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# MODULATION TRANSFER FUNCTION (MTF) MEASUREMENT TECHNIQUES FOR LENSES AND LINEAR DETECTOR ARRAYS

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FOR LENSES AND LINEAR DETECTOR ARRAYS**

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## INTRODUCTION

The Multispectral Linear Array (MLA) Assessment Laboratory, at Goddard Space Flight Center, is primarily concerned with the analysis of linear detector arrays built to NASA specifications. The determination of the Modulation Transfer Function (MTF), which relates the output to the input of an optical system, is undoubtedly the most comprehensive approach to such an evaluation involving such varied parameters as resolution, sensitivity, physical geometry and noise. Expressed interchangeably in cycles, lines, or line pairs per millimeter, MTF is a measure of the resolving power of the optical system. In a system made up of non-compensatory components, the total system MTF can be arrived at by multiplying the component MTF's together. It follows then, that in order to determine the MTF of a specific component, an array for example, the final MTF of the system must be divided by the MTF's associated with the remaining components. In the direct measurement approach, the only remaining component is the imaging lens. The determination of the MTF of the imaging lens therefore represents a separate effort.

This paper shall briefly discuss the theory, some methods, and some problems which relate to the determination of the MTF for lenses and linear arrays.

## THEORY

The Modulation Transfer Function is defined as the *modulus* of the Optical Transfer Function (OTF), which, like its counterpart in electronics, contains both magnitude and phase information. If we assume that our system is relatively free of aberrations, then the Phase Transfer Function is of no special interest. While the electronic transfer function describes the ability of a circuit or electrical system to transmit *temporal* frequencies, the MTF is a description of a system's transmission of *spatial* frequencies. That is, knowing the MTF of a lens and the distribution of light intensity

across an object, we can predict the light distribution in the image plane. Indeed, every imaging system component, including the eye, photographic film, phosphors, apertures, lenses, and others (even focus errors, vibration and other degradations) can be treated as a spatial filter. In general, the MTF is a monotonically decreasing function of spatial frequency (analogous to the low-pass filter in electronics).

The modulation of a wave form is defined as

$$\text{MTF} = (I_{\max} - I_{\min}) / (I_{\max} + I_{\min}) \quad (1)$$

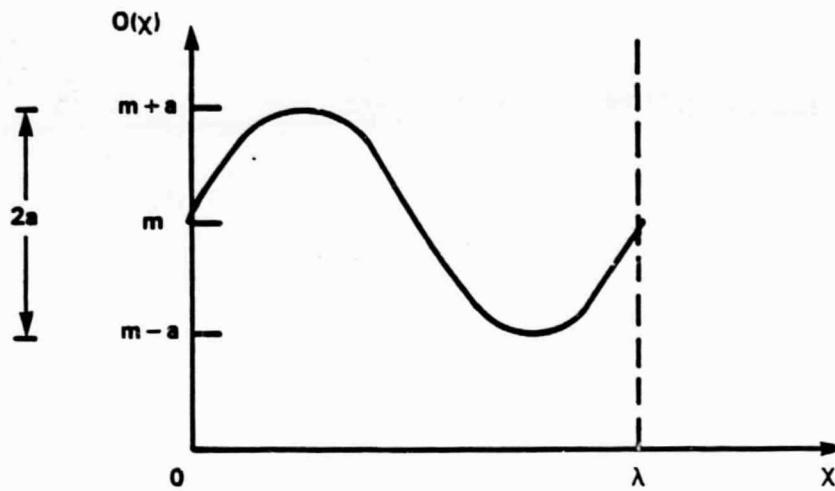
The numerator represents the depth of the modulation, while the division by the mean normalizes the modulation with respect to the background at zero spatial frequency.

Take  $m$  as the mean intensity of the background and  $a$  as the amplitude of a sinusoidal intensity function, superimposed on this background. As in electronics, any linear imaging system will output a sinusoid of equal or reduced amplitude and possibly altered phase when the input is a sinusoid, taking magnification into account. To determine the MTF of an optical system, the input would ideally be a target with a sinusoidal distribution (Fig. 1) where  $m$  is the mean and  $a$  is the amplitude of the sinusoid. Alternatively, Eq. 1 may be expressed as

$$\text{MTF} = ((m+a) - (m-a)) / ((m+a) + (m-a)) = a/m. \quad (2)$$

When the image modulation is divided by the object modulation, the result is the value of the MTF at that spatial frequency. In a properly designed target, the amplitudes and background remain constant at all spatial frequencies.

