. To provide additional information on the surface and meteorological parameters, the mirror-reflected power,  $P_m$ , is considered as a possible candidate.  $P_m$  corresponds to the portion of the power scattered by the surface, intercepted by the precipitation and returned to the surface where it is scattered a final time back to the antenna. Under a number of simplifications a closed form expression for  $P_m$  is obtained. The dependence of  $P_m$  on the backscattering cross section,  $\sigma^{\circ}$ , was found to depend upon the magnitude of the quantity  $H_j \Gamma / \sqrt{\sigma^{\circ}} \rho_0$ , where  $\Gamma^2$  is the Fresnel reflectivity of the surface,  $H_j$  is the height of the reflection layer and  $\rho_0$  is the radius of the FOV. When this quantity is much greater than one,  $P_m$  is directly proportional to  $\sigma^{\circ}$ ; if it is much less than one,  $P_m$  is independent of  $\sigma^{\circ}$ . Algorithms based on these limiting cases are proposed for the estimation of path attenuation, rainfall rate and  $\sigma^{\circ}$ .

The levels of  $P_m$  to noise power at nadir over the ocean indicate that detectable signal levels can be obtained from satellite altitudes over a fairly wide rain rate interval. This remains true even with a modest antenna dimension of 1 m. Thus, the method may provide an important auxiliary measurement scheme for a space-borne meteorological radar for the estimation of rain and the backscattering cross section of the surface. Estimates of this latter quantity will in turn permit one to investigate the dependence of the mean square surface slope on wind speed in the presence of rain. When the peak transmitted power is reduced from 10 kw to 1 kw, however, the range of detectable rain rates is reduced considerably. Furthermore, over land the decrease in  $\Gamma^2$  and  $\sigma^{\circ}$  relative to the ocean background indicates that even at 10 kw, the mirror reflected power as measured from a low earth orbiter will be smaller than the receiver noise power for most rain rates and radar wavelengths.

In the derivation of  $P_m$  a number of simplifying assumptions were made both to make the problem tractable and to understand the approximate dependence of  $P_m$  on meteorological

and surface scattering parameters. Although a qualitative discussion of the error sources is given, quantitative results are needed to check the range of validity of the expressions for  $P_m$  and to compare the relative magnitudes of side-lobe and other multi-path contributions that arrive at the antenna simultaneous with  $P_m$ .



## FIGURE CAPTIONS

- Fig. 1: Schematic of the mirror-reflected echo. The mirror-reflection echos which are returned subsequent to the surface return in 1c occur by means of a double reflection from the surface as shown in 1b.
- Fig. 2: Curves of the signal to noise ratios for returns from the ocean,  $P_g$ , the precipitation viewed directly at a range just above the surfaces  $P_d$ , and the mirror-reflected return,  $P_m$ , from a 3 km height. Radar parameters are given in Table 1.
- Fig. 3: Same as Fig. 2 except for an antenna diameter of 1 m.
- Fig. 4: Schematic of a scattering path that arrives at the receiver simultaneous with the mirror-reflected return.
- Fig. 5: Bistatic cross section of the ocean for nadir incidence. Inset table shows the conical angle from which 50% of the total power is received.
- Fig. A1: Scattering geometry of a ray incident from the radar to the ocean surface element  $\Delta A_1$ , scattered to the precipitation volume  $\Delta V$ , and returned to the surface element  $\Delta A_2$  where it is scattered a final time back to the radar.
- Fig. A2: Schematic of the precipitation volume (ellipsoidal shell) that contributes to the mirror-reflected power for two fixed surface scattering elements.

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## APPENDIX I

### Table of Symbols

$\lambda$	-	Wavelength
$P_T$	-	Peak transmitted power
$P_t$	-	Transmitted power waveform
$t$	-	Transmitted pulse duration
$L/2$	-	Range resolution
$H_0$	-	Orbit altitude
$H_j$	-	Height of rain layer
$H_s$	-	Height of storm
$\sigma^0$	-	Ocean cross-section at nadir
$\sigma_b(\hat{\Gamma}, \hat{\Theta})$	-	Ocean normalized bistatic cross-section: incident wave in the $\hat{\Gamma}$ direction, scattered wave in the direction $\hat{\Theta}$ .
$\theta_b$	-	3 db beamwidth measured from the peak to the half power point.
$A_e$	-	Effective antenna area
$G_0$	-	Antenna gain along beam axis
$G_t(\hat{\Gamma})$	-	Antenna gain pattern along the $\hat{\Gamma}$ direction on transmit
$G_r(\hat{\Theta})$	-	Antenna gain pattern along the $\hat{\Theta}$ direction on receive
$\eta(\hat{\Gamma}, \hat{\Theta})$	-	Bistatic rain reflectivity
$\eta$	-	Backscattered rain reflectivity
$R$	-	Path-averaged rain rate
$k$	-	Attenuation coefficient
$Z$	-	Rain reflectivity factor
$A$	-	Path-integrated attenuation
$P_m$	-	Mirror reflected echo power

### Table of Symbols

$P_d$	-	Echo power directly scattered from precipitation.
$P_g$	-	Echo power directly scattered from ocean surface.
$N$	-	Receiver noise power
$c$	-	Speed of light in free space

