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TWO DEGREE-OF-FREEDOM FLUTTER SOLUTION FOR A PERSONAL COMPUTER

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SUMMARY

A computer program has been written for a personal computer which can be used to make two-degree-of-freedom (bending and torsion) flutter calculations by utilizing two-dimensional Theodorsen aerodynamics. The program may be used to approximately account for Mach number (compressibility) effects and aspect ratio (finite span) effects. This report contains the equations of motion, a program listing, user instructions, and test cases.

INTRODUCTION

Flutter is a self-excited dynamic oscillation produced by a coupling of inertial and elastic forces with aerodynamic forces resulting from elastic deformations of an aircraft wing. These oscillations could result in significant structural damage. Consequently, flutter must be taken into account during aircraft design; that is, the wing must be designed so that it will not flutter within the operating envelope of the airplane.

The types of vibrations that an airplane wing could experience in flight are illustrated in Figure 1.

If the wing is disturbed at flight conditions below the flutter boundary, the ensuing motion is a damped sinusoidal decaying oscillation as shown in Figure 1a. This is a stable condition. At the flutter boundary, any disturbance will cause the wing to oscillate at a constant amplitude as shown in Figure 1b. This is a neutrally stable condition. At conditions above the flutter boundary, a disturbance will produce a divergent oscillation as shown in Figure 1c. This is an unstable condition.

The purpose of this paper is to describe a computer program written for a personal, or home, computer that can be used to analyze aircraft wings for two-degree-of-freedom (bending and torsion) flutter. The equations of Theodorsen and Garrick (references 1 and 2) are implemented in this program. The analysis is a two-dimensional, two-degree-of-freedom, representative-section method. Although this method is not new, it is sufficiently accurate for many present day applications.

The method employed is adequate for moderate-to-high aspect ratio wings and unswept or slightly swept wings. However, the analysis has certain limitations. The aerodynamics are only applicable at subsonic velocities. The analysis as implemented in the computer program does not include the effects of concentrated masses such as wing mounted engines and fuel tanks. It also does not include the effects of aerodynamic control surfaces such as ailerons. These limitations are very important to note. Furthermore, the results should only be used as a guide, not as the final authority to determine whether or not a new or old wing design is safe from flutter.

This report presents documentation of the computer program. Also presented are the derivation of the equations, user instructions, program listing, and test cases which serve as illustrative examples.

SYMBOLS

a	non-dimensional distance from midchord to elastic axis, positive aft
b	semichord, c/2, ft
c	chord, ft
e_1	span efficiency factor, function of taper ratio
f_h	fundamental frequency in plunge or bending, Hz
f_α	fundamental frequency in pitch or torsion, Hz
g	artificial damping
g_h, g_α	structural damping in plunge and pitch, respectively
h	vertical deflection of the wing in plunge, ft
$\dot{h}$	plunge velocity of deflecting wing, ft/sec
$\ddot{h}$	plunge acceleration of deflecting wing, ft/sec^2
i	$\sqrt{-1}$, imaginary coefficient
k	reduced frequency, $b\omega/V$
m	mass per unit length of span, slugs/ft
q	dynamic pressure, $\rho V^2/2$, lb/ft^2
r_α	radius of gyration of the wing, non-dimensional
s	full wing span, ft
x_{CG}	distance of center of gravity from leading edge of wing, ft
X_{EA}	distance of elastic axis from leading edge of wing, ft
x_α	distance of center of gravity location aft of the elastic axis, non-dimensional
A	area of the wing, ft^2
AR	aspect ratio
$\overline{A}$, $\overline{B}$	flutter determinant coefficients
C	Theodorsen circulation function
CG%	center of gravity location, percent chord
C_{L_α}	lift curve slope, per radian
$\overline{D}$, $\overline{E}$	flutter determinant coefficients
EA%	elastic axis location, percent chord
F	real part of Theodorsen's circulation function
G	imaginary part of Theodorsen's circulation function
H	altitude, ft
I_α	pitch inertia per unit length of span, slug-ft^2/ft
L'	lift per unit length of span, lb/ft
L_h	lift due to plunge, non-dimensional
L_α	lift due to pitch, non-dimensional
M	Mach number
M'	moment per unit length of span, ft-lb/ft
M_h	moment due to plunge, non-dimensional
M_α	moment due to pitch, non-dimensional
S_α	wing imbalance, slug-ft.
V	velocity, ft/sec
V_a	speed of sound, ft/sec
V_e	equivalent airspeed, ft/sec

α angle of attack, wing deflection about elastic axis in pitch, radians

$\dot{\alpha}$ pitch velocity of wing deflection about elastic axis, radians/sec

$\ddot{\alpha}$ pitch acceleration of wing deflection about elastic axis, radians/sec^2

β Mach number correction factor

μ mass ratio, $m/\pi\rho b^2$, nondimensional

ρ air density, slugs/ft^3

ω coupled frequency, rad/sec

ω_h fundamental plunge (bending) frequency, rad/sec

ω_α fundamental pitch (torsion) frequency, rad/sec

Ω variable of substitution, complex frequency

Ω_R real part of complex frequency

Ω_I imaginary part of complex frequency

EQUATIONS OF MOTION

The flutter analysis used in this program is a two-dimensional, two-degree-of-freedom, representative-section method. The term "two-dimensional" applies to the aerodynamics that are utilized in the program. A two-dimensional wing has a constant lift and moment along the span. "Two-degree-of-freedom" means that the wing is free to deform or vibrate in two different motions. In this case, the motions are bending and torsion. Bending, or plunge, is the spanwise bending of the wing about the root. Torsion, or pitch, is the chordwise twisting of the wing about the rotation point or elastic axis of a beam-like wing. These motions are assumed to be simple harmonic or sinusoidal. "Representative section" means the wing characteristics can be represented by the properties and motions of a unit section of the wing. This section is chosen to be located at the three-quarter span station. The elasticity of the wing can be represented as two springs as shown attached to the wing section in Figure 2. If the wing is given a displacement, the wing will oscillate in simple harmonic motion (sinusoidal oscillations) at frequencies associated with the two degrees of freedom. Flutter occurs when these two modes of motion become coupled and extract energy from the airstream to produce a self-excited oscillation.

The governing differential equations for a two degree-of-freedom flutter problem are derived in reference 3 and repeated here as equations 1 and 2.

$$m\ddot{h} + S_\alpha\ddot{\alpha} + \frac{m\omega_h^2 g_h}{\omega}\dot{h} + m\omega_h^2 h = L' \qquad (1)$$

$$S_\alpha\ddot{h} + I_\alpha\ddot{\alpha} + \frac{I_\alpha\omega_\alpha^2 g_\alpha}{\omega}\dot{\alpha} + I_\alpha\omega_\alpha^2\alpha = M' \qquad (2)$$

These generalized forces are functions of the bending deflection h and the torsional twist α. These forces are defined explicitly in reference 3 and repeated here as equations 3 and 4.

$$L' = \pi\rho b^3\omega^2\{L_h h/b + [L_\alpha - (1/2 + a)L_h]\alpha\} \quad (3)$$

$$M' = \pi\rho b^4\omega^2\{[M_h - (1/2 + a)L_h]h/b + [M_\alpha - (1/2 + a)(L_\alpha + M_h) + (1/2 + a)^2 L_h]\alpha\} \quad (4)$$

where L_h, L_α, M_h, M_α are defined by reference 3 as

$$L_h = 1 - 2i(1/k)(F + iG) \quad (5)$$

$$L_\alpha = 1/2 - i(1/k)[1 + 2(F + iG)] - 2(1/k)^2(F + iG) \quad (6)$$

$$M_h = 1/2 \quad (7)$$

$$M_\alpha = 3/8 - i(1/k) \quad (8)$$

These are termed Theodorsen aerodynamics because of their reliance on the Theodorsen circulation function.

$$C = F + iG \quad (9)$$

The actual function is determined by several Bessel functions and other complicated mathematics. However, an approximation is given in reference 4 that is dependent upon the reduced frequency, k.

$$C(k) = 1 - \frac{0.165}{1 - \frac{0.0455}{k}i} - \frac{0.335}{1 - \frac{0.3}{k}i}$$

Theodorsen assumed several things in the derivation of the aerodynamics. The major assumption was that the wing is two-dimensional. From this assumption follows the assumption of infinite span. To further simplify the aerodynamics, incompressible flow was assumed. However, both finite-span and compressible-flow corrections can be included to determine their effect on the flutter characteristics. These corrections are defined in the following text.

The assumption of an infinite span is that no wing tip exists and the lift distribution is constant along the span of the wing. The lift curve slope of a two-dimensional wing is actually greater than the lift curve slope for a finite aspect ratio wing at the three quarter span location. For this reason the infinite aspect ratio assumption causes conservative results. However, for finite-span analysis, the equation for estimating the average lift curve slope is derived in reference 5 as

$$C_{L_\alpha} = \frac{C_{L_{\alpha_\infty}}}{1 + \frac{C_{L_{\alpha_\infty}}}{\pi e_1 AR}} \tag{11}$$

where $C_{L_{\alpha_\infty}}$ is C_{L_α} for infinite aspect ratio.

hence

$$L_\alpha = \frac{L_{\alpha_\infty}}{1 + \frac{C_{L_{\alpha_\infty}}}{\pi e_1 AR}} \tag{12}$$

where

L_{α_∞} = original value for L_α from the Theodorsen aerodynamics

$C_{L_{\alpha_\infty}}$ = 2π from theoretical analysis

e_1 = .85 for untapered wings

If the aspect ratio is assumed to be infinity as Theodorsen did, the corrected C_{L_α} is equal to $C_{L_{\alpha_\infty}}$.

Furthermore, a reduced lift means a reduced moment. Because the moment is equal to the lift multiplied by a distance, the moment is adjusted by the relationship

$$M_\alpha = \frac{L_\alpha}{L_{\alpha_\infty}} M_{\alpha_\infty} \tag{13}$$

where the value M_{α_∞} is the original value calculated by the Theodorsen aerodynamics.

The assumption of incompressible flow means that the effects of Mach number are not considered in the calculations. The correction factor for compressible flow, which can be used below a Mach number of 0.8, is derived in Reference 6 as

$$\beta = \frac{1}{\sqrt{1-M^2}} \tag{14}$$

All Theodorsen aerodynamics are multiplied by this correction factor.

