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IMPROVEMENT OF CONDITIONS FOR MESHING SPIRAL BEVEL GEARS

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16. Abstract This article analyzes the effect of axial pinion displacement on gear meshing conditions during cutting and correction of the rolling chain gear ratio. The so-called inverse problem-solving method is used.

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IMPROVEMENT OF CONDITIONS FOR MESHING SPIRAL BEVEL GEARS

F.I. Litvin, Kai Go

In the previous article [1], the authors proved analytically that meshing spiral bevel gears cut without correcting the set-up of the gear-cutting machine has the following deficiencies: /87*

- the operating line on the tooth's surface, which defines the position of the bearing pattern, is diagonal, and the gear ratio is variable.

E. Waldgaber [2] showed that eliminating these deficiencies requires correcting the adjustment of a pinion to be cut and the kinematic set-up of the gear-cutting machine.

V. N. Kedrinskiy and K. M. Pismanik [3] developed several methods for correcting machine set-up, among which the most commonly used are 1) changing the axial displacement of the change gear and the gear ratio of the machine's rolling chain and 2) changing the axial and hypoid displacements of the gear and the gear ratio of the rolling chain. They also proposed approximation equations for determining these corrections.

This article analyzes the effect of axial pinion displacement on gear meshing conditions during cutting and correction of the rolling chain gear ratio. The so-called inverse problem-solving method, presented in the authors' work [3] and in F. L. Litvin's work [5], is used.

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*Numbers in the margin indicate pagination of the foreign text.

1. Coordinate System

Each crown wheel $p_i (i = 1,2)$ has a coordinate system x_{pi}, y_{pi}, z_{pi} (figures 1 and 2). Two indices (i = 1, 2) are

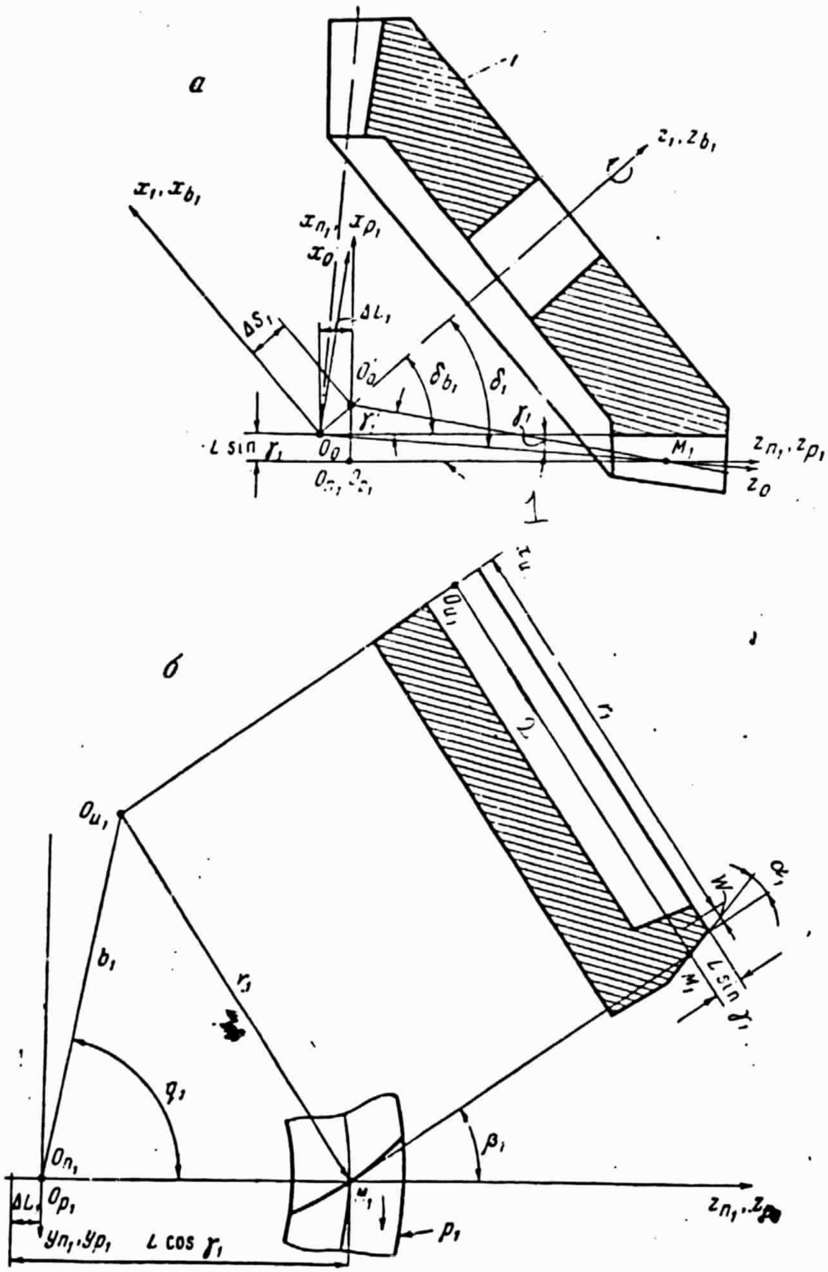

Figure 1. System of coordinates corresponding to cutting a pinion. 1 - Adjusting plane; 2 - Section along O_u, M_i.

introduced, because cutting pinion 1 and gear 2 using different generating surfaces (cf. figures 1 and 2) is considered separately.

The coordinate system x_{ni}, y_{ni}, z_{ni} is an auxiliary fixed coordinate system in which the rotation of crown wheel p_i is given. When teeth are being cut, crown wheel p_i /88 rotates around axis x_{ni}. The symbol Ψ_i is the current angle of rotation. The plane $x_{ni} = 0$, the locating plane, is parallel to the plane tangent to the root cone of the gear being cut. The forming radius of the cutting head r_i and angle β_i between the tangent to the longitudinal line of the tooth at the midpoint M_i and axis z_{pi} are set in the locating plane (figures 1, b, 2, b). Cutting head adjustment parameters q_i and b_i are determined from functions (3.7), (3.8), and (3.9), introduced below. When

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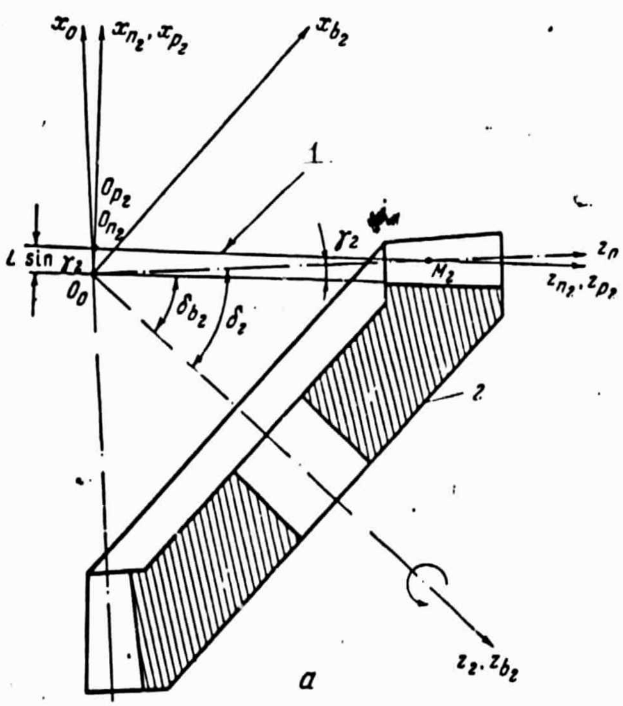
a)



