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HEAT TRANSFER IN PIPES

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16. Abstract

This study determines the heat transfer from hot water to a cold copper pipe in laminar and turbulent flow condition. The mean flow-through velocity in the pipe, relative test length and initial temperature in the vessel were varied extensively during tests. Measurements confirm Nusselt's theory for large test lengths in laminar range. A new equation is derived for heat transfer for large starting lengths which agrees satisfactorily with measurements for large starting lengths. Test results are compared with the new Prandtl equation for heat transfer and correlated well. Test material for 200- to 400-diameter test length is represented at four different vessel temperatures.

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HEAT TRANSFER IN PIPES

Th. Burbach

Introduction

"Heat transfer" is the heat exchange which takes place between a fluid and a solid when they have different temperatures. Heat transfer occurs through two different processes: through conduction and through convection of heat. While a heat transfer through unordered movement of molecules occurs in heat conductance, heat transfer by convection takes place through movement of many larger aggregates, which change location and carry their heat with them. In this latter process, one differentiates between "free" and "forced" convection. If a flow condition results solely from gravity of unequal densities occurring from temperature differences, then one speaks of "free convection". If, on the other hand, the velocity field is determined by pressure differences under diminishing density differences, then we are dealing with "forced convection". In this study we shall investigate only heat transfer with forced flow. The great interest in technology for heat transfer explains the large number of investigations carried out in this field. Records of experiments on heat transfer with heated gases and steam have been made by Groeber [1], Josse [2], Jordan [3], Poensgen [4], Tietschel [5] and Nusselt [6]. There are fewer investigations on heat exchange with fluids flowing through /48* pipes. In this direction, the works of Stanton [7]. Soennecken [8] and Stender [9] should be mentioned. While test results gained by various experimenters on gases agree fairly well, this is not the case for measurements on fluids. This can be explained only through theoretical treatment of heat

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*Numbers in the margin indicate pagination in the foreign text.

transfer using similarity observations. Such an investigation shows that heat transfer is independent of Reynolds number $\frac{\bar{u} d}{\nu}$ ($\bar{u}$ = mean velocity in the pipe, d = pipe diameter, v = kinematic viscosity), of the relative test length z/d (z = distance of measuring point from intake), of the relative roughness $\frac{\varepsilon}{d}$ (c approximately = height of roughness element), and of a substance value $\sigma = \frac{\lambda}{c\mu}$ (λ = heat conduction, c = specific heat, μ = viscosity). It has a value of almost 1 in gases, but varies greatly with temperature in fluids. Thus, a variable plays a smaller role with gases, a fact which has simplified relationships and theory. In 1910, Prandtl [10] already formulated such a theory for the situation $\sigma \neq 1$. He succeeded in deriving a formula for heat transfer in gases. An expansion of this equation also for the situation σ 1, i.e., for any fluid, is given in Chapter 2 of this study using a concept from Prof. Schiller based on similarity observations. New experiments, discussed in Chapter 3, were needed to test this formula for longer test lengths and to answer the question: "What is the influence of substance value σ and, further, of relative test length on heat transfer?"

Chapter One
Theoretical Observation

It is customary, for all practical questions of heat transfer to assume a heat transfer number α which is defined by:

$$Q = \alpha (\bar{\theta} - \theta_w) \tag{1}$$

(Q = transferred amount of heat per unit of time and surface, θ = mean fluid temperature, θ_W = wall temperature). In this formula, amount of heat Q is further determined by the equation:

$$Q = \lambda_w \left(\frac{\partial \theta}{\partial y}\right)_w \tag{2}$$

in which w means that the value of the temperature gradient is set for $\frac{\partial \theta}{\partial y}$ and that pipe wall temperature and/or the temperature of the fluid which is in contact with it and equal to it is set for heat conductivity λ. Since the mean fluid and pipe wall temperature can be measured so that $\bar{\theta} -- \theta_w$ may be considered as known, using (1) and (2) and eliminating Q, one arrives at a suitable formula for heat transfer α, when an easily measurable dimension can be assumed for the value of in equation (2).

Since differential equations of impulse conduction and convection and of temperature conduction and convection are similar in fluids, $\frac{\partial \theta}{\partial y}$ can be determined. Reynolds first ascertained the similarity of these two processes, and, based on this, he succeeded in deriving an equation for heat transfer. We shall use such similarity observations here to investigate more generally the question: "When is a temperature field similar to a velocity field?" Prandtl found this to be the case in a system[1] where $\sigma = 1$. In this way, however, we obtain only an equation for α when $\sigma = 1$. If, in the same way, seeking a velocity field similar to the temperature field, we wish to obtain an equation for any σ, this will not be possible without additional assumptions, and then only approximately. Below we shall see that experiences with velocity distribution of turbulent atom flow permit us to make such assumptions [11, 12]. First, we must formulate the requirement that temperature dispersion and velocity dispersion at the wall are similar and/or that the dimensionless gradients are equal at the wall. This condition is written as: /50

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[1] Since the velocity field is only a function of the Reynolds number, another system could be used instead of this one for the velocity field.

$$\frac{\left(\frac{\partial u_1}{\partial y}\right)_w \cdot r_1}{\bar{u}_1} = \frac{\left(\frac{\partial \theta}{\partial y}\right)_w \cdot r_2}{\bar{\theta} - \theta_w} \qquad (3)$$

If the relationship between pressure drop and velocity gradient at the wall is introduced into equation 30:

$$-\text{grad}_x p = \frac{2\mu_w}{r_1}\left(\frac{\partial u_1}{\partial y}\right)_w, \qquad (4)$$

then the following is obtained, taking (2) into consideration:

$$\frac{-\text{grad}_x' p \cdot r_1^2}{2\mu_w \cdot \bar{u}_1} = \frac{Q \cdot r_2}{\lambda_w (\bar{\theta} - \theta_w)}. \qquad (5)$$

This equation states that a very definite relationship must exist between pressure decrease and amount of heat per unit of time and surface. In a stationary situation mean velocity maintains its value along the pipe while mean temperature of the fluid eventually nears the wall temperature as the result of the heat released outwardly during flow. According to Prandtl, as much new heat has to be created by heat sources inside the fluid section as the pipe loses, for the temperature field also to attain inertia condition along the pipe, that is, for it to attain a complete pattern with regard to the stationary condition. If we call the yield of these heat sources per volume q, we obtain the following relationship between Q and q: /51

$$2 r_2 \pi Q = r_2^2 \pi q \text{ oder : } Q = \frac{r_2}{2} q. \qquad (6)$$

Using (6) one can now write equation (5) in the following form:

$$\frac{-\text{grad}_x p}{q} = \frac{r_2^2 \bar{u}_1 \mu_w}{r_1^2 (\bar{\theta} - \theta_w) \lambda_w} \qquad (7)$$

Equation 7 represents the relationship which must be satisfied between pressure gradient and heat yield inside the fluid and/or converted amounts of heat, so that gradients on the wall are "similar."

This relationship can be obtained from differential equations of impulse conduction and convection and of temperature conduction and convection formulated in dimensionless variables u*, x*, etc. They are given here:

$$\frac{\varrho_1 \overline{u}_1^2}{r_1}\left(u_1^* \frac{\partial u_1^*}{\partial x_1^*} + v_1^* \frac{\partial u_1^*}{\partial y_1^*} + \ldots\right) = -\operatorname{grad}_x p + \frac{\mu \overline{u}_1}{r_1^2}\left(\frac{\partial^2 u_1^*}{\partial x_1^{*2}} + \ldots\right)$$

$$\frac{c \varrho_2 \overline{u}_2 (\overline{\theta} - \theta_w)}{r_2}\left(u_2^* \frac{\partial \theta^*}{\partial x_2^*} + v_2^* \frac{\partial \theta^*}{\partial y_2^*} + \ldots\right) = q + \frac{\lambda(\overline{\theta} - \theta_w)}{r_2^2}\left(\frac{\partial^2 \theta^*}{\partial x_2^{*2}} + \ldots\right) \tag{8}$$

The first equation is true for the velocity field of a flow, the second for the temperature field of a second flow which is similar to the first field.

With equation (7) a comparison of the right side of both differential equations, which contains the viscosity and/or heat conduction component and relate primarily to the processes near the wall apparently because of the laminarity there, shows immediately that one obtains the relationship originating there between grad_x p and q for both components of the right side through the similarity statement.

All the differential equations show, further, that condition (7) can be satisfied only when the following ratio is simultaneously true:

$$\frac{-\operatorname{grad}_x p}{q} = \frac{\varrho_1 \overline{u}_1^2 \cdot r_2}{r_1 c \varrho_2 \overline{u}_2 (\theta - \theta_w)}. \tag{7a}$$

This statement is acceptable only under the condition that the velocity fields of both flows are similar, that is, that at corresponding points, $u^*_1 = u^*_2$, etc., everywhere. Then and only then can the convective components act everywhere like the expressions in front of the parentheses. The condition mentioned above simply means that the Reynolds number in both cases are must be the same. This does not give us answer to our question. As a result, we make the obvious assumption that we may limit the aforementioned requirement of similarity of both velocity fields to the turbulent atom, since the inertia components involved here are the most essential item. The requirement that the velocity profile be similar in the turbulent atom, independent of the Reynolds number, is actually satisfied to great extents for the Reynolds number. It suffices to point to the reliability of the Prandtl-Karman 1/7 exponent law of velocity distribution. It justifies the similarity statement of equation (7a).

Since (7) and (7a) must be satisfied simultaneously, if velocity and temperature fields are to be entirely similar, the following condition must be observed:

$$\frac{r_2^2 \bar{u}_1 \mu_w}{r_1^2 (\bar{\Theta}-\Theta_w) \lambda_w} = \frac{\varrho_1 \bar{u}_1^2 r_2}{r_1 c \varrho_2 \bar{u}_2 (\bar{\Theta}-\Theta_w)} \quad \text{or}$$

or

$$\left(\frac{\varrho \bar{u} r}{\mu_w}\right)_1 = \left(\frac{c \varrho \bar{u} r}{\lambda_w}\right)_2$$

$$\text{Reynolds}_1 = \text{Peclet}_2 \tag{9}$$

That is: the velocity field of a flow (system 1) is similar to the temperature field of another flow (system 2) if the Reynolds number for system 1 is equal to the Peclet number for system 2. Using this similarity law: "Reynolds = Peclet" makes it easy to establish a formula for heat transfer which is true for all fluids. The foregoing considerations showed that the per-unit volume and -time heat produced, q, must stand in a definite ratio to pressure gradients to guarantee similarity of both profiles. According to equation 7a:

$$q = -\text{grad}_x p \frac{r_1 c \varrho_2 \bar{u}_2 (\bar{\Theta}-\Theta_w)}{\varrho_1 \bar{u}_1^2 \cdot r_2} \tag{10}$$

Considering equations (1) and (6) we obtain the following equation from (10) for α:

$$\alpha = -\text{grad}_x p \frac{c \varrho_2 \bar{u}_2 r_1}{2 \varrho_1 \bar{u}_1^2} \tag{11}$$

so that for the dimensionless heat transfer number $\frac{\alpha d}{\lambda}$ there results: /53

$$\frac{\alpha d}{\lambda} = \frac{\psi}{4}(\text{Pe}')\frac{c \rho_1 \bar{u}_1 \cdot d}{\lambda}$$

$$\frac{\alpha d}{\lambda} = \frac{\psi}{4}(\text{Pé}) \cdot \text{Pé}, \qquad (12)$$

in which ψ represents the resistance coefficients. Using Blasius' value for $\psi = \frac{0{,}1582}{\sqrt[4]{\text{Pé}}}$, we obtain:

$$\frac{\alpha d}{\lambda_w} = 0{,}03955 \left(\frac{\bar{u}\, d\, \bar{\rho}\, \bar{c}}{\lambda_w}\right)^{\frac{3}{4}} \qquad (12a)$$

The indices in this formula should indicate for which temperatures with great temperature differences the separate material values are to be inserted--according to the meaning they take on from heat transfer.

Equations (12) and (12a) represent the heat transfer law for fluids which we are seeking. Special equation (12a) is, of course, only valid when the conditions underlying the derivation are satisfied. The distance from the test length to the measuring point is primarily long enough here to guarantee the validity of Blasius' law. An experimental proof of this theoretical formula can be expected only from heat transfer numbers, which are measured far enough from the intake.

Chapter 2: Experimental Investigations /54

a) Description of the Experiment Apparatus

The test apparatus depicted in Figure 1 has proven after lengthy preliminary tests to be very suited to answer the questions posed in our introduction and to test the heat transfer laws which we have just derived. The test apparatus permits heat transfer numbers to be measured in very different distances from the intake.

