NASA Technical Memorandum 86340

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Lawrence W. Townsend and John W. Norbury

MARCH 1985

NASA

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Lawrence W. Townsend
Langley Research Center
Hampton, Virginia

John W. Norbury
Old Dominion University
Norfolk, Virginia

National Aeronautics
and Space Administration

Scientific and Technical Information Branch

1985

Introduction

As the era of a permanently manned Space Station begins to unfold, the necessity for improved radiation protection methods for the astronauts who will man it increases in importance. For the first time in the history of the manned space program, the combination of long-duration missions and career astronauts will cause the high-energy heavy-ion component of galactic cosmic rays to assume major radiobiological significance. To properly assess spacecraft and personal shielding requirements and to properly determine radiation exposures of critical organs, methods for estimating the radiation fields produced by relativistic heavy nuclei incident upon thick layers of absorbers of different composition are required. To meet these requirements, an extensive program for the development of theoretical methods for predicting relativistic heavy-ion interaction and transport is in progress at the Langley Research Center. Concurrently, experimental verifications of these models are obtained by researchers at Lawrence Berkeley Laboratory using state-of-the-art equipment (ref. 1) and techniques (refs. 2 and 3).

In previous work (refs. 4 to 8), a simple theory of heavy-ion fragmentation based upon a two-stage abrasion-ablation model has been developed for use in estimating the fragmentation parameters required for input into an accurate heavy-ion transport theory under concurrent development (refs. 3 and 9 to 11). Since the abrasion formalism calculates only the cross section for removal of a given number of abraded nucleons without specifying their identities as either neutrons or protons, a prescription for calculating the charge dispersions of the prefragments (the projectile pieces remaining after abrasion) is needed in order to calculate final isotopic and/or elemental production cross sections from the ablation process. Previously (refs. 6 to 8), two methods of determining prefragment charge dispersions have been utilized in the Langley fragmentation model: (1) the hypergeometric distribution, which treats the neutron and proton distributions as completely uncorrelated (ref. 12), and (2) the giant dipole resonance (GDR) distribution based upon the zero-point vibrations of the giant dipole resonance of the projectile nucleus (ref. 13). Both methods were used to calculate element production cross sections for 1.88 GeV/nucleon iron nuclei fragmenting in carbon, silver, and lead targets, and were also used to estimate isotope production cross sections for 213 MeV/nucleon argon nuclei fragmenting in a carbon target. When estimating element production cross sections (ref. 6), better agreement between theory and experiment was obtained for the GDR method, whereas the hypergeometric distribution yielded the best predictions for the isotope production cross sections (ref. 7). These later results, from reference 7, are displayed in figure 1, where it is apparent that the GDR calculations are too narrow in width. In this work, attempts to broaden the distribution widths of these cross sections are made by incorporating a physically correct charge dispersion model based upon the Langley quantum mechanical abrasion theory (ref. 5) rather than the geometric one used by Morrissey and coworkers (ref. 13). As will become apparent, the replacement of the geometric abrasion charge dispersion model by a quantum mechanical one yields only slight improvement in the agreement between theory and experiment for production cross sections of some isotopes produced in collisions of 213 MeV/nucleon ^{40}Ar with carbon targets.

Symbols

A_F	prefragment mass number
A_P	projectile nuclear mass number
A_T	target nuclear mass number
$B(e)$	average slope parameter of nucleon-nucleon scattering amplitude, fm^2
b	projectile impact parameter, fm
$C(\mathrm{y})$	Pauli correlation function
e	two-nucleon kinetic energy in their center of mass frame, GeV
GDR	giant dipole resonance
J	droplet model coefficient (25.76 MeV)
m	number of abraded nucleons
$\bar{m}$	mean number of abraded nucleons
N	total number of nuclear neutrons
N_j	renormalization coefficient for jth prefragment
n	number of abraded neutrons
$P(\mathrm{b})$	probability of not removing a single nucleon by abrasion
$Pr(Z_j, A_j)$	prefragment charge-to-mass dispersion function
Q	droplet model coefficient (11.9 MeV)
u	defined in equation (12)
Δx	defined in equation (14)
y	two-nucleon relative position, fm
Z	total number of nuclear protons