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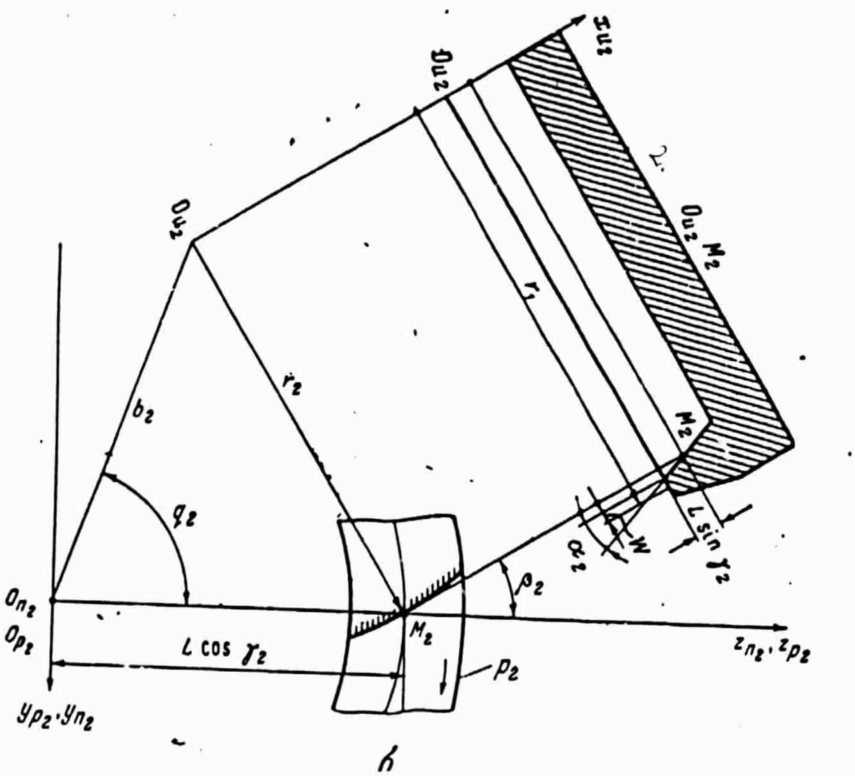

Figure 2. Coordinate system corresponding to cutting a wheel. 1 - Locating plane; 2 - Section O_{u2} M^2

the cutting head turns around axis O_{ui} (cf. figure 1, b, 2, b), a bevel surface results, which is also crown. /90

When a pinion is being cut, it assumes axial shift $0_0 0'_0$ (cf. figure 1), where 0_0 is the point of intersection of the axes of the pinion and the gear during meshing in gearing, and $0'_0$ is the point of intersection of axis z_1 of pinion rotation during cutting with rotation axis x_{ni} for crown wheel p_i. The gear is cut without axial shift (cf. figure 2), and 0_0 is both the point of intersection of pinion and gear axes when they mesh and the point of intersection of axis z_2 of the gear when it is cut and axis of rotation x_{ni} of crown wheel p_2.

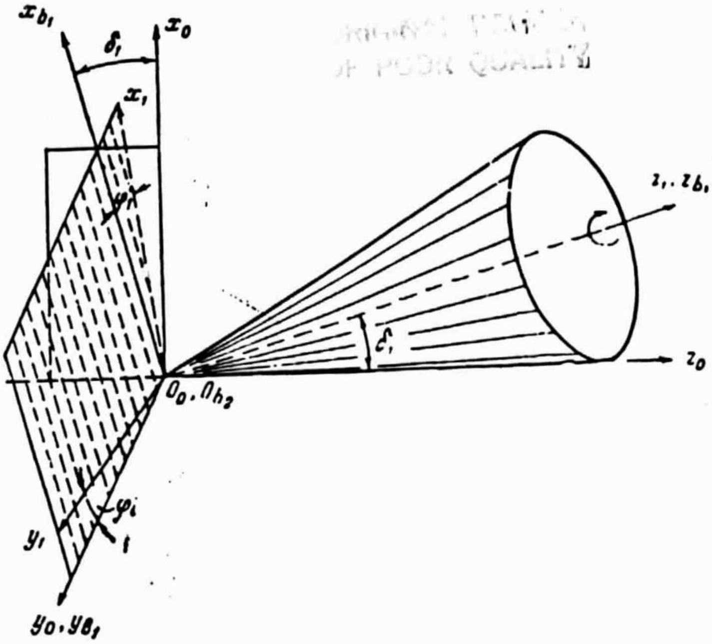

Figure 3. Solving equations for the relationship between coordinate systems x_{bi}, y_{bi}, z_{bi} and x_0, y_0, z_0.

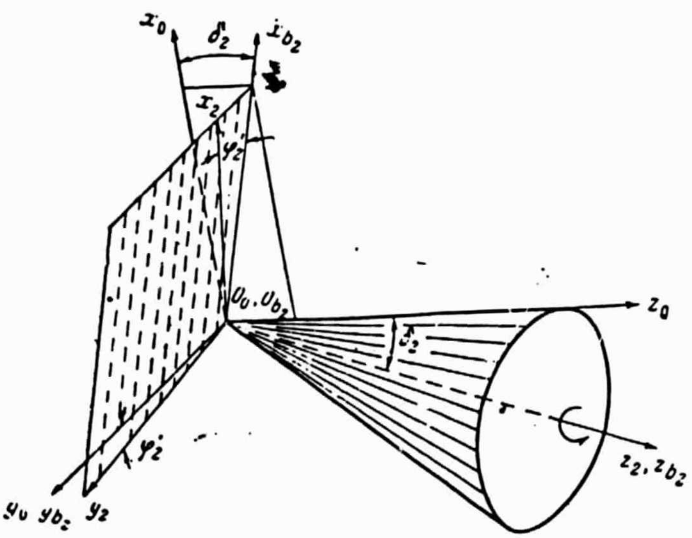

Figure 4. Solving equations for the relationship between coordinate systems x_{b2}, y_{b2}, z_b2 and x_0, y_0, z_0.

The fixed coordinate system x_0, y_0, z_0 considers the meshing of the pinion and the wheel. Axis 0_0z_0 coincides with the overall generatrix of the initial bevel of the pinion and gear, and 0_0 is, as already mentioned, the point of intersection of their axes of rotation. System x_0, y_0, z_0 is turned by angle γ_i of the tooth edge around y_{ni}, and coordinate origin 0_0 relative to 0_{ni} is shifted by the value: /91

$$x_{n_1} = L \sin \gamma_1; \; y_{n_1} = 0; \; z_{n_1} = -\Delta L_1; \qquad \text{(cf. figure 1)}$$

$$x_{n_2} = -L \sin \gamma_2; \; y_{n_2} = z_{n_2} = 0. \qquad \text{(cf. figure 2)}$$

Here $L = 0_0M_1 = 0_0M_2$ is the midpoint of the generatrix of the initial cone.

The gear being cut, i, has a coordinate system x_i, y_i, z_i (i = 1, 2). Pinion and gear, when meshing, rotate respectively around axes z_{bi} and z_{b2}, which coincide with axes z_1 and z_2. Auxiliary fixed coordinate system x_{bi}, y_{bi}, z_{bi}, presented in figures 3 and 4, can also be used to illustrate the transition to this system from system x_0, y_0, z_0.

2. Gear Tooth Surface Equations

When a gear is being cut, axis O_0z_0 is the instantaneous rotation axis in relative motion. Determining the line of contact of the crown and cut surfaces requires that the normal to the surfaces at points on the line of contact must intersect axis 0_0z_0.

