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ABSTRACT

With the application to weld defects in mind, the interaction problem between a planar crack and a flat inclusion in an elastic solid is considered. The elastic inclusion is assumed to be sufficiently thin so that the thickness distribution of the stresses in the inclusion may be neglected. The problem is reduced to a system of four integral equations having Cauchy type dominant kernels. The stress intensity factors are calculated and tabulated for various crack-inclusion geometries and the inclusion to matrix modulus ratios, and for general homogeneous loading conditions away from the crack-inclusion region.

1. Introduction

In studying the strength and fracture of structural solids it is often necessary to take into account, among other factors, the effect of the imperfections in the material. Generally such imperfections are in the form of either geometric discontinuities or material inhomogeneities. For example, in welded joints, various shapes of voids, cracks, notches and regions of lack of fusion may be mentioned as examples for the former and variety of inclusions for the latter. From a viewpoint of fracture mechanics two important classes of imperfections are the planar flaws which may be idealized as cracks and relatively thin inhomogeneities which may be represented by flat inclusions.

The correct way of modeling an inclusion would perhaps be to consider it as an elastic continuum fully bonded to the surrounding matrix. In this case, however, the crack-inclusion problems are generally difficult and only simple geometries and orientations can be treated analytically (see, for

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example, [1], [2]). A simple feature of such crack-inclusion interaction problems is that generally the stress intensity factors are magnified if the stiffness of the inclusion is less than that of the matrix and are diminished if the inclusion is stiffer than the matrix. For certain types of "flat" inclusions a simpler way of modeling may be to represent them as either a membrane with no bending stiffness or a perfectly rigid plane stiffener with negligible thickness. In these problems one may use the basic body force solution as the Green's function to derive the related integral equations. On the other hand, since the flat inclusion with an elastic modulus smaller than that of the matrix would itself have a behavior similar to a crack, it needs to be modeled basically as a "cavity" rather than a "stiffener".

Even though the technical literature on cracks, voids and inclusions which exist in the material separately is quite extensive, the problems of interaction between cracks and inclusions do not seem to be as widely studied. Such problems may be important in studying, for example, the micromechanics of fatigue and the fracture in welded joints. In this paper a simple model for flat elastic inclusions is presented and the crack-inclusion interaction problem is considered for various relative orientations.

2. Integral Equations of the Problem

The plane strain or the generalized plane stress interaction problem under consideration is described in Fig. 1. It is assumed that the boundaries of the medium are sufficiently far away from the crack-inclusion region so that their effect on the stress state perturbed by the crack and the inclusion may be neglected and the plane may be considered as being infinite.

Referring to Fig. 1 we define the following unknown functions

$$g_1(x_1) = \frac{\partial}{\partial x_1}[v_1(x_1,+0)-v_1(x_1,-0)] \ , \ (a<x_1<b) \ , \tag{1}$$

$$h_1(x_1) = \frac{\partial}{\partial x_1}[u_1(x_1,+0)-u_1(x_1,-0)] \ , \ (a<x_1<b) \ , \tag{2}$$

$$g_2(x_2) = \frac{\partial}{\partial x_2}[v_2(x_2,+0)-v_2(x_2,-0)] \ , \ (c<x_2<d) \ , \tag{3}$$

$$h_2(x_2) = \frac{\partial}{\partial x_2}[u_2(x_2,+0)-u_2(x_2,-0)] \ , \ (c<x_2<d) \tag{4}$$

where u and v are, respectively, x and y components of the displacement vector in the coordinate systems shown in the figure. It is assumed that the inclusion fills a flat cavity the initial thickness of which is $h_0(x)$ which is "small" compared to its length $2a_1$. It is also assumed that the thickness variation of the stresses and the strain ε^i_{xx} in the inclusion are negligible. Thus, for the plane strain case, from the Hooke's Law we obtain the following stress-strain relations in the inclusion

$$\varepsilon^i_{yy}(x_1) = \frac{1-\nu_0-2\nu_0^2}{E_0(1-\nu_0)}\,\sigma^i_{yy}(x_1)\ ,\ \varepsilon^i_{xy}(x_1) = \frac{1}{2\mu_0}\,\sigma^i_{xy}(x_1)\ , \tag{5}$$

where E_0, ν_0, μ_0 are the elastic constants of the inclusion. Now, by observing that

$$\varepsilon^i_{yy}(x_1) \cong [v_1(x_1,+0)-v_1(x_1,-0)]/h_0(x_1)\ , \tag{6}$$

$$2\varepsilon^i_{xy}(x_1) \cong [u_1(x_1,+0)-u_1(x_1,-0)]/h_0(x)\ , \tag{7}$$

and

$$E_0 = 2\mu_0(1+\nu_0)\ ,\ \kappa_0 = 3-4\nu_0\ , \tag{8}$$

from (1), (2) and (5)-(8) we find

$$\sigma^i_{yy}(x_1) = \frac{\kappa_0+1}{\kappa_0-1}\,\frac{\mu_0}{h_0(x_1)}\int_a^{x_1} g_1(t)dt\ , \tag{9}$$

$$\sigma^i_{xy}(x_1) = \frac{\mu_0}{h_0(x_1)}\int_a^{x_1} h_1(t)dt\ . \tag{10}$$

If we let the medium to be uniformly loaded away from the crack-inclusion region as shown in Fig. 1, for the stress components along the x_1 and x_2 axes we obtain

$$\sigma^{1\infty}_{yy}(x_1,0) = \sigma^\infty_{yy}\ ,\ \sigma^{1\infty}_{xy}(x_1,0) = \sigma^\infty_{xy}\ , \tag{11}$$

$$\sigma^{2\infty}_{yy}(x_2,0) = \sigma^\infty_{yy}\cos^2\theta + \sigma^\infty_{xx}\sin^2\theta - 2\sigma^\infty_{xy}\,\sin\theta\cos\theta\ , \tag{12}$$

$$\sigma_{xy}^{2\infty}(x_2,0) = (\sigma_{yy}^{\infty}-\sigma_{xx}^{\infty})\sin\theta\cos\theta + \sigma_{xy}^{\infty}(\cos^2\theta-\sin^2\theta) \ . \tag{13}$$

From the basic dislocation solution given in, for example, [3], referred to the coordinate system x_1,y_1 the stress state at a point (x_1,y_1) in the plane due to the displacement derivatives g_1,h_1 defined by (1) and (2) may be expressed as

$$\sigma_{xx}^{11}(x_1,y_1) = \int_a^b [G_{xx}(x_1,y_1,t)g_1(t) + H_{xx}(x_1,y_1,t)h_1(t)]dt \ , \tag{14}$$

$$\sigma_{yy}^{11}(x_1,y_1) = \int_a^b [G_{yy}(x_1,y_1,t)g_1(t) + H_{yy}(x_1,y_1,t)h_1(t)]dt \ , \tag{15}$$

$$\sigma_{xy}^{11}(x_1,y_1) = \int_a^b [G_{xy}(x_1,y_1,t)g_1(t) + H_{yy}(x_1,y_1,t)h_1(t)]dt \ , \tag{16}$$