Z_F	number of fragment protons
Z_P	total number of projectile protons
z	number of abraded protons
z_o	longitudinal position of projectile center of mass, fm
α_z	defined in equation (11)
ξ_T	collection of constituent relative coordinates for target, fm
ρ_P	projectile nuclear density, fm^{-3}
ρ_T	target nuclear density, fm^{-3}
$\sigma(e)$	average nucleon-nucleon total cross section, mb
$\sigma_{\text{abr}}(Z,A)$	cross section for production of nucleus of type (Z,A) by abrasion, mb
σ_F	fragmentation cross section, mb
σ_m	cross section for abrading m nucleons, mb

Theoretical Development

In order to ultimately calculate nuclear fragmentation cross sections, the cross section for forming a particular prefragment of mass A_j and charge Z_j must be specified (refs. 6 and 7). In terms of the cross section for abrading m nucleons, σ_m, the prefragment formation cross section is written as

$$\sigma_{\text{abr}}(Z_j,A_j) = Pr(Z_j,A_j)\ \sigma_m \tag{1}$$

where $Pr(Z_j,A_j)$ is the prefragment charge-to-mass dispersion function (to be specified) and σ_m is given by (ref. 6)

$$\sigma_m = \binom{A_P}{m} \int d^2\mathbf{b}\ [1-P(\mathbf{b})]^m\, P(\mathbf{b})^{A_F} \tag{2}$$

where $P(\mathbf{b})$ is the probability as a function of impact parameter of not removing a nucleon in the collision, and A_F, the residual mass (prefragment) number, is

$$A_F = A_P - m \tag{3}$$

Within the context of eikonal scattering theory, an optical model potential approximation to the exact nucleus-nucleus multiple-scattering series, which includes Pauli correlation effects, yields (ref. 6)

$$P(\mathbf{b}) = \exp\left\{-A_T\ \sigma(e)\,[2\pi\ B(e)]^{-3/2} \int dz_o \times \int d^3\boldsymbol{\xi}_T\ \rho_T(\boldsymbol{\xi}_T) \int d^3\mathbf{y}\ \rho_P(\mathbf{b}+\mathbf{z}_o+\mathbf{y}+\boldsymbol{\xi}_T) \times [1-C(\mathbf{y})]\ \exp\left[\frac{-y^2}{2B(e)}\right]\right\} \tag{4}$$

Methods for determining the appropriate nuclear distributions $\rho_i (i = P,T)$, constituent-averaged nucleon-nucleon cross sections $\sigma(e)$, nucleon-nucleon scattering slope parameter $B(e)$, and Pauli correlation function $C(\mathbf{y})$ are described in reference 6 and references cited therein.

Hypergeometric Charge-to-Mass Dispersion

The hypergeometric charge-to-mass dispersion function is based on the assumption that there is no correlation at all between neutron and proton distributions. Therefore, unphysical results such as abrading all neutrons or protons from a nucleus while leaving the remaining fragment intact are possible. If z of the original Z projectile nucleus protons are abraded along with n of the original N projectile neutrons, then the hypergeometric distribution yields

$$Pr(Z_j,A_j) = \frac{\binom{N}{n}\binom{Z}{z}}{\binom{A_P}{m}} \tag{5}$$

where

$$A_P = N + Z \tag{6}$$

and

$$m = n + z \tag{7}$$

with

$$Z_j = Z - z \tag{8}$$

and

$$A_j = A_P - m \tag{9}$$

As an alternative to the hypergeometric distribution, Morrissey et al. (ref. 13) proposed a charge-to-mass dispersion distribution based upon the zero-point vibrations of the giant dipole resonance of the projectile nucleus. In this model, equation (5) becomes

$$Pr\left(Z_j, A_j\right) = N_j \left(2\pi\alpha_z^2\right)^{-1/2} \exp\left\{\frac{-\left[Z_j - A_j\left(Z/A_P\right)\right]^2}{2\alpha_z^2}\right\} \tag{10}$$