In our article [1], tooth surface Σ_2 was determined with the following equations:

$$\begin{aligned}
x_2 &= (r_2 \operatorname{ctg} \alpha_2 - u_2 \cos \alpha_2 + L \sin \gamma_2) a_1^{(2)} + u_2 \sin \alpha_2 \sin \vartheta_2 b_1^{(2)} + \\
&\quad + (u_2 \sin \alpha_2 \cos \vartheta_2 + b_2) c_1^{(2)}; \\
y_2 &= (r_2 \operatorname{ctg} \alpha_2 - u_2 \cos \alpha_2 + L \sin \gamma_2) a_2^{(2)} + u_2 \sin \alpha_2 \sin \vartheta_2 b_2^{(2)} + \\
&\quad + (u_2 \sin \alpha_2 \cos \vartheta_2 + b_2) c_2^{(2)}; \\
z_2 &= (r_2 \operatorname{ctg} \alpha_2 - u_2 \cos \alpha_2 + L \sin \gamma_2) a_3^{(2)} + u_2 \sin \alpha_2 \sin \vartheta_2 b_3^{(2)} + \\
&\quad + (u_2 \sin \alpha_2 \cos \vartheta_2 + b_2) c_3^{(2)}; \\
&[u_2 - (r_2 \operatorname{ctg} \alpha_2 + L \sin \gamma_2) \cos \alpha_2] \sin (\vartheta_2 - q_2 + \psi_2) + \\
&\quad + b_2 [\cos \alpha_2 \operatorname{tg} \gamma_2 \sin \vartheta_2 - \sin \alpha_2 \sin (q_2 - \psi_2)] = 0.
\end{aligned} \qquad (2.1)$$

The projection of the vector of the normal to surface Σ_2 was expressed as

$$\begin{aligned}
e_{x_2} &= \sin \alpha_2 a_1^{(2)} + \cos \alpha_2 \sin \vartheta_2 b_1^{(2)} + \cos \alpha_2 \cos \vartheta_2 c_1^{(2)}; \\
e_{y_2} &= \sin \alpha_2 a_2^{(2)} + \cos \alpha_2 \sin \vartheta_2 b_2^{(2)} + \cos \alpha_2 \cos \vartheta_2 c_2^{(2)}; \\
e_{z_2} &= \sin \alpha_2 a_3^{(2)} + \cos \alpha_2 \sin \vartheta_2 b_3^{(2)} + \cos \alpha_2 \cos \vartheta_2 c_3^{(2)}.
\end{aligned} \qquad (2.2)$$

In equations (2.1) and (2.2): /92

$$a_1^{(2)} = \cos\varphi_2 \cos\delta_{b_2};$$
$$a_2^{(2)} = -\sin\varphi_2 \cos\delta_{b_2};$$
$$a_3^{(2)} = -\sin\delta_{b_2};$$
$$b_1^{(2)} = \sin\varphi_2 \cos(q_2 - \psi_2) + \cos\varphi_2 \sin\delta_{b_2} \sin(q_2 - \psi_2);$$
$$b_2^{(2)} = \cos\varphi_2 \cos(q_2 - \psi_2) - \sin\varphi_2 \sin\delta_{b_2} \sin(q - \psi_2);$$
$$b_3^{(2)} = \cos\delta_{b_2} \sin(q_2 - \psi_2);$$
$$c_1^{(2)} = -\sin\varphi_2 \sin(q_2 - \psi_2) + \cos\varphi_2 \sin\delta_{b_2} \cos(q_2 - \psi_2);$$
$$c_2^{(2)} = -\cos\varphi_2 \sin(q_2 - \psi_2) - \sin\varphi_2 \sin\delta_{b_2} \cos(q_2 - \psi_2);$$
$$c_3^{(2)} = \cos\delta_{b_2} \cos(q_2 - \psi_2).$$

Crown surface Σ_{p2} and the projection of the vector of the normal to the same surface in system x_0, y_0, z_0 were expressed by the equations:

$$\begin{aligned} x_0^{(p_2)} &= (r_2 \operatorname{ctg}\alpha_2 - u_2 \cos\alpha_2 + L \sin\gamma_2)\cos\gamma_2 - \\ &- u_2 \sin\alpha_2 \sin\gamma_2 \cos(\vartheta_2 - q_2 + \psi_2) - b_2 \sin\gamma_2 \cos(q_2 - \psi_2); \\ y_0^{(p_2)} &= u_2 \sin\alpha_2 \sin(\vartheta_2 - q_2 + \psi_2) - b_2 \sin(q_2 - \psi_2); \\ z_0^{(p_2)} &= (r_2 \operatorname{ctg}\alpha_2 - u_2 \cos\alpha_2 + L \sin\gamma_2)\sin\gamma_2 + \\ &+ u_2 \sin\alpha_2 \cos\gamma_2 \cos(\vartheta_2 - q_2 + \psi_2) + b_2 \cos\gamma_2 \cos q_2; \end{aligned} \quad (2.3)$$

$$\begin{aligned} e_{x_0}^{(p_2)} &= \sin\alpha_2 \cos\gamma_2 - \cos\alpha_2 \sin\gamma_2 \cos(\vartheta_2 - q_2 + \psi_2); \\ e_{y_0}^{(p_2)} &= \cos\alpha_2 \sin(\vartheta_2 - q_2 + \psi_2); \\ e_{z_0}^{(p_2)} &= \sin\alpha_2 \sin\gamma_2 + \cos\alpha_2 \cos\gamma_2 \cos(\vartheta_2 - q_2 + \psi_2). \end{aligned} \quad (2.4)$$

In equations (2.1), (2.2), (2.3), and (2.4): u_2 and ϑ_2 are independent parameters of the crown surface; α_2 is the cutter profile angle; r_2 is the forming radius of the cutting head; q_2 and b_2 are head adjustment parameters; δ_{b2} is the gear's internal cone; γ_2 is the angle of the dedendum part of the tooth; γ_2 and ϕ_2 are the rotation angles for the crown wheel and the wheel being cut. Then

$$\varphi_2 = \psi_2 \frac{\cos\gamma_2}{\sin(\delta_{b_2} + \gamma_2)}.$$

When a pinion wheel is being cut, o_0z_0 is the instantaneous rotation axis in relative motion. Using the conclusion presented in [3], we can represent pinion tooth surface Σ_1 with the following equations:

$$z_1 = (r_1 \operatorname{ctg} \alpha_1 - u_1 \cos \alpha_1 - L \sin \gamma_1)\, a_3^{(1)} + u_1 \sin \alpha_1 \sin \vartheta_1 b_3^{(1)} + (u_1 \sin \alpha_1 \cos \vartheta_1 + b_1)\, c_3^{(1)} + |\Delta L_1| \cos \delta_{b1};$$

$$[-u_1 + (r_1 \operatorname{ctg} \alpha_1 - L \sin \gamma_1 - |\Delta L_1| \operatorname{tg} \delta_{b1}) \cos \alpha_1] \sin (\vartheta_1 - q_1 + \psi_1) + b_1 \sin \alpha_1 \sin (q_1 - \psi_1) + b_1 \cos \alpha_1 \sin \vartheta_1 \operatorname{tg} \gamma_1' = ?,$$

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$$z_1 = (r_1 \operatorname{ctg} \alpha_1 - u_1 \cos \alpha_1 - L \sin \gamma_1)\, a_3^{(1)} + u_1 \sin \alpha_1 \sin \vartheta_1 b_3^{(1)} + (u_1 \sin \alpha_1 \cos \vartheta_1 + b_1)\, c_3^{(1)} + |\Delta L_1| \cos \delta_{b1};$$

$$[-u_1 + (r_1 \operatorname{ctg} \alpha_1 - L \sin \gamma_1 - |\Delta L_1| \operatorname{tg} \delta_{b1}) \cos \alpha_1] \sin (\vartheta_1 - q_1 + \psi_1) + b_1 \sin \alpha_1 \sin (q_1 - \psi_1) + b_1 \cos \alpha_1 \sin \vartheta_1 \operatorname{tg} \gamma_1' = 0, \qquad (2.5)$$

where γ'_1 is the angle between instantaneous axis O'_0M_1 and the generatrix of the inner pinion cone.