FLUTTER DETERMINANT

The solution of the flutter equations is described in this section. After the aerodynamic forces in equations 3 and 4 are substituted into equations 1 and 2 and simple harmonic motion is assumed, the two simultaneous equations that must be solved are given by

$$\bar{A}\ h/b + \bar{B}\alpha = 0 \tag{15}$$

$$\bar{D}\ h/b + \bar{E}\alpha = 0 \tag{16}$$

where

$$\bar{A} = \mu\left[1 - \left(\frac{\omega_\alpha}{\omega}\right)^2 \left(\frac{\omega_h}{\omega_\alpha}\right)^2 (1 + ig_h)\right] + L_h \tag{17}$$

$$\bar{B} = \mu x_\alpha + L_\alpha - L_h\ (1/2 + a) \tag{18}$$

$$\bar{D} = \mu x_\alpha + 1/2 - L_h\ (1/2 + a) \tag{19}$$

$$\bar{E} = \mu r_\alpha^2[1 - (\omega_\alpha/\omega)^2 (1+ig_\alpha)] - 1/2 (1/2 + a) + M_\alpha \tag{20}$$

$$- L_\alpha(1/2 + a) + L_h(1/2 + a)^2$$

Several methods exist for solving the system of equations 15 and 16. A popular method, and the one used here, is from Theodorsen and Garrick (references 1 and 2). An artificial damping g and complex frequency Ω are introduced where

$$\Omega = (\omega_\alpha/\omega)^2(1+ig) \tag{21}$$

This term is introduced into equations 15 and 16 and the $\bar{A}$ and $\bar{E}$ expressions become

$$\bar{A} = \mu[1-(\omega_h/\omega_\alpha)^2(1+ig_h)\Omega]+L_h \tag{22}$$

$$\bar{E} = \mu r_\alpha^2[1-\Omega(1+ig_\alpha)] - 1/2(1/2+a) + M_\alpha - L_\alpha(1/2+a) + L_h(1/2+a)^2 \tag{23}$$

The $\bar{B}$ and $\bar{D}$ expressions remain unchanged.

For a non-trivial solution to exist, the determinant of the coefficients must vanish. That is,

$$\bar{A}\,\bar{E} - \bar{B}\,\bar{D} = 0 \tag{24}$$

Equation 24 is quadratic in Ω. The flutter solution occurs when Ω is real that is, when the artificial damping vanishes. Because the aerodynamic forces are parametric in reduced frequency k, equation 24 must be solved repeatedly to find the flutter solution. The individual solutions are usually plotted as shown in Figure 3 and the flutter velocity and frequency are obtained from the graph. The flutter velocity is defined as the velocity where the damping value is zero. The flutter frequency is the corresponding frequency at that velocity.

Individual values are found each time equation 24 is solved for a value of reduced frequency. These values of velocity and frequency are obtained from the two resulting complex solutions by using equations 25 through 30.

$$\Omega = \Omega_R + i\Omega_I \tag{25}$$

$$\Omega_R = (\omega_\alpha/\omega)^2 \tag{26}$$

$$\Omega_I = (\omega_\alpha/\omega)^2 g \tag{27}$$

$$g = \Omega_I/\Omega_R \tag{28}$$

$$\omega = \frac{\omega\alpha}{\sqrt{\Omega_R}} \tag{29}$$

$$V = b\omega/k \tag{30}$$

Equations 5 through 8, 12 through 14, and 22 through 30 were used as shown in the text in the "computerized" flutter solution. Equations 10 and 17 through 20 were used in a slightly different form due to the use of complex mathematics. A complete listing of the program source code is found in Appendix A.

USER INSTRUCTIONS

Of course, any computer program is useless without the knowledge of how to use it. This section explains the input parameters needed by the program, discusses how to obtain specific numerical values, and describes various input options. A description of the output is also included.

Input of Structural Properties

Structural values may be difficult to obtain in the form required by the program. However, reference 7 and the following guidelines should help determine the values of certain parameters needed for input to the computer program.

Mass per unit length of span, m. This is mass in slugs (1 slug = 32.2 lbs) of a one foot section of the wing. Assuming an untapered straight wing with constant mass properties throughout the value can be derived as

$$m = \frac{\text{mass of wing (slugs)}}{\text{s (ft)}}$$

If the wing is tapered, swept, or has non-constant mass properties, the mass is determined for a one-foot-wide representative section centered about the three-quarter semi-span. Assuming a linear distribution of mass, the mass value is given by

$$m = \frac{\text{mass of wing (slugs)}}{2s\ \text{(ft)}}$$

Center of gravity location. This is the location in percent chord of the point at which the wing balances on a knife edge as seen in Figure 4.

$$CG\% = (x_{CG}/c)\times 100$$

Wing moment of inertia about rotation point, I_α. This is the pitch inertia of a one-foot representative section of the wing measured in slug-ft^2. Experimental measurement may be done with a bifilar pendulum. (see reference 7, pp. 30-31.)

Elastic axis location. This is defined as the point about which the wing twists as illustrated in Figure 5.

$$EA\% = (x_{EA}/c)\times 100$$

Semichord length, b. This is one half the chord length of the representative section.

Natural bending and torsion frequencies f_h, f_α. These are fundamental frequencies in Hertz of the wing in bending and torsion. These are functions of mass, inertia, and stiffness. (See reference 7, pp. 31-33.)

Structural damping - bending and torsion, g_h, g_α. This is defined as any internal damping of the structure. Values of .01 to .03 are typical, however, for a solution that is conservative, a value of zero may be used.

Aspect ratio, AR. This is the ratio of span to chord defined as

$$AR = s^2/A$$

For straight, untapered wings, this reduces to

$$AR = s/c$$

For a fully two-dimensional analysis, an infinite aspect ratio should be used. Infinity however, cannot be input to a computer because of numerical difficulties. Therefore, a very large (but finite) value must be used. As very few wings have an aspect ratio greater than twenty, a value that is significantly greater than this (100,000,000 for instance) is recommended.

In making calculations to determine if a wing is free from flutter, it is prudent to assume that the aspect ratio is infinite because this assumption tends to cause conservative results.

Input for Graphic Output

The graphic input determines the plot resolution on the screen. A large scale may be needed for some solutions. However, that scale may be far too large for others. A typical plot appears in Figure 6. Guidelines for these input values follow:

Maximum frequency to be plotted. This is usually the nearest multiple of five above the torsion frequency.

Maximum velocity to be plotted. This should be about twice the expected flutter speed.

Maximum G-value. This is the maximum value of artificial damping that is plotted. A value of .05 is sufficient for most cases.

The above input will create two plots, frequency versus velocity and damping versus velocity. A typical plot has points off of the page. However, this is only undesirable if the flutter solution crossing point is off of the page.

Intermediate Values Option

As has been previously described, a flutter solution consists of solving the equations at many different reduced frequency values. Two values of velocity, coupled frequency and damping are obtained each time the equations are solved. If intermediate values are to be printed, then every value will be printed. Appendix B illustrates a solution with all values being printed.

Air Density Option

The air density that the wing flies at is very important to the flutter characteristics. A wing may flutter at one velocity and altitude yet be very stable at the same velocity and a higher altitude. Therefore, air density must be input to the problem also. Two options exist within the program for this purpose.

Option 1 - One value for air density is input. This corresponds to flying the wing at one altitude.

Option 2 - Several values for air density. The computer will calculate a solution at each air density or altitude.

If option 2 is selected, then the following values must also be input:

Number of values - This is the number of times the program must calculate a flutter solution using a different value of air density each time.

Interval between values - This is the increment to be added to the previous density value for the next calculation.

First Value - This is the starting value for the computer to calculate.

Example. Three values with an interval of .001 and a starting value of .001 are as follows: .001, .002, and .003.

Mach Number Option

If compressibility effects are desired, a Mach number must be input. The input number must be less than one. If no compressibility effects are desired, a Mach number of zero should be used.

Program Output

The following information is output by the program:

Structural parameters. The structural input parameters are printed so that the parameters can be saved and known for the flutter solution.

Graphic input parameters. The graphic input parameters are printed in order to be able to determine the plot size.

Preliminary values. These values are printed in order to know the Mach Number and air density. These values appear before each solution.

Intermediate values. These are printed only if the intermediate values option is "turned on." The values that are printed are the reduced frequency k and the corresponding values of wing frequency, velocities, and damping. These intermediate values are used to form data points which are then plotted.

Flutter values. The flutter values are printed along with an appropriate message. Because there are two sets of values (one for bending and one for torsion), the flutter conditions (frequency, velocity) are the numbers with the corresponding damping or g-value that is close to zero.

The plots. The intermediate points of frequency and damping are plotted versus velocity. The points labeled "1" in Figure 3 are one set of intermediate points. As was stated before, two solutions existed for each reduced frequency. The points labeled "2" are the solutions of the determinant for a second value of reduced frequency.

The damping curves indicate that one mode will cross the axis. The point of crossing is the corresponding velocity at which the wing will flutter. The other mode will generally become very stable as airspeed increases.

Matched Point Solution

Because the air density is an important factor in flutter calculations, an aircraft may flutter near sea level yet be perfectly safe at the same velocity when flying at ten thousand feet. Due to this fact, a flutter boundary is sometimes defined as Mach number M versus dynamic pressure q. A matched point is defined as the density condition where the flutter velocity is the same velocity determined from the product of the Mach number and the speed of sound V_a. To find a matched point solution, multiple air densities are used in flutter solutions at the same Mach number. The velocities are plotted against air density and a curve is faired through the points. Each value of air density corresponds to a particular altitude in the standard atmosphere for which the speed of sound also has a particular value. For the given Mach number a curve of velocity versus density is generated for the standard atmosphere. The point at which the two curves cross is the matched point. The corresponding velocity and air density are used to calculate dynamic pressure. When plotted as dynamic pressure against Mach number, a flutter boundary is defined. A complete matched point solution is given in Appendix C.

CONCLUDING REMARKS

A personal computer flutter solution has been written that employs modified two-dimensional Theodorsen aerodynamics for two degree-of-freedom lifting surfaces. The solution is only adequate for unswept or slightly swept wings with no concentrated masses located on the lifting surface. The aerodynamics utilized are only adequate for subsonic velocities. Control surface flutter is not addressed by the program. These limitations are further stressed by noting that the program is only to be used as a guide and not the final authority for determining if a wing is free from flutter.