where the variance (dispersion) is

$$\alpha_z = 2.619\left(\frac{u}{A_P}\right)^{1/2}\left(\frac{Z}{A_P}\right)\left(\frac{dm}{db}\right)(1+u)^{-3/4} \tag{11}$$

with

$$u = \frac{3J}{Q\left(A_P\right)^{1/3}} \tag{12}$$

In the droplet model of the nucleus, the coefficients J and Q have the nominal values of 25.76 MeV and 11.9 MeV, respectively (ref. 13). The rate of change of the number of nucleons removed as a function of impact parameter dm/db is calculated numerically by using the geometric abrasion model of reference 13. The normalization factor N_j insures that for a given value of A_j, the discrete sum over all allowed values of Z_j yields unity for the dispersion probabilities. This overall normalization is not included in the original model of reference 3 but was added by us to insure probability conservation.

Quantum Mechanical GDR Dispersion

In an attempt to improve the GDR dispersion model, the method was generalized to a fully quantum mechanical treatment by replacing the geometric abrasion model of reference 13 with the Langley abrasion model of references 5 and 6. Recall that in the geometric formalism, the variance was given by equation (11), which can be rewritten as

$$\alpha_z = \left(\frac{Z}{A_P}\right)\left(\frac{dm}{db}\right)\triangle x \tag{13}$$

where from equation (11) we find

$$\triangle x = 2.619\left(\frac{u}{A_P}\right)^{1/2}(1+u)^{-3/4} \tag{14}$$

In references 6, 7, and 13, dm/db was calculated numerically by using a geometric abrasion model. In order to calculate this quantity quantum mechanically, we note that $P(\mathbf{b})$ is the probability of not removing a nucleon by abrasion. Hence $1 - P(\mathbf{b})$ is the probability that a nucleon is removed. Clearly the mean number of nucleons removed as a function of impact parameter is then

$$\bar{m} = A_P[1 - P(\mathbf{b})] \tag{15}$$

Therefore, the rate of change of the number of nucleons removed as a function of impact parameter becomes

$$\frac{dm}{db} = -A_P\frac{dP(\mathbf{b})}{db} \tag{16}$$

where $P(\mathbf{b})$ is obtained from equation (4). In the actual calculation, values from equation (16) are obtained numerically. These values are then substituted into equation (13) to determine the dispersion. The latter is then utilized in equation (10) to obtain values for the quantum mechanical GDR charge-to-mass dispersion model.

Results

To illustrate the results of these calculation methods, cross sections for the production of sulfur, phosphorus, silicon, and aluminum isotopes from the fragmentation of 213 MeV/nucleon ^{40}Ar projectile nuclei by carbon targets are listed in table I. Displayed are theoretical predictions obtained from the fragmentation model of references 6 and 7 for a hypergeometric distribution (eq. (5)) and for the GDR distribution (eq. (10)) evaluated for both the geometric and quantum mechanical models. Also listed are experimental values estimated from the results of reference 14. As is apparent, only minor improvement in the agreement between theory and experiment is obtained for the quantum mechanical GDR model when compared with the geometric GDR model. This is clearly indicated in table II, where the χ^2 values (ref. 15) for individual element production are listed for each charge-to-mass dispersion method. From tables I and II, it is clear that the hypergeometric distribution yields much better agreement with experiment than either of the GDR methods. The rather large χ^2 value for aluminum production from the hypergeometric distribution comes mainly from the relatively insignificant ^{32}Al isotope. If this isotope is excluded, the χ^2 value reduces to 8.4 for the hypergeometric distribution, to 17.3 for the geometric GDR distribution, and to 16.7 for the quantum mechanical GDR distribution.

The results of this work confirm the findings of reference 12, in which it was noted that the hypergeometric dispersion relation yielded better agreement with experiment for predicted isotopic production cross sections than those obtained with a "complete correlation" function given by (ref. 12)

$$Pr\left(Z_j, A_j\right) = \frac{Z_P}{A_P}m \tag{17}$$

This distribution, which would be correct if the nucleus were treated as a two-component crystal, is

not used in this work. Although not presented in this work, similar conclusions are obtained for the fragmentation results of ^{16}O by Be and Pb targets with the charge-to-mass dispersion models described herein. Hence, future improvements to the Langley abrasion-ablation model will require improved methods for estimating the prefragment excitation energy as well as the incorporation of final-state interactions into the theory (ref. 16).