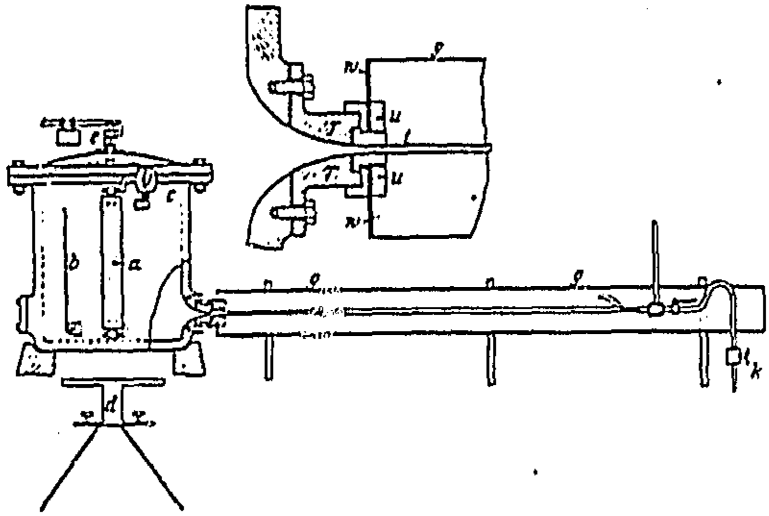

Fig. 1 and 1a: Test apparatus

A large test length makes it possible to test the theoretical formula from the previous chapter. By gradually changing the distance from the intake, the influence of the test length could by investigated. By changing the initial temperature in the vessel and the flow speed in the test pipe, tests could be carried out on the influence of the dimensions "Reynolds" and "Peclet" and $\sigma = \frac{\lambda}{c\mu}$ on heat transfer numbers. The heat flow always traveled water-to-wall, since hot water flows through a cold pipe in the test apparatus. To calculate heat transfer number, the following dimensions had to be measured in accordance with the definitions given in the previous chapter: transfered amount of heat along a given measuring section, temperature inside the pipe wall, mean fluid temperature, and mean flow velocity of the water in the pipe. The transfered amount of heat could be calculated from the variation of the mean fluid temperature for the involved measuring section. /55

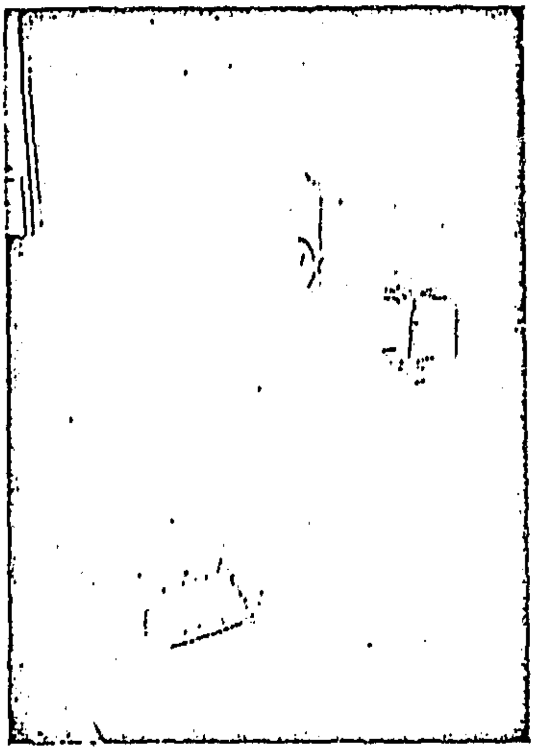

Fig. 2: Test apparatus

A gas burner in a steel cast vessel with an inner diameter of 50 cm heated water to the desired temperature. The vessel was cast by the Leipzig firm Max Jahn and possessed, as can be seen from Fig. 1 and the photographic depiction in Fig. 2, various /56
control instruments, such as: water stand glass a, manometer c, and safety valve e. A mercury thermometer b, bent at a right angle, determined water temperature in the vessel. After its installation in the vessel, the thermometer was calibrated with a normal thermometer. The large heat capacity of the vessel and a sensitive setting on the gas burner d made it possible to regulate the water temperature to 1/10th of a degree. The vessel was connected to the Institute's compressed air line (Fig. 2f) to attain starting temperatures over 100 degrees and also to obtain a large velocity range in the test pipe. The highest attainable flow-through velocity 5 m/sec was established by a high pressure of 6 atmospheres.

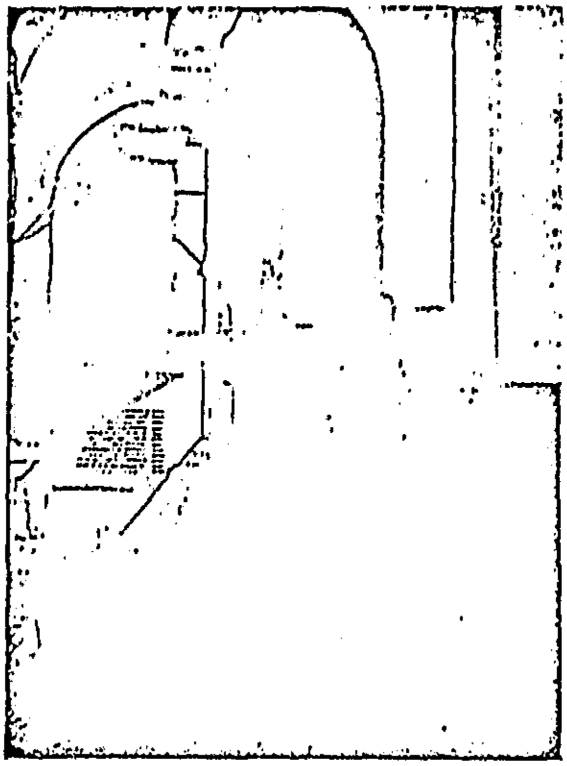

Fig. 3: Test apparatus

An electric heating device was erected to shorten the heating time, which amounted to several hours when using gas and a full vessel. Tests to heat the vessel water directly using a nickel wire resistance were unsuccessful. On the other hand, an electric circulating heater from Loki-Works in Offenbach worked very well. Kettle water traveled through a hot water pump a in Fig. 3 (produced by Schwada, Erfuhrt) to the cirulating heater (7 kW) and then flowed back to the vessel. When the desired vessel temperature was nearly reached, the heater and pump were shut off. The starting temperature was regulated by the sensitivity settings on the gas burner. The regulator on gas burner d is indicated in Fig. 1. The electric heating device is not shown, to avoid confusion in the drawing. In the photograph of the entire device in Fig. 3 the pipe lines of the circulating heater can be seen. /57

In connection with the description of the vessel, the heavy rust formation in the vessel should be mentioned here, since it can cause disturbances when the device is put on line. A lead or chrome coating of the vessel would have made it too heavy to take out of the test structure. The only solution was to paint all of the parts coming in contact with water with a rust protector on location. After a number of unsuccessful attempts with minium, lacquer paint and other rust proofers on the market, Krusta Nera enamel lacquer from the firm Heyn & Manthe, Berlin, proved to be effective as a long-term rust proofer for the vessel.

A precision brass pipe, 2 m long and with 5.0 mm l.W. and 1.0 mm wall thickness, was used for test pipe. A small intake converter (t in Fig. 1) was welded to one end. Using a clamping nut u, it was bolted to a larger intake converter T fastened securely onto the vessel. The two funnels were screwed into each other smoothly. The test pipe, as seen in Fig. 1, was placed in a long zinc trough g, which was filled with ice water during the test. It guaranteed a well-defined outside temperature for the pipe. The trough g in Fig. 1a was screwed onto the vessel at the aforementioned intake converter T. Good heat insulation at w was of particular importance. Further, it was necessary that the /58 thermic effect of the ice water begin exactly at the beginning of the pipe to permit a clear determination of the test length. Details of the apparatus, which satisfied all requirements, can be seen in Fig. 1a.

The test pipe had to be straightened very precisely during measuring, since, as tests had shown, even a small sag in the pipe had a substantial effect on heat transfer. A silver-constantan thermoelement determined pipe wall temperature. Its thermal power was measured with a sensitivity of 10^{-4} Volts by a so-called tower instrument from the firm Siemens & Halske. Installation in the pipe wall required great care, so that heat was not drawn from the measuring place by the element itself, and the geometry of the pipe wall was not altered. A short description of the construction with references to Fig. 4 is in order here, since the apparatus is thermally efficient and has

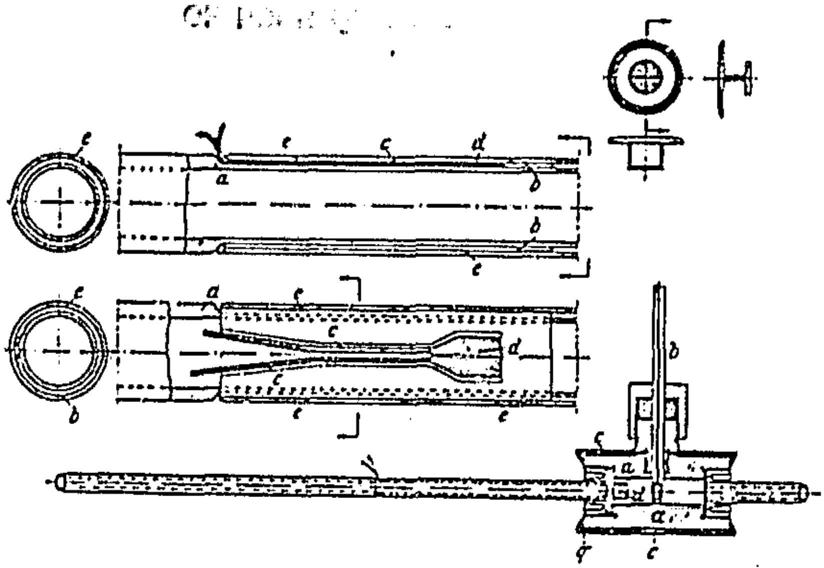

Fig. 4 (center), 4a (above), 4b (below): Measuring points.

proven itself quite well in every aspect. The two silver constantan wires, each 3/10 mm thick, were soldered to each other and the soldering bead was hammered out to a thickness of 4/10 mm. The soldering spot was cut to fit installation (d in Fig. 4, 4a, 4b). To bring this thermoelement as close as /59 possible to the inside of the pipe wall, this wire was stripped from point a to the end of the test pipe in a length of 6.5 cm with a wall thickness of 3/10 mm. A further reduction proved impossible because of strength limitations. A 4/10 mm thick, notched casing b was clamped on the tapered pipe piece. An electrically insulated thermoelement c-d fit into the notch. By clamping on another casing e the original wall thickness of 1 mm was restored throughout and the thermoelement simultaneously closed completely to the outside. This installation permitted the thermoelement wires to go 2 cm into the pipe wall so that the measuring point would not lose heat to the element itself.

A mercury thermometer measured fluid temperature in the pipe. Its scale was calibrated against a normal thermometer, so that the correct temperature was indicated when the mercury container was rinsed with water. A device, depicted in Fig. 4b, was constructed so that this thermometer indicated the mean temperature of a precisely given cross section which was necessary to determine the test length. This device should be described here briefly. A thick-walled measuring body, which also secures the thermometer b, is wrapped with insulation band c and protects the test pipe from the cross section q on up against heat losses, so that a drop in the mean temperature can no longer take place. A twisting, brass water mixer d is located in the insulated pipe piece in front of the fluid thermometer. Its form can be seen in Fig. 4a. It causes good mixing of the water so that the thermometer actually indicates the mean temperature. Two regulator valves established the desired flow-through speed. A rough setting was produced by the simple metal ball faucet shown to the right of the fluid thermometer in Fig. 1. A finely turned screw (k in Fig. 1) permitted a more exact regulating by continuously changing the flow-through resistance. The mean flow-through resistance. The mean flow-through temperature u in the test pipe could be determined by timing the flow-through mass M obtained in t sec according to the equation:

$$r^2 \pi \varrho \overline{u} = \frac{M}{t}$$

The Helmhotz Society provided the funds for the equipment /60
and apparatus described here. Without their generous assistance this project would not have been possible, and we thank them for it.

b) Execution of the Tests: Measuring Accuracy and Test Results

Before we present the results of the measurements, we shall describe the tests for determining heat transfer numbers briefly.

The actual measurements could begin after regulating the starting temperature in the vessel and filling the ice water trough. First, the stretching device with 20 kg tractive force was activated to straighten the test pipe. Then, the compressed air pressure in the vessel was set to produce approximately the desired flow-through velocity. The exact setting was regulated by the fine setting device at the end of the test pipe. A constant discharge of the safety valve, even during the test, permitted a very good constant of the mean flow-through velocity, which was of great importance to measuring accuracy. Heat exchange stabilized about 20 to 30 seconds after the measurements began. This could be recognized from the constant setting on the fluid thermometer. The water mass emerging in about 50 seconds was timed to determine the mean flow-through velocity. The indicator instrument for pipe wall temperature and the fluid thermometer were read a number of times during the timing period. The timed water mass had to be weighed on a platform scale immediately following the test, since the hot water evaporated very rapidly.

The mercury thermometer settings could be read with a precision of 1/10 degree, if the flow-through velocity in the test pipe was beyond the critical range. Such an exact determination of pipe wall temperature was not possible because of the limited sensitivity of the indicator galvanometer. Reading errors during measuring amounted to ± 3/10 degree, but multiple readings reduced this error factor by 1/10 degree. After each measurement the ice water mixture in the trough was replenished, and the melted water was removed from time to time. The length of one measurement, including all preparations, amounted to about 1/4 hour; an average of 20 tests could be made per day.