## APPENDIX II

The purpose of the appendix is to derive approximate expressions for the return power  $P_m$ .  $P_m$  corresponds to the portion of the power scattered by the surface which is intercepted by the precipitation, returned again to the surface and is scattered back to the antenna. A general ray path contributing to  $P_m$  is shown in Fig. A1 and can be written as

$$\Delta P_m(t) = \frac{P_t(t - \sum_0^3 r_i/c)}{(4\pi)^4 (r_0 r_1 r_2 r_3)^2} G_t(\hat{r}_0)$$

$$\begin{aligned} & * (\sigma_b(\hat{r}_0; \hat{r}_1) \Delta A_1) * (\eta(\hat{r}_1; \hat{r}_2) \Delta V) \\ & * (\sigma_b(\hat{r}_2; \hat{r}_3) \Delta A_2) * A_e 10^{-0.1 \int_Q k ds} \end{aligned} \quad (A1)$$

where  $Q$  is that portion of the ray path within the precipitation and where the symbols in (A1) are defined in Appendix I.

Assuming that  $\Delta A_1$ ,  $\Delta A_2$ , and  $\Delta V$  can be converted into differential area and volume elements, and using the relation  $A_e = \lambda^2 G_r/4\pi$  then  $P_m$  can be written

$$\begin{aligned} P_m(t) = & \frac{\lambda^2}{(4\pi)^5} \int_{\underline{x}_1} \int_{\underline{x}_2} \frac{G_t(\hat{r}_0) G_r(\hat{r}_3)}{r_0^2 r_3^2} d\underline{x}_1 d\underline{x}_2 \\ & \times \left\{ \int_V P_t(t - \sum_0^3 r_i/c) \frac{\sigma_b(\hat{r}_0; \hat{r}_1) \sigma_b(\hat{r}_2; \hat{r}_3) \eta(\hat{r}_1; \hat{r}_2)}{r_1^2 r_2^2} 10^{-0.1 \int_Q k ds} dV \right\} \end{aligned} \quad (A2)$$

where the volume integral is taken over the precipitation, and  $d\underline{x}_1$ ,  $d\underline{x}_2$  are elements of surface area. The range of integration on both  $\underline{x}_1$ ,  $\underline{x}_2$  is the portion of the surface illuminated by the main beam. Let the transmitted pulse be rectangular and of duration  $\tau$  with  $P_T$  the peak transmitted power

$$P_t(t) = P_T [U(t) - U(t - \tau)] \quad (A3)$$

where  $U$  is the unit step function. Therefore,

$$P_t(t - \sum_0^3 r_i/c) = P_T [U(t - T - (r_1 + r_2)/c) - U(t - T - (r_1 + r_2)/c - \tau)] \quad (A4)$$

If  $H_0 \gg \rho_0$  where  $H_0$  is the spacecraft altitude and  $\rho_0$  is the radius of the footprint, then

$$T \approx 2H_0/c + (|x_1|^2 + |x_2|^2) / 2H_0 c \quad (A5)$$

where  $|x_1|$ ,  $|x_2|$  are the distances measured from the center of the footprint to  $x_1$ ,  $x_2$ , respectively. Thus  $P_t$  is non zero for

$$2H \leq r_1 + r_2 < 2(H + L/2) \quad (A6)$$

where  $L/2$  is the range resolution, equal to  $c\tau/2$ , and

$$2H = ct - 2H_0 - (|x_1|^2 + |x_2|^2) / 2H_0 - c\tau \quad (A7)$$

The condition given by (A6) specifies the region of precipitation that contributes to  $P_m$ . In particular, for a fixed observation time and fixed  $x_1$ ,  $x_2$  the volume integral taken over the precipitation is that region contained within a prolate spheroidal 'shell' with foci at  $x_1$ ,  $x_2$  (Fig. A2).

Using a coordinate system whose origin is mid-way between  $x_1$ ,  $x_2$  and whose x-axis passes through both points, the family of prolate spheroids within this region can be expressed by the equation (Morse and Feshbach, 1953)

$$\frac{x^2}{\xi^2 - \alpha^2} + \frac{(y^2 + z^2)}{\xi^2 - \beta^2} = 1; \xi > \beta \geq \alpha \quad (A8)$$

where  $\xi_{\min} \leq \xi \leq \xi_{\max}$  with

$$\begin{aligned} \xi_{\min} &= (H^2 + \alpha^2)^{1/2} \\ \xi_{\max} &= [(H + L/2)^2 + \alpha^2]^{1/2} \end{aligned} \quad (A9)$$



The distances between the points  $\underline{x}_1$  and  $\underline{x}_2$  is given by  $2x_0$  where

$$x_0 = (\beta^2 - \alpha^2)^{1/2}$$

To simplify the expression for  $P_m$ , we make the following approximations

- (a) The bistatic rain reflectivity factor,  $\eta(\hat{r}_1; \hat{r}_2)$ , can be replaced by the backscattered rain reflectivity,  $\eta$ .
- (b) The majority of the power backscattered from the surface is directed along the vertical direction so that the attenuation can be approximated by the attenuation along the vertical direction, i.e.

$$\int_Q k ds \approx 2 \int_0^{H_s} k dz + 2 \int_0^H k dz$$

where  $H_s$  is the storm height. We assume, moreover that  $H$  can be interpreted as the height of the reflection layer and that  $H$  is nearly independent of  $\underline{x}_1, \underline{x}_2$ .