Concluding Remarks

Methods for describing the charge-to-mass dispersion distributions of projectile prefragments are described and used to determine individual isotope production cross sections for various elements produced in the fragmentation of relativistic argon nuclei by carbon targets. Although slight improvements in predicted cross sections are obtained for the quantum mechanical giant dipole resonance (GDR) distribution when compared with the predictions of the geometric GDR model, the closest agreement between theory and experiment continues to be obtained with the simple hypergeometric distribution, which treats the nucleons in the nucleus as completely uncorrelated.

NASA Langley Research Center
Hampton, VA 23665
December 13, 1984

References

1. Schimmerling, Walter; Rapkin, Marwin; Wong, Mervyn; Howard, Jerry; Kaplan, Selig N.; Spieler, Helmuth G.; Jarrett, Blair V.; and Walton, John T.: Physical Characterization of Heavy-Ion Beams. *Biology and Medicine Division Annual Report 1982–1983,* LBL-16840 (Contract DE-AC03-76SF00098), Univ. of California, Apr. 1984, pp. 77–81.
2. Schimmerling, Walter; Kaplan, Selig N.; Subramanian, T. S.; McDonald, W. John; Gabor, George; and Sadoff, Ahren J.: Measurements of W for High-Energy Heavy Ions. *Biology and Medicine Division Annual Report 1982–1983,* LBL-16840 (Contract DE-AC03-76SF00098), Univ. of California, Apr. 1984, pp. 81–83.
3. Wong, Mervyn; Schimmerling, Walter; Civello, Joe; Howard, Jerry; Wilson, John W.; Townsend, Lawrence W.; and Bidasaria, Hari B.: Transport of High-Energy Heavy Ions Through Extended Matter. *Biology and Medicine Division Annual Report 1982–1983,* LBL-16840 (Contract DE-AC03-76SF00098), Univ. of California, Apr. 1984, pp. 85–87.
4. Townsend, Lawrence W.: *Optical-Model Abrasion Cross Sections for High-Energy Heavy Ions.* NASA TP-1893, 1981.
5. Townsend, L. W.: Abrasion Cross Sections for ^{20}Ne Projectiles at 2.1 GeV/Nucleon. *Canadian J. Phys.,* vol. 61, no. 1, Jan. 1983, pp. 93–98.
6. Townsend, Lawrence W.; Wilson, John W.; Norbury, John W.; and Bidasaria, Hari B.: *An Abrasion-Ablation Model Description of Galactic Heavy-Ion Fragmentation.* NASA TP-2305, 1984.
7. Townsend, L. W.; Wilson, J. W.; and Norbury, J. W.: A Simplified Optical Model Description of Heavy Ion Fragmentation. *Canadian J. Phys.,* vol. 63, no. 2, Feb. 1985, pp. 135–138.
8. Townsend, Lawrence W.: *Ablation Effects in Oxygen-Lead Fragmentation at 2.1 GeV/Nucleon.* NASA TM-85704, 1984.
9. Wilson, John W.: *Analysis of the Theory of High-Energy Ion Transport.* NASA TN D-8381, 1977.
10. Wilson, John W.: *Heavy Ion Transport in the Straight Ahead Approximation.* NASA TP-2178, 1983.
11. Wilson, John W.; Townsend, L. W.; Bidasaria, H. B.; Schimmerling, Walter; Wong, Mervyn; and Howard, Jerry: ^{20}Ne Depth-Dose Relations in Water. *Health Phys.,* vol. 46, no. 5, May 1984, pp. 1101–1111.
12. Oliveira, Luiz F.; Donangelo, Raul; and Rasmussen, John O.: Abrasion-Ablation Calculations of Large Fragment Yields From Relativistic Heavy Ion Reactions. *Phys. Rev.,* ser. C, vol. 19, no. 3, Mar. 1979, pp. 826–833.
13. Morrissey, D. J.; Oliveira, L. F.; Rasmussen, J. O.; Seaborg, G. T.; Yariv, Y.; and Fraenkel, Z.: Microscopic and Macroscopic Model Calculations of Relativistic Heavy-Ion Fragmentation Reactions. *Phys. Rev. Lett.,* vol. 43, no. 16, Oct. 15, 1979, pp. 1139–1142.
14. Viyogi, Y. P.; Symons, T. J. M.; Doll, P.; Greiner, D. E.; Heckman, H. H.; Hendrie, D. L.; Lindstrom, P. J.; Mahoney, J.; Scott, D. K.; Van Bibber, K.; Westfall, G. D.; Wieman, H.; Crawford, H. J.; McParland, C.; and Gelbke, C. K.: Fragmentations of ^{40}Ar at 213 MeV/Nucleon. *Phys. Rev. Lett.,* vol. 42, no. 1, Jan. 1, 1979, pp. 33–36.
15. Bevington, Philip R.: *Data Reduction and Error Analysis for the Physical Sciences.* McGraw-Hill Book Co., Inc., 1969.
16. Hufner, J.; Schafer, K.; and Schurmann, B.: Abrasion-Ablation in Reactions Between Relativistic Heavy Ions. *Phys. Rev.,* ser. C, vol. 12, no. 6, Dec. 1975, pp. 1888–1898.