The sequence of measurements was already prescribed by the test apparatus. As can be seen in Fig. 1, the apparatus had only one measuring point for fluid velocity at a certain distance from the vessel, since installation of additional fluid temperature measuring points would have disturbed the construction. /61 To measure the temperature change along the pipe for calculating the transferred amount of heat, it was necessary to move the fluid temperature measuring place along the pipe, that is, to move it to another distance from the vessel. The easiest way to achieve this was to disconnect the pipe from the vessel. Since this procedure only provided a shortened distance from the vessel, the measurements proceeded in the following fashion: First, the fluid and pipe wall temperature independence of the mean flow due to velocity was determined at a constant initial temperature in the vessel and with the largest test length having 400 diameters. When this independence was ascertained at about 40 test points in the velocity range of 5-500 cm/sec, the same tests were carried out at three other vessel temperatures to increase the measuring range. Thus, the influence of initial temperature on heat transfer could be ascertained. Then, to change the relative test length, a piece of pipe was cut from the vessel, and the same measurements were carried out at the same four initial temperatures. In this way the test length was varied seven times. Each time the pipe was cut, it was thoroughly cleaned inside and outside to avoid any obstructions. Two sets of curves were obtained for each of the four initial temperatures in the vessel: the fluid and the pipe wall temperature independences of the mean flow-through velocity at seven different test lengths. In all sets of curves the abscissa indicates the flow-through velocity, the ordinate indicates the temperature of the measuring points. The parameter was the relative test length. The curves may be used to obtain the decrease in fluid and pipe wall temperature along the pipe independently of the flow-through velocity at four different initial temperatures in the vessel.

At the beginning of the tests, the pipe length up to the reference point for the fluid temperature (cross section q in Fig. 4b) amounted to 200 cm. At a pipe diameter of 0.5 cm the fluid temperature measuring point was 400 diameters away from the intake. The corresponding test length for pipe wall temperature, however, amounted to only 390 diameters, since the soldering bead of the thermoelement lay exactly 10 diameters in front of the fluid temperature measuring point. The test lengths for the other two temperature measuring points are given in Table 1.

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TABLE I.

RELATIVE TEST LENGTH FOR

Pipe Wall Temperature Measuring Point	Fluid Temperature Measuring Point
390	400
290	300
210	220
140	150
90	100
50	60
20	30

The course of the fluid and pipe wall temperature for turbulent flow condition was determined along the pipe at five different flow-through velocities for each of the four initial temperatures and recorded in the graphs in Figures 5 - 9. (Kettle temperatures 69.2 degrees, 85.5 degrees, 101.8 degrees, 118.0 degrees; velocities 100 cm/sec; 104 cm/sec, 200 cm/sec, 300 cm/sec, 500 cm/sec.) A numerical record of these results is to be found in Table II, in which test results for laminar and unsteady flow condition are given.

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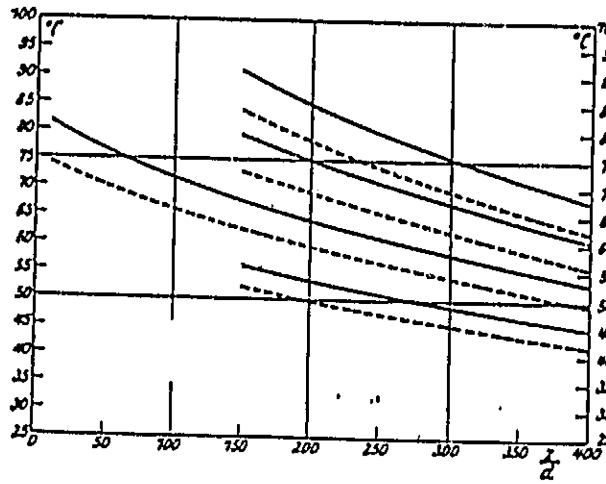

Fig. 5: Course of fluid and pipe wall temperature along pipe at 100 cm/sec mean flow-through velocity.
Solid line = fluid temperature; dotted line = pipe wall temperature; parameter: vessel water tempterature.

Since the per-second heat loss of the fluid flow in any pipe element of length dz must equal the amount of heat released outwardly at the same time, the following equation results:

$$r^2 \pi \overline{u} \overline{\varrho} \overline{c} \, d\overline{\Theta} = \alpha \, 2 r \pi \, dz \, (\overline{\Theta} - \Theta_w).$$

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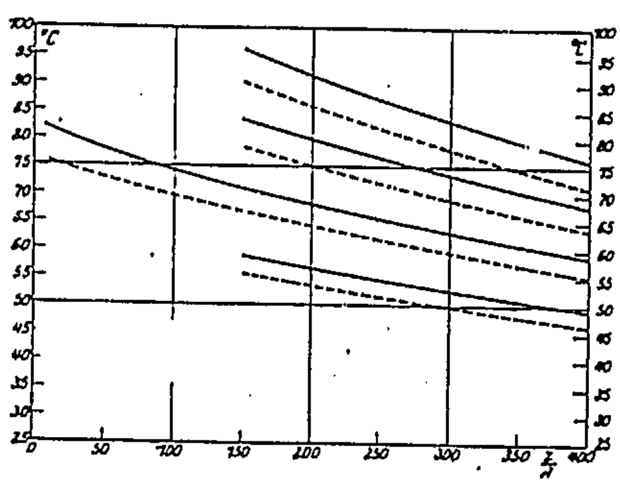

Fig. 6: Course of the fluid and pipe wall temperature along pipe at 140 cm/sec mean flow-through velocity.
Solid line = fluid temperature; dotted line = pipe wall temperature; parameter: vessel water tempterature.

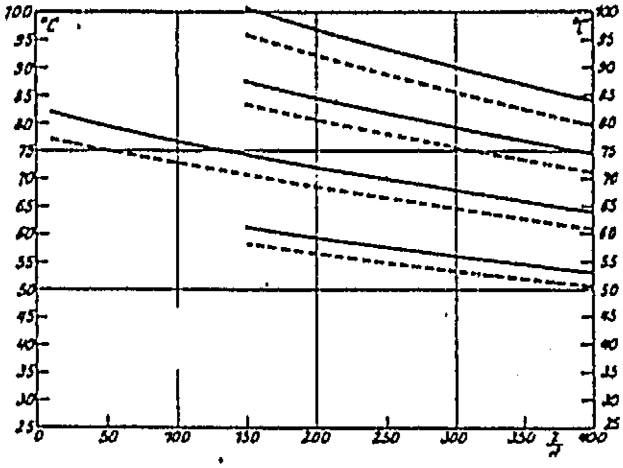

Fig. 7: Course of the fluid and pipe wall temperature along pipe at 200 cm/sec mean flow-through velocity. Solid line = fluid temperature; dotted line = pipe wall temperature; parameter: vessel water tempterature.

One obtains the following relationship for α:

$$\alpha = \frac{1}{2} r \bar{u} \bar{\varrho} \bar{c} \frac{d\bar{\Theta}}{dz} \cdot \frac{1}{(\bar{\Theta} - \Theta_w)}$$

The heat transfer numbers given in Table II are calculated according to this formula. The value of $\bar{\Theta} - \Theta_W$ can be taken directly from the curves for individual test lengths. /64
By placing a tangent on the fluid temperature curve we obtain the value $\frac{d\bar{\Theta}}{dz}$ for a certain test length. The curves had to be drawn in very large scale for this purpose to obtain enough precision. Since this curve gives the change of the fluid temperature with the pipe length, the tangent of the slope angle of the tangent is the value $\frac{d\bar{\Theta}}{dz}$ we are looking for. Of course, the scale ratio has to be taken into consideration for it.

The given α values in strict accordance with the form of their calculation are valid only for an infinitely small pipe length. A differential α is given which is valid for only a very definite test length on which the calculation is based. This is contrary to the heat transfer numbers related to data in literature where mean values are represented by a finite pipe

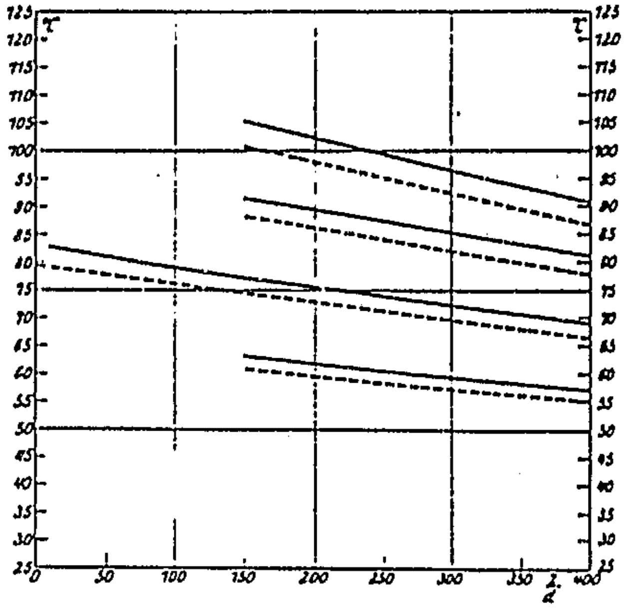

Fig. 8: Course of the fluid and pipe wall temperature along pipe at 300 cm/sec mean flow-through velocity.
Solid line = fluid temperature; dotted line = pipe wall temperature; parameter: vessel water tempterature.

length. For this reason, the procedure described here enables us to calculate heat transfer numbers for any test length and is particularly useful when the influence of the test length /65 is determined by heat transfer number. The influence of radiation was not taken into consideration in my test material, since evaluation showed this influence was smaller than 1/10%, even in the maximum situation.

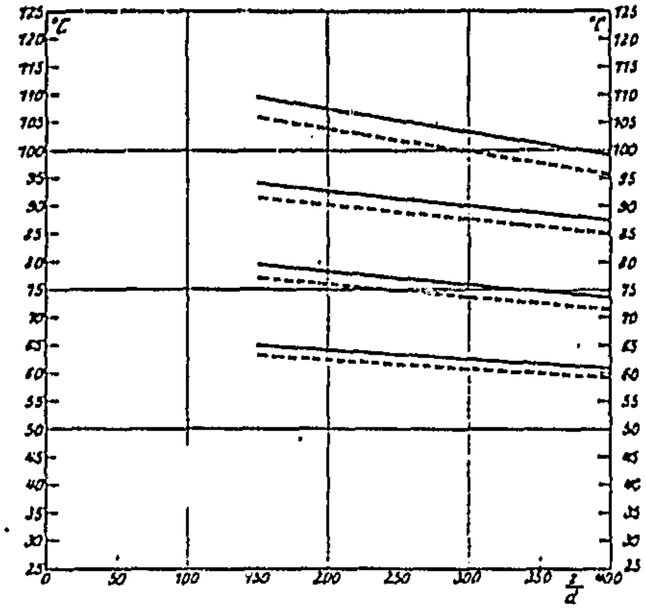

Fig. 9: Course of the fluid and pipe wall temperature along pipe at 500 cm/sec mean flow-through velocity.
Solid line = fluid temperature; dotted line = pipe wall temperature; parameter: vessel water tempterature.

c) Discussion of Measurements and Comparison to Theory

An exact theory of heat transfer for laminar flow condition was formulated by Nusselt [13] in 1910, and handled in detail in the book by Groeber [14]. Nusselt's investigation found that, for large test lengths, the dimensionless formulated heat transfer number $\frac{a \cdot d}{\lambda}$ assumes the value 3.65 independent of flow velocity.
If one plots the dimensioneless Peclet number $\frac{\bar{u} d \rho c}{\lambda}$ as abscissa /66
essentially being a measure of velocity, and the dimensionless heat transfer number $\frac{a d}{\lambda}$ as ordinate, a line parallel to the abscissa is obtained as the theoretical curve for laminar flow condition. Using the equation $\frac{a d}{\lambda} = 0{,}0395 \cdot P^{3/4}$ derived in detail in part 2, it is now possible for the turbulent condition likewise to be given a theoretical curve for large test lengths.

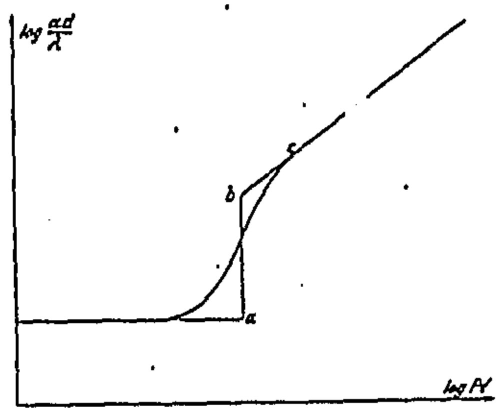

Fig. 10: Theoretic course of heat transfer numbers.