APPENDIX A

Copy of Computer Program Listing

The flutter solution was programmed on an Apple IIE computer. Printed and plotted solutions were obtained with an Epson printer with Graftrax interfaced by a Microbuffer II interface.[1] The computer was configured with 64K of internal memory and the program used less than 40K. This program was written in Applesoft BASIC. Obviously, the program will not run on other small computers unless it is modified to account for differences in other manufacturers BASIC codes. However, the conversion of the code should be relatively straightforward in most instances.

The listing of the program begins on the next page.

[1]Use of trade names does not constitute an official endorsement, either expressed or implied, by the National Aeronautics and Space Administration.

```
10  REM  FLUTTER PROGRAM -- MODULAR SUBROUTINES
20  PR# 1
30  PRINT  CHR$ (9);"1L"
     : PRINT  CHR$ (9);"80N"
40  PR# 0
50  GOSUB 1000
60  GOSUB 2000
70  GOSUB 3000
80  GOSUB 3500
90  GOSUB 4000
100  GOSUB 4500
110  PRINT
     : PRINT
     : INPUT "MACH NUMBER ? ";MCH
120  FOR L = 1 TO NR
130  PR# 1
     : PRINT  CHR$ (12)
140  PRINT "MACH NUMBER : ";MCH;"   AIR DENSITY : ";RHO(L);" SLUGS/FT^3"
150  PR# 0
160  GOSUB 500
170  GOSUB 6000
180  FOR Q = 1 TO 25
190  GOSUB 5000
200  GOSUB 7000
210  GOSUB 8000
220  GOSUB 9000
230  GOSUB 10000
240  GOSUB 11000
250  GOSUB 12000
260  GOSUB 13000
270  GOSUB 17000
280  GOSUB 14000
290  NEXT Q
300  GOSUB 15000
310  PR# 1
320  PRINT  CHR$ (9);"G2RDL"
330  TEXT
340  PR# 0
350  FOR TD = 1 TO 500
     : NEXT TD
360  NEXT L
370  INPUT "DO YOU WANT TO SEE ANOTHER MACH NUMBER ? ";R$
380  IF R$ = "Y" THEN 110
390  END
500  REM  INITIALIZE GRAPHICS
510  HGR2
520  HCOLOR= 2
530  HPLOT 0,0 TO 0,42
     : HPLOT 0,50 TO 0,191
```

```
540  FOR X = 0 TO 278 STEP 2
550  HCOLOR= 2
560  HPLOT X,42
      : HPLOT X,117
570  HCOLOR= 1
580  HPLOT (X + 1),42
      : HPLOT (X + 1),117
590  NEXT X
600  HPLOT 275,44
      : HPLOT 279,44
      : HPLOT 275,45
      : HPLOT 279,45
      : HPLOT 277,47
610  HPLOT 275,119
      : HPLOT 279,119
      : HPLOT 275,120
      : HPLOT 279,120
      : HPLOT 277,122
620  HCOLOR= 2
      : HPLOT 276,46
      : HPLOT 278,46
630  HPLOT 276,121
      : HPLOT 278,121
640  RETURN
1000  REM  DIMENSION VARIABLES
1010  DIM OMEGAF(2),K(126),KI(126),LH(2),LA(2),MOMENT(2),COEFF(3,2),VEL(2
     ),G(2,126),KN(2),KP(2)
1020  RETURN
2000  REM  SET CONSTANTS
2010 PI = 3.14159265
2020 R1 = 0.165
      :R2 = 1
      :R3 = 0.335
      :R4 = 1
2030 I1 = 0
      :I2 =  - .0455
      :I3 = 0
      :I4 =  - .3
2040  RETURN
3000  REM  GET INPUT DATA
3010  HOME
3020  INPUT "MASS PER FOOT SPAN (SLUGS) ? ";MASS
3030  INPUT " C-G LOCATION (% OF CHORD) ? ";CG
3040  INPUT "WING MOMENT OF INERTIA ABOUT ROTATION POINT (SLUG-FT^2) ? ";
     IALPHA
3050  INPUT "ELASTIC AXIS LOCATION (% OF CHORD) ? ";EA
3060  INPUT "SEMI-CHORD LENGTH (FEET) ? ";SEMCH
3070  INPUT "NATURAL BENDING FREQUENCY (HZ) ? ";OH
3080  INPUT "NATURAL TORSION FREQUENCY (HZ) ? ";OA
3090  INPUT "STRUCTURAL DAMPING - BENDING (% CRITICAL DAMPING) ? ";GH
3100  INPUT "STRUCTURAL DAMPING - TORSION (% CRITICAL DAMPING) ? ";GT
3110  INPUT " FULL SPAN ASPECT RATIO ? ";RTIO
3120  PRINT
      : PRINT
3130  HOME
3140  REM  CHECK PARAMETERS FOR CORRECTNESS
3150  PR# 1
3160  PRINT  CHR$ (12)
3170  PRINT "INPUT PARAMETERS : "
3180  PRINT
```

```
     : PRINT "MASS PER FOOT SPAN : ";MASS;" SLUGS "
3190  PRINT
3200  PRINT "CG LOCATION : ";CG;" % CHORD"
3210  PRINT
3220  PRINT "PITCH INERTIA : ";IALPHA;" SLUG-FT^2 "
3230  PRINT
3240  PRINT "ELASTIC AXIS LOCATION : ";EA;" % CHORD "
3250  PRINT
3260  PRINT "SEMI-CHORD : ";SEMCH;" FEET"
3270  PRINT
3280  PRINT "NATURAL BENDING FREQUENCY : ";OH;" HZ"
3290  PRINT
3300  PRINT "NATURAL TORSION FREQUENCY : ";OA;" HZ"
3310  PRINT
3320  PRINT "BENDING DAMPING : ";GH;" % CRITICAL DAMPING"
3330  PRINT
3340  PRINT "TORSION DAMPING : ";GT;" %CRITICAL DAMPING "
3350  PRINT
3360  PRINT "FULL SPAN ASPECT RATIO : ";RTIO
3370  PRINT
      : PRINT
      : PR# 0
3380  PRINT
      : INPUT "IS THIS CORRECT (Y/N) ? ";A$
3390  PRINT
      : PRINT
      : PRINT
3400  IF A$ <  > "Y" THEN 3000
3410 EA = EA / 100
3420 GH = GH / 100
3430 GT = GT / 100
3440 CG = CG / 100
3450  RETURN
3500  REM  GET PLOT DATA
3510  HOME
3520  INPUT "MAXIMUM VELOCITY TO BE PLOTTED (FEET/SEC)  ? ";VMAX
3530  PRINT
3540  INPUT "MAXIMUM FREQUENCY TO BE PLOTTED (HZ) ? ";FMAX
3550  PRINT
3560  INPUT "MAXIMUM G-VALUE TO BE PLOTTED ? ";GMAX
3570  PR# 1
3580  HOME
3590  PRINT
      : PRINT "MAXIMUM VELOCITY TO BE PLOTTED : ";VMAX;" FEET/SEC"
3600  PRINT
      : PRINT "MAXIMUM FREQUENCY TO BE PLOTTED : ";FMAX;" HZ"
3610  PRINT
      : PRINT "MAXIMUM G-VALUE TO BE PLOTTED : ";GMAX
3620  PRINT
      : PR# 0
3630  PRINT
      : INPUT " IS THIS CORRECT (Y/N) ? ";A$
3640  PRINT "                    "
3650  IF A$ <  > "Y" THEN 3500
3660  HOME
3670  PRINT
      : INPUT "ARE INTERMEDIATE VALUES TO BE PRINTED ? ";N$
3680  PR# 1
3690  PRINT
      : PRINT
```

```
3700  PR# 0
3710  RETURN
4000  REM  READ K-VALUES
4010  FOR I = 1 TO 25
4020  READ K(I)
4030 KI(I) = 1 / K(I)
4040  NEXT I
4050  DATA  10.00,6.00,4.00,3.00,2.00,1.50,1.20,1.00,0.80,0.66,0.60,0.56,
     0.50,0.40,0.30,0.20,0.16,0.12,0.10,0.08,0.06,0.04,0.025,0.01,0.001
4060  RETURN
4500  REM  READ IN OR GET RHO VALUES
4510  HOME
4520  PRINT "INPUT OPTIONS ON RHO : "
      : PRINT
4530  PRINT " 1. SINGLE RHO-USER DECIDED"
4540  PRINT " 2. MULTIPLE RHO'S-USER DECIDED "
4550  PRINT
      : INPUT "OPTION ? ";CH
4560  IF CH = 1 THEN 4650
4570  PRINT
      : INPUT "HOW MANY RHO VALUES DO YOU WANT TO INPUT ";NR
4580  PRINT
      : INPUT "INTERVAL BETWEEN VALUES ? ";RI
4590  PRINT
      : INPUT "BEGINNING RHO VALUE ? ";BR
4600  DIM RHO(NR)
4610  FOR L = 1 TO NR
4620 RHO(L) = BR + (L - 1) * RI
4630  NEXT L
4640  RETURN
4650 NR = 1
4660  INPUT "RHO VALUE ? ";RHO(1)
4670  RETURN
5000  REM  CALCULATE THEODORSON VALUES
5010 L1 = I2 / K(Q)
      :L2 = I4 / K(Q)
5020 H = (R1 * R2 + I1 * L1) / ((R2 ^ 2) + (L1 ^ 2))
5030 J = (R3 * R4 + I3 * L2) / ((R4 ^ 2) + (L2 ^ 2))
5040 N = 1 - H - J
5050 HI = (R2 * I1 - R1 * L1) / ((R2 ^ 2) + (L1 ^ 2))
5060 JI = (R4 * I3 - R3 * L2) / ((R4 ^ 2) + (L2 ^ 2))
5070 NI = 0 - HI - JI
5080 LH(1) = 1 + 2 * NI * KI(Q)
5090 LH(2) =  - 2 * N * KI(Q)
5100 LA(1) = 1 / 2 + 2 * KI(Q) * NI - 2 * N * KI(Q) ^ 2
5110 LA(2) =  - KI(Q) - 2 * KI(Q) * N - 2 * (KI(Q) ^ 2) * NI
5120 LR(1) = LA(1)