The projection of the unit vector of the normal to surface Σ_1 is determined with the equations:

$$e_{x_1} = \sin \alpha_1 a_1^{(1)} + \cos \alpha_1 \sin \vartheta_1 b_1^{(1)} + \cos \alpha_1 \cos \vartheta_1 c_1^{(1)};$$
$$e_{y_1} = \sin \alpha_1 a_2^{(1)} + \cos \alpha_1 \sin \vartheta_1 b_2^{(1)} + \cos \alpha_1 \cos \vartheta_1 c_2^{(1)}; \qquad (2.6)$$
$$e_{z_1} = \sin \alpha_1 a_3^{(1)} + \cos \alpha_1 \sin \vartheta_1 b_3^{(1)} + \cos \alpha_1 \cos \vartheta_1 c_3^{(1)}.$$

In equations (2.5) and (2.6)

$$a_1^{(1)} = \cos \varphi_1 \cos \delta_{b_1};$$
$$a_2^{(1)} = \sin \varphi_1 \cos \delta_{b_1};$$
$$a_3^{(1)} = \sin \delta_{b_1};$$
$$b_1^{(1)} = -\sin \varphi_1 \cos (q_1 - \psi_1) - \cos \varphi_1 \sin \delta_{b_1} \sin (q_1 - \psi_1);$$
$$b_2^{(1)} = \cos \varphi_1 \cos (q_1 - \psi_1) - \sin \varphi_1 \sin \delta_{b_1} \sin (q_1 - \psi_1);$$
$$b_3^{(1)} = \cos \delta_{b_1} \sin (q_1 - \psi_1);$$
$$c_1^{(1)} = \sin \varphi_1 \sin (q_1 - \psi_1) - \cos \varphi_1 \sin \delta_{b_1} \cos (q_1 - \psi_1);$$
$$c_2^{(1)} = -\cos \varphi_1 \sin (q_1 - \psi_1) - \sin \varphi_1 \sin \delta_{b_1} \cos (q_1 - \psi_1);$$
$$c_3^{(1)} = \cos \delta_{b_1} \cos (q_1 - \psi_1).$$

where $\text{где } \psi_1 = \varphi_1 \dfrac{\cos\gamma_1'}{\sin(\vartheta_{b_1}+\gamma_1')}$.

Equations for crown surface Σ_{pi} and of the projections of the vector of the normal to this same surface in the system x_0, y_0, z_0 are written:

$$\begin{aligned}
x_0^{(p_1)} &= (r_1 \operatorname{ctg}\alpha_1 - u_1\cos\alpha_1 - L\sin\gamma_1)\cos\gamma_1 + u_1\sin\alpha_1\sin\gamma_1\cos(\vartheta_1 - q_1 + \psi_1) + \\
&\quad + b_1\sin\gamma_1\cos(q_1 - \psi_1) + |\Delta L_1|\sin\gamma_1; \\
y_0^{(p_1)} &= u_1\sin\alpha_1\sin(\vartheta_1 - q_1 + \psi_1) - b_1\sin(q_1 - \psi_1); \\
z_0^{(p_1)} &= -(r_1\operatorname{ctg}\alpha_1 - u_1\cos\alpha_1 - L\sin\gamma_1)\sin\gamma_1 + u_1\sin\alpha_1\cos\gamma_1\cos(\vartheta_1 - q_1 + \psi_1) + \\
&\quad + b_1\cos\gamma_1\cos(q_1 - \psi_1) + |\Delta L_1|\cos\gamma_1; \qquad (2.7) \\
&[-u_1 + (r_1\operatorname{ctg}\alpha_1 - L\sin\gamma_1 - |\Delta L_1|\operatorname{tg}\delta_{b_1})\cos\alpha_1]\sin(\vartheta_1 - q_1 + \psi_1) + \\
&\quad + b_1\sin\alpha_1\sin(q_1 - \psi_1) + b_1\cos\alpha_1\sin\vartheta_1\operatorname{tg}\gamma_1' = 0;
\end{aligned}$$

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$$\begin{aligned}
e_{x_0}^{(p_1)} &= \sin\alpha_1\cos\gamma_1 + \cos\alpha_1\sin\gamma_1\cos(\vartheta_1 - q_1 + \psi_1); \\
e_{y_0}^{(p_1)} &= \cos\alpha_1\sin(\vartheta_1 - q_1 + \psi_1); \qquad (2.8) \\
e_{z_0}^{(p_1)} &= -\sin\alpha_1\sin\gamma_1 + \cos\alpha_1\cos\gamma_1\cos(\vartheta_1 - q_1 + \psi_1).
\end{aligned}$$

3. Selecting Parameters for Cutting Heads and Machine Adjustment

When parameters are being selected, as noted in [3], gear tooth surfaces must be instantaneously mated when they touch at the midpoint. This requires that both crown surfaces and pinion and gear tooth surfaces touch each other at midpoint M_1 (i =1, 2) (cf. figures 1 and 2), which is the point of intersection of axes 0_0z_0 and $0_{ni}z_{ni}$. Axis 0_0z_0 coincides with the overal generatrix of normal cones of bevel gears and is their axis of rotation in relative motion.

The crown surfaces and the surfaces of the teeth to be cut can be brought to touch at the midpoint M_1(i = 1, 2) by the appropriate setting of the machine's rolling chain. This requires that the gear ratio be calculated so that the instantanecus rotation axis in relative motion of the crown wheel and the wheels to be cut passes through midpoint M_1.

On the basis of this condition and referring to structures in figures 1 and 2, we obtain the following for the gear

$$i_{p_2 2} = \frac{\sin \delta_2}{\cos \gamma_2}, \tag{3.1}$$

for the pinion

$$i_{p_1 1} = \frac{\sin (\delta_{b_1} + \gamma_1')}{\cos \gamma_1'}. \tag{3.2}$$

Here angle γ'_1 is calculated from the equation (cf. figure 1)

$$\operatorname{tg} \gamma_1' = \frac{L \sin \gamma_1 + |\Delta L_1| \operatorname{tg} \delta_{b_1}}{L \cos \gamma_1 - |\Delta L_1|}. \tag{3.3}$$

In addition, if generating surfaces touch each other at point M_i, and the instantaneous rotation axis of rotation passes through M_1, the surfaces of the teeth will alco touch each other at point M_i.

This requires that the radius-vector of the crown surfaces and the normal's unit vector at point M_i be equal, i.e.

$$\bar{r}_0^{(p_1)} = \bar{r}_0^{(p_2)}, \ \bar{e}_0^{(p_1)} = \bar{e}_0^{(p_2)}. \tag{3.4}$$

On the basis of these equations and (2.3), (2.4), (2.7), and (2.8) for crown surfaces and unit vectors of normals in the x_0, y_0, z_0 system, when $\Psi_1 = \Psi_2 = 0$, we obtain: /35

$$\begin{gathered}
(r_1 \operatorname{ctg} \alpha_1 - u_1 \cos \alpha_1 - L \sin \gamma_1) \cos \gamma_1 + u_1 \sin \alpha_1 \sin \gamma_1 \cos (\vartheta_1 - q_1) + \\
+ b_1 \sin \gamma_1 \cos q_1 + |\Delta L_1| \sin \gamma_1 = (r_2 \operatorname{ctg} \alpha_2 - u_2 \cos \alpha_2 + L \sin \gamma_2) \cos \gamma_2 - \\
- u_2 \sin \alpha_2 \sin \gamma_2 \cos (\vartheta_2 - q_2) - b_2 \sin \gamma_2 \cos q_2; \\
u_1 \sin \alpha_1 \sin (\vartheta_1 - q_1) - b_1 \sin q_1 = u_2 \sin \alpha_2 \sin (\vartheta_2 - q_2) - b_2 \sin q_2, \\
-(r_1 \operatorname{ctg} \alpha_1 - u_1 \cos \alpha_1 - L \sin \gamma_1) \sin \gamma_1 + u_1 \sin \alpha_1 \cos \gamma_1 \cos (\vartheta_1 - q_1) + \\
+ b_1 (\cos \gamma_1 \cos q_1 + |\Delta L_1| \cos \gamma_1) = (r_2 \operatorname{ctg} \alpha_2 - u_2 \cos \alpha_2 + L \sin \gamma_2) \sin \gamma_2 + \\
+ u_2 \sin \alpha_2 \cos \gamma_2 \cos (\vartheta_2 - q_2) + b_2 \cos \gamma_2 \cos q_2;
\end{gathered} \tag{3.5}$$

$$\begin{gathered}
\sin \alpha_1 \cos \gamma_1 + \cos \alpha_1 \sin \gamma_1 \cos (\vartheta_1 - q_1) = \sin \alpha_2 \cos \gamma_2 - \cos \alpha_2 \sin \gamma_2 \cos (\vartheta_2 - q_2); \\
\cos \alpha_1 \sin (\vartheta_1 - q_1) = \cos \alpha_2 \sin (\vartheta_2 - q_2); \\
-\sin \alpha_1 \sin \gamma_1 + \cos \alpha_1 \cos \gamma_1 \cos (\vartheta_1 - q_1) = \sin \alpha_2 \sin \gamma_2 + \cos \alpha_2 \cos \gamma_2 \cos (\vartheta_2 - q_2).
\end{gathered} \tag{3.6}$$