In a logarithmic coordinate system with Peclet as abscissa and $\frac{\alpha d}{\lambda}$ as ordinate, this curve is a straight one. Its slope angle φ is tangent φ = 3/4. If both straight lines are entered in a logarithmic coordinate system, the following theoretical picture, given in Fig. 10, is obtained for heat transfer with laminar and turbulent flow. For small Peclet values left of point a in Fig. 10 laminar flow prevails and, for this reason, $\frac{\alpha \cdot d}{\lambda}$ is constant here. For large Peclet numbers right of point b the turbulent flow is completely developed and $\frac{\alpha \cdot d}{\lambda}$ is proportional to 3/4 of the Peclet exponent. For the boundary situation of very great test lengths, the transfer from laminar to turbulent values must occur very acutely, as represented in Fig. 10 by the straight line a--b. A totally, correspondingly abrupt transfer is /67 obtained in a resistance coefficient-Reynolds number diagram for the increase of resistance coefficients resulting from laminar t turbulent values for large test lengths [15]. For smaller test lengths the transfer does not occur so suddenly, but is rather more or lessed blurred. For even with laminar flow intake disturbances will not yet be completely faded, and these cause the heat transfer number to move above the theoretical horizontal at an earlier point. On the other hand, turbulence is not yet fully developed in small Reynolds numbers, also as the

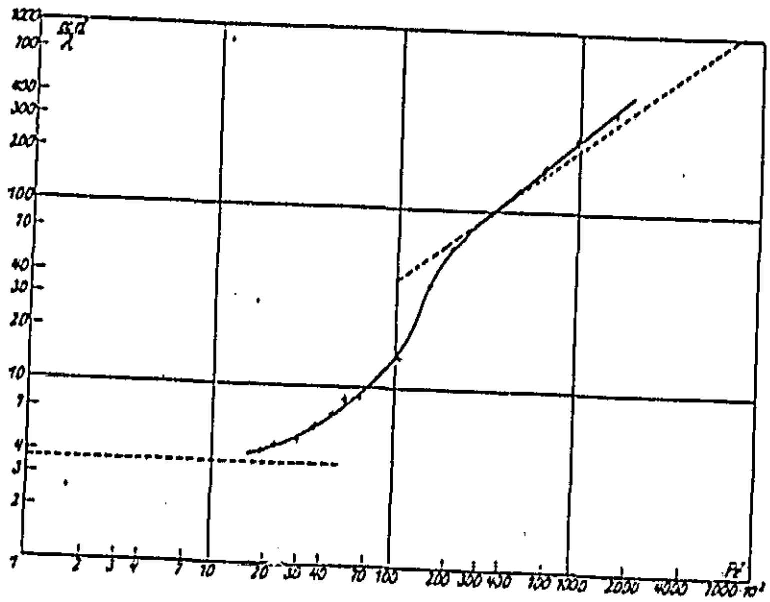

Fig. 11: Experimental values for z/d = 400 with both theoretical straight lines.

result of the short intake. Therefore, the theoretical straight line is attained only at higher values of Pe = $\frac{Re_y}{\sigma}$. This causes the heat transfer numbers left of point c to have to curve down from the turbulent straight line to come closer to the laminar straight line. This consideration necessarily leads to a curve with a turning point for the transfer range. This curve is drawn into Fig. 10. An experimental confirmation of this theoretical concept should be expected only from measurements which are made at a large distance from the input. For this reason, Fig. 11 contains my measuring results for the greatest test length of 400 diameters together with both theoretical straight lines. /68

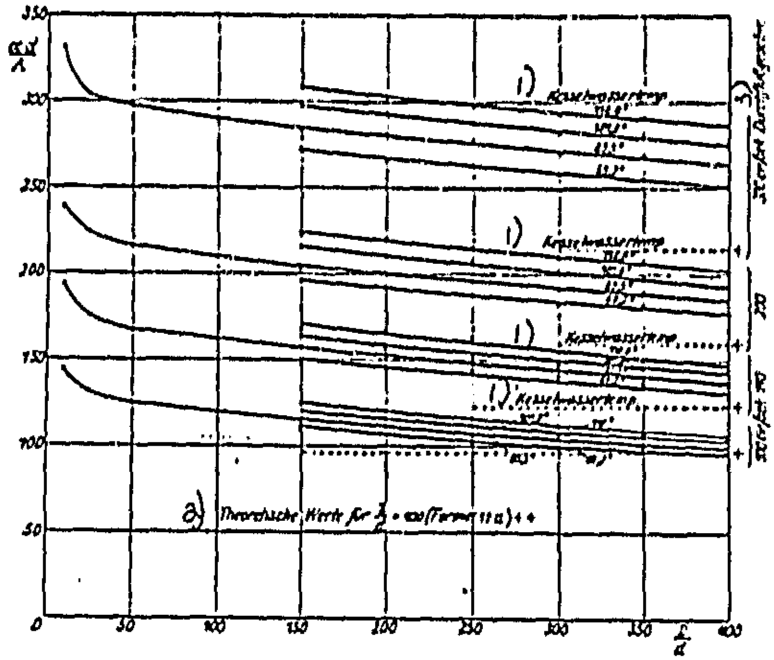

Fig. 12: Experimental course of dimensionless heat transfer number along pipe at four velocities and four vessel temperatures. Key: 1) Kettle water temperature
2) Theoretical value for z/d = 400 (formula 12 a) + +
3) Flow-through velocity

One recognizes immediately that the test results confirm theoretical considerations. In laminar range the heat transfer numbers do not reach the theoretical value completely, however, a gradual approximation to the straight line seems sure. In complete correspondence with the previous considerations, the test values lie on a curve with a turning point in the transfer range. The fact that the test points for the turbulent flow condition do not agree completely with the theoretical straight line probably has to do with severe thermic disturbances coming from the vessel and not faded completely at these test lengths and for that reason causing an increase in heat /69
transfer numbers. The correctness of this interpretation is verified by Fig. 12, which reproduces the experimentally determined cause of the heat transfer number with the pipe length at four vessel temperatures. It can be seen clearly that the heat transfer numbers at high vessel temperatures, that is, with greater thermic disturbances, assume higher values as test length increases and come closer and closer to the theoretical

values. The course of these curves, therefore, allows us toexpect that the calculated and the measured test values also agree for higher vessel temperatures with correspondingly larger test lengths. The values calculated here lose their meaning with smaller test lengths, since, as corresponding measurements showed, the Blasius law is no longer valid for them. The values in these cases are given only to emphasize the approximation of test values to theoretical values. For test lengths which are not too short, measured deviations from Blasius' law are so small that they do not suffice to explain the large number of heat transfer numbers observed as deviations from theoretical law. It is quite apparent that the development of the final temperature profile still must be influenced substantially by the dimensionless $\sigma = \frac{\lambda}{c\mu}$. It is easy to overlook the fact that, ceteris paribus, greater specific heat requires a higher test length for the temperature profile. This probably explains the fact that measurements for gases ($\sigma = 1$) are substantially less able to show this effect than are our measurements for liquids, for which the value of σ is substantially smaller than 1.

Before we consider my measurements on short test lengths, we shall compare a quite recently proposed theory of Prandtl's [16] on heat transfer for turbulent pipe flow and my tests at 400 diameters test length. As already pointed out in the anticipation to this study, this new theory from Prandtl compliments his earlier work, in which the relationship of velocity on the inside of the boundary level to the middle velocity in the pipe still remained undetermined. In his new theory Prandtl now finds the following formula for the dimensionless heat transfer number:

/70

$$\frac{\alpha \cdot d}{\lambda} = 0{,}0395 \frac{(2\mathrm{Rey})^{\frac{3}{4}}}{\sigma + 1{,}6\,(1-\sigma)\,\mathrm{Rey}^{-\frac{1}{8}}}$$

According to Prandtl's own words, the number factor 1.6 in the denominator is very uncertain in this equation and most likely may be determined from heat transfer observations. If one follows this suggestion and ascertains this number factor from my test material with 400-diameter test length, then a value of 0.4 is obtained. In Fig. 13 the Prandtl equation with this number factor is given for the σ values 1/4, 1/2, and 1. Furthermore, my test value for σ = 1/4 is included. As one sees, the measuring points correspond quite well to the Prandtl curve for σ = 1/4.

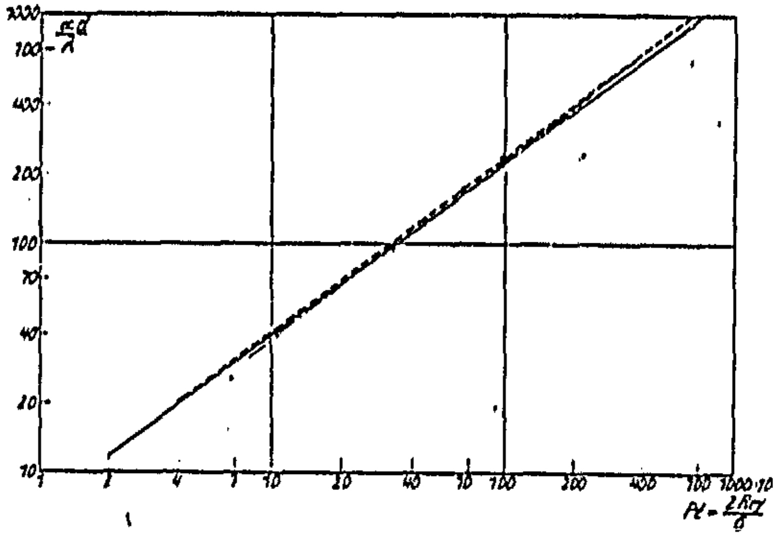

Fig. 13: Formula from Prandtl.
Number factor 0.40; for σ = 1/4 -- . --; σ = 1/2 - - - - ;
for σ = 1 ----- identical with Schiller.
Experimental values for σ = 1/4 + + +

Further, it is also very noteworthy that, for nearly all possible σ values from 1/4 to 1 with this number factor, the Prandtl curves hardly differ from the theorectical straight lines developed here. Both theoretical curves for every number factor for σ = 1 agree, as can be seen immediately from Prandtl's formula. As seen, as well, from Fig. 13, the maximum deviation of the Prandtl curves amounts to 8% for the remaining σ values in the large Peclet range of 6000 to 600,000. This small deviation is present, however, only for the number factor 0.4. Figures 14 and 15 show clearly how inserting other number values into the Prandtl equation moves this deviation up or down. /71

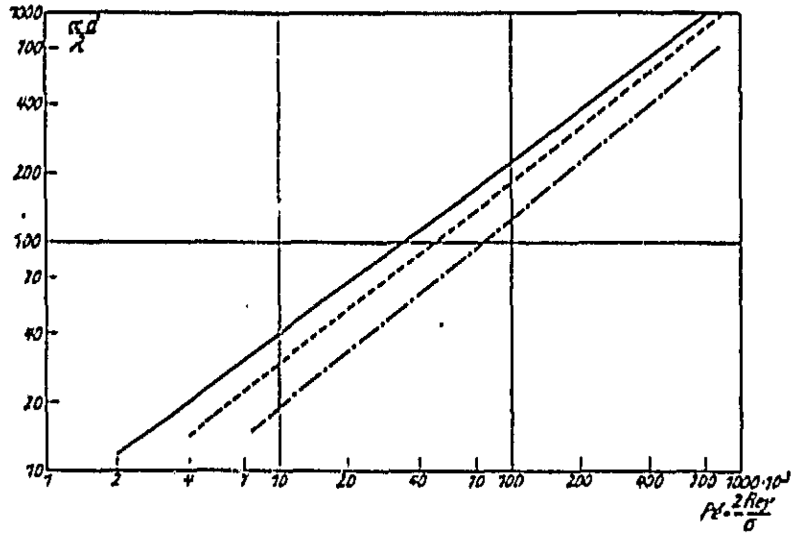

Fig. 14: Formula from Prandtl.
Number factor 1.50; for σ = 1/4 -- . --; σ = 1/2 - - - - ;
for σ = 1 ----- identical with Schiller.
Experimental values for σ = 1/4 + + +

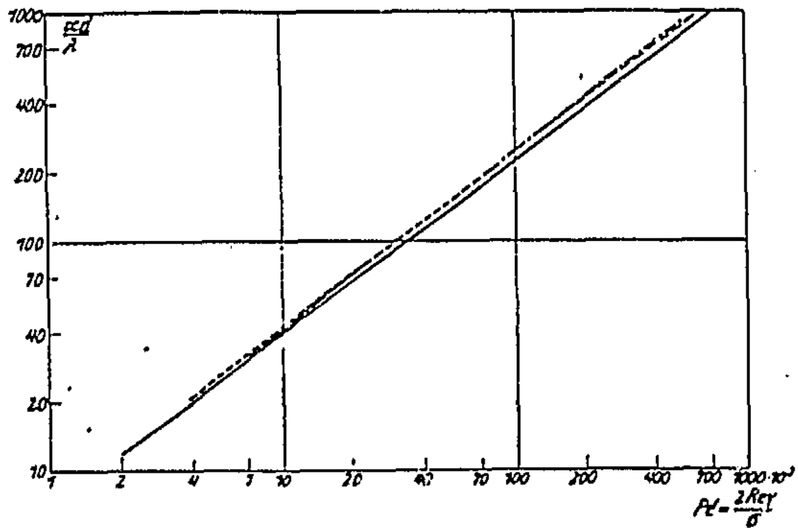

Fig. 15: Formula from Prandtl.
Number factor 0.30; for σ = 1/4 -- . --; σ = 1/2 - - - - ;
for σ = 1 ----- identical with Schiller.
Experimental values for σ = 1/4 + + +

In Fig. 14 the theoretically resulting number value 1.6 is /72
inserted, in Fig. 15 the value 0.3. From these curves we see, as

well, that the influence of σ on heat transfer with the Prandtl curves is very much dependent on the dimension of the number factor. This is contrary to the formula given here, in which σ occurs only in the combination $\frac{Rey}{\sigma}$. One advantage of our theory, it seems to me, is that there is no uncertain factor in it, as is the case with Prandtl's theory. Only new tests with large test lengths can ascertain which of the two formulas better represents the truth. These tests also must measure the pressure drop to determine the valid resistance law and must demonstrate increased measuring precision.

My results, also for smaller test lengths, are reproduced in Fig. 16 (Table). Not only the two theoretical straight lines are given, but also partial comparisons of test results from Stender and Stanton. I cannot explain why Stender and Stanton's test results are substantially lower than my measurements. Substantially closer to our results are Nusselt's test results for compressed air and other gases which practically agree with our measurements for $\frac{z}{d} = 400$. This might seem surprising since the mean test lengths in Nusselt's tests amounted to only about 50 diameters. Nusselt's values are lower than ours relative to the same test lengths. The reason for this may lie in the extension of the starting stretch by reducing the σ value in our experiment.