- (c) The spacecraft or aircraft altitude is taken to be much greater than the radius of the main-beam footprint:  $H_0 \gg \rho_0$ . This implies that the 'ray' paths  $\underline{r}_0, \underline{r}_3$  shown in Fig. (A1) are nearly vertical so that

$$\sigma_b(\hat{r}_0; \hat{r}_1) \approx \sigma_b(-\hat{z}; \hat{r}_1) = f(\theta_1)$$

$$\sigma_b(\hat{r}_2; \hat{r}_3) \approx \sigma_b(\hat{r}_2; \hat{z}) = f(\theta_2)$$

where the angles  $\theta_1, \theta_2$  are defined in Fig. (A2).

- (d) The bistatic cross section of the surface for incident energy along the vertical which is scattered at an angle  $\theta$  is given by the scalar Kirchhoff approximation for a very rough surface (Ishimaru: 1978):

$$f(\theta) = \frac{2m\Gamma^2}{(1 + \cos \theta)^2} e^{-\frac{m}{2} \tan^2 \theta/2} \quad (A10)$$

where  $\Gamma^2$  is the Fresnel reflectivity of the surface and  $m = \ell^2/2\beta_0^2$  where  $\ell$  is the correlation length of the surface and  $\beta_0$  is the rms surface height.

(e) Assuming a Gaussian antenna pattern, the antenna gains are written

$$G_t(\hat{r}_0) = G_0 e^{-(\gamma\theta_1/\theta_b)^2} \approx G_0 e^{-\beta^2 \rho_1^2}$$

$$G_r(\hat{r}_3) = G_0 e^{-(\gamma\theta_2/\theta_b)^2} \approx G_0 e^{-\beta^2 \rho_2^2}$$

where  $\theta_b$  is the beamwidth measured from the maximum to the half power point and where

$$G_0 = \pi^2/(2\theta_b)^2$$

$$\gamma = 0.831$$

$$\beta = \gamma/\theta_b H_0 \approx \gamma/\rho_0$$

The quantities  $\rho_1, \rho_2$  are the respective distances from the center of the field of view (FOV) to the scattering centers  $\underline{x}_1, \underline{x}_2$  on the surface (i.e.  $\rho_1 = |\underline{x}_1|, \rho_2 = |\underline{x}_2|$ ) while  $\rho_0$  is the radius of the FOV.

The validity of these approximations is discussed in the text. With these assumptions (A2) can be written

$$P_m(t) = \frac{\lambda^2 G_0^2 P_T \eta}{(4\pi)^5 H_0^4} 10^{-0.2(A_n + A)} J \quad (A11)$$

where

$$J = \int_{\underline{x}_1} \int_{\underline{x}_2} e^{-\beta^2(\rho_1^2 + \rho_2^2)} I(\underline{x}_1, \underline{x}_2) d\underline{x}_1 d\underline{x}_2 \quad (A12)$$

$$I(\underline{x}_1, \underline{x}_2) = \int_V \frac{f(\theta_1) f(\theta_2)}{r_1^2 r_2^2} dV \quad (A13)$$

$$A_n = \int_0^{H_s} k dz; \quad A = \int_0^H k dz \quad (A14)$$

where the observation time  $t$  corresponds to the reflection layer height,  $H$  and where  $f(\theta)$  is given by (A10).

To express the volume integration of (A13) as an iterated integral, we first note from Fig. A2 that  $\cos \theta_2 = z/r_2$ ,  $\cos \theta_1 = z/r_1$  and therefore  $\cos \theta_2 = r_1 \cos \theta_1 / r_2$ .

For any point  $(x, y, z)$  within the spheroidal 'shell' shown in Fig. A2 we have

$$r_1 = [(x + x_0)^2 + y^2 + z^2]^{1/2}$$

$$r_2 = [(x - x_0)^2 + y^2 + z^2]^{1/2}$$

where as before the origin of the coordinate system is taken midway between the two scattering centers where the x-axis passes through  $x_1$  and  $x_2$ . Therefore,

$$r_1 = (r_2^2 + 4xx_0)^{1/2} \quad (\text{A15})$$

Choosing a spherical coordinate system with an origin at  $x = x_0$ , an arbitrary point  $(x, y, z)$  in the reference system can be expressed by

$$\begin{aligned} x &= x_0 + r_2 \sin \theta_2 \cos \phi_2 \\ y &= r_2 \sin \theta_2 \sin \phi_2 \\ z &= r_2 \cos \theta_2 \end{aligned} \quad (\text{A16})$$

Substituting (A16) into (A15) gives

$$r_1 = r_2 F \quad (\text{A17})$$

where

$$F = \left[ 1 + \frac{4x_0}{r_2} \sin \theta_2 \cos \phi_2 + \frac{4x_0^2}{r_2^2} \right]^{1/2} \quad (\text{A18})$$

so that

$$\cos \theta_1 = \cos \theta_2 / F \quad (\text{A19})$$

Using (A19) and (A10) the function  $f(\theta_1)$  can be written in terms of  $r_2, \phi_2, \theta_2$ . Explicitly,  $f(\theta_1) = \tilde{f}(r_2, \phi_2, \theta_2)$  where

$$\tilde{f}(r_2, \phi_2, \theta_2) = \frac{2m \Gamma^2}{(1 + \cos \theta_2/F)^2} \exp \left[ -\frac{m}{2} (1 - \cos^2 \theta_2/F)/(1 - \cos \theta_2/F)^2 \right] \quad (\text{A20})$$