where

$$\begin{aligned}
G_{xx}(x,y,t) &= A(t-x)[(t-x)^2-y^2] \ , \\
G_{yy}(x,y,t) &= A(t-x)[3y^2+(t-x)^2] \ , \\
G_{xy}(x,y,t) &= Ay[y^2-(t-x)^2] \ , \\
H_{xx}(x,y,t) &= Ay[y^2+3(t-x)^2] \ , \\
H_{yy}(x,y,t) &= Ay[y^2-(t-x)^2] \ , \\
H_{xy}(x,y,t) &= A(t-x)[(t-x)^2-y^2] \ ,
\end{aligned} \tag{17}$$

$$A(x,y,t) = \frac{2\mu}{(1+\kappa)} \frac{1}{[(t-x)^2+y^2]^2} \ ,$$

and μ and κ are the elastic constants of the medium ($\mu=E/2(1+\nu)$, $\kappa=3-4\nu$ for plane strain and $\kappa=(3-\nu)/(1+\nu)$ for generalized plane stress). Similarly, referred to the axes x_2,y_2 the stress state σ_{ij}^{22}, $(i,j=x,y)$ in the plane due to g_2,h_2 may be obtained from (14)-(17) by substituting (c,d) for (a,b) and (x_2,y_2) for (x_1,y_1) and (g_2,h_2) for (g_1,h_1).

The integral equations to determine the unknown functions g_1, h_1, g_2, and h_2 may be obtained from the following traction boundary conditions along ($y_1=0$, $a<x_1<b$) and ($y_2=0$, $c<x_2<d$):

$$\sigma_{yy}^{11}(x_1,0) + \sigma_{yy}^{12}(x_1,0) + \sigma_{yy}^{1\infty}(x_1,0) = \sigma_{yy}^{i}(x_1) \ , \ (a<x_1<b) \ , \tag{18}$$

$$\sigma_{xy}^{11}(x_1,0) + \sigma_{xy}^{12}(x_1,0) + \sigma_{xy}^{1\infty}(x_1,0) = \sigma_{xy}^{i}(x_1) \ , \ (a<x_1<b) \ , \tag{19}$$

$$\sigma_{yy}^{22}(x_2,0) + \sigma_{yy}^{21}(x_2,0) + \sigma_{yy}^{2\infty}(x_2,0) = 0 \ , \ (c<x_2<d) \ , \tag{20}$$

$$\sigma_{xy}^{22}(x_2,0) + \sigma_{xy}^{21}(x_2,0) + \sigma_{xy}^{2\infty}(x_2,0) = 0 \ , \ (c<x_2<d) \ , \tag{21}$$

where all except the coupling stresses in the second column are given by (9)-(17). The coupling stresses have the following meaning: $\sigma_{yy}^{12}(x_1,0)$ is the normal stress on $y_1=0$ plane due to the displacement derivatives $g_2(x_2)$ and $h_2(x_2)$ and $\sigma_{yy}^{21}(x_2,0)$ is the normal stress on $y_2=0$ plane due to g_1, h_1, etc. Thus, after making the necessary stress transformations similar to (12) and (13), we obtain

$$\sigma_{yy}^{12}(x_1,0) = \int_c^d [G_{yy}^{12}(x_1,t)g_2(t) + H_{yy}^{12}(x_1,t)h_2(t)]dt \ , \tag{22}$$

$$\sigma_{xy}^{12}(x_1,0) = \int_c^d [G_{xy}^{12}(x_1,t)g_2(t) + H_{xy}^{12}(x_1,t)h_2(t)]dt \ , \tag{23}$$

$$\sigma_{yy}^{21}(x_2,0) = \int_a^b [G_{yy}^{21}(x_2,t)g_1(t) + H_{yy}^{21}(x_2,t)h_1(t)]dt \ , \tag{24}$$

$$\sigma_{xy}^{21}(x_2,0) = \int_a^b [G_{xy}^{21}(x_2,t)g_1(t) + H_{xy}^{21}(x_2,t)h_1(t)]dt \ , \tag{25}$$

where from

$$\sigma_{yy}^{12}(x_1,0) = \sigma_{yy}^{22}(x_2,y_2)\cos^2\theta + \sigma_{xx}^{22}\sin^2\theta + \sigma_{xy}^{22}\sin 2\theta \tag{26}$$

calculated at $x_2=x_1\cos\theta$, $y_2=-x_1\sin\theta$ we have

$$G_{yy}^{12}(x_1,t) = G_{yy}(x_1\cos\theta,-x_1\sin\theta,t)\cos^2\theta + G_{xx}(x_1\cos\theta,-x_1\sin\theta,t)$$

$$+ G_{xy}(x_1\cos\theta,\ x_1\sin\theta,t)\sin 2\theta \ , \qquad (27)$$

$$H_{yy}^{12}(x_1,t) = H_{yy}(x_1\cos\theta,-x_1\sin\theta,t)\cos^2\theta + H_{xx}(x_1\cos\theta,-x_1\sin\theta,t)$$

$$+ H_{xy}(x_1\cos\theta,-x_1\sin\theta,t)\sin 2\theta \ . \qquad (28)$$

Similar expressions for the remaining kernels in (23)-(25) are obtained by using the stress transformations

$$\sigma_{xy}^{12}(x_1,0) = [\sigma_{xx}^{22}(x_2,y_2)-\sigma_{yy}^{22}(x_2,y_2)]\sin\theta\cos\theta$$

$$+ \sigma_{xy}^{22}(x_2,y_2)(\cos^2\theta-\sin^2\theta) \ , \ (x_2=x_1\cos\theta,\ y_2=-x_1\sin\theta) \ , \qquad (29)$$

$$\sigma_{yy}^{21}(x_2,0) = \sigma_{yy}^{11}(x_1,y_1)\cos^2\theta + \sigma_{xx}^{11}(x_1,y_1)\sin^2\theta$$

$$- \sigma_{xy}^{11}(x_1,y_1)\sin 2\theta,\ (x_1=x_2\cos\theta,\ y_1=x_2\sin\theta) \ , \qquad (30)$$

$$\sigma_{xy}^{21}(x_2,0) = [\sigma_{yy}^{11}(x_1,y_1)-\sigma_{xx}^{11}(x_1,y_1)]\sin\theta\cos\theta$$

$$+ \sigma_{xy}^{11}(x_1,y_1)(\cos^2\theta-\sin^2\theta),\ (x_1=x_2\cos\theta,\ y_1=x_2\sin\theta) \ . \qquad (31)$$