Establishing a heat transfer law also for small test lengths would advance technology greatly, since, in practice, primarily heat transfer with small test lengths occurs. This is very difficult, however, since the number of independent dimensions influencing the heat transfer multiplies with the transfer. Besides the relative test length, another variable is the dimension of the intake disturbances governed by free convection. For a numerical determination of these disturbances the velocity field with free convection must be known. Nusselt [17] has shown that this velocity field is dependent on the following two /73
dimensionless factors:

on $\sigma = \frac{\lambda}{c \mu}$ and Grashof $Gr = \frac{a^3 g (T_w - T_{fl}) \beta}{\nu^2}$
(a for us = about vessel radius, g = earth's acceleration, T_W -- T_{fl} = temperature difference between vessel wall and fluid, β = expansion coefficient, ν = kinematic viscosity).

To simplify matters, only the large "Grashof" will be considered for calculating the intake disturbances in the vessel. Since the heat transfer formula must be converted into the law derived in Chapter 1 before it can be established for the boundary case of infinitely large test lengths, it is written here in the following form:

$$\frac{\alpha d}{\lambda} = 0{,}0395 \left(\frac{\bar{u} d \bar{\varrho} \bar{c}}{\lambda} \right)^{\frac{3}{4}} \cdot e^{0{,}02275 \cdot 10^{-6} \frac{Gr \cdot d}{x}} \tag{13}$$

The values of "Grashof" at the four vessel temperatures were determined by measurements of the temperature difference between vessel wall and fluid and are given in Table 3 with the number material needed for calculation. The number factor in the exponent of equation (13) was determined empirically. Using this equation, it was possible to represent the test results for 200- to 400-diameter test lengths for all four vessel temperatures with sufficient precision. The values of $\frac{\alpha d}{\lambda}$ calculated using formula (13) are given in the last column of Table 2. Due to the complexities of the relationships in the starting area, the equation imparted here, in spite of its dimensionlessness, serves only for orientation and, as such, has no universal validity.

Summary of the Results

1. In this study, the heat transfer from hot water to a cold copper pipe in laminar and turbulent flow condition was determined. The mean flow-through velocity in the pipe, the relative test length and the initial temperature in the vessel were varied extensively during the tests. The measurements confirm Nusselt's theory for large test lengths in laminar range.

2. By establishing a similarity law for a temperature and velocity field for turbulent flow condition, a new equation was derived for heat transfer for large test lengths. This equation agrees satisfactorily with the measurements for large test lengths.

3. The test results also were compared with the new Prandtl formula for heat transfer. The value of the number factor, still uncertain in that equation, resulted from measurements at 0.4. Measured values and those calculated with this number agreed very well.

4. The influence of intake disturbances could be determined by the dimensionless "Grashof". Using an equation, it was possible to represent the test material for 200- to 400-diameter test length at four different vessel temperatures.

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Tables.

Table II.*

$\bar{u}$	$\bar{\theta}$	θ_w	$\bar{\theta} - \theta_w$	$\bar{\rho} \cdot \bar{c}$	$\frac{d\bar{\theta}}{dx}$	α	$\bar{c}$
$\frac{cm}{sec}$	C°	C°	C°	$\frac{cal}{cm^3\,C°}$	$\frac{C°}{cm}$	$\frac{cal}{cm^2\,sec\,C°}$	$\frac{cal}{g\,C°}$
			$x/d = 400$				
5,0	6,6	4,2	2,4	1,0022	0,044	0,0115	1,0025
6,0	9,2	6,0	3,2	1,0012	0,053	0,0125	1,0016
8,0	12,9	8,7	4,2	0,9996	0,057	0,0130	1,0005
10,0	16,4	11,1	5,3	0,9985	0,069	0,0162	0,9997
12,5	20,0	13,7	6,3	0,9972	0,077	0,0191	0,9990
15,0	23,2	16,2	7,0	0,9960	0,088	0,0235	0,9985
18,0	26,7	18,4	8,3	0,9947	0,091	0,0245	0,9981
30,0	30,2	21,4	8,8	0,9934	0,098	0,0415	0,9978
45,0	34,0	28,9	5,1	0,9919	0,096	0,1072	0,9976
60,0	38,0	34,1	3,9	0,9904	0,092	0,1750	0,9975
			$x/d = 350$				
5,0	7,9	5,5	2,4	1,0017	0,054	0,0141	1,0020
6,0	10,5	7,4	3,1	1,0007	0,064	0,0155	1,0012
8,0	14,5	10,3	4,2	0,9992	0,070	0,0166	1,0001
10,0	18,2	12,8	5,4	0,0978	0,083	0,0192	0,9993
12,5	22,0	16,6	6,4	0,9965	0,089	0,0216	0,9986
15,0	25,5	17,8	7,7	0,9952	0,100	0,0242	0,9982
18,0	29,1	19,8	9,3	0,9938	0,099	0,0238	0,9979
30,0	32,6	23,8	8,8	0,9925	0,106	0,0448	0,9976
45,0	36,5	31,3	5,2	0,9910	0,105	0,1127	0,9975
60,0	40,3	36,2	4,1	0,9895	0,099	0,1790	0,9975
			$x/d = 300$				
5,0	9,5	7,1	2,4	1,0011	0,069	0,0180	1,0015
6,0	12,3	9,2	3,1	1,0000	0,079	0,0191	1,0007
8,0	16,3	12,1	4,2	0,9985	0,086	0,0204	0,9997
10,0	20,5	14,8	5,7	0,9970	0,097	0,0212	0,9989
12,5	24,3	17,6	6,7	0,9956	0,097	0,0225	0,9983
15,0	28,0	19,7	8,3	0,9942	0,106	0,0238	0,9980
18,0	31,6	21,6	10,0	0,9928	0,102	0,0227	0,9977
30,0	35,3	26,5	8,8	0,9914	0,113	0,0478	0,9975
45,0	39,1	34,0	5,1	0,9900	0,110	0,1200	0,9975
60,0	42,8	38,6	4,2	0,9685	0,104	0,1836	0,9975
			$x/d = 250$				
5,0	11,4	9,0	2,4	1,0004	0,086	0,0224	1,0010
6,0	14,4	11,1	3,3	0,9992	0,098	0,0223	1,0001
8,0	18,6	14,1	4,5	0,9977	0,109	0,0242	0,9992
10,0	23,0	16,9	6,1	0,9961	0,111	0,0226	0,9985
12,5	26,8	19,7	7,1	0,9947	0,113	0,0247	0,9981
15,0	30,6	21,9	8,7	0,9932	0,110	0,0236	0,9978
18,0	34,0	23,8	10,2	0,9919	0,112	0,0245	0,9976

* T.N.: Commas in numerical entries represent decimal points.

Tables.

Table II

λ_w	η	$\bar{\nu}$	$Pé = \frac{\bar{u} d \overline{\rho c}}{\lambda_w}$	$2Re_y = \frac{\bar{u} d}{\bar{\nu}}$	$\sigma = \frac{\lambda_w}{\bar{c}\,\bar{\mu}}$	$\Lambda = \frac{\alpha \cdot d}{\lambda_w}$
$\frac{cal}{cm\ sec\ C°}$	$\frac{g}{cm^3}$	$\frac{cm^2}{sec}$	—	—	—	exp. —
			$x/d = 400$			
0,001342	0,9999	0,01446	1607	173,0	0,0926	4,26
0,001349	0,9998	0,01340	2228	224,0	0,1005	4,62
0,001359	0,9994	0,01207	2942	331,0	0,1127	5,00
0,001368	0,9989	0,01100	3650	455	0,1245	5,93
0,001379	0,9982	0,01007	4520	620	0,1373	6,92
0,001388	0,9975	0,00936	5380	801	0,1489	8,45
0,001397	0,9960	0,00865	6410	1040	0,1625	8,78
0,001409	0,9956	0,00803	10570	1870	0,1766	14,72
0,001439	0,9944	0,00742	15500	3032	0,1955	37,2
0,001460	0,9929	0,00687	20360	4370	0,2146	60,0
			$x/d = 350$			
0,001347	0,9998	0,01390	1600	179,8	0,0968	5,23
0,001354	0,9996	0,01290	2217	232,5	0,1050	5,73
0,001365	0,9992	0,01156	2927	346,0	0,1181	6,10
0,001375	0,9986	0,01052	3630	475,0	0,1310	6,97
0,001356	0,9978	0,00960	4492	651,0	0,1449	7,81
0,001395	0,9969	0,00888	5350	845,0	0,1579	8,70
0,001403	0,9959	0,00820	6370	1097	0,1722	8,48
0,001419	0,9948	0,00764	10455	1964	0,1872	15,8
0,001449	0,9935	0,00706	15390	3190	0,2071	38,9
0,001468	0,9921	0,00658	20200	4560	0,2255	61,0
			$x/d = 300$			
0,001353	0,9997	0,01326	1850	188,2	0,1017	6,65
0,001361	0,9995	0,01228	2204	244,0	0,1108	7,02
0,001372	0,9989	0,01103	2910	362,5	0,1246	7,45
0,001383	0,9981	0,00936	3600	502,0	0,1393	7,07
0,001394	0,9972	0,00912	4460	686,0	0,1535	8,08
0,001402	0,9962	0,00840	5320	893,0	0,1680	8,48
0,001410	0,9952	0,00780	6340	1153	0,1820	8,08
0,001429	0,9939	0,00724	10400	2071	0,1990	16,70
0,001459	0,9925	0,00672	15270	3350	0,2193	41,20
0,001477	0,9911	0,00629	20085	4770	0,2377	62,10
			$x/d = 250$			
0,001360	0,9996	0,01257	1638	199	0,1082	8,24
0,001368	0,9992	0,01160	2190	259	0,1178	8,14
0,001381	0,9985	0,01040	2890	385	0,1331	8,75
0,001391	0,9975	0,00940	3580	532	0,1485	8,14
0,001402	0,9966	0,00863	4438	724	0,1633	8,82
0,001411	0,9955	0,00797	5280	941	0,1784	8,35
0,001419	0,9944	0,00743	6290	1210	0,1925	8,64

Tables.

Table II

$\bar{u}$	$\bar{\theta}$	θ_w	$\bar{\theta}-\theta_w$	$\bar{\rho}\cdot\bar{c}$	$\frac{d\bar{\theta}}{dz}$	α	$\bar{c}$
$\frac{cm}{sec}$	C°	C°	C°	$\frac{cal}{cm^3\,C^0}$	$\frac{C^0}{cm}$	$\frac{cal}{cm^2\,sec\,C^0}$	$\frac{cal}{g\,C^0}$
			z/d = 250				
30,0	38,1	29,4	8,7	0,9904	0,125	0,0534	0,9975
45,0	42,0	36,7	5,3	0,9889	0,122	0,128	0,9975
60,0	45,5	41,0	4,5	0,9875	0,114	0,188	0,9975
			z/d = 200				
5,0	13,6	11,2	2,4	0,9990	0,103	0,0268	1,0003
6,0	16,8	13,4	3,4	0,9984	0,111	0,0244	0,9990
8,0	21,4	16,6	4,8	0,9967	0,120	0,0262	0,9988
10,0	25,8	19,3	6,5	0,9951	0,122	0,0234	0,9982
12,5	29,7	22,2	7,5	0,9936	0,135	0,0280	0,9979
15,0	33,5	24,4	9,1	0,9921	0,132	0,0270	0,9976
18,0	36,9	26,3	10,6	0,9908	0,135	0,0284	0,9975
30,0	41,4	32,4	9,0	0,9891	0,143	0,0589	0,9975
45,0	45,1	39,7	5,4	0,9876	0,134	0,138	0,9975
60,0	48,3	43,7	4,6	0,9864	0,122	0,190	0,9976
			z/d = 150				
5,0	16,6	14,2	2,4	0,9985	0,134	0,0348	0,9996
6,0	20,0	16,6	3,4	0,9972	0,163	0,0359	0,9990
8,0	24,8	19,8	5,0	0,9954	0,176	0,0350	0,9983
10,0	29,3	22,5	6,8	0,9937	0,187	0,0342	0,9979
12,5	33,3	25,2	8,1	0,9922	0,180	0,0345	0,9976
15,0	36,9	27,2	9,7	0,9908	0,177	0,0339	0,9975
18,0	40,7	28,9	11,8	0,9894	0,160	0,0339	0,9975
30,0	45,5	35,7	9,8	0,9875	0,180	0,0660	0,9975
45,0	48,8	42,8	6,0	0,9862	0,160	0,148	0,9977
60,0	51,5	46,5	5,0	0,9852	0,137	0,203	0,9979
			z/d = 100				
100	71,8	66,0	5,8	0,9769	0,181	0,381	1,0003
140	74,3	69,7	4,6	0,9759	0,140	0,520	1,0007
200	76,8	73,0	3,8	0,9748	0,100	0,660	1,0011
300	78,9	76,1	2,8	0,9739	0,072	0,939	1,0015
			z/d = 50				
100	76,8	70,0	6,8	0,9748	0,224	0,402	1,0011
140	78,1	72,8	5,3	0,9742	0,168	0,540	1,0014
200	79,6	75,2	4,4	0,9736	0,127	0,703	1,0016
300	80,8	77,8	3,0	0,9730	0,080	0,974	1,0018
			z/d = 25				
100	79,6	72,5	7,1	0,9738	0,247	0,424	1,0016
140	80,4	74,6	5,8	0,9732	0,194	0,570	1,0018
200	81,1	76,4	4,7	0,9729	0,141	0,730	1,0019
300	81,9	78,6	3,3	0,9725	0,090	0,995	1,0020

Tables.

