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# APPROXIMATE NEUTRAL POINT OF SUBSONIC CANARD AIRCRAFT

James D. Phillips

## SUMMARY

The purpose of this report is to derive an approximate formula for the position of the neutral point in canard aircraft. This formula accounts for the aerodynamic interference between the wing (rear wing) and the canard (forward wing). The report is organized into three main sections: determination of the canard downwash derivative, $\epsilon_{\alpha_c}$; determination of the canard and wing liftslopes including the aerodynamic interference; and calculation of the neutral point position.

## SYMBOLS

| | |
|---|---|
| $AR$ | aspect ratio, the ratio of wingspan to average chord |
| $a_c$ | lift curve slope of the canard alone |
| $a_w$ | lift curve slope of the wing alone |
| $C_{L_c}$ | canard lift coefficient based upon canard planform area |
| $C_{L_w}$ | wing lift coefficient based upon wing planform area |
| $C_{L_{\alpha c}}$ | lift curve slope of the canard including wing interference |
| $C_{L_{\alpha w}}$ | lift curve slope of the wing including canard interference |
| $i_c$ | geometric incidence of the canard at aircraft zero angle of attack ($\alpha = 0$) |

| | |
|---|---|
| $i_w$ | geometric incidence of the wing at aircraft zero angle of attack ($\alpha = 0$) |
| $L$ | lift force |
| $L_\alpha$ | rate of change of lift with angle of attack |
| $l$ | distance between the canard aerodynamic center and the wing aerodynamic center |
| $M_\alpha$ | rate of change of total aircraft moment about the CG with angle of attack |
| $P_1, P_2$ | pressure distributions over arbitrary lifting surfaces, condition 2 with the flow reversed |
| $q$ | dynamic pressure |
| $S$ | total aircraft planform area |
| $S_c$ | canard planform area |
| $S_w$ | wing planform area |
| $x_{np}$ | horizontal distance of the neutral point aft of the canard aerodynamic center |
| $\alpha$ | aircraft angle of attack relative to an arbitrary reference axis, usually parallel to the fuselage centerline |
| $\alpha_1, \alpha_2$ | angles of attack of arbitrary lifting surfaces, condition 2 with the flow reversed |
| $\alpha_c$ | actual angle of attack of the canard including wing upwash |
| $\alpha_w$ | actual angle of attack of the wing including canard downwash |
| $\epsilon_{\alpha_c}$ | rate of change of *downwash* at the wing due to positive change in the canard angle of attack |

$\epsilon_{\alpha_w}$ rate of change of *upwash* at the canard due to positive change in the wing angle of attack

$\bar{\epsilon}$ dimensionless downwash, $\epsilon/\alpha_i$, downwash is defined as negative

$\left(\frac{\partial \epsilon}{\partial \alpha}\right)_w$ rate of change of downwash at a virtual horizontal tail placed the same distance downstream from the wing as the canard is placed forward of the wing

subscripts:

$c$ canard or front wing

$w$ wing or rear wing

1 condition 1 in applying the Reverse Flow Theorem

2 condition 2 is with the flow reversed

## CANARD DOWNWASH DERIVATIVE

In order to calculate the static longitudinal stability of a canard aircraft, it is necessary to find the rate of change of downwash angle at the wing for a change in angle of attack of the canard, which is designated $\epsilon_{\alpha_c}$.

For a conventional configuration with the horizontal tailspan much less than the wingspan, the downwash angle can be assumed approximately constant across the horizontal tailspan. The downwash derivative is then linearly related to the downwash angle by the following equation:

$$\frac{\partial \epsilon}{\partial \alpha} = \frac{\epsilon}{C_L} C_{L_\alpha} = \frac{\bar{\epsilon}(x,z)}{\pi AR} C_{L_\alpha} \tag{1}$$

where $C_{L_\alpha}$ refers to the three-dimensional lift slope of the wing alone and is given, for example, by equation (3.47b) (p. 137 in ref. 1) and

$\bar{\epsilon}$ can be found from equation (26) or equation (29) (pp. 14,18 in ref. 2).

The canard problem is more difficult because constant downwash at the wing due to the canard cannot be assumed. In fact, the influence of the canard ranges from upwash outboard of the canard wing tip to theoretically infinite velocities if the trailing vortices impinge on the wing. To be treated properly, the downwash field of the canard should be integrated over the wingspan.

Fortunately, the Reverse Flow Theorem allows the difficult canard problem to be transformed into the easier conventional airplane problem. The Reverse Flow Theorem states that for two lifting surfaces, the influence of the first on the second is the same as the influence of the second on the first with flow in the opposite direction. Mathematically, in its most general form (ref. 3)

$$\int \alpha_1 P_2 dS = \int \alpha_2 P_1 dS \tag{2}$$

where $dS$ is a differential surface area. $\alpha_1, P_1$ refer to some angle of attack distribution and its associated lift distribution with the flow in one direction and $\alpha_2, P_2$ refer to some *other* angle of attack distribution and associated lift distribution with the flow in the reverse direction. The pressure distributions must be related to the angle of attack distributions by the linear potential flow equations.