Thus, from (14)-(25) and (29)-(31) it follows that

$$G_{xy}^{12}(x_1,t) = [G_{xx}(x,y,t)-G_{yy}(x,y,t)]\sin\theta\cos\theta$$

$$+ G_{xy}(x,y,t)\cos 2\theta,\ (x=x_1\cos\theta,\ y=-x_1\sin\theta) \ , \qquad (32)$$

$$H_{xy}^{12}(x_1,t) = [H_{xx}(x,y,t)-H_{yy}(x,y,t)]\sin\theta\cos\theta$$

$$+ H_{xy}(x,y,t)\cos 2\theta,\ (x=x_1\cos\theta,\ y=-x_1\sin\theta) \ , \qquad (33)$$

$$G_{yy}^{21}(x_2,t) = G_{yy}(x,y,t)\cos^2\theta + G_{xx}(x,y,t)\sin^2\theta$$

$$- G_{xy}(x,y,t)\sin 2\theta \ , \ (x=x_2\cos\theta, \ y=x_2\sin\theta) \ , \tag{34}$$

$$H_{yy}^{21}(x_2,t) = H_{yy}(x,y,t)\cos^2\theta + H_{xx}(x,y,t)\sin^2\theta$$

$$- H_{xy}(x,y,t)\sin 2\theta \ , \ (x=x_2\cos\theta, \ y=x_2\sin\theta) \ , \tag{35}$$

$$G_{xy}^{21}(x_2,t) = [G_{yy}(x,y,t)-G_{xx}(x,y,t)]\sin\theta\cos\theta$$

$$+ G_{xy}(x,y,t)\cos 2\theta \ , \ (x=x_2\cos\theta, \ y=x_2\sin \ ) \ , \tag{36}$$

$$H_{xy}^{21}(x_2,t) = [H_{yy}(x,y,t)-H_{xx}(x,y,t)]\sin\theta\cos\theta$$

$$+ H_{xy}(x,y,t)\cos 2\theta \ , \ (x=x_2\cos\theta, \ y=x_2\sin\theta) \ . \tag{37}$$

From (18)-(21) the integral equations of the problem may then be obtained as

$$\frac{1}{\pi}\int_a^b \frac{1}{t-x_1}\, g_1(t)dt + \int_a^{x_1} G(x_1)g_1(t)dt + c_o \int_c^d G_{yy}^{12}(x_1,t)g_2(t)dt$$

$$+ c_o \int_c^d H_{yy}^{12}(x_1,t)h_2(t)dt = -c_o\sigma_{yy}^{\infty} \ , \ (a<x_1<b) \ , \tag{38}$$

$$\frac{1}{\pi}\int_a^b \frac{1}{t-x_1}\, h_1(t)dt + \int_a^{x_1} H(x_1)h_1(t)dt + c_o \int_c^d G_{xy}^{12}(x_1,t)g_2(t)dt$$

$$+ c_o \int_c^d H_{xy}^{12}(x_1,t)h_2(t)dt = -c_o\sigma_{xy}^{\infty}, \ (a<x_1<b) \ , \tag{39}$$

$$c_o \int_a^b G_{yy}^{21}(x_2,t)g_1(t)dt + c_o \int_a^b H_{yy}^{21}(x_2,t)h_1(t)dt + \frac{1}{\pi}\int_c^d \frac{1}{t-x_2}\, g_2(t)dt$$

$$= -c_o(\sigma_{yy}^{\infty}\cos^2\theta+\sigma_{xx}^{\infty}\sin^2\theta-\sigma_{xy}^{\infty}\sin 2\theta), \ (c<x_2<d) \ , \tag{40}$$

$$c_o \int_a^b G_{xy}^{21}(x_2,t)g_1(t)dt + c_o \int_a^b H_{xy}^{21}(x_2,t)h_1(t)dt + \frac{1}{\pi}\int_c^d \frac{1}{t-x_2} h_2(t)dt$$

$$= -c_o[(\sigma_{yy}^{\infty}-\sigma_{xx}^{\infty})\sin\theta\cos\theta + \sigma_{xy}^{\infty}\cos 2\theta], \ (c<x_2<d) \ , \tag{41}$$

where

$$c_o = \frac{1+\kappa}{2\mu} \ , \ G(x_1) = -\frac{\mu_o(\kappa+1)(\kappa_o+1)}{2\mu(\kappa_o-1)} \frac{1}{h_o(x_1)} \ ,$$

$$H(x_1) = -\frac{\mu_o(\kappa+1)}{2\mu} \frac{1}{h_o(x_1)} \ . \tag{42}$$

If there is no crack in the medium, $g_2=0=h_2$, the integral equations uncouple and (38) and (39) give the unknown functions g_1 and h_1. For example, if the inclusion has an elliptic cross-section given by

$$h_o(x) = b_o\sqrt{1-x^2} \ , \tag{43}$$

(38) becomes

$$\frac{1}{\pi}\int_{-1}^{1} \frac{g_1(t)}{t-x} dt - \int_{-1}^{x} \frac{c_1}{\sqrt{1-x^2}} g_1(t)dt = -c_o \ \sigma_{yy}^{\infty} \tag{44}$$

where

$$c_1 = \frac{\mu_o(1+\kappa)(1+\kappa_o)}{2\mu b_o(\kappa_o-1)} \ , \tag{45}$$

and without any loss in generality it is assumed that $a=-1$, $b=1$, $x_1=x$. The solution of (44) is found to be

$$g_1(t) = -\frac{c_o\sigma_{yy}^{\infty}}{1+c_1} \frac{t}{\sqrt{1-t^2}} \ , \ (-1<t<1) \tag{46}$$

which, for $\mu_o=0$ reduces to the well-known crack solution. By using the following definition of the stress intensity factor

$$k_1(t) = -\lim_{x\to 1} \frac{2\mu}{1+\kappa} \sqrt{2(1-x)} \ g_1(x) \ , \tag{47}$$

from (46) it follows that

$$k_1(1) = \frac{\sigma_{yy}^{\infty}}{1+c_1} \quad . \tag{48}$$

Similarly, in the absence of a crack from (39), (42) and (43) it may be shown that

$$h_1(t) = -\frac{c_o\sigma_{xy}^{\infty}}{1+c_2}\frac{t}{\sqrt{1-t^2}} \ , \ (-1<t<1) \tag{49}$$

$$k_2(1) = \frac{\sigma_{xy}^{\infty}}{1+c_2} \ , \ c_2 = \frac{\mu_o(1+\kappa)}{2\mu b_o} \ . \tag{50}$$

As another special case if we assume that the stiffness of the inclusion $\mu_o=0$, then the functions G and H defined by (42) vanish and the integral equations (38)-(41) reduce to that of two arbitrarily oriented cracks shown in Fig. 1.

3. Stress Intensity Factors

In the linearly elastic medium under consideration the intensity of the stress state around the end points of the crack and the inclusion is governed by the singular behavior of the displacement derivatives g_1, g_2, h_1 and h_2 which are defined by (1)-(4). If we assume the following standard definition of Modes I and II stress intensity factors

$$k_1(a) = \lim_{x_1\to a} \sqrt{2(a-x_1)}\ \sigma_{yy}^{1}(x_1,0) \ , \tag{51}$$

$$k_2(a) = \lim_{x_1\to a} \sqrt{2(a-x_1)}\ \sigma_{yy}^{1}(x_1,0) \ , \tag{52}$$

$$k_1(c) = \lim_{x_2\to c} \sqrt{2(c-x_2)}\ \sigma_{yy}^{2}(x_2,0) \ , \text{ etc. } , \tag{53}$$

and observe that the system of integral equations (38)-(41) which has simple Cauchy type kernels has a solution of the form

$$g_i(t) = \frac{G_i(t)}{\sqrt{(b-t)(t-a)}} , \; h_i(t) = \frac{H_i(t)}{\sqrt{(d-t)(t-c)}} , \; (i=1,2) , \tag{54}$$

from (38)-(41) and (51)-(54) it can be shown that

$$k_1(a) = \frac{2\mu}{1+\kappa} \lim_{x_1 \to a} \sqrt{2(x_1-a)} \, g_1(x_1) , \tag{55}$$

$$k_1(b) = - \frac{2\mu}{1+\kappa} \lim_{x_1 \to b} \sqrt{2(b-x_1)} \, g_1(x_1) , \tag{56}$$

$$k_2(a) = \frac{2\mu}{1+\kappa} \lim_{x_1 \to a} \sqrt{2(x_1-a)} \, h_1(x_1) , \tag{57}$$

$$k_2(b) = - \frac{2\mu}{1+\kappa} \lim_{x_1 \to b} \sqrt{2(b-x_1)} \, h_1(x_1) . \tag{58}$$

The stress intensity factors $k_i(c)$ and $k_i(d)$, $(i=1,2)$ may be expressed in terms of g_2 and h_2 by means of equations similar to (55)-(58).