(continued)

λ_w	ϱ	ν	$Pé = \frac{\bar{u} d \overline{\varrho c}}{\lambda_w}$	$2 Rey = \frac{\bar{u} d}{\nu}$	$\sigma = \frac{\lambda_w}{c \mu}$	$A = \frac{\alpha d}{\lambda_w}$
$\frac{cal}{cm\ sec\ C^0}$	$\frac{g}{cm^3}$	$\frac{cm^2}{sec}$	—	—	—	exp.
			$z/d = 250$			
0,001441	0,0920	0,00085	10300	2190	0,2124	18,5
0,001470	0,0914	0,00038	15140	3530	0,2330	43,5
0,001487	0,0900	0,00600	19920	5000	0,2510	63,2
			$z/d = 200$			
0,001369	0,0993	0,01184	1825	211	0,1157	9,79
0,001378	0,0988	0,01090	2173	275	0,1266	8,87
0,001390	0,0979	0,00975	2670	410	0,1431	9,41
0,001401	0,0968	0,00882	3550	567	0,1597	8,34
0,001412	0,0957	0,00810	4395	772	0,1755	9,90
0,001421	0,0945	0,00750	5240	1000	0,1910	9,50
0,001429	0,0933	0,00702	6240	1260	0,2055	9,94
0,001453	0,0917	0,00645	10200	2325	0,2279	20,3
0,001481	0,0901	0,00604	15000	3725	0,2464	40,6
0,001497	0,0888	0,00573	19760	5240	0,2650	65,5
			$z/d = 150$			
0,001381	0,0988	0,01034	1807	229	0,1264	12,6
0,001390	0,0982	0,01006	2150	298	0,1366	12,9
0,001403	0,0971	0,00902	2837	444	0,1563	12,5
0,001414	0,0959	0,00818	3510	611	0,1740	12,1
0,001424	0,0946	0,00753	4355	830	0,1907	12,1
0,001432	0,0933	0,00701	5190	1070	0,2061	11,8
0,001439	0,0919	0,00653	6165	1378	0,2228	11,8
0,001460	0,0900	0,00600	10100	2500	0,2475	23,2
0,001494	0,0886	0,00568	14850	3960	0,2668	49,5
0,001508	0,0873	0,00545	19600	5500	0,2810	67,3
			$z/d = 100$			
0,001580	0,0767	0,00407	30800	12260	0,399	120,0
0,001601	0,0752	0,00395	42700	17720	0,415	162,4
0,001614	0,0737	0,00382	60400	26200	0,434	210,5
0,001620	0,0725	0,00373	89900	40200	0,448	289,0
			$z/d = 50$			
0,001602	0,0737	0,00382	30400	13090	0,430	125,3
0,001613	0,0730	0,00376	42300	18610	0,441	167,4
0,001622	0,0720	0,00369	60000	27100	0,452	216,5
0,001633	0,0713	0,00364	89500	41200	0,461	298,0
			$z/d = 25$			
0,001612	0,0720	0,00369	30200	13550	0,449	131,4
0,001620	0,0715	0,00365	42000	19170	0,457	175,8
0,001627	0,0711	0,00362	59800	27600	0,462	224,2
0,001636	0,0706	0,00359	89200	41800	0,469	304,0

Tables.

Table II

$\dot{u}$	$\overline{\Theta}$	Θ_w	$\overline{\Theta}-\Theta_w$	$\overline{\rho}\cdot\overline{c}$	$\frac{d\overline{\Theta}}{dz}$	a	$\overline{c}$
$\frac{cm}{sec}$	C°	C°	C°	$\frac{cal}{cm^3\,C^0}$	$\frac{C^0}{cm}$	$\frac{cal}{cm^2\,sec\,C^0}$	$\frac{cal}{g\,C^0}$
			z/d = 10				
100	81,5	74,2	7,3	0,9727	0,260	0,407	1,0019
140	81,9	75,8	6,1	0,9725	0,226	0,630	1,0020
200	82,2	77,2	5,0	0,9724	0,160	0,779	1,0020
300	82,6	79,1	3,5	0,9722	0,104	1,065	1,0021
			z/d = 400				
100	45,1	41,9	3,2	0,9876	0,074	0,286	0,9975
100	53,2	49,2	4,0	0,9845	0,098	0,302	0,9980
100	60,7	56,0	4,7	0,9816	0,123	0,321	0,9987
100	67,9	62,4	5,5	0,9786	0,150	0,334	0,9997
			z/d = 350				
100	47,0	43,7	3,3	0,9869	0,078	0,292	0,9976
100	55,8	51,7	4,1	0,9835	0,104	0,312	0,9982
100	64,1	59,1	5,0	0,9802	0,130	0,333	0,9992
100	71,6	66,0	5,6	0,9770	0,157	0,343	1,0002
			z/d = 300				
100	49,1	45,7	3,4	0,9861	0,083	0,301	0,9977
100	58,5	54,3	4,2	0,9824	0,110	0,322	0,9985
100	67,0	62,4	5,2	0,9787	0,144	0,339	0,9990
100	75,6	69,7	5,9	0 9753	0,173	0,358	1,0009
			z/d = 250				
100	51,3	47,7	3,6	0,9853	0,092	0,315	0,9979
100	61,2	56,9	4,3	0,9814	0,116	0,331	0,9988
100	71,2	65,8	5,4	0,9772	0,157	0,355	1,0002
100	80,1	73,9	6,2	0,9733	0,192	0,377	1,0017
			z/d = 200				
100	53,6	49,9	3,7	0,9844	0,098	0,326	0,9980
100	64,3	59,7	4,6	0,9801	0,131	0,349	0,9992
100	75,1	69,5	5,6	0,9755	0,171	0,373	1,0008
100	85,2	78,6	6,6	0,9710	0,215	0,396	1,0026
			z/d = 150				
100	56,1	52,2	3,9	0,9834	0,109	0,343	0,9982
100	67,9	62,7	5,2	0,9768	0,155	0,365	0,9997
100	79,5	72,9	6,6	0,9736	0,190	0,350	1,0017
100	90,8	83.9	6,9	0,9684	0,237	0,416	1,0037
			z/d = 400				
140	49,2	46,4	2,8	0,9860	0,063	0,388	0,9977
140	58,9	55,4	3,5	0,9823	0,086	0,423	0,9985
140	67,9	63,8	4,1	0,9786	0,108	0,451	0,9997
140	76,1	71,2	4,9	0,9751	0,136	0,473	1,0010

Tables.

λ_w	$\bar{\rho}$	$\bar{\nu}$	$Pé = \frac{\bar{u} d \bar{\rho} \bar{c}}{\lambda_w}$	$2 Rey = \frac{\bar{u} d}{\bar{\nu}}$	$\sigma = \frac{\lambda_w}{\bar{c} \bar{\mu}}$	$A = \frac{\alpha d}{\lambda_w}$	A calculated	
$\frac{cal}{cm\ sec\ C^0}$	$\frac{g}{cm^3}$	$\frac{cm^2}{sec}$	—	—	—	exp.	Formula 12a	13
z/d = 10								
0,001618	0,9708	0,00360	30100	13890	0,462	144,1		
0,001625	0,9706	0,00359	41900	19500	0,465	194,0		
0,001630	0,9704	0,00358	59700	27900	0,468	239,0		
0,001638	0,9701	0,00356	89100	42200	0,473	331,0		
z/d = 400								
0,001490	0,9902	0,00604	33150	8280	0,250	90,0	97,2	105,0
0,001519	0,9565	0,00530	32400	9435	0,291	99,2	95,5	108,1
0,001540	0,9625	0,00472	31720	10590	0,334	103,6	94,0	113,7
0,001572	0,9789	0,00426	31100	11060	0,376	106,1	92,0	121,4
z/d = 350								
0,001497	0,9894	0,00565	32900	8550	0,259	97,4	96,8	105,6
0,001529	0,9853	0,00509	32150	9825	0,305	102,0	95,0	109,6
0,001559	0,9810	0,00450	31400	11110	0,354	106,6	93,3	115,9
0,001560	0,9768	0,00407	30770	12260	0,399	108,0	91,9	125,2
z/d = 300								
0,001505	0,9884	0,00506	32750	8840	0,269	100,0	96,3	106,8
0,001539	0,9830	0,00466	31900	10245	0,321	104,6	94,4	111,6
0,001572	0,9791	0,00429	31100	11050	0,375	107,7	92,6	119,3
0,001601	0,9745	0,00368	30430	12885	0,424	111,7	91,1	130,8
z/d = 250								
0,001513	0,9874	0,00545	32550	9175	0,282	103,9	95,8	108,5
0,001550	0,9825	0,00469	31650	10655	0,377	106,9	93,9	114,7
0,001585	0,9770	0,00410	30600	12190	0,396	112,0	92,0	124,7
0,001617	0,9717	0,00366	30050	13655	0,455	116,6	90,3	139,2
z/d = 200								
0,001522	0,9863	0,00526	32350	9510	0,294	107,0	95,4	
0,001561	0,9809	0,00446	31360	11155	0,356	111,8	93,2	
0,001600	0,9748	0,00390	30470	12820	0,421	116,3	91,2	
0,001636	0,9665	0,00346	29630	14450	0,488	120,8	89,3	
z/d = 150								
0,001531	0,9851	0,00506	32100	9885	0,308	112,0	94,9	
0,001573	0,9769	0,00428	31100	11660	0,376	115,9	92,6	
0,001613	0,9721	0,00370	30150	13510	0,448	108,6	90,5	
0,001657	0,9648	0,00325	29200	15380	0,527	125,4	88,4	
z/d = 400								
0,001508	0,9884	0,00565	45800	12380	0,270	128,6	123,8	133,6
0,001544	0,9837	0,00485	44500	14430	0,324	136,8	121,2	137,3
0,001577	0,9789	0,00428	43400	16350	0,377	143,1	116,9	143,9
0,001606	0,9741	0,00385	42450	18180	0,428	147,3	117,0	153,4

Tables.

Table II

$\bar{u}$	$\bar{\Theta}$	Θ_w	$\bar{\Theta}-\Theta_w$	$\rho \cdot \bar{c}$	$\frac{d\bar{\Theta}}{dx}$	α	$\bar{c}$
$\frac{cm}{sec}$	C°	C°	C°	$\frac{cal}{cm^3\ C°}$	$\frac{C°}{cm}$	$\frac{cal}{cm^2\ sec\ C°}$	$\frac{cal}{g\ C°}$
x/d = 350							
140	51,0	48,1	2,9	0,9854	0,070	0,416	0,9976
140	61,0	57,4	3,6	0,9814	0,092	0,435	0,9955
140	70,7	66,3	4,4	0,9774	0,120	0,466	1,0001
140	79,6	74,6	5,0	0,9735	0,144	0,490	1,0016
x/d = 300							
140	52,9	49,9	[illegible]	0,9840	0,075	0,430	0,9960
140	63,3	59,6	[illegible]	0,9805	0,096	0,454	0,9990
140	73,6	69,1	4,5	0,9762	0,125	0,475	1,0006
140	83,2	78,2	5,0	0,9719	0,150	0,510	1,0023
x/d = 250							
140	54,6	51,6	3,0	0,9839	0,077	0,441	0,9961
140	65,8	62,0	3,8	0,9795	0,103	0,465	0,9994
140	76,7	72,0	4,7	0,9746	0,135	0,490	1,0011
140	87,1	82,0	5,1	0,9701	0,162	0,536	1,0030
x/d = 200							
140	56,6	53,6	3,2	0,9831	0,083	0,446	0,9963
140	68,4	64,5	3,9	0,9784	0,110	0,463	0,9998
140	80,0	75,2	4,8	0,9734	0,143	0,508	1,0017
140	91,5	85,9	5,6	0,9650	0,184	0,557	1,0036
x/d = 150							
140	58,8	55,5	3,4	0,9823	0,090	0,455	0,9965
140	71,1	67,0	4,1	0,9772	0,120	0,501	1,0002
140	83,6	78,6	5,0	0,9717	0,157	0,534	1,0023
140	96,2	90,2	6,0	0,9657	0,200	0,503	1,0016
x/d = 400							
200	53,3	50,8	2,5	0,9845	0,055	0,541	0,9960
200	64,1	61,1	3,0	0,9802	0,071	0,560	0,9992
200	74,7	71,2	3,5	0,9757	0,089	0,620	1,0008
200	84,0	79,6	4,4	0,9715	0,120	0,663	1,0024
x/d = 350							
200	54,7	52,1	2,6	0,9839	0,059	0,558	0,9981
200	66,0	62,8	3,2	0,9794	0,078	0,596	0,9994
200	77,0	73,3	3,7	0,9747	0,097	0,638	1,0012
200	87,0	82,6	4,4	0,9702	0,124	0,684	1,0030
x/d = 300							
200	56,3	53,5	2,8	0,9833	0,064	0,563	0,9963
200	68,0	64,7	3,3	0,9785	0,082	0,606	0,9997
200	79,4	75,7	3,7	0,9736	0,100	0,657	1,0016
200	90,2	85,6	4,6	0,9686	0,133	0,700	1,0036

Tables.