In electronic systems theory, the signal input to a system is *convolved* with a function known as the "impulse response" of that system to obtain the signal output. Fourier theory states that the Fourier transform of the input signal can be multiplied by the system frequency response (i.e. — the



$$O(x) = m + a \sin(2\pi f x)$$

frequency  $f = 1/\lambda$

Figure 1. Sinusoidal intensity function with zero spatial phase angle.

Fourier Transform of the impulse response) to find the Fourier Transform of the output signal. In other words, convolution in the time domain is equivalent to multiplication in the frequency domain. Analogously, every imaging system has a characteristic "point-spread function", which is the light distribution in the image plane when the object is a point. Since this discussion will cover linear arrays, which lie along one dimension only, the point-spread function will be replaced by its linear counterpart, the "line-spread function,"  $L(x)$ .

The image,  $I(x')$ , formed by any optical system is the superposition of individual images of each object point,  $O(x)$ , where  $x'$  is measurement in image space and  $x$  is in object space. The multiplication of every point in the object function by the system line spread function, summed over all the object points, is a *convolution* operation and describes the image intensity function

$$I(x') = \int_{-\infty}^{\infty} O(x) L(x' - x) dx \quad (3)$$

The Fourier Transform of the line spread function is the MTF.

Continuing the analogy between electrical and optical, the Fourier Transform of the object intensity function can be multiplied by the Fourier Transform of the system's line spread function (MTF) to yield the Fourier Transform of the image intensity. Therefore, given the spatial analog of the impulse function (a slit) for an object, which in the limit has a Fourier Transform of unity, and measuring the Fourier Transform of the image intensity, one directly determines the system MTF.

Therefore, the MTF of a system can typically be determined from two experimental approaches: either by directly measuring the image modulation and dividing it by the known object modulation using an object with a sinusoidal luminance distribution, or by measuring the system line spread function and taking the Fourier transform.

#### MEASUREMENT THEORY

In order to determine an array MTF from a measurement of its output, one must, of course, first know the nature of the input signal to the array. If the MTF of both the object and of the imag-

ing lens are known, one can deduce the MTF of the array from the output of the system:

$$(MTF_{array}) = (MTF_{output}) / [(MTF_{object}) \times (MTF_{lens})] \quad (4)$$

Given an object to view with a known modulation, the array MTF can be determined if the MTF of the lens is known.

The MTF is a multi-parametric function, depending on magnification, field angle, f-stop, focus, spectral range, and many other factors. It is thus not always possible to rely on the MTF data published by the manufacturer, unless you are confident that you can duplicate precisely the manufacturer's measurement system set-up.

#### LENS MTF MEASUREMENT

There are two usual approaches for determining the MTF of a lens—directly through the use of bar targets, or indirectly by transforming the line-spread function. There are benefits and shortcomings to both methods: There is the scarcity and cost of sinusoidal targets of high enough spatial frequency for lens testing, and generally a transmissive periodic target is not useable over a wide spectral range. However, square wave bar targets are commonly available and can be used in lieu of a sinusoidal target. One would arrive at the square wave MTF which can then be mathematically converted to the sine wave MTF using a series approximation. In addition, using bar targets is conceptually and experimentally simple. On the other hand, the determination of the line-spread function enables the calculation of the entire MTF frequency curve, as opposed to one discrete frequency at a time. We have used both methods.

Theoretically, to output the line-spread function for a lens would require the input of a slit of infinitesimal width. In practice, even to use a very narrow slit would so attenuate the light as to make detection difficult. But the MTF can be calculated with an acceptably large energy input if a knife edge is used as the object, and its image intensity is differentiated with respect to distance.



This follows from the fact that the derivative of a step function is the impulse function which can be identified as the line spread function.

Thus, by imaging an acceptably sharp edge with a test lens and differentiating this image, one determines the line-spread function of the lens; taking the Fourier Transform of this and normalizing the data to zero spatial frequency yields the MTF of lens.

A common method of scanning this image for analysis is through the use of a narrow slit, mounted in front of a detector, onto which the image has been focused (Fig. 2a). The light incident on the slit falls on the detector and if this slit/detector combination is scanned across the image plane, while sampling at appropriate intervals, a one-dimensional representation of the image can be stored by a computer. The slit itself must be viewed as another optical element in the system, with which an MTF is associated. This MTF can be calculated by taking the Fourier Transform of the slit function, which turns out to be the sinc function. If the detector is large enough with respect to the slit, the light will fall on enough detector area to minimize non-uniformity error. If, however, the light spreads out too widely behind the slit, the detector itself becomes a limiting aperture and thus acts as a spatial filter, lowering the MTF values, and must be included in the calculations.