4. Results

The integral equations (38)-(41) are solved by using the technique described in [4] and the stress intensity factors are calculated from (55)-(58) and from similar expressions written for the crack. For various crack-inclusion geometries and stiffness ratios μ_0/μ (μ_0 being the shear modulus of the inclusion) the calculated results are given in Tables 1-6. The main interest in this paper is in relatively "thin" and flat inclusions. Hence in the numerical analysis it is assumed that the thickness h_0 is constant. Table 1 shows the normalized stress intensity factors in a plane which contains a crack equal in size and coplanar with an inclusion and subjected to uniform tension and shear away from the crack-inclusion region (Fig. 2a). The inclusion model used in this analysis is basically a crack the surfaces of which are held together by an elastic medium of shear modulus μ_0. Thus, for $\mu_0=0$ one recovers the two crack solution. It may be observed that for $\mu_0>0$ there is a significant reduction in the stress intensity factors around the end points $x_1=a$ and $x_1=b$ (Fig. 2a). In Table 1 the variables are the stiffness ratio μ_0/μ and the thickness of the inclusion h_0/a_1 with the spacing $a/a_1 = 0.01$ being constant, where $2a_1$ is the length of the inclusion (Fig. 2a). Similar results calculated by assuming that $h_0/a_1 = 1/20$ and a/a_1 is variable are shown in Table 2.

For various values of the stiffness ratio μ_0/μ and fixed values of the inclusion thickness ($h_0/a_1=1/20$) and the distance a ($a/a_1=0.1$), the effect of the angle θ on the crack tip stress intensity factors are given in Table 3. The geometry and the loading condition away from the crack-inclusion region are shown in Fig. 2b. In this example, too, it is assumed that the inclusion and the crack are of equal length ($a_2=a_1$). For the special case of $\mu_0=0$, that is, for the case of two cracks of equal lengths oriented at an angle θ the stress intensity factors are given in Table 4.

The stress intensity factors for the symmetric crack-inclusion geometries shown in Figures 3a and 3b are given in Table 5, where the length ratio a_2/a_1 is assumed to be the variable. In both examples the inclusion (half) length a_1 is used as the normalizing length parameter and the relative distance c/a_1 (Fig. 3a) or a/a_1 (Fig. 3b) is assumed to be constant.

Table 6 gives the stress intensity factors for a crack perpendicular to the inclusion where, referring to Fig. 1, $\theta=\pi/2$, $a=0$, $\mu_0=\mu/20$ and $c/a_1=0.05$ are fixed and a_2 is variable.

It should be noted that since the superposition is valid, the tables give the stress intensity factors for the most general homogeneous loading conditions away from the crack-inclusion region. Also, the tables give the stress intensity factors which are normalized with respect to $\sigma_{ij}^{\infty}\sqrt{a_1}$ where $2a_1$ is the length of the inclusion and $(i,j)=(x,y)$, (Fig. 1). The notation used in the tables is

$$k_{1a} = \frac{k_1(a)}{\sigma_{ij}^{\infty}\sqrt{a_1}}\ ,\ k_{2a} = \frac{k_2(a)}{\sigma_{ij}^{\infty}\sqrt{a_1}}\ ,\ k_{1c} = \frac{k_1(c)}{\sigma_{ij}^{\infty}\sqrt{a_1}}\ ,\ \text{etc.} \tag{59}$$

where k_1 and k_2 are, respectively, Modes I and II stress intensity factors defined by equations such as (51)-(53) and calculated from the expressions such as (55)-(58).

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Table 1. Modes I and II stress intensity factors for the case of a crack located in the plane of the inclusion in a medium subjected to σ_{yy}^{∞} or σ_{xy}^{∞} away from the crack-inclusion region (Fig. 2a); c=-a, d=-b, $a/a_1=0.01$, $k_{1c}=k_1(c)/\sigma_{yy}^{\infty}\sqrt{a_1}$, $k_{1d}=k_1(d)/\sigma_{yy}^{\infty}\sqrt{a_1}$, $k_{2c}=k_2(c)/\sigma_{xy}^{\infty}\sqrt{a_1}$, $k_{2d}=k_2(d)/\sigma_{xy}^{\infty}\sqrt{a_1}$, $k_{1a}=k_1(a)/\sigma_{yy}^{\infty}\sqrt{a_1}$, $k_{2a}=k_2(a)/\sigma_{xy}^{\infty}\sqrt{a_1}$, $k_{1b}=k_1(b)/\sigma_{yy}^{\infty}\sqrt{a_1}$, $k_{2b}=k_2(b)/\sigma_{xy}^{\infty}\sqrt{a_1}$, $a_1=(b-a)/2$.

	$\frac{2h_o}{b-a}$	μ_o/μ							
		0	0.05	0.1	0.25	0.5	1.0	2.0	5.0
k_{1b}	0.01	1.2063	.1578	.1031	.0535	.0303	.0163	.0085	.0035
	0.02	1.2063	.2320	.1578	.0888	.0535	.0303	.0163	.0068
	0.1	1.2063	.5146	.3713	.2320	.1578	.1031	.0634	.0303
	0.2	1.2063	.6836	.5146	.3323	.2320	.1578	.1031	.0535
k_{1a}	0.01	2.9642	.5725	.3908	.2104	.1207	.0654	.0342	.0140
	0.02	2.9642	.7941	.5725	.3404	.2104	.1207	.0654	.0276
	0.1	2.9642	1.5036	1.1620	.7941	.5725	.3908	.2478	.1207
	0.2	2.9642	1.8803	1.5036	1.0636	.7941	.5725	.3908	.2104
k_{1c}	0.01	2.9642	1.1795	1.1045	1.0479	1.0255	1.0132	1.0067	1.0027
	0.02	2.9642	1.2952	1.1795	1.0870	1.0480	1.0255	1.0132	1.0054
	0.1	2.9642	1.7825	1.5321	1.2952	1.1795	1.1045	1.0583	1.0255
	0.2	2.9642	2.0764	1.7825	1.4645	1.2952	1.1795	1.1045	1.0479
k_{1d}	0.01	1.2063	1.0116	1.0063	1.0027	1.0014	1.0007	1.0004	1.0001
	0.02	1.2063	1.0211	1.0116	1.0051	1.0027	1.0014	1.0007	1.0003
	0.1	1.2063	1.0693	1.0432	1.0211	1.0116	1.0063	1.0033	1.0014
	0.2	1.2063	1.1019	1.0693	1.0366	1.0211	1.0116	1.0063	1.0027
k_{2b}	0.01	1.2063	.3106	.2159	.1275	.0810	.0482	.0269	.0117
	0.02	1.2063	.4368	.3106	.1910	.1275	.0810	.0482	.0221
	0.1	1.2063	.8214	.6500	.4368	.3106	.2159	.1459	.0810
	0.2	1.2063	.9673	.8214	.5946	.4368	.3106	.2159	.1275
k_{2a}	0.01	2.9642	1.0075	.7480	.4743	.3122	.1900	.1076	.0470
	0.02	2.9642	1.3214	1.0075	.6747	.4743	.3122	.1900	.0885
	0.1	2.9642	2.1749	1.8071	1.3214	1.0075	.7480	.5345	.3122
	0.2	2.9642	2.4785	2.1749	1.6847	1.3214	1.0075	.7480	.4743
k_{2c}	0.01	2.9642	1.4272	1.2691	1.1366	1.0778	1.0425	1.0225	1.0093
	0.02	2.9642	1.6463	1.4272	1.2298	1.1366	1.0778	1.0425	1.0182
	0.1	2.9642	2.3136	2.0183	1.6463	1.4272	1.2691	1.1622	1.0778
	0.2	2.9642	2.5619	2.3136	1.9221	1.6463	1.4272	1.2691	1.1366
k_{2d}	0.01	1.2063	1.0330	1.0188	1.0085	1.0045	1.0023	1.0012	1.0005
	0.02	1.2063	1.0549	1.0330	1.0156	1.0085	1.0045	1.0023	1.0010
	0.1	1.2063	1.1292	1.0954	1.0549	1.0330	1.0188	1.0103	1.0045
	0.2	1.2063	1.1583	1.1292	1.0846	1.0549	1.0330	1.0188	1.0085