λ_w	$\bar{\rho}$	$\bar{\nu}$	$Pé = \frac{\bar{u} d \bar{\rho} \bar{c}}{\lambda_w}$	$2Re_y = \frac{\bar{u} d}{\bar{\nu}}$	$n = \frac{\lambda_w}{\bar{c} \mu}$	$A = \frac{\alpha \cdot d}{\lambda_w}$	A calculated	
$\frac{cal}{cm\ sec\ C°}$	$\frac{g}{cm^3}$	$\frac{cm^2}{sec}$	—	—	—	exp. —	Formula 12a	13
				z/d = 350				
0,001515	0,9870	0,00518	45500	12770	0,260	137,2	123,2	134,7
0,001552	0,9820	0,00470	44250	14885	0,330	141,1	120,7	139,3
0,001587	0,9773	0,00412	43100	16950	0,394	140,9	116,3	147,1
0,001620	0,9720	0,00369	42000	18970	0,451	151,2	110,0	158,2
				z/d = 300				
0,001522	0,9867	0,00533	45260	13130	0,290	141,2	122,8	136,3
0,001561	0,9814	0,00454	43920	15415	0,351	145,3	120,0	141,9
0,001598	0,9750	0,00398	42700	17585	0,411	148,7	117,5	151,5
0,001634	0,9698	0,00354	41600	19770	0,475	150,0	115,2	165,5
				z/d = 250				
0,001530	0,9857	0,00517	45000	13530	0,301	144,0	122,2	138,5
0,001570	0,9801	0,00439	43050	15940	0,365	148,0	119,4	140,0
0,001610	0,9738	0,00382	42320	18320	0,433	152,2	116,7	156,3
0,001649	0,9672	0,00336	41150	20700	0,503	163,3	114,3	176,5
				z/d = 200				
0,001537	0,9847	0,00501	44760	13970	0,312	146,1	121,7	142,3
0,001580	0,9780	0,00425	43300	16470	0,380	153,0	118,7	152,5
0,001622	0,9718	0,00367	42000	19070	0,455	116,0	116,0	169,7
0,001665	0,9643	0,00322	40700	21730	0,535	167,1	113,2	194,7
				z/d = 150				
0,001544	0,9837	0,00485	44500	14430	0,324	147,3	121,2	
0,001590	0,9771	0,00410	43000	17070	0,397	157,5	118,1	
0,001636	0,9695	0,00352	41520	19860	0,478	163,1	115,0	
0,001682	0,9610	0,00307	40150	22800	0,568	167,6	112,2	
				z/d = 400				
0,001526	0,9865	0,00529	64500	16900	0,293	177,3	160,1	173,1
0,001566	0,9810	0,00450	62600	22220	0,355	165,0	156,5	177,4
0,001606	0,9750	0,00392	60700	25500	0,420	193,0	153,0	185,1
0,001640	0,9693	0,00350	59200	28500	0,4825	202,0	150,1	196,8
				z/d = 350				
0,001531	0,9858	0,00518	64250	19300	0,300	182,1	159,0	174,5
0,001574	0,9800	0,00438	62200	22820	0,367	189,3	155,8	179,8
0,001615	0,9736	0,00381	60300	26230	0,435	197,6	152,2	189,3
0,001652	0,9673	0,00338	58650	29580	0,504	207,0	149,1	203,3
				z/d = 300				
0,001536	0,9850	0,00505	64000	19790	0,309	183,1	159,1	176,6
0,001581	0,9789	0,00427	61850	23400	0,378	192,3	155,1	183,3
0,001624	0,9721	0,00370	59900	27000	0,451	202,2	151,4	195,2
0,001664	0,9652	0,00327	58150	30580	0,525	210,2	148,1	212,7

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Tables.

Table II

$\bar{u}$	$\bar{\Theta}$	Θ_w	$\bar{\Theta}-\Theta_w$	$\bar{\rho}\cdot\bar{c}$	$\frac{d\bar{\Theta}}{dz}$	α	$\bar{c}$
$\frac{cm}{sec}$	C°	C°	C°	$\frac{cal}{cm^3\,C°}$	$\frac{C°}{cm}$	$\frac{cal}{cm^2\,sec\,C°}$	$\frac{cal}{g\,C°}$
				z/d = 250			
200	57,8	55,0	2,8	0,9827	0,060	0,560	0,9964
200	70,1	66,7	3,4	0,9777	0,087	0,625	1,0000
200	82,0	78,2	3,8	0,9725	0,100	0,678	1,0020
200	93,5	88,8	4,7	0,9671	0,140	0,721	1,0043
				z/d = 200			
200	59,4	56,6	2,8	0,9821	0,068	0,596	0,9966
200	72,2	68,8	3,4	0,9767	0,089	0,640	1,0003
200	84,7	80,7	4,0	0,9712	0,114	0,692	1,0025
200	97,0	92,2	4,8	0,9653	0,148	0,744	1,0050
				z/d = 150			
200	61,2	58,3	2,9	0,9813	0,072	0,610	0,9968
200	74,4	70,8	3,6	0,9758	0,096	0,652	1,0007
200	87,6	83,4	4,2	0,9699	0,124	0,715	1,0030
200	100,8	95,9	4,9	0,9633	0,155	0,760	1,0058
				z/d = 400			
300	57,2	55,2	2,0	0,9830	0,042	0,774	0,9984
300	69,3	66,7	2,6	0,9780	0,059	0,833	0,9999
300	81,3	78,1	3,2	0,9728	0,079	0,900	1,0019
300	90,9	86,8	4,1	0,9683	0,108	0,956	1,0037
				z/d = 350			
300	58,3	56,2	2,1	0,9825	0,045	0,789	0,9985
300	70,8	68,2	2,6	0,9774	0,061	0,859	1,0010
300	83,3	80,1	3,2	0,9719	0,081	0,922	1,0023
300	93,7	89,5	4,2	0,9670	0,113	0,976	1,0043
				z/d = 300			
300	59,4	57,2	2,2	0,9820	0,046	0,804	0,9986
300	72,4	69,7	2,7	0,9767	0,064	0,868	1,0004
300	85,3	82,1	3,2	0,9709	0,082	0,933	1,0026
300	96,5	92,3	4,2	0,9656	0,115	0,992	1,0049
				z/d = 250			
300	60,6	58,4	2,2	0,9816	0,049	0,820	0,9987
300	74,0	71,3	2,7	0,9760	0,066	0,893	1,0006
300	87,4	84,2	3,2	0,9700	0,084	0,955	1,0030
300	99,4	95,2	4,2	0,9640	0,118	1,017	1,0055
				z/d = 200			
300	61,9	59,6	2,3	0,9811	0,052	0,831	0,9989
300	75,6	72,9	2,7	0,9753	0,067	0,907	1,0009
300	89,4	86,2	3,2	0,9690	0,086	0,976	1,0034
300	102,3	98,1	4,2	0,9625	0,121	1,040	1,0061

Tables.

λ_w	$\bar{\varrho}$	$\bar{\nu}$	$Pé = \frac{\bar{u} d \bar{\varrho} \bar{c}}{\lambda_w}$	$2\,Rey = \frac{\bar{u} d}{\bar{\nu}}$	$\sigma = \frac{\lambda_w}{c\,\mu}$	$A = \frac{\alpha d}{\lambda_w}$	Λ calculated	
$\frac{cal}{cm\ sec\ C^{0}}$	$\frac{g}{cm^3}$	$\frac{cm^2}{sec}$	—	—	—	exp.	Formula 12a	13
				$z/d = 250$				
0,001542	0,9843	0,00494	63700	20230	0,316	188,0	155,6	179,7
0,001589	0,9777	0,00415	61500	24085	0,392	190,7	154,5	188,9
0,001634	0,9705	0,00358	59450	27910	0,470	207,3	150,6	204,3
0,001670	0,9629	0,00316	57000	31640	0,548	215,1	147,1	227,0
				$z/d = 200$				
0,001549	0,9835	0,00481	63400	20780	0,328	192,3	156,0	164,7
0,001597	0,9704	0,00405	61100	24650	0,403	200,4	153,7	197,4
0,001644	0,9688	0,00347	59000	28800	0,488	210,5	149,7	219,0
0,001690	0,9604	0,00305	57050	32780	0,575	220,0	140,0	251,1
				$z/d = 150$				
0,001556	0,9825	0,00468	63000	21360	0,339	196,0	157,3	
0,001605	0,9752	0,00394	60750	25370	0,417	203,2	153,0	
0,001655	0,9670	0,00336	58550	29750	0,506	216,0	148,9	
0,001705	0,9577	0,00294	56400	34000	0,602	223,0	144,8	
				$z/d = 400$				
0,001543	0,9846	0,00498	95600	30120	0,315	251,0	215,0	232,5
0,001589	0,9781	0,00420	92300	35720	0,387	262,0	209,4	237,3
0,001634	0,9710	0,00361	89300	41550	0,465	275,2	204,3	247,2
0,001668	0,9647	0,00324	87050	46300	0,532	286,5	200,4	262,8
				$z/d = 350$				
0,001547	0,9840	0,00490	95300	30620	0,321	255,0	214,5	234,5
0,001595	0,9772	0,00411	91850	36500	0,397	269,0	208,7	240,8
0,001642	0,9697	0,00354	88750	42370	0,477	280,5	203,4	253,0
0,001679	0,9626	0,00315	86400	47620	0,552	290,7	199,3	271,7
				$z/d = 300$				
0,001551	0,9835	0,00481	95000	31200	0,328	259,0	214,0	237,5
0,001601	0,9763	0,00403	91500	37220	0,407	271,0	208,1	240,0
0,001650	0,9684	0,00345	88250	43480	0,492	283,0	202,5	261,0
0,001690	0,9608	0,00306	85650	49030	0,573	294,0	196,0	284,5
				$z/d = 250$				
0,001550	0,9826	0,00472	94650	31760	0,336	263,5	213,4	241,9
0,001607	0,9754	0,00396	91100	37880	0,416	278,0	207,4	253,6
0,001658	0,9671	0,00337	87700	44500	0,507	288,0	201,6	272,4
0,001702	0,9587	0,00298	84950	50330	0,593	298,5	196,8	303,7
				$z/d = 200$				
0,001561	0,9821	0,00464	94250	32330	0,343	266,0	212,7	248,7
0,001613	0,9745	0,00387	90700	38760	0,427	281,0	206,7	265,4
0,001666	0,9657	0,00330	87200	45450	0,520	293,2	200,7	293,7
0,001713	0,9567	0,00289	84250	51900	0,617	303,5	195,6	336,5

Tables.

Table II

$\bar{u}$	$\bar{\Theta}$	Θ_w	$\bar{\Theta}-\Theta_w$	$\bar{\rho}\cdot\bar{c}$	$\frac{d\bar{\Theta}}{dz}$	a	$\bar{c}$
$\frac{cm}{sec}$	C°	C°	C°	$\frac{cal}{cm^3\ C°}$	$\frac{C°}{cm}$	$\frac{cal}{cm^2\ sec\ C°}$	$\frac{cal}{g\ C°}$
				z/d = 150			
300	63,1	60,8	2,3	0,9806	0,053	0,852	0,9990
300	77,2	74,5	2,7	0,9746	0,068	0,920	1,0012
300	91,5	88,3	3,2	0,9680	0,088	0,998	1,0039
300	105,3	101,0	4,3	0,9008	0,127	1,065	1,0068
				z/d = 400			
500	61,1	59,5	1,6	0,9814	0,029	1,112	0,9988
500	73,8	71,7	2,1	0,9760	0,042	1,220	1,0006
500	87,6	85,3	2,3	0,9699	0,050	1,317	1,0031
500	99,2	95,7	3,5	0,9642	0,083	1,428	1,0054
				z/d = 350			
500	61,8	60,2	1,6	0,9811	0,030	1,149	0,9989
500	74,9	72,7	2,2	0,9756	0,045	1,247	1,0008
500	88,8	86,4	2,4	0,9693	0,053	1,338	1,0033
500	101,3	97,8	3,5	0,9630	0,064	1,444	1,0059
				z/d = 300			
500	62,6	60,9	1,7	0,9808	0,032	1,155	0,9990
500	76,0	73,7	2,3	0,9751	0,047	1,246	1,0010
500	90,1	87,7	2,4	0,9687	0,053	1,337	1,0036
500	103,3	99,8	3,5	0,9619	0,065	1,460	1,0063
				z/d = 250			
500	63,5	61,7	1,8	0,9804	0,033	1,124	0,9991
500	77,2	74,9	2,3	0,9746	0,048	1,271	1,0012
500	91,4	89,0	2,4	0,9680	0,054	1,361	1,0038
500	105,4	101,9	3,5	0,9607	0,086	1,476	1,0065
				z/d = 200			
500	64,3	62,5	1,8	0,9801	0,035	1,180	0,9992
500	78,4	76,0	2,4	0,9740	0,050	1,270	1,0014
500	92,7	90,3	2,4	0,9675	0,056	1,410	1,0041
500	107,5	104,0	3,5	0,9595	0,068	1,509	1,0073
				z/d = 150			
500	65,1	63,3	1,8	0,9798	0,030	1,224	0,9992
500	79,6	77,2	2,4	0,9736	0,051	1,294	1,0016
500	94,2	91,6	2,6	0,9657	0,061	1,418	1,0044
500	109,6	106,0	3,6	0,9583	0,092	1,530	1,0078

Tables.