      :LR(2) = LA(2)
5130  FOR Z = 1 TO 2
5140 LA(Z) = (1 / (1 + 2 / (.85 * RTIO))) * LR(Z)
5150  NEXT Z
5160 MOMENT(1) = 3 / 8
5170 MOMENT(2) =  - KI(Q)
5180  FOR Z = 1 TO 2
5190 MOMENT(Z) = LA(Z) / LR(Z) * MOMENT(Z)
5200  NEXT Z
5210 BETA = 1 /  SQR (1 - MCH ^ 2)
5220  FOR X = 1 TO 2
5230 LH(X) = LH(X) * BETA
5240 LA(X) = LA(X) * BETA
```

```
5250 MOMENT(X) = MOMENT(X) * BETA
5260  NEXT X
5270  RETURN
6000  REM  SET INTERMEDIATE CONSTANTS
6010 MU = MASS / (PI * RHO(L) * SEMCH ^ 2)
6020  IF MU > 4 THEN 6050
6030  PR# 1
     : PRINT "WARNING - LOW MASS RATIO MAY INVALIDATE EQUATIONS ";
6040  PR# 0
6050 XALPHA = 2 * CG - 2 * EA
6060 RALPHA2 = IALPHA / (MASS * (SEMCH ^ 2))
6070 PSI = MU * RALPHA2
6080 PHI =  - 1 / 2 + 2 * EA
6090 DLTA = (OH / OA) ^ 2
6100 TAU = MU * XALPHA
6110  RETURN
7000  REM  SET COEFFICIENTS
7010 COEFF(1,1) = MU * DLTA * PSI * (1 - GH * GT)
7020 COEFF(1,2) = MU * DLTA * PSI * (GT + GH)
7030 COEFF(2,1) = MU * DLTA * (PHI / 2 - MOMENT(1) + LA(1) * PHI - LH(1) *
     PHI ^ 2 - PSI - LA(2) * GH * PHI + GH * LH(2) * PHI ^ 2 + MOMENT(2) *
     GH) + PSI * (LH(2) * GT - LH(1) - MU)
7040 COEFF(2,2) = MU * DLTA * (PHI * LA(2) - PHI ^ 2 * LH(2) - MOMENT(2) +
     PHI / 2 * GH - MOMENT(1) * GH + LA(1) * PHI * GH - GH * LH(1) * PHI ^
     2 - PSI * GH) + PSI * ( - LH(1) * GT - LH(2) - MU * GT)
7050 COEFF(3,1) = PSI * LH(1) + MOMENT(1) * LH(1) + MU * PSI - MU * PHI /
     2 + MU * MOMENT(1) - MU * LA(1) * PHI + MU * LH(1) * PHI ^ 2 - MOMEN
     T(2) * LH(2) - TAU ^ 2 + 2 * LH(1) * PHI - 1 / 2 * LA(1) - TAU / 2 -
     TAU * LA(1)
7060 COEFF(3,2) = LH(2) * (PSI - PHI / 2 - LA(1) * PHI + LH(1) * PHI ^ 2)
      + (LH(1) + MU) * (MOMENT(2) - LA(2) * PHI + LH(2) * PHI ^ 2) + LH(2
     ) * PHI * (TAU + LA(1) - LH(1) * PHI) - (LA(2) - LH(2) * PHI) * (TAU
      + 1 / 2 - LH(1) * PHI)
7070  RETURN
8000  REM  SET 2ND INTERMEDIATE VALUES
8010 DR = COEFF(2,1) ^ 2 - COEFF(2,2) ^ 2 - 4 * COEFF(1,1) * COEFF(3,1) +
     4 * COEFF(1,2) * COEFF(3,2)
8020 DI = 2 * COEFF(2,1) * COEFF(2,2) - 4 * COEFF(1,1) * COEFF(3,2) + 4 *
     COEFF(1,2) * COEFF(3,1)
8030 SI =  SQR (( - DR +  SQR (DR ^ 2 + DI ^ 2)) / 2)
8040 SR = DI / (2 * SI)
8050  RETURN
9000  REM  SET 3RD INTERMEDIATE VALUES
9010 Y = (( - COEFF(2,1) + SR) * 2 * COEFF(1,1) + ( - COEFF(2,2) + SI) *
     2 * COEFF(1,2)) / (4 * (COEFF(1,1) ^ 2 + COEFF(1,2) ^ 2))
9020 Z = (( - COEFF(2,1) - SR) * 2 * COEFF(1,1) + ( - COEFF(2,2) - SI) *
     2 * COEFF(1,2)) / (4 * (COEFF(1,1) ^ 2 + COEFF(1,2) ^ 2))
9030 U = (2 * COEFF(1,1) * ( - COEFF(2,2) + SI) - ( - COEFF(2,1) + SR) *
     2 * COEFF(1,2)) / (4 * (COEFF(1,1) ^ 2 + COEFF(1,2) ^ 2))
9040 W = (2 * COEFF(1,1) * ( - COEFF(2,2) - SI) - ( - COEFF(2,1) - SR) *
     2 * COEFF(1,2)) / (4 * (COEFF(1,1) ^ 2 + COEFF(1,2) ^ 2))
9050  RETURN
10000  REM  SOLVE FOR FLUTTER FREQUENCIES
10010  IF Y < 0 THEN 10100
10020  IF Z < 0 THEN 10100
10030 OMEGAF(1) = OA /  SQR (Y)
10040 OMEGAF(2) = OA /  SQR (Z)
10050  RETURN
10100  TEXT
10110  PRINT "NEGATIVE NUMBER UNDER SQUARE ROOT WHEN TRYING TO SOLVE FOR
```

```
     FLUTTER FREQUENCY ";
10120  PRINT
     : PRINT "CALCULATIONS ABORTED";
10130  END
11000  REM  SOLVE FOR FLUTTER VELOCITY
11010 VEL(1) = SEMCH * OMEGAF(1) * 2 * PI / K(Q)
11020 VEL(2) = SEMCH * OMEGAF(2) * 2 * PI / K(Q)
11030  RETURN
12000  REM  SET 4TH INTERMEDIATE VALUES
12010 C1 = (OA / OMEGAF(1)) ^ 2
12020 C2 = (OA / OMEGAF(2)) ^ 2
12030  RETURN
13000  REM  SOLVE FOR G-VALUES
13010 G(1,Q) = U / C1
13020 G(2,Q) = W / C2
13030  RETURN
14000  REM  PRINT OUT PRELIMINARY DATA
14010  IF N$ <  > "Y" THEN 14040
14020  PR# 1
14030  PRINT
14040  PRINT "K-VALUE : ";K(Q)
14050  PRINT
     : PRINT "FREQUENCIES (HZ) : ";OMEGAF(1);"   ";OMEGAF(2)
14060  PRINT
     : PRINT "VELOCITIES (F/S) : ";VEL(1);"   ";VEL(2)
14070  PRINT
     : PRINT "G-VALUE      : ";G(1,Q);"   ";G(2,Q)
14080  PRINT
     : PRINT
14090  RETURN
15000  REM  DWELL ON FLUTTER
15010 MR = 0
15020 RL = 0
15030  FOR E = 1 TO 2
15040 F1 = 0
15050  FOR M = 1 TO 25
15060 S1 = G(E,M)
     :S2 = G(E,M + 1)
15070  IF S1 < 0 AND S2 > 0 AND F1 = 0 THEN  GOSUB 15400
15080  NEXT M
15090  NEXT E
15100  IF KN(1) > KN(2) THEN R = 1
15110  IF KN(2) > KN(1) THEN R = 2
15120 KL = KN(R)
     :KH = KP(R)
15130  FOR Q = 26 TO 125
15140 K(Q) = (KL + KH) / 2
15150 KI(Q) = 1 / K(Q)
15160  GOSUB 5000
15170  GOSUB 7000
15180  GOSUB 8000
15190  GOSUB 9000
15200  GOSUB 10000
15210  GOSUB 11000
15220  GOSUB 12000
15230  GOSUB 13000
15240  GOSUB 17000
15250  GOSUB 16000
15260  IF MR = 1 THEN 15310
15270  IF G(1,Q) > 0 AND G(2,Q) > 0 THEN KH = K(Q)
```

```
15280  IF G(1,Q) < 0 AND G(2,Q) < 0 THEN KL = K(Q)
15290  IF (G(1,Q) > 0 AND G(2,Q) < 0) OR (G(1,Q) < 0 AND G(2,Q) > 0) THEN
     KH = K(Q)
15300  NEXT Q
15310  RETURN
15400 F1 = 1
      :KN(E) = K(M)
      :KP(E) = K(M + 1)
      : RETURN
16000  REM  PRINT DWELLING DATA
16010  IF N$ <  > "Y" THEN 16040
16020  PR# 1
16030  PRINT
16040  PRINT "K-VALUE  : ";K(Q)
16050  PRINT
      : PRINT "FREQUENCIES : ";OMEGAF(1);"   ";OMEGAF(2)
16060  PRINT
      : PRINT "VELOCITIES  : ";VEL(1);"   ";VEL(2)
16070  PRINT
      : PRINT "G-VALUES    : ";G(1,Q);"   ";G(2,Q)
16080  IF RL = 1 THEN 16170
16090  IF  ABS (VEL(1) - VA) <  = 1 OR  ABS (VEL(2) - VB) <  = 1 THEN 161
     20
16100 VA = VEL(1)
      :VB = VEL(2)
16110  GOTO 16190
16120 RL = 1
      :MR = 1
16130  PR# 1
16140  PRINT
16145  IF N$ = "Y" THEN  PRINT  CHR$ (12)
16150  PRINT "THE AIRCRAFT WILL FLUTTER NEAR THE FOLLOWING VALUES "
16160  GOTO 16030
16170  PRINT
16180  PRINT GMAX * ( - 1);"   STABLE          DAMPING    UNSTABLE     ";GM
     AX;"  O FREQ (HZ)  ";FMAX;
16190  RETURN
17000  REM  PLOT DATA
17010 X1 =  INT (VEL(1) / VMAX * 280)
17020 X2 =  INT (VEL(2) / VMAX * 280)
17030 Y1 =  INT (G(1,Q) / GMAX * 75)
17040 Y2 =  INT (G(2,Q) / GMAX * 75)
17050 Z1 =  INT (OMEGAF(1) / FMAX * 42)
17060 Z2 =  INT (OMEGAF(2) / FMAX * 42)
17070 Y1 = 117 - Y1
      :Y2 = 117 - Y2
      :Z1 = 42 - Z1
      :Z2 = 42 - Z2
17080  IF X1 > 279 THEN 17160
17090  IF Z1 < 0 THEN 17140
17100  IF X1 = (2 *  INT (X1 / 2)) THEN  GOTO 17120
17110  HCOLOR= 1
      : GOTO 17130
17120  HCOLOR= 2
17130  HPLOT X1,Z1
17140  IF Y1 < 50 OR Y1 > 191 THEN 17160
17150  HPLOT X1,Y1
17160  IF X2 > 279 THEN 17240
17170  IF Z2 < 0 THEN 17220
17180  IF X2 = (2 *  INT (X2 / 2)) THEN  GOTO 17200
```

```
17190  HCOLOR= 1
     : GOTO 17210
17200  HCOLOR= 2
17210  HPLOT X2,Z2
17220  IF Y2 < 50 OR Y2 > 191 THEN 17240
17230  HPLOT X2,Y2
17240  RETURN
```