Table 2. Modes I and II stress intensity factors for the case of a crack located in the plane of the inclusion in a medium subjected to σ_{yy}^{∞} or σ_{xy}^{∞} away from the crack-inclusion region (Fig. 2a); c=-a, d=-b, h_o/a_1=1/20.

	$\frac{2a}{b-a}$	μ_o/μ							
		0	0.05	0.1	0.25	0.5	1.0	2.0	5.0
k_{1b}	0.01	1.2063	.3713	.2611	.1578	.1031	.0635	.0366	.0163
	0.5	1.0517	.3544	.2513	.1527	.0998	.0615	.0354	.0158
	1	1.0280	.3493	.2479	.1508	.0986	.0607	.0350	.0156
	2	1.0125	.3453	.2452	.1492	.0976	.0601	.0347	.0154
k_{1a}	0.01	2.9642	1.1620	.8751	.5725	.3908	.2478	.1454	.0654
	0.5	1.1125	.3877	.2768	.1693	.1110	.0685	.0395	.0176
	1	1.0480	.3604	.2564	.1563	.1023	.0630	.0364	.0162
	2	1.0176	.3481	.2474	.1506	.0985	.0607	.0350	.0156
k_{1c}	0.01	2.9642	1.5321	1.3433	1.1795	1.1045	1.0583	1.0313	1.0132
	0.5	1.1125	1.0229	1.0130	1.0057	1.0030	1.0015	1.0008	1.0003
	1	1.0480	1.0096	1.0054	1.0024	1.0012	1.0006	1.0003	1.0001
	2	1.0176	1.0035	1.0020	1.0009	1.0004	1.0002	1.0001	1.0000
k_{1d}	0.01	1.2063	1.0432	1.0253	1.0116	1.0063	1.0033	1.0017	1.0007
	0.5	1.0517	1.0104	1.0058	1.0026	1.0013	1.0007	1.0003	1.0001
	1	1.0280	1.0056	1.0031	1.0014	1.0007	1.0004	1.0002	1.0001
	2	1.0125	1.0025	1.0014	1.0006	1.0003	1.0002	1.0001	1.0000
k_{2b}	0.01	1.2063	.6500	.4845	.3106	.2159	.1459	.0943	.0481
	0.5	1.0517	.6031	.4576	.2979	.2084	.1412	.0914	.0467
	1	1.0280	.5925	.4503	.2938	.2057	.1395	.0903	.0461
	2	1.0125	.5849	.4449	.2905	.2035	.1380	.0893	.0456
k_{2a}	0.01	2.9642	1.8071	1.4340	1.0075	.7480	.5345	.3601	.1900
	0.5	1.1125	.6498	.4971	.3272	.2302	.1567	.1017	.0520
	1	1.0480	.6081	.4636	.3035	.2129	.1446	.0937	.0479
	2	1.0176	.5889	.4483	.2930	.2053	.1393	.0902	.0461
k_{2c}	0.01	2.9642	2.0183	1.7299	1.4272	1.2691	1.1623	1.0937	1.0425
	0.5	1.1125	1.0523	1.0344	1.0172	1.0095	1.0050	1.0026	1.0011
	1	1.0480	1.0222	1.0145	1.0072	1.0040	1.0021	1.0011	1.0004
	2	1.0176	1.0081	1.0053	1.0026	1.0014	1.0008	1.0004	1.0002
k_{2d}	0.01	1.2063	1.0954	1.0637	1.0330	1.0188	1.0104	1.0055	1.0023
	0.5	1.0517	1.0239	1.0157	1.0078	1.0043	1.0023	1.0012	1.0005
	1	1.0280	1.0129	1.0084	1.0042	1.0023	1.0012	1.0006	1.0003
	2	1.0125	1.0057	1.0038	1.0019	1.0010	1.0005	1.0003	1.0001

Table 3. The effect of angular orientation θ and the modulus ratio μ_0/μ on the stress intensity factors in a medium under general in-plane loading (Fig. 1); c=a, d=b, $2h_0/(b-a)=1/20$, $2a/(b-a)=0.1$.

σ^∞	k	$\theta°$ 30	60	90	120	150	180
		$\mu_0/\mu = 0.05$					
σ_{xx}^∞	k_{1c}	0.2624	0.8047	1.0961	0.8097	0.2654	0
	k_{2c}	-0.4711	-0.4636	0.0163	0.4737	0.4585	0
	k_{1d}	0.2560	0.7618	1.0106	0.7562	0.2518	0
	k_{2d}	-0.4378	-0.4253	0.0122	0.4432	0.4383	0
σ_{yy}^∞	k_{1c}	0.6402	0.2232	-0.0311	0.2749	0.8366	1.1094
	k_{2c}	0.4596	0.4217	-0.0483	-0.5019	-0.4771	0
	k_{1d}	0.7052	0.2221	-0.0109	0.2568	0.7702	1.0250
	k_{2d}	0.4105	0.3981	-0.0386	-0.4636	-0.4493	0
σ_{xy}^∞	k_{1c}	-0.5020	-0.5895	0.2839	1.1440	1.0302	0
	k_{2c}	0.3394	-0.5681	-1.0010	-0.3793	0.7098	1.2367
	k_{1d}	-0.9072	-0.8566	0.0354	0.9049	0.8903	0
	k_{2d}	0.4353	-0.5284	-0.9911	-0.4631	0.5521	1.0567
		$\mu_0/\mu = 0.1$					
σ_{xx}^∞	k_{1c}	0.2552	0.7786	1.0613	0.7908	0.2608	0
	k_{2c}	-0.4593	-0.4546	0.0095	0.4610	0.4512	0
	k_{1d}	0.2534	0.7570	1.0066	0.7540	0.2512	0
	k_{2d}	-0.4366	-0.4291	0.0072	0.4395	0.4366	0
σ_{yy}^∞	k_{2c}	0.6535	0.2334	-0.0181	0.2628	0.8003	1.0643
	k_{2c}	0.4533	0.4238	-0.0293	-0.4758	-0.4605	0
	k_{1d}	0.7248	0.2350	-0.0058	0.2540	0.7615	1.0143
	k_{2d}	0.4219	0.4145	-0.0215	-0.4506	-0.4425	0
σ_{xy}^∞	k_{1c}	-0.6023	-0.6717	0.1849	1.0482	0.9749	0
	k_{2c}	0.3956	-0.5401	-0.9996	-0.4197	0.6414	1.1599
	k_{1d}	-0.8892	-0.8588	0.0230	0.8910	0.8817	0
	k_{2d}	0.4617	-0.5172	-0.9943	-0.4762	0.5343	1.0374

Table 3 - cont.