The procedure for determining the MTF of a lens using the transform method is as follows:

1. Move detector/slit incrementally.
2. Sample detector output with voltmeter.
3. Repeat 1 and 2 until an unambiguous recording of the edge is realized.
4. Take the derivative of this edge scan with respect to distance to yield the line-spread function.
5. Take the Fourier Transform of this line-spread function to yield the MTF of the system.
6. Divide this system value by the calculated Fourier Transform of the slit function to yield the lens MTF.

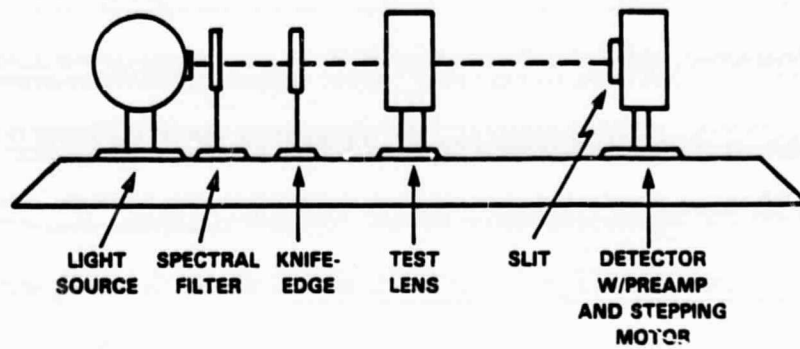


Figure 2a. Optical Bench for determining the lens MTF.

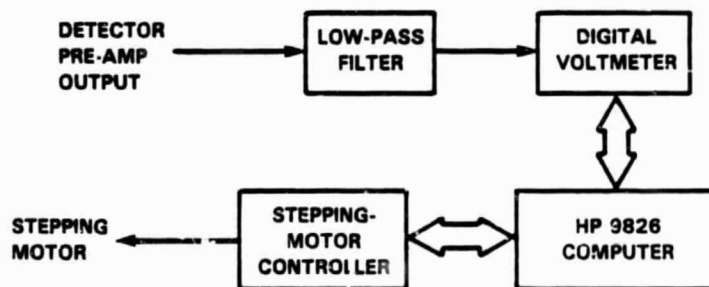


Figure 2b. Lens MTF measurement and control system.

This lens MTF measurement system is relatively straight forward, and when fully automated with a computer and translator, takes about ten seconds to complete a scan and compute the MTF with an HP series 200 desktop computer. Detailed below are the basic components of the system

Source — This should provide incoherent, uniform illumination at the object. The MLA lab uses an 8 inch integrating sphere with a 150-watt quartz bulb, although a bare bulb may be used if spatial nonuniformity effects are not too noticeable.

Object — A good quality single-edged razor blade should be sufficient if checked under a microscope for flaws.

Scanning Slit — Use either a moveable or fixed-width slit, with very sharp edges. The dimensions must be quantifiable, so that its MTF can be calculated mathematically. Rigidly attach the slit close enough to the detector so that they can move together and so that light passing through the slit does not overfill the detector. If this requirement is not met, then the detector becomes the limiting aperture and its MTF must be included in the calculations.

Detector — The MLA Lab has used, for visible and near-infrared wavelengths, a silicon UV-440 photodiode, manufactured by Reticon EG&G Div. The detector is circular, with an approximate area of 1 square centimeter. Any shape of detector will do, as long as it is large enough to accommodate the slit before it and its areal response is relatively uniform. For wavelengths from 1 to 2.5 microns (the short wave infrared, SWIR, region), we have used a thermoelectrically cooled lead-sulfide detector.

In a typical system, the output from the detector is fed to a low-noise pre-amp and then to a low-pass filter, which essentially eliminates all but the DC portion of the signal, before going to a digital voltmeter (Fig. 2b). The voltmeter output is sampled by a computer, which also controls the stepping motor. This data is then differentiated and the MTF determined.