TABLE I. ISOTOPE PRODUCTION CROSS SECTIONS FOR THE REACTION $^{40}Ar + ^{12}C \rightarrow {}^{A}Z + X$

[Incident kinetic energy is 213 MeV/nucleon]

Isotope produced	Isotope production cross sections, mb			
	Hypergeometric model (ref. 7)	Geometric GDR model (ref. 7)	Quantum mechanical GDR model	Experiment (ref. 14)
^{37}S	7	0	0	10 ± 5
^{36}S	31	6	6	19 ± 8
^{35}S	34	27	25	27 ± 10
^{34}S	60	81	81	61 ± 20
^{33}S	35	44	43	29 ± 10
^{32}S	10	6	6	9 ± 4
^{31}S	.2	.03	.04	1 ± 0.5
^{36}P	0.3	0	0	
^{35}P	9	1	1	
^{34}P	15	2	2	11 ± 4
^{33}P	27	25	25	25 ± 10
^{32}P	30	40	40	41 ± 10
^{31}P	16	22	22	31 ± 9
^{30}P	12	3	12	9 ± 3
^{29}P	0	0	0	
^{34}Si	0.3	0	0	0.3 ± 0.15
^{33}Si	2.6	0	0	1.3 ± 0.6
^{32}Si	9	1	2	8 ± 3
^{31}Si	12	3	4	18 ± 8
^{30}Si	31	35	35	49 ± 11
^{29}Si	29	38	38	41 ± 10
^{28}Si	21	26	25	21 ± 8
^{27}Si	.4	.2	.2	2.3 ± 1
^{32}Al	0.3	0	0	0.08 ± 0.04
^{31}Al	1.1	0	0	1.1 ± 0.4
^{30}Al	5.7	1	1	4 ± 1.8
^{29}Al	8	4	5	16 ± 6
^{28}Al	19	19	19	31 ± 10
^{27}Al	22	24	24	44 ± 11
^{26}Al	13	15	14	16 ± 6

TABLE II. TOTAL CHI-SQUARE VALUES FOR ELEMENTS LISTED IN TABLE I

Element produced	Chi-square value		
	Hypergeometric distribution	Geometric GDR distribution	Quantum mechanical GDR distribution
Sulfur	6.1	14.2	13.9
Phosphorus	12.3	16.3	13.3
Silicon	13.1	24.2	23.7
Aluminum	38.6	21.3	20.7

