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APPLICATION OF THE MHD ENERGY PRINCIPLE TO MAGNETOSTATIC ATMOSPHERES

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Abstract

We apply the MHD energy principle to the stability of a magnetized atmosphere which is bounded below by much denser fluid, as is the solar corona. We treat the two fluids as ideal; the approximation which is consistent with the energy principle, and use the dynamical conditions that must hold at a fluid-fluid interface to show that if vertical displacements of the lower boundary are permitted, then the lower atmosphere must be perturbed as well. However, displacements which do not perturb the coronal boundary can be properly treated as isolated perturbations of the corona alone.

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I. INTRODUCTION

Studies of the equilibrium and stability of magnetized plasma in a gravitational field are important to many areas of astrophysics, including solar and stellar physics (reviewed by Priest, 1982, and Rosner et al., 1984). The stability of structures in the solar corona is relevant to understanding the onset of eruptive activity, as well as the necessary conditions for equilibrium. Because even the simplest models of coronal features are sufficiently inhomogeneous that solving the full mode problem is very difficult, many studies of coronal MHD stability have used the energy principle method of Bernstein et al. (1958; hereafter BFKK) to determine stability without calculating the modes themselves.

Coronal magnetic fieldlines are thought to be connected to the lower solar atmosphere (chromosphere and photosphere), which is much denser than the corona, and ultimately to extend into the solar interior. Rather than considering the stability of the composite system consisting of hot gas and cooler underlying material, most studies of coronal MHD stability have imposed a boundary at the coronal base and have treated the lower atmosphere only through its influence on the boundary conditions.

Several different assumptions about the boundary conditions on $\xi_{\parallel}$, the component of the fluid displacement $\vec{\xi}$ parallel to the magnetic field $\vec{B}$, have been made in the literature. Schindler et al. (1983) chose $\vec{\xi} = 0$, as if the photosphere were a rigid boundary. Einaudi and van Hoven (1981) imposed parity constraints on $\xi_{\parallel}$ that allow $\xi_{\parallel} \neq 0$. Hood (1984a,b) did not explicitly restrict $\xi_{\parallel}$ at all.

In this paper, we discuss the influence of the photospheric boundary condition on stability by assuming that both the upper and lower atmosphere are ideal fluids. It is clear that material in the solar atmosphere does not always behave adiabatically. Radiative processes, thermal conduction, and some form of heating all play roles in the structure. Mass flow is often present. The upper and lower fluid treatment in this paper is an idealization, but it is an idealization which is consistent with the MHD energy principle, and should be a good approximation as long as the MHD time-scales are rapid compared to the timescale on which mass exchange occurs between the two fluids. Equilibria with flows, and non-adiabatic perturbations, cannot be studied with the ideal MHD energy principle.

We find that for coronal stability problems in which gravitational stratification is included, the effect of nonzero $\xi_\parallel$ is to force the lower atmosphere to be perturbed. This arises in a natural way from the conditions at the boundary between the two fluids. Therefore, in order to derive necessary and sufficient conditions for the stability of the corona as an isolated system, $\xi_\parallel$ must vanish on the lower boundary.

In Section II, we use the MHD energy principle to demonstrate the existence of a surface integral and a perturbation of the lower atmosphere when $\xi_\parallel$ is not zero at the boundary. In Section III, we discuss the effect of the boundary terms on various results in the literature. Section IV is a discussion together with conclusions.

II. BOUNDARY CONDITIONS AND ENERGY PRINCIPLE ANALYSIS

We first describe the equilibrium model, including the conditions which must be fulfilled at the coronal base. We then derive the corresponding conditions in the presence of small perturbations. Finally, we use the MHD energy principle of BFKK to assess the effect of the boundary conditions on stability. Ideal MHD (adiabatic, inviscid, infinite electrical conductivity) is assumed to hold throughout.

a) Boundary conditions

The equation of mechanical equilibrium in a stratified atmosphere is

$$\nabla \cdot \left[\vec{B}\,\vec{B} - \bar{\bar{I}}(p + B^2/2)\right] + \rho\vec{g} = 0 \tag{1}$$

where P, $\vec{B}$, ρ, and $\vec{g}$ are the gas pressure, magnetic field, gas density, and gravitational acceleration, respectively. For a surface of discontinuity with normal direction $\hat{n}$ in the fluid, it follows from $\nabla \cdot B = 0$ and from equation (1) that

$$\langle B_n \rangle = 0 \tag{2a}$$

$$\langle B_n B_t \rangle = 0 \tag{2b}$$

$$\left\langle (B_n^2 - B_t^2)/2 - P \right\rangle = 0 \tag{2c}$$

where the notation <S> refers to the jump in quantity S across the discontinuity and the subscripts n and t refer to normal and tangent directions to the surface, respectively. Note that if $B_n = 0$, B_t may be discontinuous, but if $B_n \neq 0$, B_n, B_t, and P are each continuous separately. These conditions are discussed, e.g. by Roberts (1967).