Invariably, the learning curve which accompanies the realization of the system is often fraught with minor problems. Listed here are some of the most typical:

1. FOCUS — Focus is of primary importance; it is self-evident that an out of focus image will seriously degrade the MTF results. The depth-of-focus of a lens, which for a given wavelength depends upon the  $f\#$ , may be on the order of several microns. Therefore, great precision is required to focus properly — a visual inspection is usually not satisfactory. To increase the focusing resolution, an automated focusing algorithm was developed. This included a second high-resolution stepping motor, mounted under the detector assembly, and operating along the optical axis to optimize the image sharpness.

2. ALIGNMENT OF SLIT WITH EDGE — In order that we treat the convolution of the edge and the scanning slit as a "step function" and a "slit function", they must be adjusted such that they are vertically aligned. It is obvious that even a slight misalignment here would broaden the output edge response and thus degrade the MTF. The slit must be adjusted through rotation with respect to the reticle of an alignment microscope which has already been aligned to the knife edge, or vice versa.

3. MAGNIFICATION — When an image is magnified, the spatial scale is spread out, and the frequency scale is correspondingly compressed. For example, an image may have a spatial frequency distribution as graphed in Figure 3. Suppose that the image is enlarged by a factor of ten, the spatial frequencies will be reduced by a factor of ten, and the frequency distribution would now range from 0 to 20 rather than the 200 cycles per millimeter shown.

Note that the only change was in the scale. The image plane at a 10x magnification is ten times the scale of the object plane. It should be noted that some lenses are "optimized" at a certain magnification. If one referred the frequency scale to the object plane for a lens at different magnifications, the graph of the MTF curves might appear as shown in Figure 4.

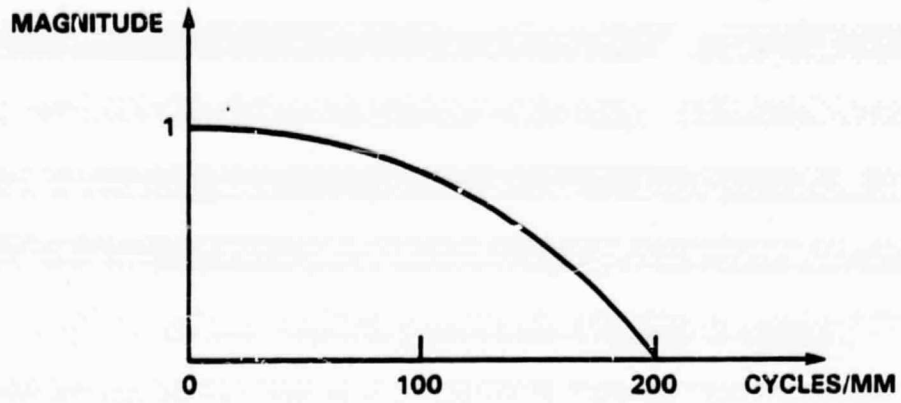


Figure 3. Spatial frequency distribution of image (1x).

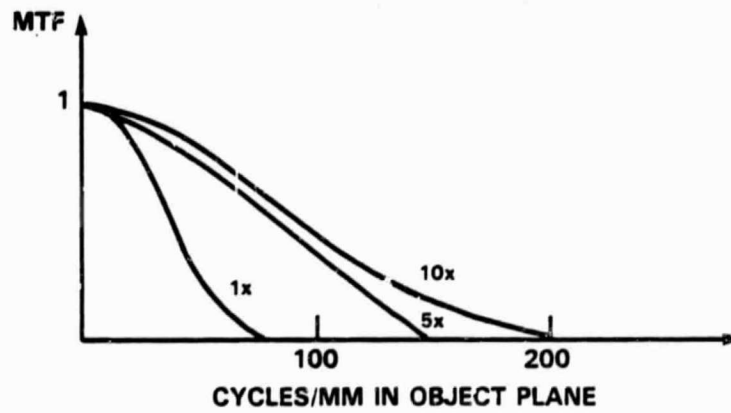


Figure 4. Lens MTF curves for different magnifications.

Manufacturer's specifications should indicate whether the spatial frequency scale refers to the object or image plane. If there is no mention of this, then it is reasonable to assume that the spatial frequencies refer to the image plane.

Note that if one is using the lens whose MTF is depicted in Figure 4 at 10x, no frequencies in the image plane would exceed 20 cycles/mm, and at 5x, no image frequencies would exceed 15 cycles/mm.