		$\mu_o/\mu = 0.5$					
σ_{xx}^{∞}	k_{1c}	0.2478	0.7537	1.0157	0.7622	0.2535	0
	k_{2c}	-0.4414	-0.4405	0.0019	0.4418	0.4391	0
	k_{1d}	0.2509	0.7517	1.0017	0.7511	0.2503	0
	k_{2d}	-0.4341	-0.4322	0.0017	0.4347	0.4341	0
σ_{yy}^{∞}	k_{1c}	0.7013	0.2427	-0.0045	0.2523	0.7620	1.0158
	k_{2c}	0.4381	0.4288	-0.0078	-0.4448	-0.4407	0
	k_{1d}	0.7446	0.2469	-0.0012	0.2510	0.7527	1.0033
	k_{2d}	0.4312	0.4292	-0.0048	-0.4371	-0.4353	0
	k_{1c}	-0.7657	-0.8011	0.0517	0.9166	0.8971	0
	k_{2c}	0.4738	-0.5057	-0.9981	-0.4766	0.5420	1.0479
	k_{1d}	-0.8712	-0.8639	0.0061	0.8726	0.8702	0
	k_{2d}	0.4910	-0.5046	-0.9987	-0.4938	0.5094	1.0105
		$\mu_o/\mu = 2$					
σ_{xx}^{∞}	k_{1c}	0.2484	0.7504	1.0041	0.7535	0.2510	0
	k_{2c}	-0.4356	-0.4354	0.0003	0.4356	0.4349	0
	k_{1d}	0.2503	0.7505	1.0004	0.7503	0.2501	0
	k_{2d}	-0.4333	-0.4328	0.0004	0.4335	0.4333	0
σ_{yy}^{∞}	k_{1c}	0.7317	0.2473	-0.0012	0.2505	0.7531	1.0042
	k_{2c}	0.4330	0.4314	-0.0022	-0.4363	-0.4352	0
	k_{1d}	0.7487	0.2492	-0.0003	0.2503	0.7507	1.0009
	k_{2d}	0.4326	0.4321	-0.0012	-0.4341	-0.4336	0
σ_{xy}^{∞}	k_{1c}	-0.8318	-0.8460	0.0146	0.8801	0.8748	0
	k_{2c}	0.4947	-0.5001	-0.9989	-0.4932	0.5122	1.0139
	k_{1d}	-0.8674	-0.8655	0.0016	0.8678	0.8672	0
	k_{2d}	0.4976	-0.5013	-0.9997	-0.4983	0.5026	1.0029

Table 4. Interaction of two cracks (Fig. 2b); $\mu_0/\mu_1=0$, c=a, d=b, 2a/(b-a) = 0.1.

		θ°					
		30	60	90	120	150	180
σ^{∞}_{xx}	k_{1a}	0.1834	-0.0122	-0.1604	-0.1271	-0.0361	0
	k_{2a}	0.1293	0.0928	0.2122	0.2877	0.1946	0
	k_{1b}	-0.1471	-0.1373	-0.0666	-0.0113	0.0024	0
	k_{2b}	0.1825	0.2323	0.2104	0.1371	0.0588	0
	k_{1c}	0.3637	1.0032	1.2370	0.8684	0.2790	0
	k_{2c}	-0.5576	-0.4950	0.0577	0.5191	0.4810	0
	k_{1d}	0.3073	0.8057	1.0308	0.7633	0.2536	0
	k_{2d}	-0.3956	-0.3708	0.0477	0.4591	0.4441	0
σ^{∞}_{yy}	k_{1a}	0.5843	0.9140	1.2370	1.3954	1.4643	1.4914
	k_{2a}	-0.1912	-0.0242	-0.0577	-0.1080	-0.0730	0
	k_{1b}	0.9210	1.0081	1.0308	1.0567	1.0994	1.1220
	k_{2b}	0.0215	-0.0427	-0.0477	-0.0168	0.0054	0
	k_{1c}	0.4051	-0.1004	-0.1604	0.3999	1.1491	1.4914
	k_{2c}	0.6195	0.4264	-0.2122	-0.6987	-0.6027	0
	k_{1d}	0.4666	0.0652	-0.0666	0.2821	0.8481	1.1220
	k_{2d}	0.1916	0.1811	-0.2104	-0.5795	-0.5082	0
σ^{∞}_{xy}	k_{1a}	0.1842	0.7402	0.6381	0.3381	0.1384	0
	k_{2a}	1.1741	1.1315	1.0152	1.1777	1.4058	1.4914
	k_{1b}	0.4327	0.1938	0.0748	0.0610	0.0532	0
	k_{2b}	0.5851	0.7960	0.9950	1.1104	1.1305	1.1220
	k_{1c}	-0.4402	-0.4311	0.6381	1.4876	1.2302	0
	k_{2c}	0.3095	-0.6671	-1.0152	-0.2462	0.9347	1.4914
	k_{1d}	-1.1414	-0.8951	0.0748	0.9554	0.9234	0
	k_{2d}	0.1531	-0.6362	-0.9950	-0.4219	0.6115	1.1220

Table 5. Stress intensity factors for the case of a crack perpendicular to the inclusion, $\mu_0/\mu=1/20$, $h_0/a_1=1/20$.