Therefore (A13) becomes

$$I(\underline{x}_1, \underline{x}_2) = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \int_{r=r_A}^{r_B} \frac{f(\theta) \tilde{f}(r, \phi, \theta)}{r^2 F^2} \sin \theta \, dr \, d\phi \, d\theta \quad (\text{A21})$$

The limits on  $r$  can be found by substituting equations (A10) into (A8) and solving  $r_2$  when  $\zeta = \zeta_{\min}, \zeta_{\max}$ . This procedure gives

$$\begin{aligned} r_A(\phi, \theta) &= (H^2 - x_0^2)/(H + x_0 \sin \theta \cos \phi) \\ r_B(\phi, \theta) &= [(H + L/2)^2 - x_0^2]/[(H + L/2) + x_0 \sin \theta \cos \phi] \end{aligned} \quad (\text{A22})$$

In spite of the simplifications made, the integral (A12) with  $I$  given by (A21) is formidable. To further simplify (A21) we use the fact that a good approximation to  $I(\underline{x}_1, \underline{x}_2)$  is

$$I(\underline{x}_1, \underline{x}_2) = I_s e^{-\alpha^2 |\underline{x}_1 - \underline{x}_2|^2} \quad (\text{A23})$$

where

$$I_s = \frac{\pi L \Gamma^2}{H} \sigma^0 \quad (\text{A24})$$

$$\alpha^2 = 1/a_e^2 = \sigma^0/8 \Gamma^2 H^2 \quad (\text{A25})$$

and where  $\sigma^0$  is the normalized backscattering cross section of the surface for nadir incidence.

To understand qualitatively why  $I(\underline{x}_1, \underline{x}_2)$  can be approximated by this function we first note that as  $\underline{x}_2 \rightarrow \underline{x}_1$ , (A21) becomes

$$I(\underline{x}_1, \underline{x}_1) \equiv I_s = 2\pi \int_{\theta=0}^{\pi/2} \int_{r=H}^{H+L/2} \frac{f^2(\theta)}{r^2} \sin \theta \, dr \, d\theta \quad (\text{A26})$$

Making the change of variable  $u = \tan^2(\theta/2)$  and using (A10), (A26) can be integrated to give

$$I_s = 2\pi(H^{-1} - (H + L/2)^{-1}) \frac{(\Gamma^2 m)^2}{2} \left\{ \frac{1 - 4e^{-m}}{m} + \frac{2(1 - 2e^{-m})}{m^2} + \frac{2(1 - e^{-m})}{m^3} \right\} \quad (\text{A27})$$

From Ishumaru (1978), we have

$$m = 2\sigma^0/\Gamma^2$$

Since for most sea states  $m \gg 1$ , we can neglect terms of order  $m^{-2}$ ,  $m^{-3}$ , and  $e^{-m}$ . Furthermore, assuming that  $H \gg L/2$  then  $I_s$  reduces to (A24).

The second feature of  $I(\underline{x}_1, \underline{x}_2)$ , i.e., its dependence on  $|\underline{x}_1 - \underline{x}_2|$  and  $u_e$  can be understood by noting that the integrand of  $I(\underline{x}_1, \underline{x}_2)$  is proportional to  $f(\theta_1) f(\theta_2)$  where

$$f(\theta_1) f(\theta_2) \propto \exp \left[ -\frac{m}{2} (\tan^2(\theta_1/2) + \tan^2(\theta_2/2)) \right]$$

Letting  $\theta_1 = \theta_2 = \theta$  we obtain  $f^2(\theta) \propto e^{-1}$  when  $\tan \theta = 2\sqrt{m}/(m-1)$ . The angle  $\theta$  can be related to the distance between the scattering centers,  $u_e$ , by noting that

$$u_e = 2H \tan \theta$$

or, using the above result for  $\tan \theta$  and  $m = 2\sigma^0/\Gamma^2 \gg 1$ , then

$$u_e = 4H\Gamma / \sqrt{2\sigma^0}$$

Returning to (A23) and substituting this equation into (A12) and expressing  $\underline{x}_1, \underline{x}_2$  in cylindrical coordinates we obtain

$$J = I_s \int_{\rho_1=0}^{\infty} \int_{\rho_2=0}^{\infty} e^{-(\alpha^2 + \beta^2)(\rho_1^2 + \rho_2^2)} \rho_1 \rho_2 \left\{ \int_{\phi_1=-\pi}^{\pi} \int_{\phi_2=-\pi}^{\pi} e^{2\alpha^2 \rho_1 \rho_2 \cos(\phi_1 - \phi_2)} d\phi_1 d\phi_2 \right\} d\rho_1 d\rho_2 \quad (\text{A28})$$

To perform the integrals over  $\phi_1, \phi_2$  we make the transformation

$$\begin{aligned} u &= \phi_1 - \phi_2 \\ v &= \phi_2 \end{aligned}$$

The double integral over  $\phi_1, \phi_2$  then becomes

$$I_1 = \int_{v=-\pi}^{\pi} \int_{u=-\pi-v}^{\pi-v} e^{2\alpha^2 \rho_1 \rho_2 \cos u} du dv$$

but,

$$\int_{u=-v-\pi}^{\pi-v} e^{p \cos u} du = 2 \int_0^{\pi} e^{p \cos u} du = 2\pi I_0(p)$$

where  $I_0$  is the modified Bessel function of the first kind, zeroth order.