4. SPECTRAL FILTERING — The MTF of a lens is also dependent on the wavelengths of the radiation involved. In general, a broad spectrum will focus less sharply than a narrow one. Therefore, a narrowing of the bandwidth, which reduces the chromatic aberration, should result in an improved MTF. Most lenses are designed for use in a limited spectral range. For example, the MTF of a normal 50 mm camera lens would tend to decrease rapidly in the region above 700 nm.

5. FIELD ALIGNMENT — One of the most important parameters in MTF measurement is field alignment, which indicates where the image was with respect to the optical axis when the MTF was measured. The MTF decreases as the image is moved away from the optical axis (Fig. 5). A ".5 field," for example, is on the order of 20 degrees off-axis for a typical camera lens. Visual alignment, however, should be accurate to within several degrees, which should not greatly affect the MTF.

6. APERTURE STOP — The MTF changes noticeably with the f-number of the lens since each lens' zone has its own optical characteristics. Most lenses are optimized at a certain f-number, resulting in the best MTF.

7. COHERENCY — An edge scan containing a "ripple" (Fig. 6) usually indicates the presence of some coherency in the illuminating source. Perhaps the best source for incoherent white light is a barium-sulfate coated integrating sphere, which acts as a light mixer.

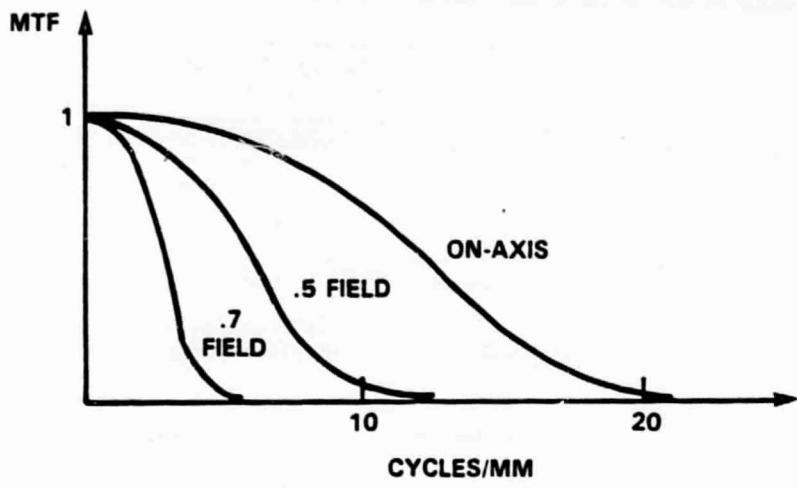


Figure 5. Lens MTF curves for different field positions.

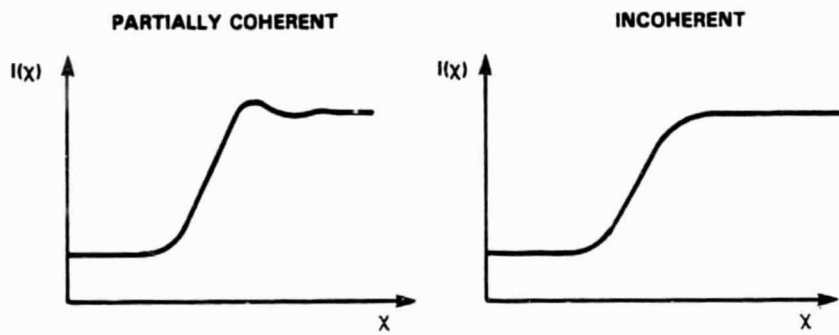


Figure 6. Edge scans with incoherent and partially coherent light.

8. VIBRATION — Often the vibration by laboratory equipment, resulting in image or detector motion, will degrade the measured MTF markedly. An optical bench that is damped or isolated from external vibrations is therefore highly desirable.

#### ARRAY MTF MEASUREMENT

Figure 7 shows the general surface geometry of a linear array having a high fill factor, i.e., very little space between contiguous elements.

There are two conceptually simple techniques involving the edge trace that have been used by the MLA laboratory to determine linear array MTF. In the first, individual array elements (pixels) are scanned across the image of the knife edge, and their MTF's are computed and are averaged to ascertain the nominal MTF of the entire array. If the uniformity of the array is known, a second technique may be employed wherein the knife edge is imaged onto the array, and the total array output is treated as an edge scan having a sampling interval equal to the pitch.

The first measurement system, similar to that for the lens MTF (Fig. 2) wherein the slit/detector is replaced by the array, involves a stepped-sampling of the data from one element.