APPENDIX B

Sample Flutter Problem for Computer Program

Due to the fact that errors can occur in entering the program in the computer, a sample case is given which can be used to determine if the program is working correctly. The sample problem appears in reference 4 pp. 219-224. The listed values for input are,

$$\mu = 76$$
$$a = -0.15$$
$$x_\alpha = .25$$
$$r_\alpha^2 = .388$$
$$b = 5 \text{ inches}$$
$$\omega_\alpha = 64.1 \text{ radians/sec}$$
$$\omega_h = 55.9 \text{ radians/sec}$$
$$g_h = g_\alpha = 0$$

As most of these parameters are not in the units required as input for the program, the units are converted as follows:

$$b = .4167 \text{ ft}$$
$$f_h = \omega_h/2\pi = 8.9 \text{ Hz}$$
$$f_\alpha = \omega_\alpha/2\pi = 10.2 \text{ Hz}$$
$$EA\% = (1/2 + a/2)\times 100 = 42.5\%$$
$$CG\% = EA\% + (x_\alpha/2 \times 100) = 55\%$$

Assuming sea level air density

$$\rho = .00237 \text{ slugs/ft}^3$$
$$m = \mu\pi\rho b^2 = .098 \text{ slugs/ft-span}$$

Therefore,

$$I_\alpha = r_\alpha^2 m b^2 = .0066 \text{ slug-ft}^2\text{/ft-span}$$

Furthermore, incompressible flow and infinite span are assumed. Therefore,

$$M = 0$$
$$AR = 100{,}000{,}000 \text{ (Essentially infinite)}$$

The input parameters appear in Table I.

The full solution including the intermediate values is presented herein beginning on page 24. The solution is also plotted in Figure 6. The flutter solution is found by tracking one root of the quadratic to the crossing of the axis. The two successive k-values which have damping values on opposite sides of the axis are then averaged to determine the next k-value to be used. The solution process continues until the velocity difference for two successive k-values is less than one foot per second. In other words, after first calculating that the flutter solution is between k-values of 0.3 and 0.2, the program then iterates to find that the flutter solution is at a k-value of 0.28125. The flutter velocity is 89.3 ft/sec. The difference of this soluiton from the one found in reference 4 is less than one percent. The reason for this difference may be due to the aerodynamic approximation of the program. The aerodynamics utilized by reference 4 are the actual values calculated from the Bessel functions whereas the algorithm used in the program is an approximation of these functions.

It is also interesting to note that the calculations at k-values of 0.01 and 0.001 indicate a second instability at about 173 ft/sec where both the frequency and damping of one of the modes goes to zero. This is the static divergence speed. The divergence phenomena is discussed in textbooks such as references 3, 4, and 6.

MACH NUMBER : 0   AIR DENSITY : 2.37E-03 SLUGS/FT^3

K-VALUE : 10

FREQUENCIES (HZ) : 12.4688466   7.90074515

VELOCITIES (F/S) : 3.26459755   2.06857571

G-VALUE : -7.63529253E-05   -1.87306349E-03

K-VALUE : 6

FREQUENCIES (HZ) : 12.4655681   7.90116384

VELOCITIES (F/S) : 5.43956527   3.44780889

G-VALUE : -1.41403715E-04   -3.12198597E-03

K-VALUE : 4

FREQUENCIES (HZ) : 12.4588868   7.90205072

VELOCITIES (F/S) : 8.15497465   5.17229385

G-VALUE : -2.52936026E-04   -4.68383308E-03

K-VALUE : 3

FREQUENCIES (HZ) : 12.4494208   7.90331766

VELOCITIES (F/S) : 10.8650383   6.8974975

G-VALUE : -4.12481448E-04   -6.24684599E-03

K-VALUE : 2

FREQUENCIES (HZ) : 12.4221688   7.90697101

VELOCITIES (F/S) : 16.2618817   10.3510289

G-VALUE : -9.33100061E-04   -9.37824872E-03

K-VALUE : 1.5

FREQUENCIES (HZ) : 12.3835824   7.9121559

VELOCITIES (F/S) : 21.6151576   13.8104219

G-VALUE : -1.8052455E-03   -.0125208874

K-VALUE : 1.2

FREQUENCIES (HZ) : 12.333302 7.91894171

VELOCITIES (F/S) : 26.9092435 17.2778328

G-VALUE : -3.10577846E-03 -.0156811715

K-VALUE : 1

FREQUENCIES (HZ) : 12.2709408 7.9274217

VELOCITIES (F/S) : 32.127818 20.7556018

G-VALUE : -4.88165899E-03 -.0188676793

K-VALUE : .8

FREQUENCIES (HZ) : 12.1539559 7.94358842

VELOCITIES (F/S) : 39.7769097 25.9974119

G-VALUE : -8.45986823E-03 -.0237219461

K-VALUE : .66

FREQUENCIES (HZ) : 11.9981948 7.96581205

VELOCITIES (F/S) : 47.5965354 31.600175

G-VALUE : -.0133534084 -.0290210464

K-VALUE : .6

FREQUENCIES (HZ) : 11.8942858 7.98119105

VELOCITIES (F/S) : 51.902764 34.8273015

G-VALUE : -.0165571556 -.0321504883

K-VALUE : .56

FREQUENCIES (HZ) : 11.8050037 7.99481965

VELOCITIES (F/S) : 55.1926781 37.3786845

G-VALUE : -.0192229777 -.0346779895

K-VALUE : .5

FREQUENCIES (HZ) : 11.6273907 8.02324407

VELOCITIES (F/S) : 60.8857458 42.0129684

G-VALUE : -.0242034231 -.0394240991

K-VALUE : .4

FREQUENCIES (HZ) : 11.1289104 8.11506943

VELOCITIES (F/S) : 72.8443754 53.1172543

G-VALUE : -.0351844704 -.052067462

K-VALUE : .3

FREQUENCIES (HZ) : 9.99785211 8.44082058

VELOCITIES (F/S) : 87.2546984 73.6659481

G-VALUE : -.0347203235 -.0898817964

K-VALUE : .2

FREQUENCIES (HZ) : 8.81288397 8.56418574

VELOCITIES (F/S) : 115.36961 112.113898

G-VALUE : .243699161 -.437955103

K-VALUE : .16

FREQUENCIES (HZ) : 8.45132548 8.12400021

VELOCITIES (F/S) : 138.295552 132.939277

G-VALUE : .332460844 -.561335102

K-VALUE : .12

FREQUENCIES (HZ) : 7.85576763 7.2834035

VELOCITIES (F/S) : 171.399974 158.911927

G-VALUE : .382571844 -.632265805

K-VALUE : .1

FREQUENCIES (HZ) : 7.42124838 6.5961167

VELOCITIES (F/S) : 194.303371 172.699746

G-VALUE : .366175506 -.610284063

K-VALUE : .08

FREQUENCIES (HZ) : 6.93017177 5.60964015

VELOCITIES (F/S) : 226.807484 183.58973

G-VALUE : .285409318 -.506082019

K-VALUE : .06

FREQUENCIES (HZ) : 6.69811934 4.23294678

VELOCITIES (F/S) : 292.283971 184.711921

G-VALUE : .147128775 -.334656658

K-VALUE : .04

FREQUENCIES (HZ) : 6.76022733 2.75350606

VELOCITIES (F/S) : 442.49124 180.230967

G-VALUE : .0668321095 -.219908692

K-VALUE : .025

FREQUENCIES (HZ) : 6.80672942 1.68681855

VELOCITIES (F/S) : 712.856061 176.657357

G-VALUE : .0351797178 -.147910484

K-VALUE : .01

FREQUENCIES (HZ) : 6.83071834 .66366487

VELOCITIES (F/S) : 1788.42094 173.760956

G-VALUE : .01282117 -.0636466531

```
K-VALUE : 1E-03

FREQUENCIES (HZ) : 6.57653004    .0661068688

VELOCITIES (F/S) : 17218.6928    173.081223

G-VALUE      : 1.64635228E-03   -6.47606803E-03


K-VALUE  : .25

FREQUENCIES : 9.13877632    8.80154616

VELOCITIES  : 95.7086978    92.1769493

G-VALUES    : .102772118    -.256309331

K-VALUE  : .275

FREQUENCIES : 9.44658006    8.70630525

VELOCITIES  : 89.9384262    82.8904628

G-VALUES    : 5.42088915E-03   -.142427175

K-VALUE  : .2875

FREQUENCIES : 9.7390505    8.55501294

VELOCITIES  : 88.691528    77.9087416

G-VALUES    : -.0240007964    -.106496951

K-VALUE  : .28125

FREQUENCIES : 9.59222603    8.62878413

VELOCITIES  : 89.2956372    80.3267954

G-VALUES    : -.012886655    -.120744428



THE AIRCRAFT WILL FLUTTER NEAR THE FOLLOWING VALUES

K-VALUE  : .28125

FREQUENCIES : 9.59222603    8.62878413

VELOCITIES  : 89.2956372    80.3267954

G-VALUES    : -.012886655    -.120744428
```

Note: When the program is executed, a plot of the results similar to the one shown in Figure 6 will be output following the listing.