Therefore,

$$I_1 = 4\pi^2 I_0(2\alpha^2 \rho_1 \rho_2) \quad (\text{A29})$$

Substituting (A29) into (A28) gives

$$J = 4\pi^2 I_s \int_{\rho_2=0}^{\infty} \int_{\rho_1=0}^{\infty} I_0(2\alpha^2 \rho_1 \rho_2) e^{-(\alpha^2 + \beta^2)(\rho_1^2 + \rho_2^2)} \rho_1 \rho_2 d\rho_1 d\rho_2 \quad (\text{A30})$$

Carrying out the remaining integrations (Gradshteyn and Ryzhik, 1965) yields

$$J = \frac{\pi^2 I_s}{\beta^2 [2\alpha^2 + \beta^2]} \quad (\text{A31})$$

Substituting (A31) into (A11) and using the definitions of  $\alpha, \beta$  and  $G_0$  gives equation (6) of the text.

To calculate the ground return we assume that beam-limited conditions hold and we choose, as before, a Gaussian antenna gain. We obtain

$$P_g = \frac{P_T G_0^2 \sigma^0 \lambda^2}{2^7 \pi^2 \beta^2 H_0^4} 10^{-0.2 A_n}$$

Using  $G_0 = \pi^2/4\theta_b^2$  and the definition of  $\beta$ , yields equation (2) of the text. The expression for the direct rain return, equation (3) is essentially equivalent to that derived by Probert-Jones (1962) for a rotationally symmetric antenna pattern.

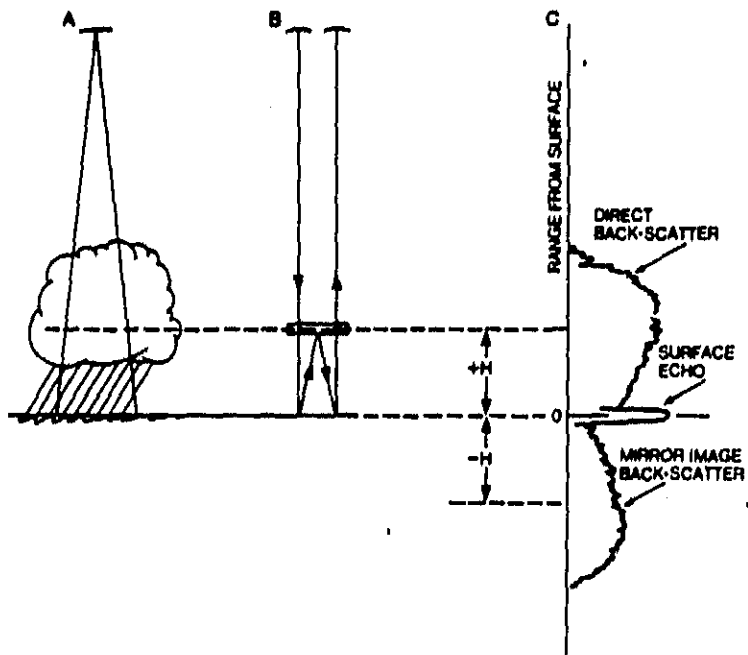


Figure 1. Schematic of the mirror-reflected echo. The mirror-reflected echos which are returned subsequent to the surface return in 1c occur by means of a double reflection from the surface as shown in 1b.