The constructions in figures 1 and 2 show that parameters for setting up the cutting head are related to equations

$$\operatorname{ctg} q_1 = \frac{L\cos\gamma_1 - |\Delta L_1| - r_1\sin\beta_1}{r_1\cos\beta_1}. \tag{3.7}$$

$$\operatorname{ctg} q_2 = \frac{L\cos\gamma_2 - r_2\sin\beta_2}{r_2\cos\beta_2}; \tag{3.8}$$

$$b_i = \frac{r_i\cos\beta_i}{\sin q_i}. \tag{3.9}$$

For midpoint M_i $u_i = r_i/\sin\alpha_i$, $\vartheta_i = 90° - \beta_i + q_i$. After substituting b_i, u_i, and ϑ_i in equations (3.5) we can be sure that $\bar{r}_0^{(p_1)} = \bar{r}_0^{(p_2)}$. Substituting ϑ_i into equations (3.6), we obtain the equation

$$\begin{aligned} \sin\alpha_2\cos\gamma_2 - \cos\alpha_2\sin\gamma_2\sin\beta_2 &= \sin\alpha_1\cos\gamma_1 + \cos\alpha_1\sin\gamma_1\sin\beta_1,\\ \cos\alpha_2\cos\beta_2 &= \cos\alpha_1\cos\beta_1,\\ \sin\alpha_2\sin\gamma_2 + \cos\alpha_2\cos\gamma_2\sin\beta_2 &= -\sin\alpha_1\sin\gamma_1 + \cos\alpha_1\cos\gamma_1\sin\beta_1 \end{aligned} \tag{3.10}$$

System (3.10) contains four unknowns: α_1, α_2, β_1, and β_2; of the three equations in this system, only two are independent.

The missing equations for calculating the unknowns are derived on the basis of the fact that the values for angles α_1, and α_2 are calculated with a well-known approximated form used in tooth-cutting practice:

$$\alpha_i = \alpha \pm \frac{1}{2}\sin\beta\,(\operatorname{tg}\gamma_2 + \operatorname{tg}\gamma_1)\frac{180°}{\pi}, \tag{3.11}$$

where α is the nominal cutter profile angle; β is the nominal tooth inclination.

Here and henceforth, a superscript will indicate gear cutting (i = 2), a subscript -- pinion cutting (i = 1).

After α_1 is calculated, angles β_1 and β_2 must be found /96
from equations (3.10). After transformation, we obtain

$$\sin\beta_1 = \frac{\sin\alpha_2 - \sin\alpha_1\cos(\gamma_2+\gamma_1)}{\cos\alpha_1\sin(\gamma_2+\gamma_1)}, \tag{3.12}$$

$$\sin\beta_2 = \frac{\sin\alpha_2\cos(\gamma_2+\gamma_1) - \sin\alpha_1}{\cos\alpha_2\sin(\gamma_2+\gamma_1)}. \tag{3.13}$$

The forming radii are determined from the equation (cf. figures 3 and 4):

$$r_i = r_n \mp \frac{w}{2} \mp L\sin\gamma_i \operatorname{tg}\alpha_i, \tag{3.14}$$

where r_n is the nominal cutting head radius; W is the cutter set.

4. Determining Operating Line and Instantaneous Gear Ratio

Solving this problem requires finding equations for the surfaces of gear and pinion teeth and projections of the unit vector for the normals to the surfaces in the fixed system x_0, y_0, z_0.

This requires that we use the derivative of the matrices M_{0bi}, M^*_{bii}, which express the transition from x_i, y_i, z_i to x_0, y_0, z_0 (cf. figures 3 and 4):

$$M_{ob_i} M^*_{b_i i} = \left\| \begin{matrix} \cos\delta_i\cos\varphi_i' & \mp\cos\delta_i\sin\varphi_i' & \mp\sin\delta_i & 0 \\ \pm\sin\varphi_i' & \cos\varphi_i' & 0 & 0 \\ \pm\sin\delta_i\cos\varphi_i' & -\sin\delta_i\sin\varphi_i' & \cos\delta_i & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \right\| \tag{4.1}$$

Using equations (4.1), (2.1), (2.2), (2.5), and (2.6), after transformation we obtain:

$$x_0^{(1)} = (r_1 \operatorname{ctg}\alpha_1 - u_1\cos\alpha_1 - L\sin\gamma_1)A_1^{(1)} + u_1\sin\alpha_1\sin\vartheta_1 B_1^{(1)} + (u_1\sin\alpha_1\cos\vartheta_1 + b_1)C_1^{(1)} + |\Delta L_1|D_1^{(1)};$$

$$y_0^{(1)} = (r_1 \operatorname{ctg}\alpha_1 - u_1\cos\alpha_1 - L\sin\gamma_1)A_2^{(1)} + u_1\sin\alpha_1\sin\vartheta_1 B_2^{(1)} + (u_1\sin\alpha_1\cos\vartheta_1 + b_1)C_2^{(1)} + |\Delta L_1|D_2^{(1)};$$

$$z_0^{(1)} = (r_1 \operatorname{ctg}\alpha_1 - u_1\cos\alpha_1 - L\sin\gamma_1)A_3^{(1)} + u_1\sin\alpha_1\sin\vartheta_1 B_3^{(1)} + (u_1\sin\alpha_1\cos\vartheta_1 + b_1)C_3^{(1)} + |\Delta L_1|D_3^{(1)};$$

$$-[u_1 - (r_1 \operatorname{ctg}\alpha_1 - L\sin\gamma_1 - |\Delta L_1|\operatorname{tg}\delta_{b_1})\cos\alpha_1]\sin(\vartheta_1 - q_1 + \psi_1) + b_1[\cos\alpha_1 \operatorname{tg}\gamma_1'\sin\vartheta_1 + \sin\alpha_1\sin(q_1 - \psi_1)] = 0; \tag{4.2}$$

$$x_0^{(2)} = (r_2 \operatorname{ctg}\alpha_2 - u_2\cos\alpha_2 + L\sin\gamma_2)A_1^{(2)} + u_2\sin\alpha_2\sin\vartheta_2 B_1^{(2)} + (u_2\sin\alpha_2\cos\vartheta_2 + b_2)C_1^{(2)};$$

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$$y_0^{(2)} = (r_2 \operatorname{ctg}\alpha_2 - u_2\cos\alpha_2 + L\sin\gamma_2)A_2^{(2)} + u_2\sin\alpha_2\sin\vartheta_2 B_2^{(2)} + (u_2\sin\alpha_2\cos\vartheta_2 + b_2)C_2^{(2)};$$

$$z_0^{(2)} = (r_2 \operatorname{ctg}\alpha_2 - u_2\cos\alpha_2 + L\sin\gamma_2)A_3^{(2)} + u_2\sin\alpha_2\sin\vartheta_2 B_3^{(2)} + (u_2\sin\alpha_2\cos\vartheta_2 + b_2)C_3^{(2)}; \tag{4.3}$$

$$[u_2 - (r_2 \operatorname{ctg}\alpha_2 + L\sin\gamma_2)\cos\alpha_2]\sin(\vartheta_2 - q_2 + \psi_2) + b_2[\cos\alpha_2 \operatorname{tg}\gamma_2\sin\vartheta_2 - \sin\alpha_2\sin(q_2 - \psi_2)] = 0;$$

$$e_{x_0}^{(i)} = \sin\alpha_i A_1^{(i)} + \cos\alpha_i\sin\vartheta_i B_1^{(i)} + \cos\alpha_i\cos\vartheta_i C_1^{(i)};$$

$$e_{y_0}^{(i)} = \sin\alpha_i A_2^{(i)} + \cos\alpha_i\sin\vartheta_i B_2^{(i)} + \cos\alpha_i\cos\vartheta_i C_2^{(i)};$$

$$e_{z_0}^{(i)} = \sin\alpha_i A_3^{(i)} + \cos\alpha_i\sin\vartheta_i B_3^{(i)} + \cos\alpha_i\cos\vartheta_i C_3^{(i)}. \tag{4.4}$$