In applying the Reverse Flow Theorem to the canard problem, it will be convenient to chose the angle of attack distributions as shown in figure 1. The left side of figure 1 shows flow from the left passing over the canard at unit angle of attack and affecting the wing which is at zero angle of attack. In condition 2 (the right side of fig. 1), flow is from the right, passing over the wing at unit angle of attack and affecting the canard (now horizontal tail) at zero angle of attack. Applying equation (2)

$$\int_c \alpha_{1_c} P_{2_c} dS + \int_w \alpha_{1_w} P_{2_w} dS = \int_c \alpha_{2_c} P_{1_c} dS + \int_w \alpha_{2_w} P_{1_w} dS \tag{3}$$

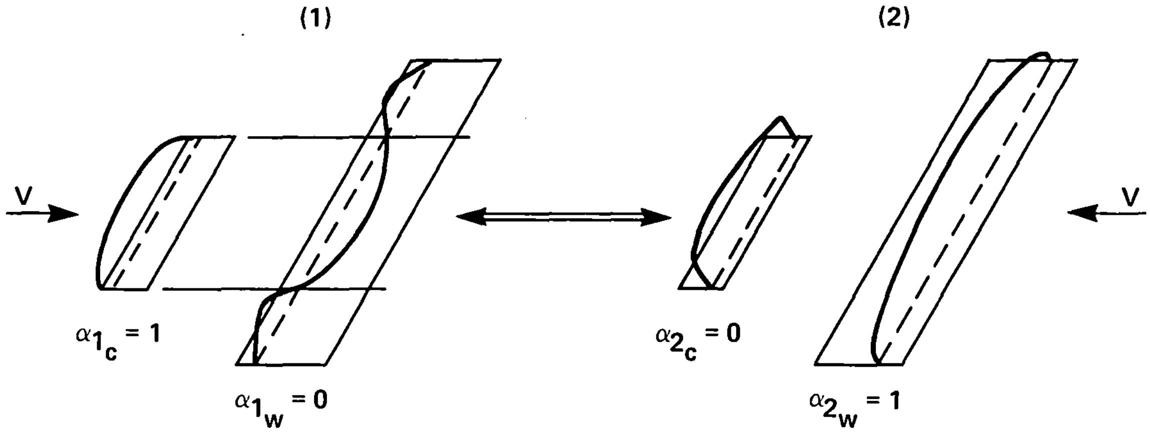


Figure 1. - Reverse flow theorem for canards.

Substituting the angle of attack distributions from figure 1:

$$\int_c P_{2_c} dS = \int_w P_{1_w} dS \tag{4}$$

The left-hand side of equation (3), the integral of the pressure over the canard due to deflecting the wing (represented by the right-hand side of fig. 1) is the same as the product of the canard liftslope and the downwash rate of the wing. Similarly, the right-hand side of equation (3), the integral of the pressure over the wing due to deflecting the canard (left-hand side of fig. 1) is the same as the product of the wing liftslope and the downwash rate of the canard (the desired quantity). Mathematically this is

$$\begin{aligned} \int_c P_{2_c} dS &= L_{\alpha_c} \left(\frac{\partial \epsilon}{\partial \alpha}\right)_w &= a_c S_c q \left(\frac{\partial \epsilon}{\partial \alpha}\right)_w \\ \int_w P_{1_w} dS &= L_{\alpha_w} \epsilon_{\alpha_c} &= a_w S_w q \epsilon_{\alpha_c} \end{aligned} \tag{5}$$

Substituting into equation (3) and solving for $\epsilon_{\alpha_c}$

$$\epsilon_{\alpha_c} = \frac{a_c S_c}{a_w S_w} \left(\frac{\partial \epsilon}{\partial \alpha}\right)_w = a_c \frac{S_c}{S_w} \frac{\bar{\epsilon}(x, z)}{\pi A R_w} \tag{6}$$

where $x$ is downstream of the wing according to the Reverse Flow Theorem.

The upwash effect of the wing on the canard, $\epsilon_{\alpha_w}$, can be directly calculated using equation (1)

$$\epsilon_{\alpha_w} = a_w \frac{\bar{\epsilon}(x, z)}{\pi AR_w} \tag{7}$$

where $x$ is upstream of the wing.

## CANARD AND WING LIFT SLOPES

With values for $\epsilon_{\alpha_c}$ and $\epsilon_{\alpha_w}$, the wing and canard lift slopes (including interference) can be calculated using the following approximate method.

If the true angles of attack at the wing and canard (including interference) can be found, then the lift on each surface is given by the following simple equations.

$$\begin{aligned} L_c &= a_c \alpha_c q S_c \\ L_w &= a_w \alpha_w q S_w \end{aligned} \tag{8}$$

These surface angles of attack are made up of three parts: the aircraft angle of attack relative to an arbitrary axis (usually the fuselage center-line), the incidence of the surface above its zero lift angle of attack, and interference terms due to the upwash or downwash of the other surface. Symbolically

$$\begin{aligned} \alpha_c &= \alpha + i_c + \epsilon_{\alpha_w} \alpha_w + \cdots \\ \alpha_w &= \alpha + i_w - \epsilon_{\alpha_c} \alpha_c + \cdots \end{aligned} \tag{9}$$

where $\epsilon_{\alpha_c}$ and $\epsilon_{\alpha_w}$ are carefully defined so that they would normally be positive quantities. The value of this choice can be clearly seen in equations (9). A fundamental aspect of canard aircraft is that the wing *increases* the average angle of attack at the canard and the canard *decreases* the average angle of attack at the wing.