	σ^∞	k	a_2/a_1			
			0.1	0.5	1.0	5.0
Fig. 3a $a=-b=-a_1$ $c/a_1=0.1$	σ_{xx}^∞	$k_{1a}=k_{1b}$	-0.0088	-0.0479	-0.0938	-0.1449
		$k_{2a}=-k_{2b}$	-0.0058	-0.0820	-0.1428	-0.2729
		k_{1c}	+1.0636	1.1611	1.1572	1.1256
		k_{1d}	1.0320	1.0245	1.0109	1.0029
	σ_{yy}^∞	$k_{1a}=k_{1b}$	0.3424	0.3441	0.3441	0.3438
		$k_{2a}=-k_{2b}$	0.0006	0.0039	0.0039	0.0033
		k_{1c}	-0.1220	-0.0896	-0.0632	-0.0255
		k_{1d}	-0.0988	-0.0116	0.0067	0.0021
	σ_{xy}^∞	$k_{1a}=-k_{1b}$	-0.0004	-0.0162	-0.0850	-0.5164
		$k_{2a}=k_{2b}$	0.5703	0.5162	0.4502	0.4199
		k_{2c}	-0.7288	-0.9533	-1.0730	-1.2431
		k_{2d}	-0.7856	-1.0338	-1.0638	-1.0200
Fig. 3b $c=-d=-a_2$ $a/a_1=0.1$	σ_{xx}^∞	k_{1a}	0.0208	-0.1238	-0.2149	-0.2773
		k_{1b}	0.0006	0.0100	0.0234	-0.1170
		$k_{1c}=k_{1d}$	1.0037	1.0053	1.0101	1.0026
		$k_{2c}=-k_{2d}$	-0.0011	-0.0074	-0.0107	-0.0045
	σ_{yy}^∞	k_{1a}	0.3476	0.3543	0.3764	0.3057
		k_{1b}	0.3416	0.3418	0.3416	0.3469
		$k_{1c}=k_{1d}$	0.1584	-0.0186	-0.0324	-0.0048
		$k_{2c}=-k_{2d}$	-0.0353	0.0460	0.0406	0.0073
	σ_{xy}^∞	k_{2a}	0.6514	0.5903	0.4304	0.0544
		k_{2b}	0.5808	0.6066	0.6315	0.3702
		$k_{1c}=-k_{1d}$	-0.4813	-0.2431	-0.1012	-0.0010
		$k_{2c}=k_{2d}$	-1.3694	-0.9632	-0.9372	-0.9946

Table 6. Stress intensity factors for a crack perpendicular to the inclusion (Fig. 1); $\theta=\pi/2$, a=0, 2c/(b-a)=0.05, $\mu_o/\mu=1/20$, $2h_o/(b-a)=0.05$.

σ^∞	k	a_2/a_1			
		0.1	0.5	1.0	5.0
σ^∞_{xx}	k_{1a}	.0399	.2055	.3675	1.1277
	k_{2a}	.0128	.0418	.0555	.1125
	k_{1b}	.0005	.0035	-.0081	-.0715
	k_{2b}	.0021	.0402	.1107	.3050
	k_{1c}	1.0762	1.1674	1.1729	1.1435
	k_{2c}	.0162	-.0056	-.0311	-.0740
	k_{1d}	1.0310	1.0274	1.0143	1.0018
	k_{2d}	.0207	.0212	.0115	-.0015
σ^∞_{yy}	k_{1a}	.3574	.3716	.3791	.3884
	k_{2a}	.0092	.0283	.0390	.0533
	k_{1b}	.3414	.3411	.3418	.3456
	k_{2b}	.0001	-.0010	-.0036	-.0062
	k_{1c}	-.0490	-.0607	-.0514	-.0250
	k_{2c}	-.3157	-.2298	-.1863	-.0933
	k_{1d}	-.0468	-.0250	-.0084	-.0009
	k_{2d}	-.1943	-.0830	-.0464	-.0048
σ^∞_{xy}	k_{1a}	.0887	.3231	.4952	1.1795
	k_{2a}	.6265	.7947	.9710	1.9112
	k_{1b}	.0002	.0001	.0079	.2709
	k_{2b}	.5805	.5910	.5713	.4743
	k_{1c}	1.1620	.6411	.4373	.1825
	k_{2c}	-1.0423	-1.1380	-1.1889	-1.2670
	k_{1d}	.6504	.1454	.0426	.0045
	k_{2d}	-.9292	-.9710	-1.0075	-1.0117