In equations (4.2), (4.3), and (4.4):

$$A_1^{(i)} = \cos\delta_i\cos(\varphi_i' - \varphi_i)\cos\delta_{b_i} + \sin\delta_i\sin\delta_{b_i};$$

$$A_2^{(i)} = \pm\sin(\varphi_i' - \varphi_i)\cos\delta_{b_i};$$

$$A_3^{(i)} = \pm\sin\delta_i\cos(\varphi_i' - \varphi_i)\cos\delta_{b_i} + \cos\delta_i\sin\delta_{b_i};$$

$$B_1^{(i)} = \pm\cos\delta_i\cos(\varphi_i' - \varphi_i)\sin\delta_{b_i}\sin(q_i - \psi_i) \mp \cos\delta_i\sin(\varphi_i' - \varphi_i)\cos(q_i - \psi_i) \mp \sin\delta_i\cos\delta_{b_i}\sin(q_i - \psi_i);$$

$$B_2^{(i)} = \sin(\varphi_i' - \varphi_i)\sin\delta_{b_i}\sin(q_i - \psi_i) + \cos(\varphi_i' - \varphi_i)\cos(q_i - \psi_i);$$

$$B_3^{(i)} = \sin\delta_i\cos(\varphi_i' - \varphi_i)\sin\delta_{b_i}\sin(q_i - \psi_i) - \sin\delta_i\sin(\varphi_i' - \varphi_i)\cos(q_i - \psi_i) + \cos\delta_i\cos\delta_{b_i}\sin(q_i - \psi_i);$$

$$C_1^{(i)} = \pm\cos\delta_i\cos(\varphi_i' - \varphi_i)\sin\delta_{b_i}\cos(q_i - \psi_i) \pm \cos\delta_i\sin(\varphi_i' - \varphi_i)\sin(q_i - \psi_i) \mp \sin\delta_i\cos\delta_{b_i}\cos(q_i - \psi_i);$$

$$C_2^{(i)} = \sin(\varphi_i' - \varphi_i)\sin\delta_{b_i}\cos(q_i - \psi_i) - \cos(\varphi_i' - \varphi_i)\sin(q_i - \psi_i);$$
$$C_3^{(i)} = \sin\delta_i\cos(\varphi_i' - \varphi_i)\sin\delta_{b_i}\cos(q_i - \psi_i) +$$
$$+ \sin\delta_i\sin(\varphi_i' - \varphi_i)\sin(q_i - \psi_i) + \cos\delta_i\cos\delta_{b_i}\cos(q_i - \psi_i);$$
$$D_1^{(1)} = -\cos\delta_1\cos(\varphi_1' - \varphi_1)\sin\delta_{b_1} + \sin\delta_1\cos\delta_{b_1};$$
$$D_2^{(1)} = \sin(\varphi_1' - \varphi_1)\sin\delta_{b_1};$$
$$D_3^{(1)} = \sin\delta_1\cos(\varphi_1' - \varphi_1)\sin\delta_{b_1} + \cos\delta_1\cos\delta_{b_1}.$$

Here and in equation (4.1), the superscript indicates cutting a gear (i = 2), the subscript -- cutting a pinion (i = 1).

We know from analytical geometry [4] that coefficients for the transformation equations for rectangular coordinate systems are related by the functions:

$$[A_1^{(i)}]^2 + [A_2^{(i)}]^2 + [A_3^{(i)}]^2 = 1;$$
$$[B_1^{(i)}]^2 + [B_2^{(i)}]^2 + [B_3^{(i)}]^2 = 1;$$
$$[C_1^{(i)}]^2 + [C_2^{(i)}]^2 + [C_3^{(i)}]^2 = 1; \qquad (4.5)$$

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$$A_1^{(i)}B_1^{(i)} + A_2^{(i)}B_2^{(i)} + A_3^{(i)}B_3^{(i)} = 0;$$
$$B_1^{(i)}C_1^{(i)} + B_2^{(i)}C_2^{(i)} + B_3^{(i)}C_3^{(i)} = 0;$$
$$C_1^{(i)}A_1^{(i)} + C_2^{(i)}A_2^{(i)} + C_3^{(i)}A_3^{(i)} = 0. \qquad (4.6)$$

In addition, it is easy to be sure that $A_j^{(1)}$, $B_j^{(1)}$, $C_j^{(1)}$ and $D_j^{(1)}$ (j = 1, 2, 3) are related by equations:

$$A_1^{(1)}D_1^{(1)} + A_2^{(1)}D_2^{(1)} + A_3^{(1)}D_3^{(1)} = 0;$$
$$B_1^{(1)}D_1^{(1)} + B_2^{(1)}D_2^{(1)} + B_3^{(1)}D_3^{(1)} = 0;$$
$$C_1^{(1)}D_1^{(1)} + C_2^{(1)}D_2^{(1)} + C_3^{(1)}D_3^{(1)} = 0;$$
$$[D_1^{(1)}]^2 + [D_2^{(1)}]^2 + [D_3^{(1)}]^2 = 1. \qquad (4.7)$$

To calculate the contact point of the gear and pinion teeth, we have to use the following system of equations:

$$x_0^{(1)} = x_0^{(2)}; \quad e_{x_0}^{(1)} = e_{x_0}^{(2)};$$
$$y_0^{(1)} = y_0^{(2)}; \quad e_{y_0}^{(1)} = e_{y_0}^{(2)};$$
$$z_0^{(1)} = z_0^{(2)}; \quad e_{z_0}^{(1)} = e_{z_0}^{(2)};$$

$$[-u_1 + (r_1 \operatorname{ctg} \alpha_1 - L \sin \gamma_1 - |\Delta L_1| \operatorname{tg} \delta_{b_1}) \cos \alpha_1] \sin (\vartheta_1 - q_1 + \psi_1) + $$
$$+ b_1 [\cos \alpha_1 \operatorname{tg} \gamma_1' \sin \vartheta_1 + \sin \alpha_1 \sin (q_1 - \psi_1)] = 0;$$
$$[u_2 - (r_2 \operatorname{ctg} \alpha_2 + L \sin \gamma_2) \cos \alpha_2] \sin (\vartheta_2 - q_2 + \psi_2) +$$
$$+ b_2 [\cos \alpha_2 \operatorname{tg} \gamma_2 \sin \vartheta_2 - \sin \alpha_2 \sin (q_2 - \psi_2)] = 0. \tag{4.8}$$

From equations (4.8), it is obvious that

$$[x_0^{(1)}]^2 + [y_0^{(1)}]^2 + [z_0^{(1)}]^2 = [x_0^{(2)}]^2 + [y_0^{(2)}]^2 + [z_0^{(2)}]^2 \tag{4.9}$$

$$e_{x_0}^{(1)} x_0^{(1)} + e_{y_0}^{(1)} y_0^{(1)} + e_{z_0}^{(1)} z_0^{(1)} = e_{x_0}^{(2)} x_0^{(2)} + e_{y_0}^{(2)} y_0^{(2)} + e_{z_0}^{(2)} z_0^{(2)}. \tag{4.10}$$