Neglecting the higher order terms, equations (9) are a system of two linear equations in two unknowns. Solving for $\alpha_c$ and $\alpha_w$

$$\begin{aligned} \alpha_c &= \frac{\alpha(1+\epsilon_{\alpha_w}) + i_c + \epsilon_{\alpha_w} i_w}{1+\epsilon_{\alpha_c}\epsilon_{\alpha_w}} \\ \alpha_w &= \frac{\alpha(1-\epsilon_{\alpha_c}) + i_w - \epsilon_{\alpha_c} i_c}{1+\epsilon_{\alpha_c}\epsilon_{\alpha_w}} \end{aligned} \tag{10}$$

Noting in equations (8) that $C_{L_c} = a_c\alpha_c$ and $C_{L_w} = a_w\alpha_w$ and differentiating equations (10) with respect to the aircraft angle of attack yields the desired result

$$\begin{aligned} C_{L_{\alpha_c}} &= \frac{a_c(1+\epsilon_{\alpha_w})}{1+\epsilon_{\alpha_c}\epsilon_{\alpha_w}} \\ C_{L_{\alpha_w}} &= \frac{a_w(1-\epsilon_{\alpha_c})}{1+\epsilon_{\alpha_c}\epsilon_{\alpha_w}} \end{aligned} \tag{11}$$

Observe that interference increases the effective lift slope of the canard and decreases the effective lift slope of the wing.

## NEUTRAL POINT POSITION

The neutral point is defined as the moment center for which a change in the aircraft angle of attack will result in no change in total moment. If the aircraft center of gravity is located at this point, the aircraft will be neutrally stable in the longitudinal axis. This means a disturbance from trim angle of attack will result in no restoring moment. If the center of gravity is ahead of the neutral point, the aircraft will be statically stable; if it is aft of the neutral point, the aircraft will be statically unstable. Mathematically, the neutral point is that point for which

$$M_\alpha = 0 \tag{12}$$

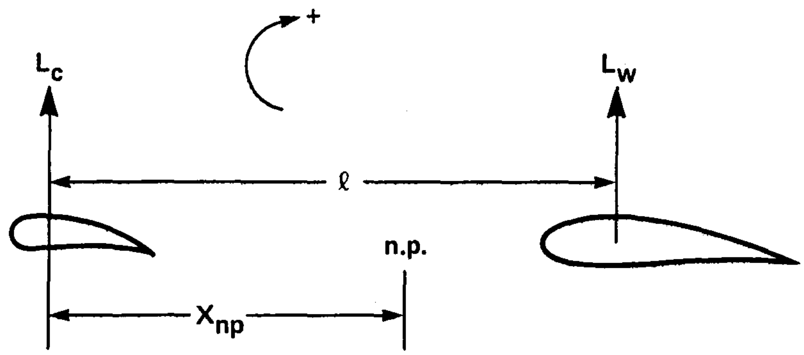


Figure 2. - Neutral point of the canard configuration.

Figure 2 illustrates the situation for a canard aircraft. Taking moments about the neutral point and differentiating with respect to aircraft angle of attack yields

$$C_{L_{\alpha_c}} q S_c x_{np} - C_{L_{\alpha_w}} q S_w (l - x_{np}) = 0 \tag{13}$$

Solving for $x_{np}/l$ yields

$$\frac{x_{np}}{l} = \frac{C_{L_{\alpha_w}} S_w}{C_{L_{\alpha_c}} S_c + C_{L_{\alpha_w}} S_w} \tag{14}$$

which can be rewritten as

$$\frac{x_{np}}{l} = \left[1 + \frac{C_{L_{\alpha_c}} S_c}{C_{L_{\alpha_w}} S_w}\right]^{-1} \tag{15}$$

Substituting from equations (11) yields the final result

$$\frac{x_{np}}{l} = \left[1 + \frac{a_c}{a_w}\left(\frac{S_c}{S_w}\right)\frac{(1 + \epsilon_{\alpha_w})}{(1 - \epsilon_{\alpha_c})}\right]^{-1} \tag{16}$$

## CONCLUDING REMARKS

Summarizing, to estimate the neutral point of a canard configuration, first use equation (26) or equation (29) of reference 2 to find $\bar{\epsilon}$ forward of the wing at the canard position and an equal distance aft of the wing (to apply the Reverse Flow Theorem). Next, use equations (6) and (7) to calculate $\epsilon_{\alpha_c}$ and $\epsilon_{\alpha_w}$, respectively. Finally, use equation (16) to estimate the neutral point.

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