The array, aligned perpendicularly to the knife edge, is stepped incrementally across the image, and the output from only one element is recorded (treated as an individual detector). The resulting edge scan is differentiated, the Fourier transform is taken, and the MTF of the lens is divided out to yield the array element MTF. Several such measurements are taken over various sections of the array. From these measurements, one can then determine if the array can be characterized by an average MTF.

The second method is more straightforward, and avoids the use of a stepping motor. The system can be the same as diagrammed above — only the data processing is different. The knife edge is imaged onto a section of the array, and the entire output (as opposed to just one element) of



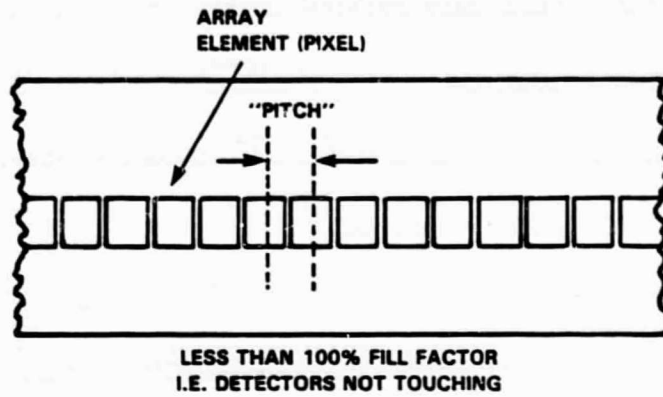


Figure 7. Simplified linear array surface geometry.

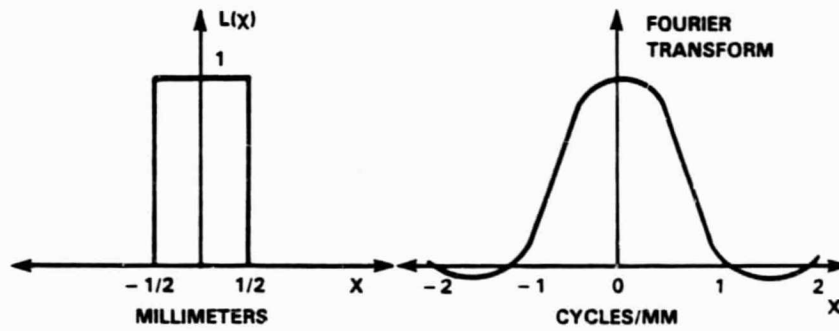


Figure 8. Line spread function and MTF of a rectangular aperture.

that section is treated as an edge scan as if generated by the first method, the sampling increment being the center-to-center distance between elements. One major problem that is often encountered is the non-uniformity of the array elements, which may create too much irregularity in the edge trace. The non-uniformity must be treated as a separate problem.

The other general problems associated with array MTF measurement are basically the same as those discussed for the lens, with one notable exception. Focusing the edge of the array is much easier since one can usually see the array output on an oscilloscope — micrometer adjustment until the edge appears sharpest is a quick and relatively accurate method.

#### VERIFYING RESULT

Verification of system accuracy is one of the last steps in developing an MTF bench. It is advisable to test a lens or array under conditions for which MTF data is available, in order to more readily spot system errors.

If MTF data is unavailable, then theoretical MTF calculations should be considered to establish, at least, a "best-case" MTF for the test component.

For a lens, the ideal MTF (diffraction-limited) is a function of the wavelength of the light and the aperture. The theoretically best MTF for a linear array is that of an aperture having the size and shape of an array element. In other words, an ideal, perfectly uniform linear array has an MTF which can be calculated by taking the Fourier transform of a single element's line-spread function (Fig. 8), typically resulting in the sinc function.

#### CONCLUSION

The Modulation Transfer Function is acknowledged to be a useful tool in evaluating the quality of imaging systems, and this paper has attempted to make known some practical information which should be more available than it generally is concerning MTF measurement. By no means have all

of the measurement technique variations been covered here; the edge-scan method, which was emphasized, is one out of many and is widely used due to its simplicity and practicality. The described variation on this method, which eliminates the need for stepping motors to take advantage of the sampling nature of linear arrays, is also useful for other non-single point detectors such as emulsions. Those in search of more involved discussions of MTF theory may refer to standard advanced optical texts.

We would like to express our appreciation to Mitchell Finkel for his comments on this work and to William L. Barnes for his support.