Substituting equations (4.2) and (4.3) into (4.9); (4.2), (4.3), and (4.4) into (4.10), and taking into account functions (4.5), (4.6), and (4.7), after transformation we obtain:

$$(r_1 \operatorname{ctg} \alpha_1 - u_1 \cos \alpha_1 - L \sin \gamma_1)^2 + (u_1 \sin \alpha_1 \sin \vartheta_1)^2 + (u_1 \sin \alpha_1 \cos \vartheta_1 + b_1)^2 +$$
$$+ |\Delta L_1|^2 + 2 |\Delta L_1| [u_1 \sin \alpha_1 \cos (\vartheta_1 - q_1 + \psi_1) + b_1 \cos (q_1 - \psi_1)] =$$
$$= (r_2 \operatorname{ctg} \alpha_2 - u_2 \cos \alpha_2 + L \sin \gamma_2)^2 + (u_2 \sin \alpha_2 \sin \vartheta_2)^2 + (u_2 \sin \alpha_2 \sin \vartheta_2 + b_2)^2, \tag{4.11}$$

$$\sin \alpha_1 (r_1 \operatorname{ctg} \alpha_1 - L \sin \gamma_1) + b_1 \cos \alpha_1 \cos \vartheta_1 + |\Delta L_1| \cos \alpha_1 \cos (\vartheta_1 - q_1 + \psi_1) =$$
$$= \sin \alpha_2 (r_2 \operatorname{ctg} \alpha + L \sin \gamma_2) + b_2 \cos \alpha_2 \cos \vartheta_2. \tag{4.12}$$

To determine parameters u_1, ϑ_1, Ψ_1 ϕ'_1, u_2, ϑ_2, Ψ_2, and ϕ'_2, using equations (4.8), (4.11), and (4.12) we obtain the following system of equations:

$$[-u_1 + (r_1 \operatorname{ctg} \alpha_1 - L \sin \gamma_1) \cos \alpha_1] \sin (\vartheta_1 - q_1 + \psi_1) + b_1 \cos \alpha_1 \operatorname{tg} \gamma_1' \sin \vartheta_1 +$$
$$+ b_1 \sin \alpha_1 \sin (q_1 - \psi_1) = 0; \tag{a}$$

$$\sin \alpha_1 (r_1 \operatorname{ctg} \alpha_1 - L \sin \gamma_1) + b_1 \cos \alpha_1 \cos \vartheta_1 + |\Delta L_1| \cos \alpha_1 \cos (\vartheta_1 - q_1 + \psi_1) -$$
$$- \sin \alpha_2 (r_2 \operatorname{ctg} \alpha_2 + L \sin \gamma_2) - b_2 \cos \alpha_2 \cos \vartheta_2 = 0; \tag{b}$$

$$(r_1 \operatorname{ctg} \alpha_1 - u_1 \cos \alpha_1 - L \sin \gamma_1)^2 + (u_1 \sin \alpha_1 \sin \vartheta_1)^2 + (u_1 \sin \alpha_1 \cos \vartheta_1 + b_1)^2 +$$
$$+ (\Delta L_1)^2 + 2 |\Delta L_1| [u_1 \sin \alpha_1 \cos (\vartheta_1 - q_1 + \psi_1) + b_1 \cos (q_1 - \psi_1)] - (r_2 \operatorname{ctg} \alpha_2 -$$
$$- u_2 \cos \alpha_2 + L \sin \gamma_2)^2 - (u_2 \sin \alpha_2 \sin \vartheta_2)^2 - (u_2 \sin \alpha_2 \cos \vartheta_2 + b_2)^2 = 0; \tag{c}$$

$$[u_2-(r_2\operatorname{ctg}\alpha_2+L\sin\gamma_2)\cos\alpha_2]\sin(\vartheta_2-q_2+\psi_2)+b_2\cos\alpha_2\operatorname{tg}\gamma_2\sin\vartheta_2-$$
$$-b_2\sin\alpha_2\sin(q_2-\psi_2)=0; \quad \text{(d)}$$

$$\text{(e)}$$
$$\sin\alpha_1A_1^{(1)}+\cos\alpha_1\sin\vartheta_1B_1^{(1)}+\cos\alpha_1\cos\vartheta_1C_1^{(1)}-\sin\alpha_2A_1^{(2)}-\cos\alpha_2\sin\vartheta_2B_1^{(2)}-$$
$$-\cos\alpha_2\cos\vartheta_2C_1^{(2)}=0;$$

$$\sin\alpha_1A_3^{(1)}+\cos\alpha_1\sin\vartheta_1B_3^{(1)}+\cos\alpha_1\cos\vartheta_1C_3^{(1)}-\sin\alpha_2A_3^{(2)}- \quad \text{(f)}$$
$$-\cos\alpha_2\sin\vartheta_2B_3^{(2)}-\cos\alpha_2\cos\vartheta_2C_3^{(2)}=0;$$

$$(r_1\operatorname{ctg}\alpha_1-u_1\cos\alpha_1-L\sin\gamma_1)A_1^{(1)}+u_1\sin\alpha_1\sin\vartheta_1B_1^{(1)}+(u_1\sin\alpha_1\cos\vartheta_1+$$
$$+b_1)+|\Delta L_1|D_1^{(1)}=(r_2\operatorname{ctg}\alpha_2-u_2\cos\alpha_2+L\sin\gamma_2)A_1^{(2)}+ \quad \text{(g)} \quad (4.13)$$
$$+u_2\sin\alpha_2\sin\vartheta_2B_1^{(2)}+(u_2\sin\alpha_2\cos\vartheta_2+b_2)C_1^{(2)}.$$

The system of transcendental equations (4.13) is solved by sequential approximation.

In solving system (4.13), Ψ_1 is considered fixed. Then the system has seven unknown equations, whose solution results in the calculating of the seven desired parameters u_i, ϑ_1, ϕ'_1, u_2, $\vartheta 2$' Ψ^2, and ϕ'^2.

Substituting the resulting values into equations (2.1) and (2.5), we find the coordinates of points on the operating line on the surfaces of the teeth. Coordinates x_i, y_i, z_i, found for various Ψ_1 values, define the operating line.

Substituting values for u_i, ϑ_1, Ψ_1, ϕ'_1, u_2, ϑ_2, Ψ_2, and ϕ'_2 into equations (4.2), (4.3), and (4.4), we find the coordinates of the point on the dedendum line of contact and the projection of the unit vector for the normal at this point.

Let us introduce an equation for finding the instantaneous gear ratio for gears.

In fixed system x_0, y_0, z_0, the equation for the normal to tooth surface contact point M_0^K is expressed as (cf. figure 5):

$$\frac{X_0 - x_0^K}{e_{x_0}^K} = \frac{Y_0 - y_0^K}{e_{y_0}^K} = \frac{Z_0 - z_0^K}{e_{z_0}^K}, \qquad (4.14)$$

where X_0, Y_0, Z_0 are the coordinates of the current normal point; x_0^K, y_0^K, z_0^K are the coordinates of point M_0^K of contact in system x_0, y_0, z_0, determined with equations (4.2) and (4.3); e_{xe}^K, e_{ye}^K, e_{ze}^K are projections of the unit vector of the normal at the point of contact in this system, determined from equations (4.4). /100

At point N_K, where the normal intersects the plane of gear axes, $Y_0 = 0$. After substituting $Y_0 = 0$ into equation (4.14), we obtain

$$X_0 = x_0^K - \frac{e_{x_0}^K}{e_{y_0}^K} y_0^K,$$

$$Z_0 = z_0^K - \frac{e_{z_0}^K}{e_{y_0}^K} y_0^K. \qquad (4.15)$$

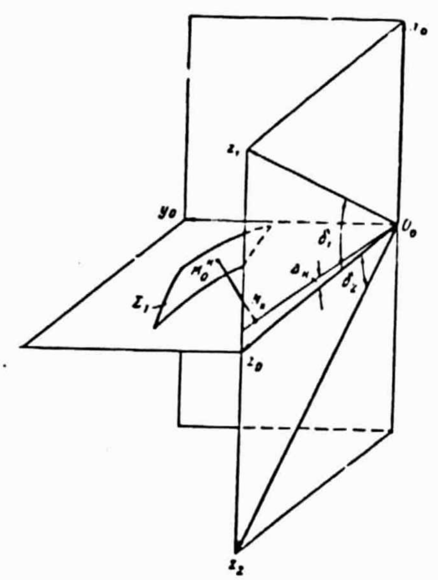

Figure 5. For solving the equation to determine instantaneous gear ratio.