APPENDIX C

A matched point solution is presented in this section. The solution is calculated for four different Mach numbers and the solution is presented in terms of dynamic pressure, altitude and equivalent airspeed.

The wing to be analyzed is taken from reference 3, page 203. An aspect ratio of eight was assumed. The input appears in table II. Air density option two was chosen and calculations were made for six value of air density. The interval between values was 0.0004 and the starting value was 0.0004. The speed of sound at each air density was obtained from a standard atmosphere chart and then multiplied by the appropriate Mach number to obtain the coreresponding velocity.

The results for each Mach number are in Table III.

Results at M = .4. When the flutter velocity is plotted versus air density as shown in Figure 7, it is seen that the aircraft does not flutter at M = .4 at any altitude above sea level. Therefore, at M = .4 the aircraft is safe from flutter.

Results at M = .5. The velocity is plotted versus air density in Figure 8 the curve intersects the Mach 0.5 curve at an air density value of .0021 slugs/ft^3 and a velocity of 550 ft/sec.

Therefore,

$$q = 1/2\rho V^2 = 317.6 \text{ lb/ft}^2$$

The altitude H, in feet, can be calculated for altitudes less than about 40,000 feet using the following equation which was derived from equations obtained from reference 5.

$$H = 145450.\left[1-\left(\frac{\rho}{.00237}\right)^{.23496}\right]$$

Thus for ρ = .0021 the altitude is H = 4075 ft.
The equivalent airspeed is:

$$V_e = V\sqrt{\frac{\rho}{.00237}} = 517.7 \text{ ft/sec}$$

Therefore, the aircraft is safe from flutter at M = .5 for altitudes greater than 4075 ft. If the plane was at 4075 ft., the aircraft would flutter when the equivalent airspeed is 517 ft/sec. For a more detailed description of the relevance of equivalent airspeed see reference 8.

Results at M = .6

Using the above procedure, a matched point is found by using Figure 9. The plot indicates the curves crossing at an air density of 0.0014 slugs/ft^3 and a velocity of 630 ft/sec. The flutter conditions are therefore:

$$q = 277.8 \text{ lb/ft}^2$$
$$H = 16922 \text{ ft}$$
$$V_e = 484.2 \text{ ft/sec}$$

Results at M = .8

Using Figure 10, the matched point occurs at

$$\rho = .00065 \text{ slugs/ft}^3$$
$$V = 780 \text{ ft/sec}$$

Therefore,

$$q = 197.8 \text{ lbs/ft}^2$$
$$H = 38124 \text{ ft}$$
$$V_e = 408.5 \text{ ft/sec}$$

When plotted as dynamic pressure versus Mach number as shown in Figure 11, the flutter boundary is described. However, when plotted as equivalent airspeed versus Mach number as in Figure 12, the flutter boundary takes on added meaning when the altitude lines are drawn. This plot makes it easy to visualize where the aircraft is safe in terms of altitude and equivalent air speed. The abbreviation "K" means thousand and is utilized in terms of feet (i.e., 10K means 10,000 feet altitude).

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Table I.- Input for problem in Appendix B.

INPUT PARAMETERS :

MASS PER FOOT SPAN : .098 SLUGS

CG LOCATION : 55 % CHORD

PITCH INERTIA : 6.6E-03 SLUG-FT^2

ELASTIC AXIS LOCATION : 42.5 % CHORD

SEMI-CHORD : .4167 FEET

NATURAL BENDING FREQUENCY : 8.9 HZ

NATURAL TORSION FREQUENCY : 10.2 HZ

BENDING DAMPING : 0 % CRITICAL DAMPING

TORSION DAMPING : 0 %CRITICAL DAMPING

FULL SPAN ASPECT RATIO : 100000000

MAXIMUM VELOCITY TO BE PLOTTED : 200 FEET/SEC

MAXIMUM FREQUENCY TO BE PLOTTED : 15 HZ

MAXIMUM G-VALUE TO BE PLOTTED : .05

Table II.- Input for problem in Appendix C.

```
INPUT PARAMETERS :
MASS PER FOOT SPAN : .6516 SLUGS
CG LOCATION : 46 % CHORD
PITCH INERTIA : 3.375 SLUG-FT^2
ELASTIC AXIS LOCATION : 35 % CHORD
SEMI-CHORD : 3.125 FEET
NATURAL BENDING FREQUENCY : 9.9 HZ
NATURAL TORSION FREQUENCY : 16.02 HZ
BENDING DAMPING : 0 % CRITICAL DAMPING
TORSION DAMPING : 0 %CRITICAL DAMPING
FULL SPAN ASPECT RATIO : 8

MAXIMUM VELOCITY TO BE PLOTTED : 1200 FEET/SEC
MAXIMUM FREQUENCY TO BE PLOTTED : 20 HZ
MAXIMUM G-VALUE TO BE PLOTTED : .05
```

Table III.- Results of Matched Point Problem in Appendix C

Mach Number M	Air Density ρ, slugs/ft^3	Flutter Velocity V, ft/sec	Mach Number Velocity, MV_a, ft/sec
.4	.0004	1192	387
	.0008	862	393
	.0012	717	412
	.0016	630	426
	.0020	572	438
	.0024	529	447
.5	.0004	1161	484
	.0008	840	492
	.0012	699	515
	.0016	614	533
	.0020	559	547
	.0024	516	559
.6	.0004	1118	581
	.0008	810	590
	.0012	674	618
	.0016	594	640
	.0020	539	656
	.0024	500	670
.8	.0004	977	774
	.0008	709	786
	.0012	593	824
	.0016	525	853
	.0020	480	875
	.0024	448	894

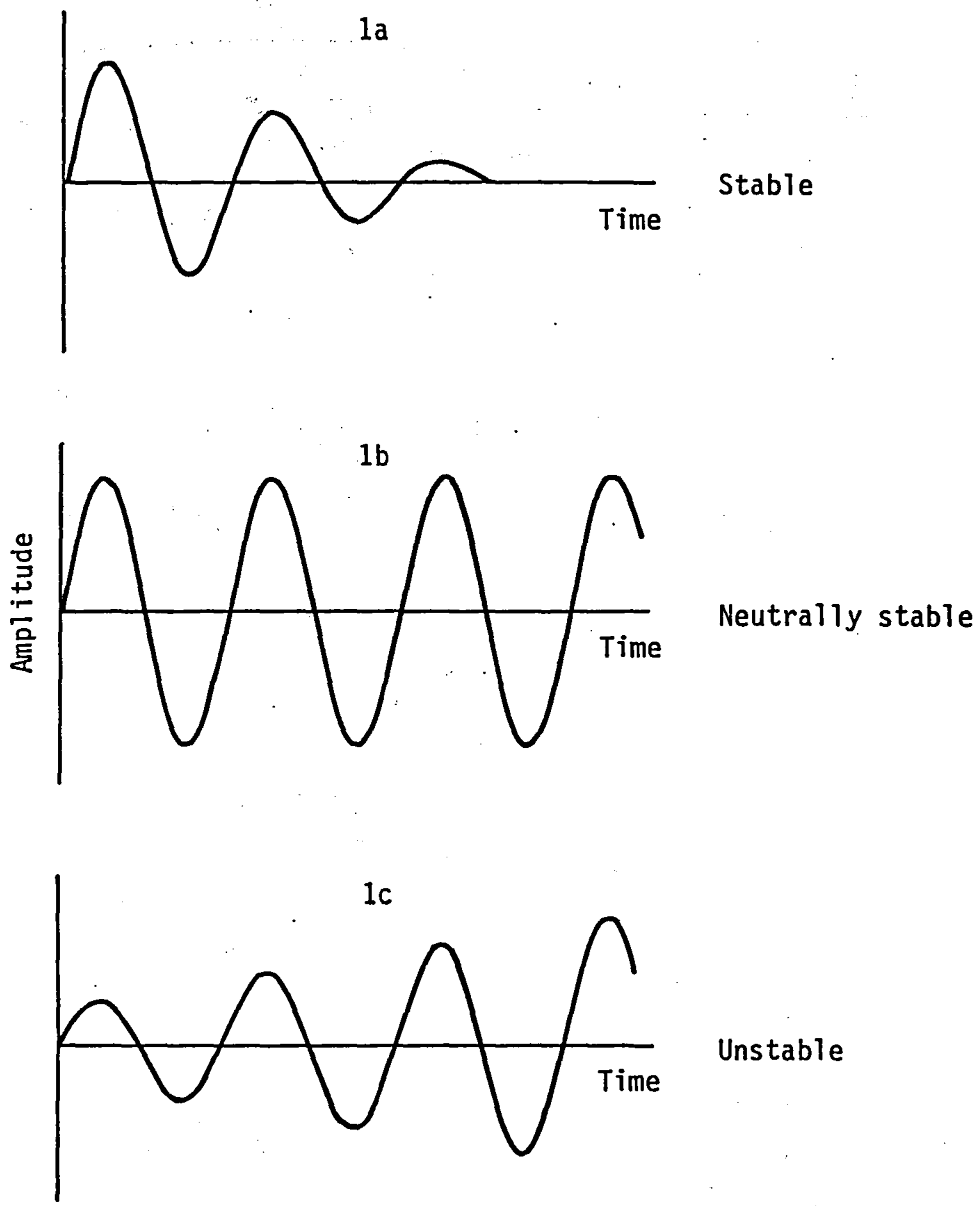

Figure 1.- Time histories of stability.