Now consider a small displacement $\vec{\xi}$ of the fluid. The Eulerian perturbations of P, $\vec{B}$, and ρ are (BFKK)

$$\delta P = -\gamma P\, \nabla \cdot \vec{\xi} - \vec{\xi} \cdot \nabla P \tag{3a}$$

$$\delta\vec{B} = \nabla \times (\vec{\xi} \times \vec{B}) \equiv \vec{Q} \tag{3b}$$

$$\delta\rho = - \nabla \cdot \rho\vec{\xi} \tag{3c}$$

Faraday's law implies that, to first order in ξ,

$$\langle\hat{n} \times (\vec{\xi} \times \vec{B})\rangle = 0 \ . \tag{4}$$

Furthermore, ξ_n must be continuous.

Conditions for the perturbed system analogous to equations (2) were derived by BFKK for a fluid-vacuum interface tangent to $\vec{B}$, and by Goedbloed (1979) for a fluid-fluid interface, again with $\vec{B} \cdot \hat{n} = 0$, while Roberts (1967) gives conditions for $\vec{B} \cdot \hat{n} \neq 0$. Equations (2) must be linearized and satisfied at the perturbed boundary, with reference to the perturbed normal. The relevant perturbations of the fluid variables here are the Lagrangian perturbations, which follow the boundary elements to their new positions. The Lagrangian perturbations ΔP, $\Delta\vec{B}$, and $\Delta\rho$ can be obtained from their Eulerian counterparts (3a,b,c) by the usual relationship for any quantity

$$\Delta s = \delta s + \vec{\xi} \cdot \nabla s \ . \tag{5}$$

As Roberts (1967) shows in detail, for the case $\vec{B} \cdot \hat{n} \neq 0$,

$$\langle\Delta\vec{B}\rangle = \langle\Delta P\rangle = 0 \tag{6}$$

Equation (6) is the analog of equations (2) above; the Lagrangian perturbations of P and $\vec{B}$ as well as P and $\vec{B}$ themselves are continuous across the interface.

b) Energy principle

The linearized equation of motion for the displacement vector $\vec{\xi}$ is

$$\rho \frac{\partial^2 \vec{\xi}}{\partial t^2} = \vec{F}(\vec{\xi}) , \tag{7}$$

where

$$\vec{F}(\vec{\xi}) = \vec{\nabla}(\gamma p \nabla \cdot \vec{\xi} + \vec{\xi} \cdot \nabla p) + (\nabla \times \vec{Q}) \times \vec{B} + (\nabla \times \vec{B}) \times \vec{Q} - \vec{g}\, \nabla \cdot \rho \vec{\xi} \tag{8}$$

and it is assumed that $\vec{g}$ is produced by external sources and remains constant. The perturbed potential energy is

$$\delta W(\vec{\xi}, \vec{\xi}) = - \frac{1}{2} \int d^3x \; \vec{\xi} \cdot \vec{F}(\vec{\xi}) \tag{9}$$

where the integral extends over the volume of the fluid. In the general case, both the photospheric and coronal fluids contribute to δW.

BFKK proved (see also Freidberg 1982) that the system is unstable if and only if $\delta W(\vec{\xi}, \vec{\xi})$ is negative for a displacement vector $\vec{\xi}$ which satisfies appropriate boundary conditions. In the most restricted sense, $\vec{\xi}$ must satisfy continuity conditions such as equations (4) and (6) and continuity of ξ_n at an interface. However, BFKK proved an extended energy principle for the plasma-vacuum problem with $\vec{B} \cdot \hat{n} = 0$. They showed that $\vec{\xi}$ need not satisfy the Lagrangian force-balance condition (equation (2.32) in their paper; equation (6) here) by proving that it is possible to correct $\vec{\xi}$ in a thin layer near the interface in a way which enables $\vec{\xi}$ to satisfy the pressure balance condition. In their construction [see also Roberts (1967)], $\vec{\xi}$ is augmented by a vector $\epsilon\vec{\eta}$ which goes to zero within a distance ϵ from the boundary. Then, the normal gradient of $\vec{\eta}$ is of order ϵ^{-1}, and the contribution of $\epsilon\vec{\eta}$ to δW_F is of order ϵ. The extended energy principle makes it possible to choose trial functions $\vec{\xi}$ for δW which do not satisfy the force balance condition. Roberts (1967) discusses the extended energy principle in cases where $\vec{B} \cdot \hat{n} \neq 0$. Since the Lagrangian force balance condition involves both $\vec{\xi}$ and its derivatives, and takes some care to satisfy, the extended energy principle is easier to work with and is used in most applications.