Angle Δ_K between axis z_0 and instantaneous axis 0_0N_K will be found with the equation

$$\operatorname{tg} \Delta_K = \frac{X_0}{Z_0} = \frac{x_0^K e_{y_0}^K - y_0^K e_{x_0}^K}{Z_0^K e_{y_0}^K - y_0^K e_{z_0}^K}. \qquad (4.16)$$

The instantaneous gear ratio for gears may be found from the equation (cf. figure 5):

$$i_{12} = \frac{\sin(\delta_2 + \Delta_K)}{\sin(\delta_2 - \Delta_K)}. \qquad (4.17)$$

Example of Calculation /101

Let us consider the case of cutting teeth on a pinion and gear with a gear ratio $i_{12} = 1$. The teeth on the gear are directed to the right; those on the pinion -- to the left. The number of teeth is $z_1 = z_2 = 20$. The face modulus $m_s = 10$ mm, tooth inclination $\beta = 35°$, meshing angle $\alpha = 20°$, toothing width B = 40 mm. Nominal cutting head radius $r_u = 152.4$ mm, cutter set W = 1.524 mm. Tooth cutting method: one-sided. Axial displacement $\Delta S_1 = -10$ mm.

Using ENIMS [expansion unknown] system geometric calculation [3] and calculating equations for setting up the machine given in point 3, we will find the parameters for meshing of gears, cutting heads, and setting up the machine. ΔL_1 is found with the equation [3] $\Delta L_1 = -\Delta S_1 \cos\delta_{b_1}$.

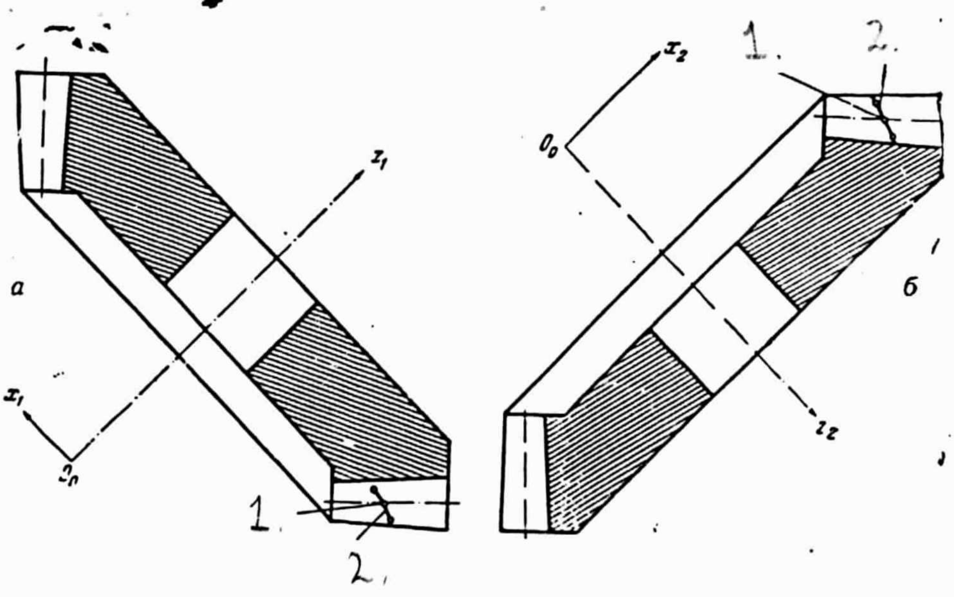

Figure 6. Projections of the operating line. 1 - Midpoint; 2 - Operating line.

Let us introduce values for solving system (4.13):

	Gear	Wheel
Average generatrix, L	121.4214	121.4214
Initial cone angle, δ_i	45°	45°
Angle of dedendum part of tooth, γ_i	4.125308°	4.125308°
Internal bevel angle, δ_{bi}	40.87469°	40.87469°
Cutter angle, α_i	17.62972°	22.37028°
Angle of spiral in locating plane, β_i	36.24563°	33.78168°
Forming radius, r_i	155.9378	148.0431
Angular adjustment, q_i	80.36620°	72.50272°
Radial adjustment, b_i	127,5611°	129.0173°
Axial displacement, ΔS_i	10	0
Rolling chain gear ratio, i_{pii}	0.7561548	0.7089436

The results of calculation using equations in system (4.13) and the coordinates of points on the operating line and of the instantaneous gear ratio for gears found according to equation (2.1), (2.5), (4.17), are given below: /102

	Current values		
Rotation angle of crown wheel, p_i, Ψ_i	-5°	0°	5°
Parameters for crown surface Σ_{pi}:			
ϑ_1	133.57°	134.12°	135.42°
u_1	520.64	514.88	509.48
Parameters for crown surface Σ_{p2}:			
ϑ_2	127.87°	128.72°	130.35°
u_2	394.42	388.95	383.47
Rotation angle of crown wheel p_2, Ψ_2	-5.28°	0°	5.47°
Pinion rotation angle, ϕ'_1	-7.65°	0°	7.82°
Gear rotation angle ϕ'_2	7.25°	0°	7.16°

Coordinates of the point on the meshing line			
$x_0^{(i)}$	-5.2	0	5.0
$y_0^{(i)}$	-11.2	0	11.9
$z_0^{(i)}$	123.5	121.4	116.6
Coordinates of the point of contact on the pinion tooth surface			
x_1	-91.7	-85.9	-79.8
y_1	1.0	0:00	1.1
z_1	83.65	85.9	86.0
Coordinates of the point of contact on the gear tooth surface			
x_2	84.4	85.9	86.8
y_2	-0.6	0	1.1
z_2	91.0	85.9	78.9
Gear ratio, i_{12}	1.002	1	0.966

These results were used to plot the projections of the operating line for pinion tooth surface on plane $x_1 0_0 y_1$ (cf. figure 6, a) and of the operating line for gear tooth surface on plane $x_2 0_0 y_2$ (cf. figure 6, b).

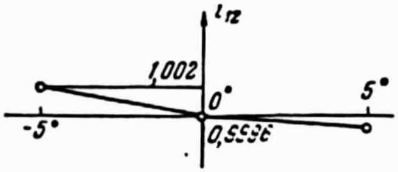

Figure 7. Graph of gear ratio function.

Figure 7 shows a graph of the change in gear ratio.

Calculations and constructions (cf. figure 6) show that, when axial displacement is introduced and the recommended method for calculating cutter head parameters and machine set-up is used, the operating lines pass through the designated midpoints on the surfaces of the teeth, and the diagonal nature of the operation line (bearing pattern) is already eliminated.

Therefore, we may draw the following conclusion. The diagonal nature of contact can be eliminated by changing displacement and gear ratio of the rolling chain. Displacement must be negative if the convex side of the gear tooth is being cut, i.e. the head should be shifted forward. Shift in this case amounts to about 10 mm, but it may be somewhat less. Optimum axial displacement should be selected on the basis of experimental calculations. In critical cases, it is advisable to program the equations given here to perform calculations on computers.

The gear ratio of the gears still remains variable. However, its deviation from nominal is less than when gears are cut without correcting machine adjustment.

REFERENCES

1. Litvin, F. L. Kai, Go., "A Study of Meshing Spiral Bevel Gears," Teoriya mashin i mekhanizmov, 1962, No. 92.

2. Waldgaber, E., Osnovy zatsepleniya konicheskikh i gipoidnykh peredach [Fundamentals of the Meshing of Bevel and Hypoid Gearing], Moscow: Mashgiz, 1948.

3. Kedrinskiy, V. N., Pismanik, K. M., Stanki dlya narezaniya konicheskikh zubchatykh koles [Machines for Cutting Bevel Gears], Moscow: Mashgiz, 1958.

4. Muskhelishvili, N. N., Kurs analiticheskoy geometrii [A Course in Analytical Geometry], Moscow: GITTL, 1947.

5. Litvin, F. L., Teoriya zubchatykh zatsepleniy [A Theory of Gear Meshing], Moscow: Fizmatgiz, 1960.