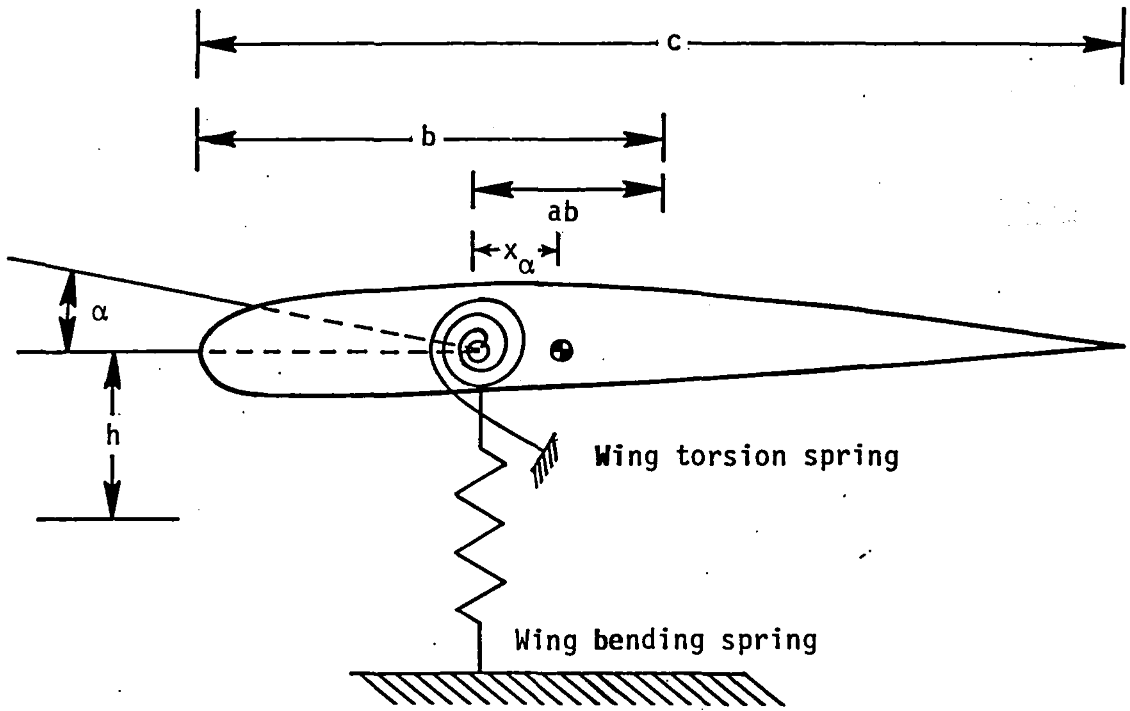

Figure 2.- Wing parameters utilized in analysis.

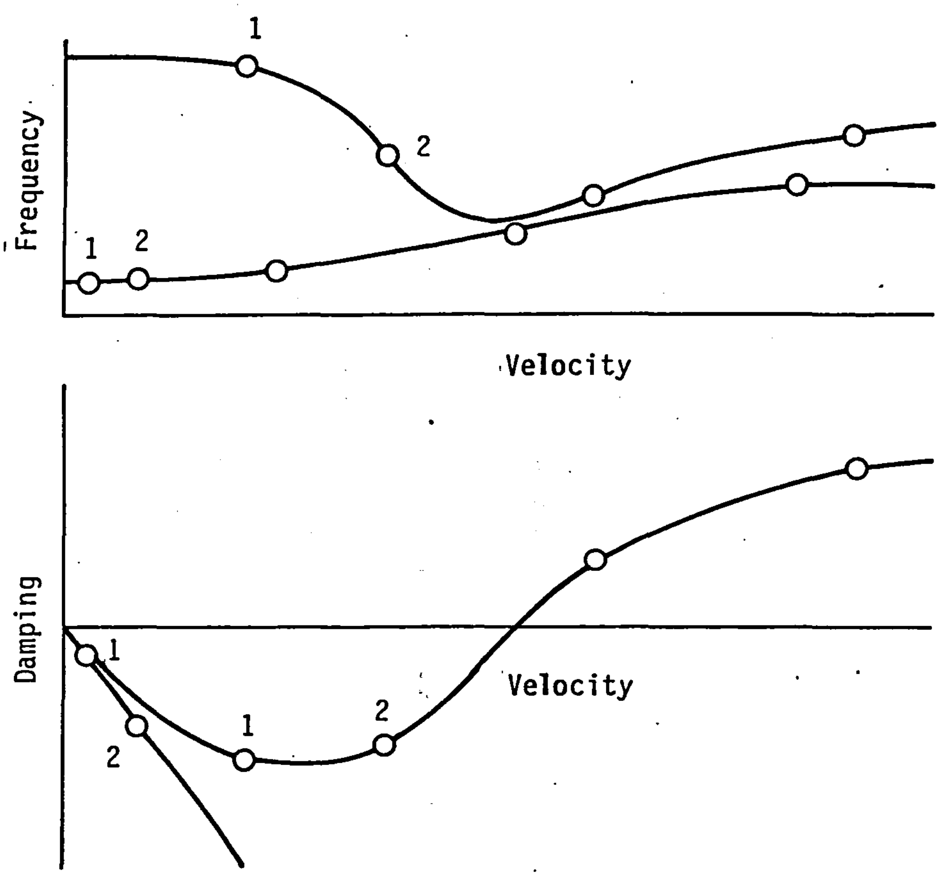

Figure 3.- Typical plots for a flutter solution.

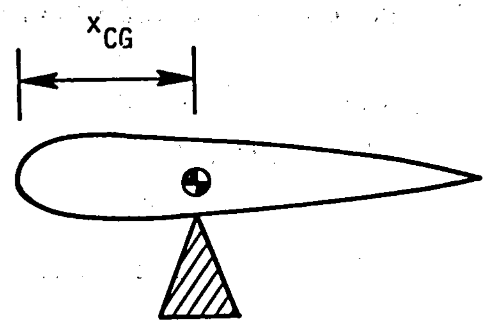

Figure 4.- Location of center of gravity of a wing.

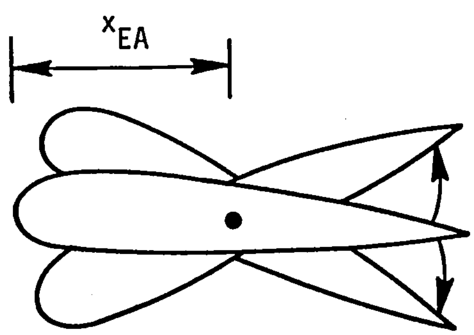

Figure 5.- Location of elastic axis of a wing.

Mach number: 0   Air density: 2.37E-03 slugs/ft^3

The aircraft will flutter near the following values

K-value: .28125

Frequencies:	9.59222603	8.62878413
Velocities :	89.2956372	80.3267954
G-values :	-.012886655	-.120744428

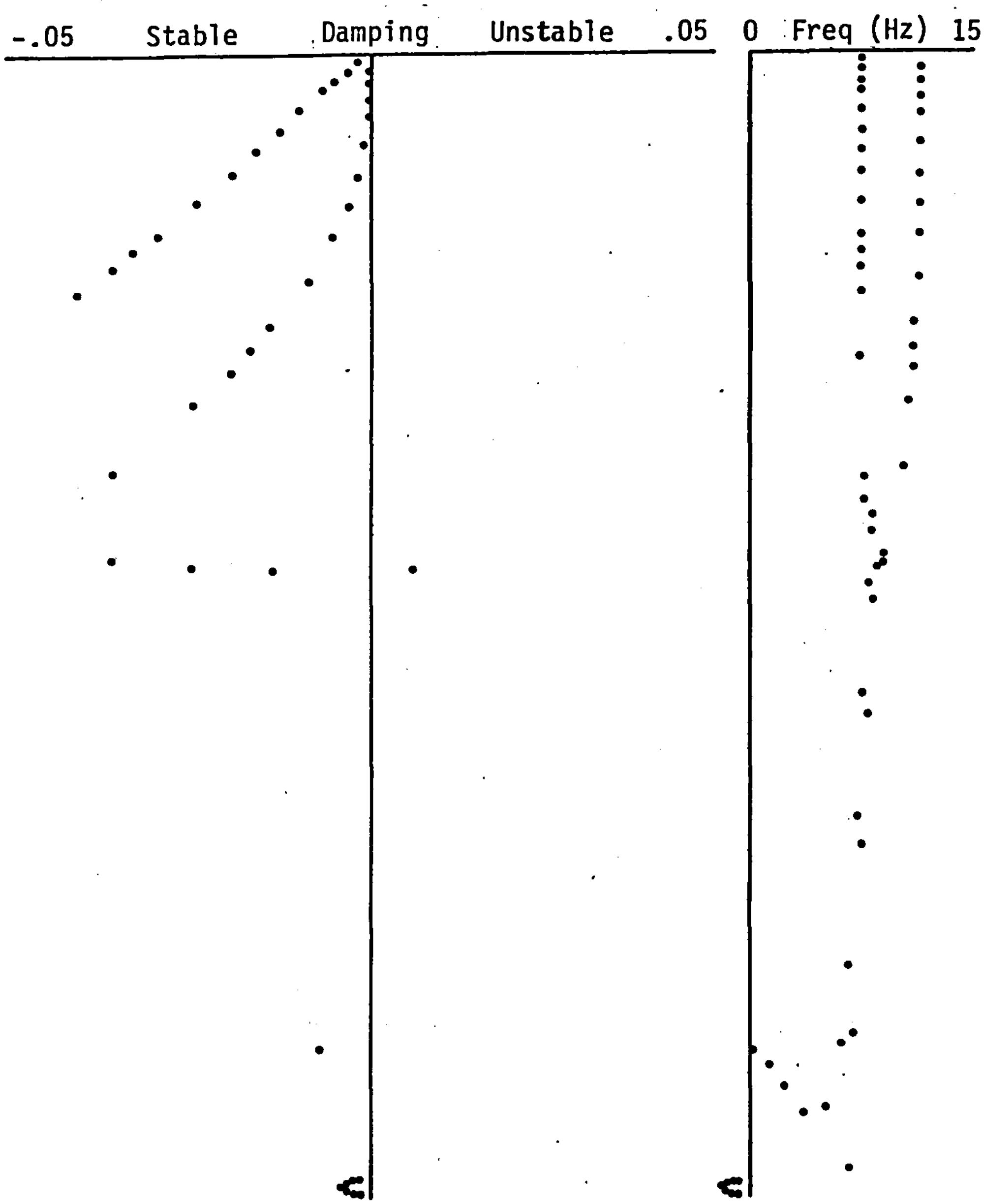

Figure 6.- Solution of sample problem in Appendix B.