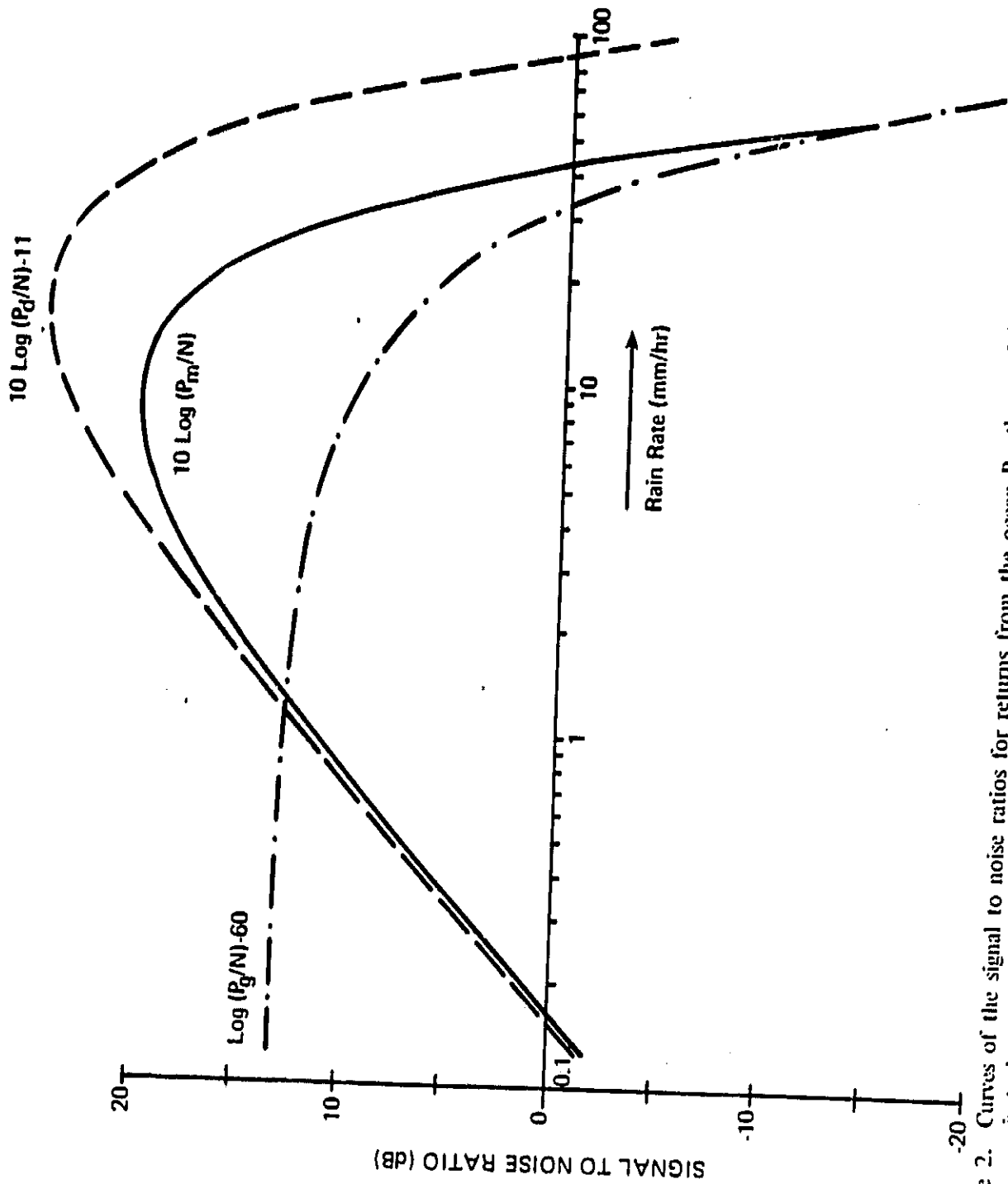


Figure 2. Curves of the signal to noise ratios for returns from the ocean P<sub>d</sub>, the precipitation viewed directly at a range just above the surface P<sub>m</sub>, and the mirror-reflected return, P<sub>g</sub>, from a 3 km height. Radar parameters are given in Table 1.