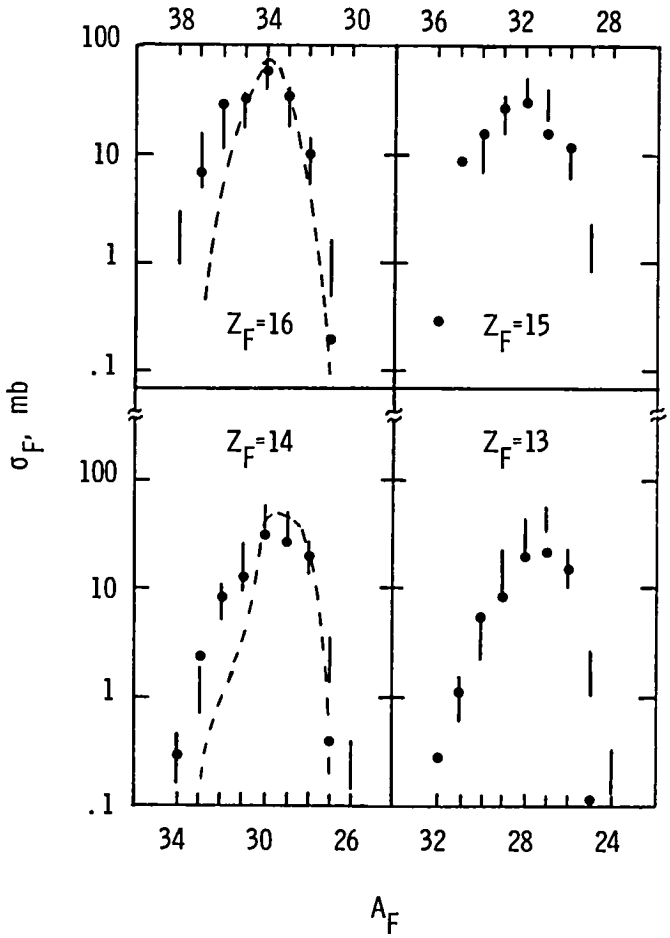

Figure 1. Production cross sections for isotopes of sulfur, phosphorus, silicon, and aluminum produced by fragmentations of 213 MeV/nucleon ^{40}Ar projectiles in carbon targets. Theoretical estimates using the hypergeometric distribution are displayed (dots) and compared with estimates obtained from the geometric giant dipole resonance distribution (dashed curves) and experimental data from Lawrence Berkeley Laboratory (error bars).

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-86340		

4. Title and Subtitle	5. Report Date
Charge-to-Mass Dispersion Methods for Abrasion-Ablation Fragmentation Models	March 1985
	6. Performing Organization Code: 199-22-76-01
7. Author(s): Lawrence W. Townsend and John W. Norbury	8. Performing Organization Report No.: L-15912
9. Performing Organization Name and Address: NASA Langley Research Center, Hampton, VA 23665	10. Work Unit No.
	11. Contract or Grant No.
12. Sponsoring Agency Name and Address: National Aeronautics and Space Administration, Washington, DC 20546	13. Type of Report and Period Covered: Technical Memorandum
	14. Sponsoring Agency Code

15. Supplementary Notes

Lawrence W. Townsend: Langley Research Center, Hampton, Virginia.
John W. Norbury: Old Dominion University, Norfolk, Virginia.

16. Abstract

Methods for describing the charge-to-mass dispersion distributions of projectile prefragments are described and used to determine individual isotope production cross sections for various elements produced in the fragmentation of relativistic argon nuclei by carbon targets. Although slight improvements in predicted cross sections are obtained for the quantum mechanical giant dipole resonance (GDR) distribution when compared with the predictions of the geometric GDR model, the closest agreement between theory and experiment continues to be obtained with the simple hypergeometric distribution, which treats the nucleons in the nucleus as completely uncorrelated.

17. Key Words (Suggested by Authors(s))	18. Distribution Statement
Heavy ions Projectile fragmentation Abrasion-ablation model Giant dipole resonance (GDR)	Unclassified—Unlimited Subject Category 73

19. Security Classif.(of this report)	20. Security Classif.(of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	7	A02

For sale by the National Technical Information Service, Springfield, Virginia 22161

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