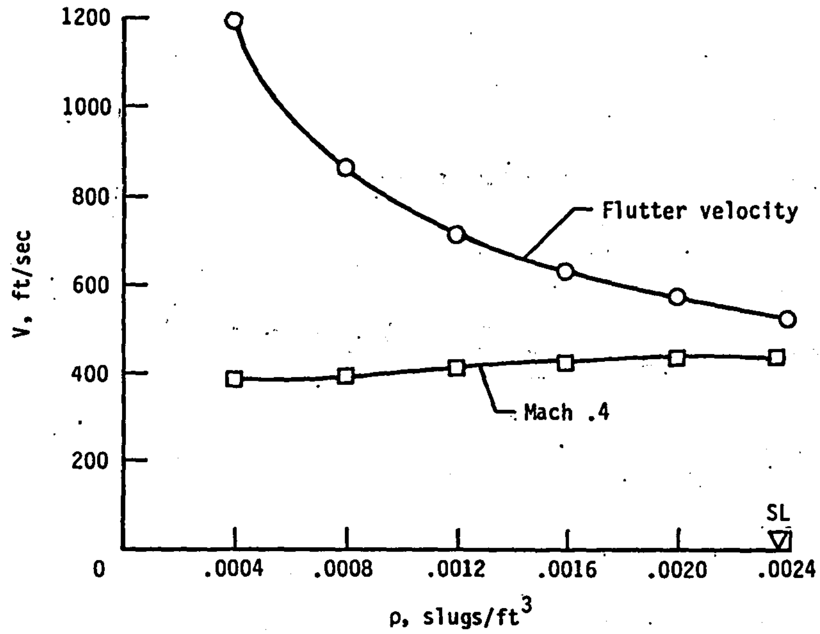

Figure 7.- Matched point for Mach .4.

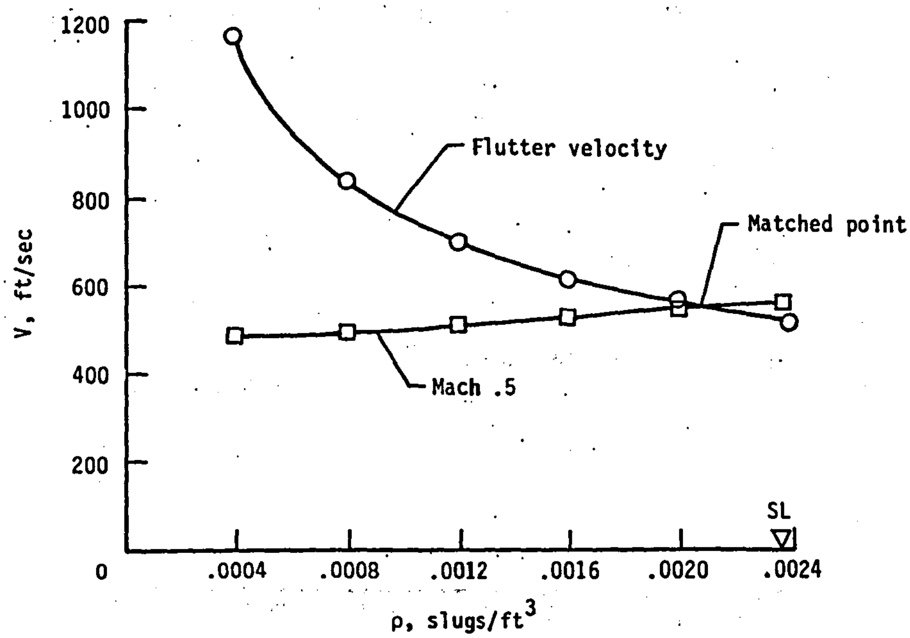

Figure 8.- Matched point for Mach .5.

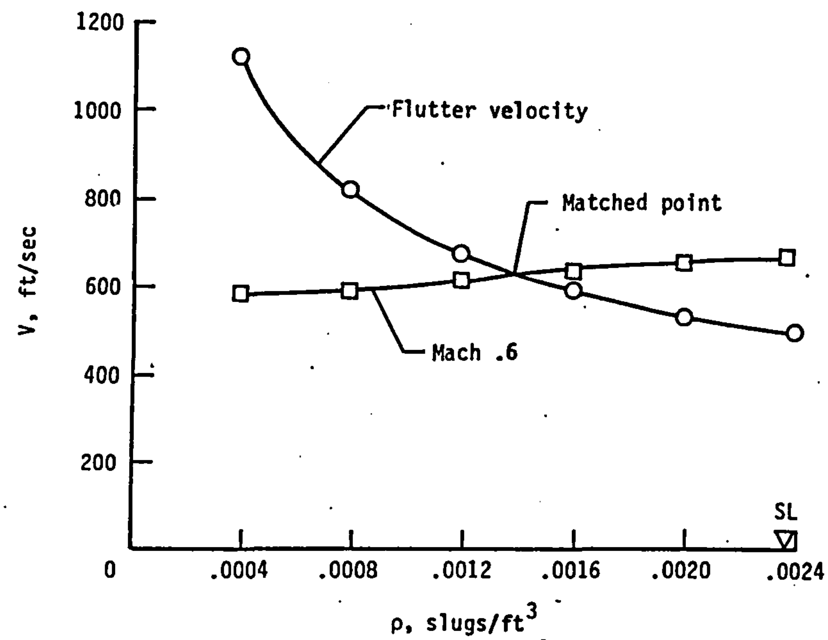

Figure 9.- Matched point for Mach .6.

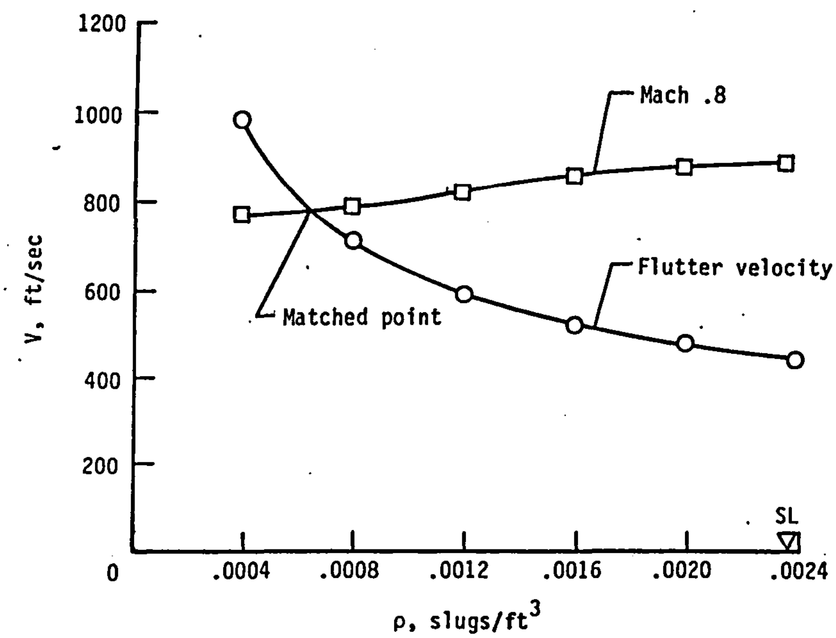

Figure 10.- Matched point for Mach .8.

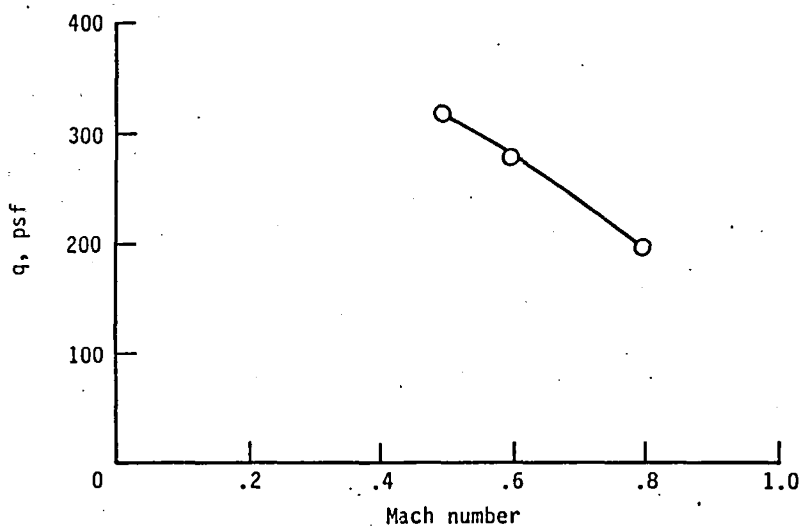

Figure 11.- Dynamic pressure flutter boundary.

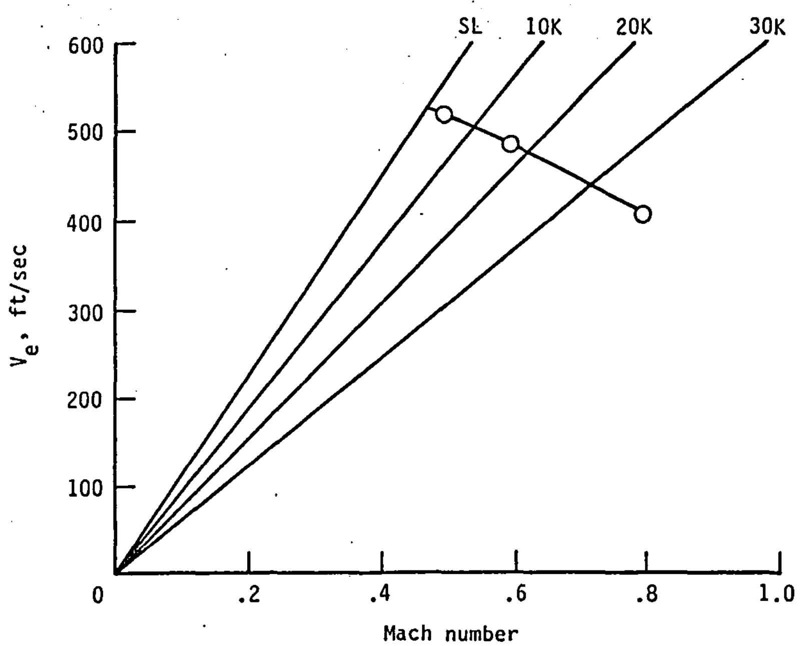

Figure 12.- Equivalent airspeed flutter boundary.

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16. Abstract

A computer programmed flutter solution has been written in the BASIC language for a personal computer. The program is for two degree-of-freedom bending-torsion flutter applications and utilizes two dimensional Theodorsen aerodynamics. The aerodynamics have been modified to include approximations for Mach number (compressibility) effects and aspect ratio (finite span) effects. The report contains a description of the program including input options, user instructions, program listing, and a test case application.

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