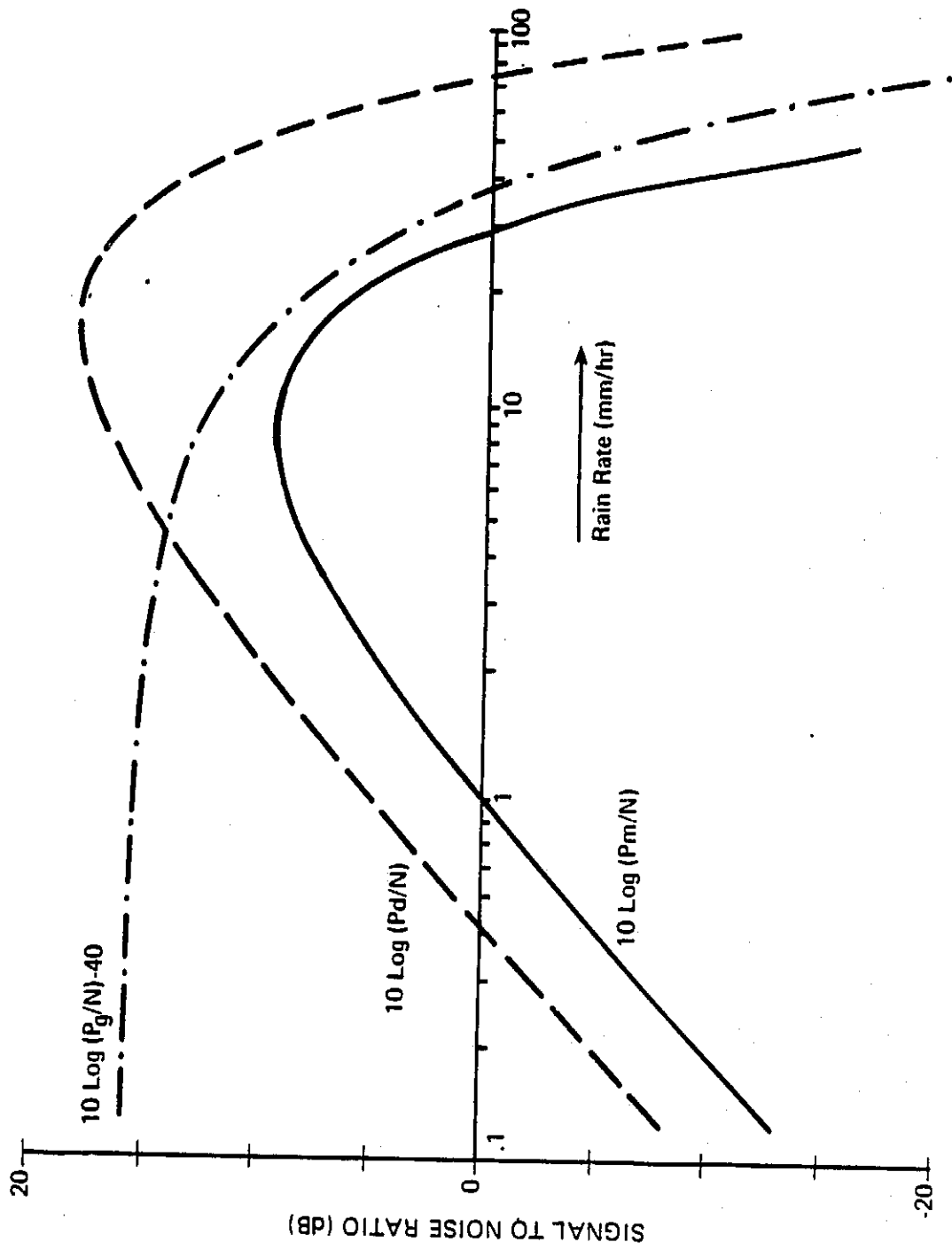


Figure 3. Same as Fig. 2 except for an antenna diameter of 1 m.

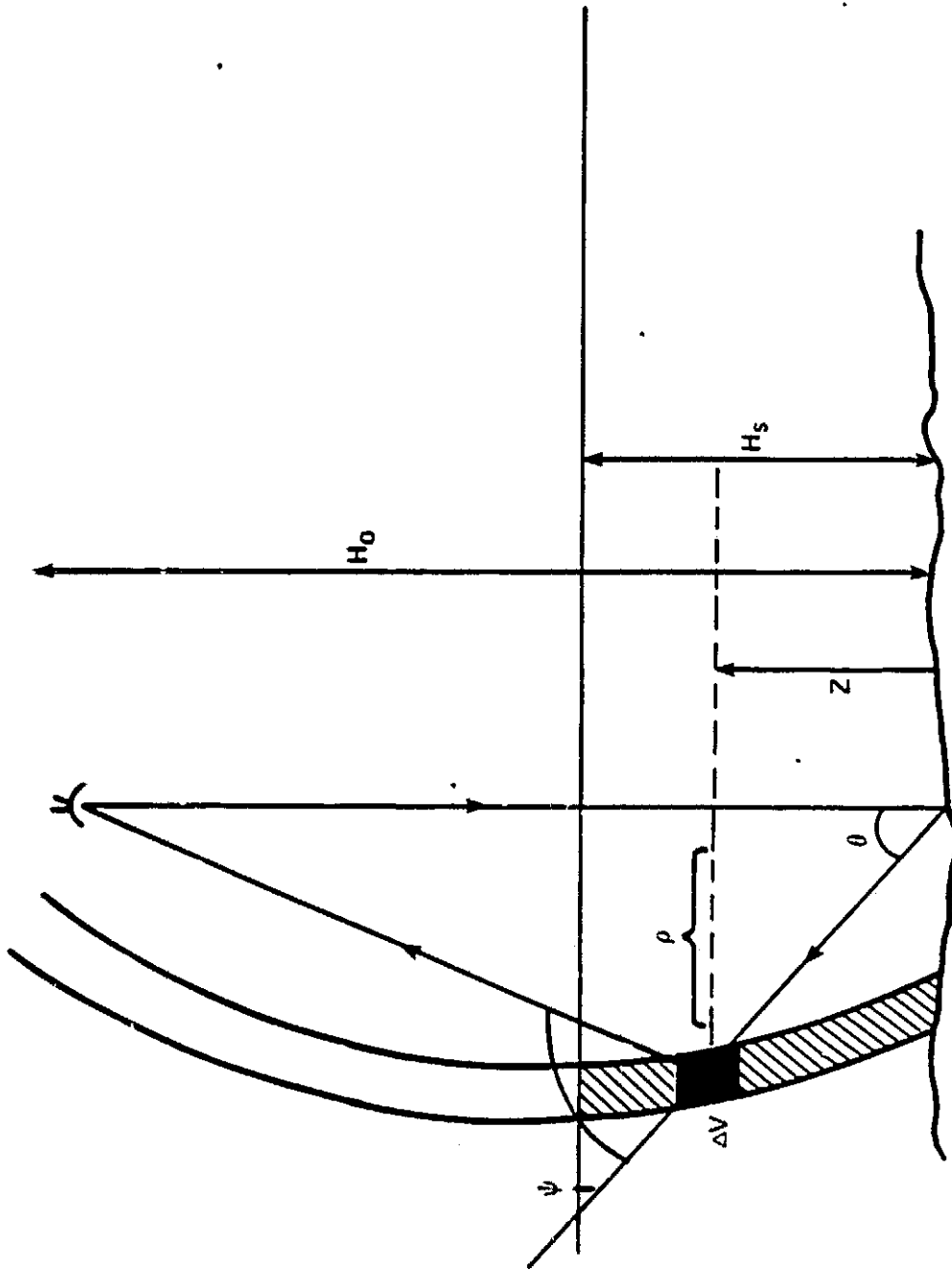


Figure 4. Schematic of a scattering path that arrives at the receiver simultaneous with the mirror-reflected return.