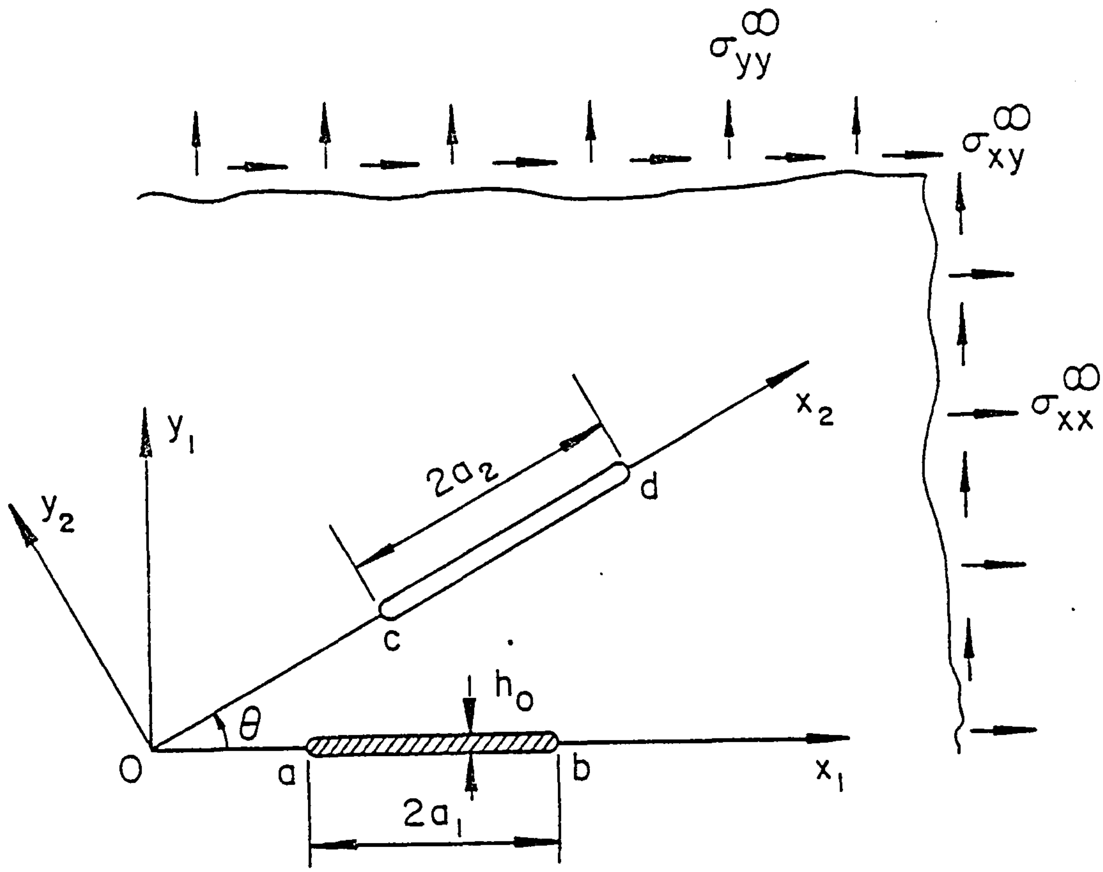

Fig. 1 The geometry of the crack-inclusion problem

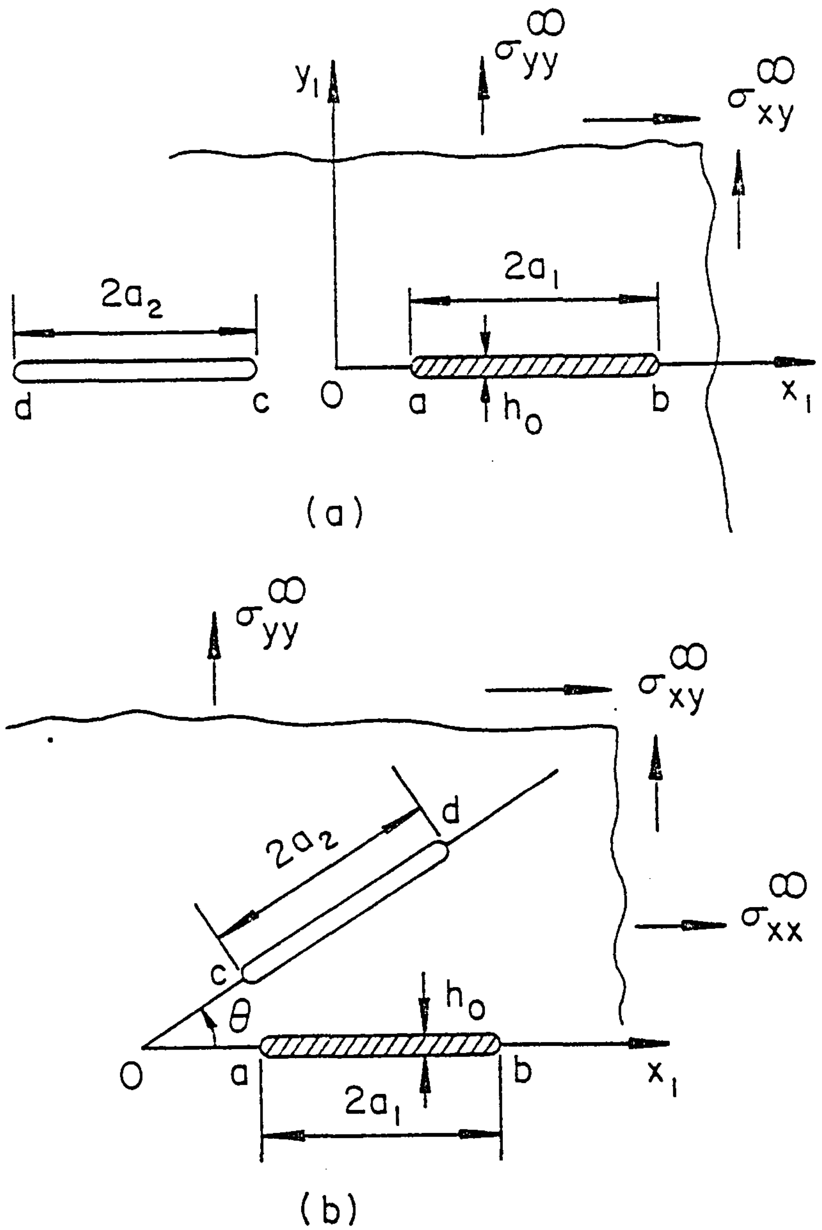

Fig. 2 Special crack-inclusion geometries used in numerical analysis

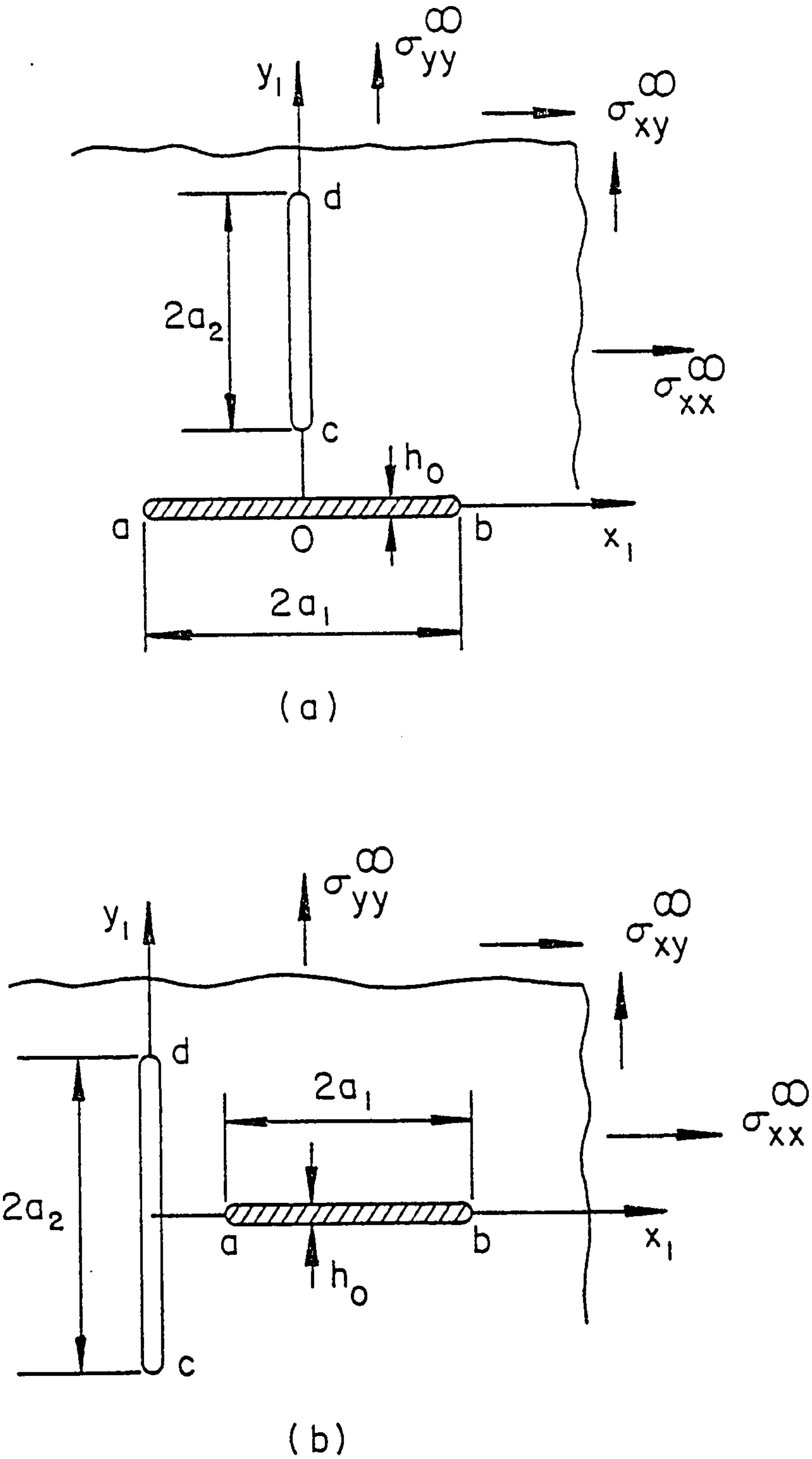

Fig. 3 Special crack-inclusion geometries used in numerical analysis

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16. Abstract

With the application to weld defects in mind, the interaction problem between a planar-crack and a flat inclusion in an elastic solid is considered. The elastic inclusion is assumed to be sufficiently thin so that the thickness distribution of the stresses in the inclusion may be neglected. The problem is reduced to a system of four integral equations having Cauchy-type dominant kernels. The stress intensity factors are calculated and tabulated for various crack-inclusion geometries and the inclusion to matrix modulus ratios, and for general homogeneous loading conditions away from the crack-inclusion region.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Fracture mechanics, stress intensity factors, crack-inclusion, weld defect	Unclassified-Unlimited Subject Category 39

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