λ_w	$\bar{\rho}$	$\bar{\nu}$	$Pé = \frac{\bar{u} d \bar{\rho} \bar{c}}{\lambda_w}$	$2Rey = \frac{\bar{u} d}{\bar{\nu}}$	$\sigma = \frac{\lambda_w}{\bar{c} \bar{\mu}}$	$\Lambda = \frac{\alpha d}{\lambda_w}$	A calculated	
$\frac{cal}{cm\ sec\ C^\circ}$	$\frac{g}{cm^3}$	$\frac{cm}{sec}$	—	—	—	exp.	Formula 12a	13
				z/d = 160				
0,001566	0,9815	0,00456	93900	32900	0,350	272,3	212,2	
0,001620	0,9735	0,00380	90250	39480	0,438	284,0	205,8	
0,001675	0,9643	0,00322	86700	40600	0,538	298,0	199,8	
0,001725	0,9545	0,00280	83500	53600	0,642	308,5	194,3	
				z/d = 400				
0,001560	0,9826	0,00470	157300	53200	0,338	350,5	312,4	337,8
0,001609	0,9755	0,00396	151600	63150	0,416	379,0	303,9	344,4
0,001662	0,9670	0,00330	145900	74400	0,510	396,0	295,3	357,3
0,001704	0,9580	0,00298	141400	83900	0,594	418,5	288,4	378,1
				z/d = 350				
0,001563	0,9822	0,00165	156050	53600	0,342	367,5	311,9	341,0
0,001612	0,9749	0,00391	151300	63900	0,423	380,5	303,4	350,2
0,001667	0,9661	0,00333	145300	75300	0,518	401,0	294,3	366,0
0,001712	0,9574	0,00292	140600	85650	0,610	421,5	287,2	391,5
				z/d = 300				
0,001566	0,9816	0,00459	156500	54500	0,346	368,5	311,2	345,5
0,001617	0,9742	0,00386	150700	64600	0,430	385,5	302,5	357,6
0,001672	0,9653	0,00327	144750	76450	0,528	399,5	293,5	378,3
0,001720	0,9560	0,00286	139750	87450	0,626	424,5	285,9	410,7
				z/d = 250				
0,001569	0,9813	0,00454	156200	55100	0,352	358,0	310,8	352,2
0,001621	0,9735	0,00380	150300	65800	0,438	392,0	301,9	369,0
0,001677	0,9644	0,00322	144300	77050	0,538	400,0	292,8	397,0
0,001728	0,9544	0,00280	139000	89300	0,643	427,0	284,7	439,2
				z/d = 200				
0,001572	0,9809	0,00449	155600	55700	0,357	376,2	310,2	362,6
0,001626	0,9728	0,00374	149700	66550	0,446	390,5	301,0	386,6
0,001683	0,9635	0,00318	143600	78650	0,548	419,0	291,8	427,0
0,001736	0,9527	0,00275	138100	90920	0,658	434,5	283,3	487,2
				z/d = 150				
0,001575	0,9805	0,00444	155500	56300	0,362	389,0	309,7	
0,001630	0,9720	0,00370	149300	67600	0,452	397,0	300,4	
0,001688	0,9624	0,00314	143150	79600	0,556	420,0	291,1	
0,001744	0,9510	0,00269	137300	92850	0,677	438,5	282,1	

TABLES.

TABLE III

CALCULATION OF "GRASHOF" AND "σ" FOR FOUR VESSEL TEMPERATURES

$\beta = 0{,}00018\ ^{\circ}C^{-1};\ g = 961\ \frac{cm}{sec^2};\ a = 25\ cm$

$T_w - T_n = 8{,}8^{\circ}$ at vessel temperature $T_n = 69.2^{\circ}$

$T_w - T_n = 9{,}5^{\circ}$ at vessel temperature $T_n = 85{,}5^{\circ}$

$T_w - T_n = 10{,}2^{\circ}$ at vessel temperature $T_n = 101{,}8^{\circ}$

$T_w - T_n = 10{,}8^{\circ}$ at vessel temperature $T_n = 118{,}0^{\circ}$

$\nu_{69,2^{\circ}} = 0{,}00420\ \frac{cm^2}{sec}$

$\nu_{85,5^{\circ}} = 0{,}00345\ \frac{cm^2}{sec}$

$\nu_{101,8^{\circ}} = 0{,}00290\ \frac{cm^2}{sec}$

$\nu_{118,0^{\circ}} = 0{,}00250\ \frac{cm^2}{sec}$

$Gr_{69,2^{\circ}} = 1{,}376 \cdot 10^9$

$Gr_{85,5^{\circ}} = 2{,}202 \cdot 10^9$

$Gr_{101,8^{\circ}} = 3{,}346 \cdot 10^9$

$Gr_{118,0^{\circ}} = 4{,}768 \cdot 10^9$

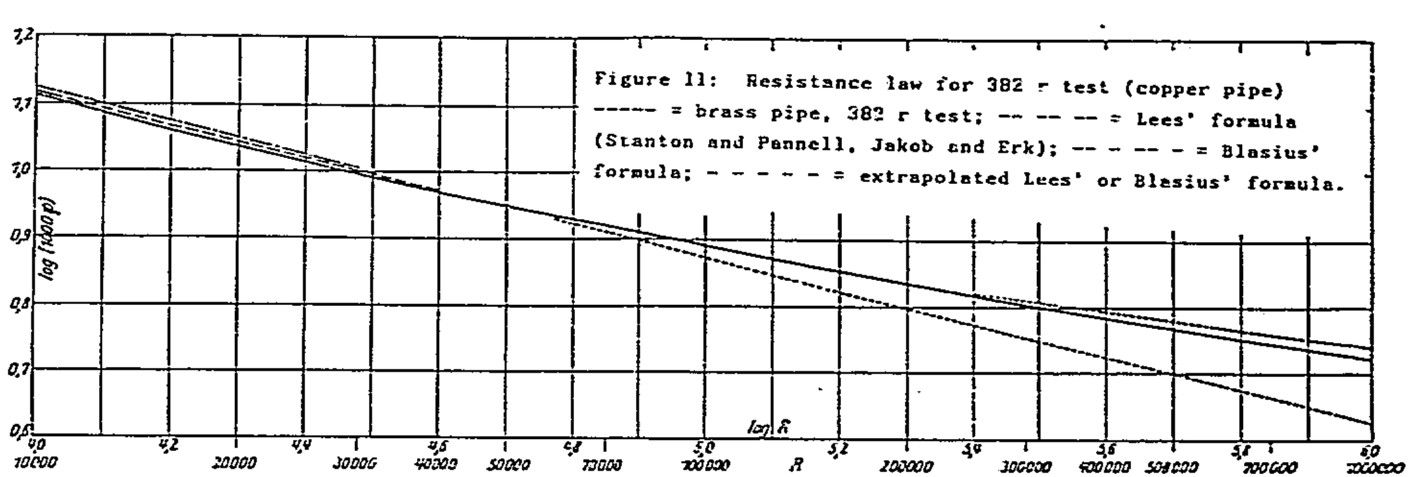

Figure 11: Resistance law for 382 r test (copper pipe) ----- = brass pipe, 382 r test; -- -- -- = Lees' formula (Stanton and Pannell, Jakob and Erk); -- - -- - = Blasius' formula; - - - - - = extrapolated Lees' or Blasius' formula.

Fig. 11. Table for Hermann

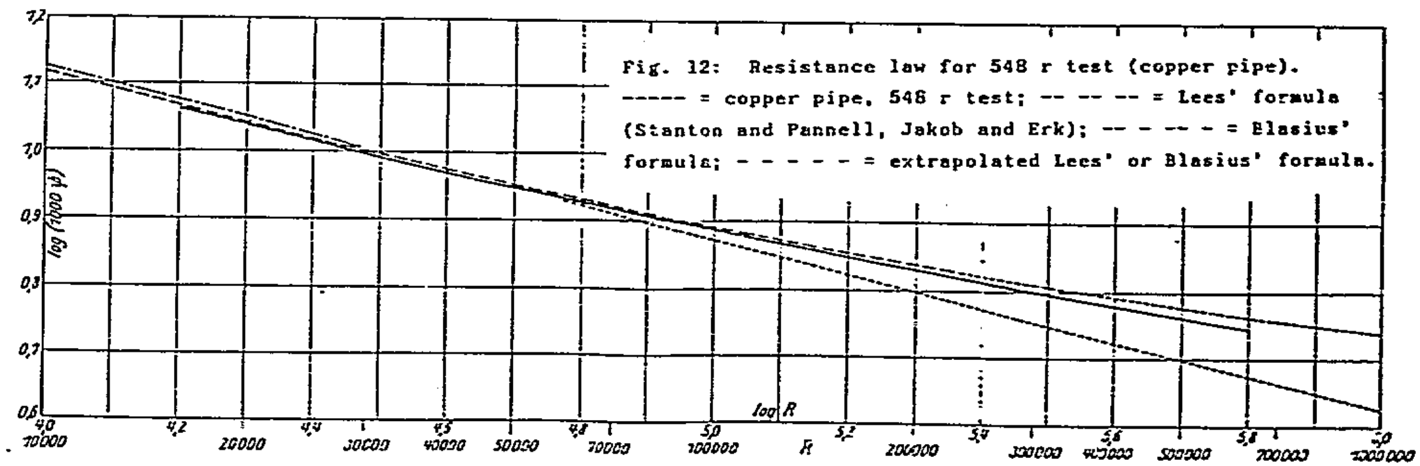

Fig. 12: Resistance law for 548 r test (copper pipe). ----- = copper pipe, 548 r test; -- -- -- = Lees' formula (Stanton and Pannell, Jakob and Erk); -- - -- - = Blasius' formula; - - - - - = extrapolated Lees' or Blasius' formula.

Fig. 12. Table for Burbach

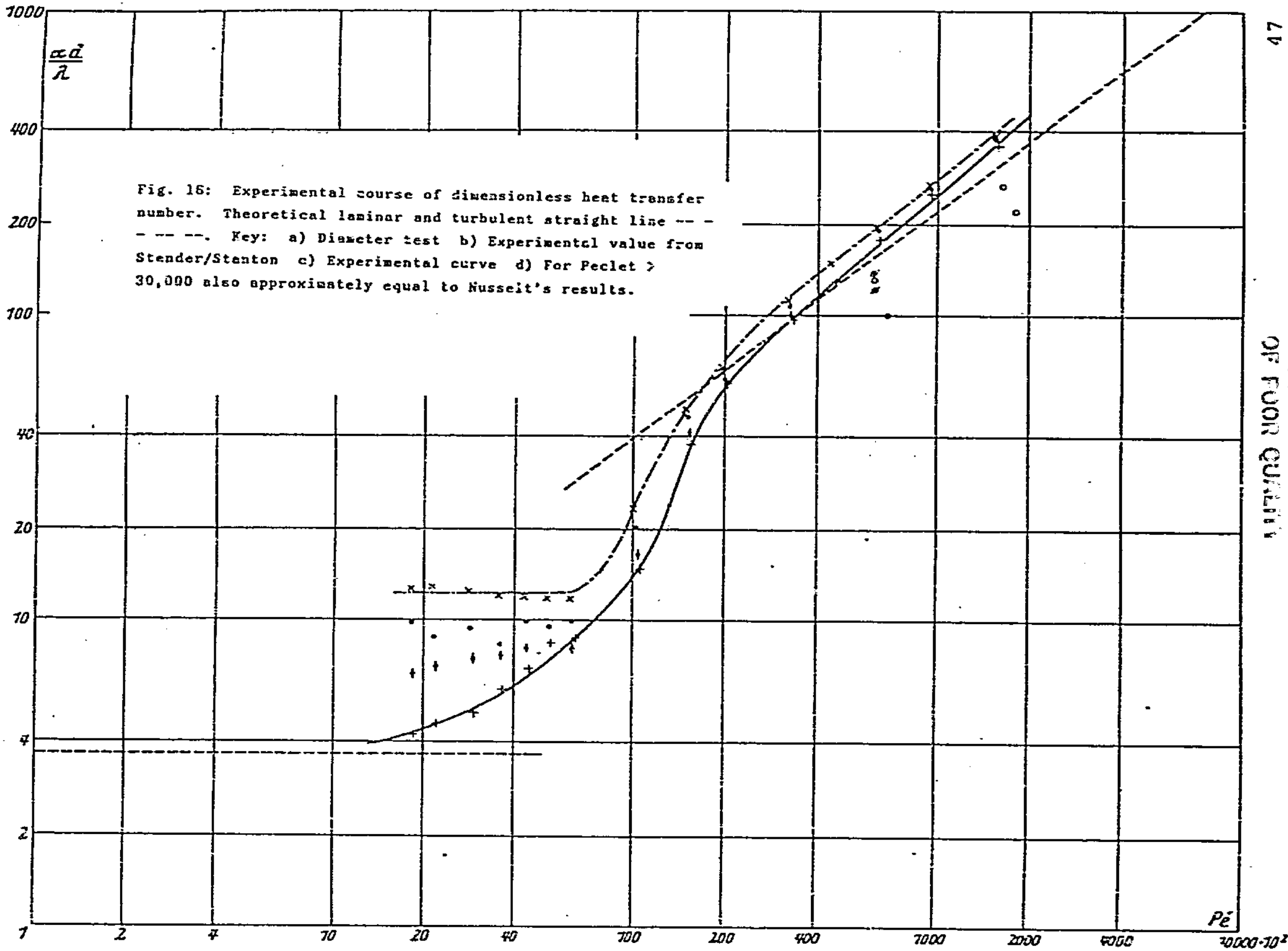

Fig. 16: Experimental course of dimensionless heat transfer number. Theoretical laminar and turbulent straight line -- - - -- --. Key: a) Diameter test b) Experimental value from Stender/Stanton c) Experimental curve d) For Peclet > 30,000 also approximately equal to Nusselt's results.