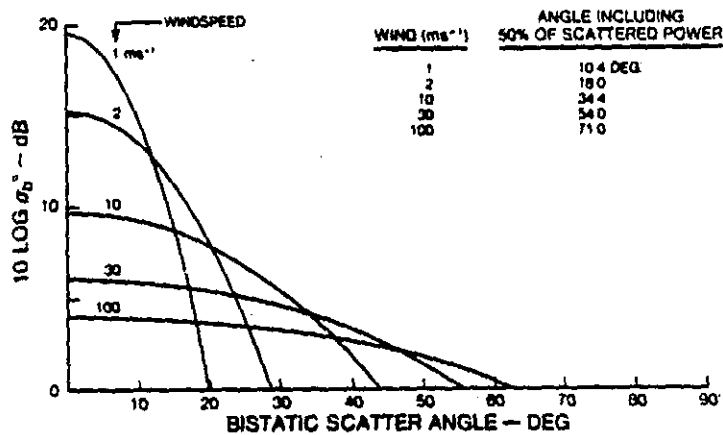


Figure 5. Bistatic cross section of the ocean for nadir incidence. Inset table shows the conical angle from which 50% of the total power is received.

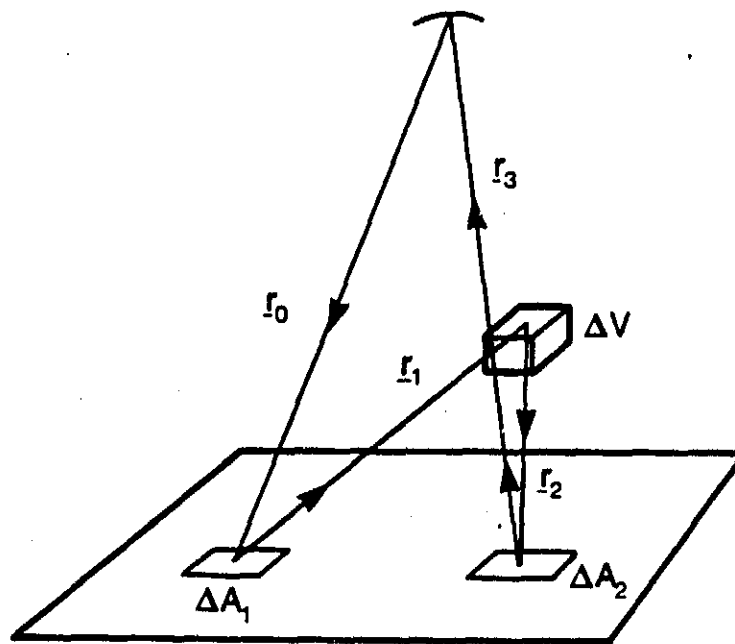


Figure A1. Scattering geometry of a ray incident from the radar to the ocean surface element  $\Delta A_1$ , scattered to the precipitation volume  $\Delta V$ , and returned to the surface element  $\Delta A_2$  where it is scattered a final time back to the radar.

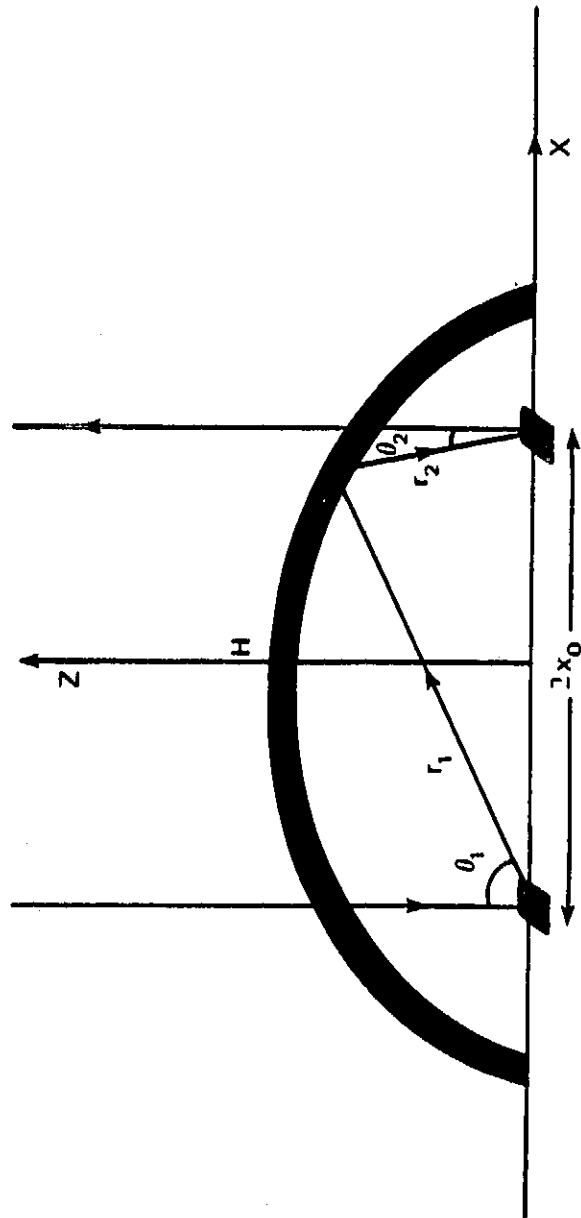


Figure A2. Schematic of the precipitation volume (ellipsoidal shell) that contributes to the mirror-reflected power for the two fixed surface scattering elements.