When the extended energy principle is written in its usual form (BFKK; Eq. 3.16), surface integrals involving the pressure balance condition appear. These integrals are related to the change in plasma potential energy caused by the PdV work done at its surface. We now consider the role of these boundary terms. According to equations (8) and (9),

$$2\delta W(\vec{\xi},\vec{\xi}) = -\int d^3x \ [\vec{\xi}\cdot\nabla(\gamma P\nabla\cdot\vec{\xi} + \vec{\xi}\cdot\nabla P) + \vec{\xi}\cdot(\nabla\times\vec{Q})\times\vec{B} + \vec{\xi}\cdot(\vec{\nabla}\times\vec{B})\times\vec{Q} - \vec{\xi}\cdot\vec{g} \ \nabla\cdot\rho\vec{\xi}] \tag{10}$$

Integrating by parts, this can be written

$$2\delta W(\vec{\xi},\vec{\xi}) = 2\delta W_F + 2\delta W_s$$

$$2\delta W_F = \int d^3x[(\nabla\cdot\vec{\xi}) \ (\gamma P \ \nabla\cdot\vec{\xi} + \vec{\xi}\cdot\nabla P) + Q^2 - \vec{\xi}\cdot(\nabla\times\vec{B})\times\vec{Q} + \vec{\xi}\cdot\vec{g} \ \nabla\cdot\rho\vec{\xi}]$$

$$2\delta W_S = -\int d^2x \left\langle (\hat{n}\cdot\vec{\xi}) \ (\gamma P \ \nabla\cdot\vec{\xi} + \vec{\xi}\cdot\nabla P) + \vec{Q}\cdot\left(\hat{n}\times(\vec{\xi}\times\vec{B})\right)\right\rangle \tag{11}$$

The surface integral δW_S is taken over the boundary between the upper and lower atmosphere, plus terms at infinity, which we assume vanish. If the horizontal extent of the structure is finite, we can impose horizontal periodic boundary conditions. The volume integral δW_F contains contributions from both fluids.

Proceeding similarly to BFKK (see also Roberts, 1967) we rewrite the surface integral in equation (11) using the boundary conditions satisfied by $\vec{\xi}$. Note that even if we use the extended energy principle, so that we allow trial functions in the volume integral δW_F which do not satisfy the boundary conditions (6), we must evaluate δW_S assuming that these conditions are satisfied. This has not always been done in the literature.

According to equation (4), $\hat{n} \times (\vec{\xi} \times \vec{B})$ is continuous at the interface. Using equations (5) and (6), ΔP and $\vec{Q} + \vec{\xi} \cdot \nabla \vec{B}$ are continuous as well, as is $\vec{\xi} \cdot \hat{n}$. These results enable us to write

$$2\delta W_S = - \oint d^2x \left\langle (\hat{n}\cdot\vec{\xi}) \; [\vec{\xi}\cdot\nabla(P + B^2/2)] - (\hat{n}\cdot\vec{B}) \; (\vec{\xi}\cdot\nabla\vec{B})\cdot\vec{\xi} \right\rangle$$

Using the equation of mechanical equilibrium, this becomes

$$2\delta W_S = \oint d^2x \left\langle (\hat{n}\cdot\vec{\xi}) \; [(\vec{B}\cdot\nabla\vec{B})\cdot\vec{\xi} + \rho\vec{\xi}\cdot\vec{g}] - (\hat{n}\cdot\vec{B}) \; (\vec{\xi}\cdot\nabla\vec{B})\cdot\vec{\xi} \right\rangle .$$

or

$$2\delta W_S = - \oint d^2x \left\langle (\hat{n}\cdot\vec{\xi})(\vec{\xi}\cdot\vec{g})\rho + [\hat{n}\times(\vec{\xi}\times\vec{B})\cdot\nabla\vec{B}]\cdot\vec{\xi} \right\rangle$$

Evidently, the second term involves only tangential derivatives of $\vec{B}$ at the interface. But the tangential derivatives of $\vec{B}$ are continuous; this term is therefore zero. If we take $\vec{g} = \hat{y}g$ and let the boundary lie in the x-z plane, then δW_S takes the final form

$$2\delta W_S = \int dx dz \, \xi_y^2 g(\rho_\ell - \rho_u) \qquad (12)$$

where ρ_ℓ and ρ_u denote the densities in the lower and upper atmosphere, respectively.

Equation (12) is exactly what is expected when one considers the Rayleigh-Taylor instability between two media of different density (e.g. Chandrasekhar, 1961). Since $\rho_\ell >> \rho_u$ in the problem considered here, the surface term is positive. Thus we have shown that for nonzero g, the presence of flow across the unperturbed fluid boundary (nonzero $\xi_\parallel$) tends to be stabilizing. The surface term given in equation (12) arises naturally from the dynamics of the problem, and must be included in any evaluation of δW.

We can write

$$\delta W = \delta W_{FC} + \delta W_{FL} + \delta W_S$$

where δW_{FC} and δW_{FL} are the contribution of the corona and the lower atmosphere to δW_F as given in equation (11), and δW_S is the surface term given in equation (12).

It is clear that if the problem of the stability of isolated coronal structures has any meaning we must be able to make δW_{FL} vanishingly small.

This requires in general, that $\vec{\xi}$ be nonzero only in an infinitesimally thin layer below the boundary. Can the argument used in deriving the extended energy principle be applied to this situation? That is, is there always a displacement $\vec{\xi}$ of the lower atmosphere which satisfies the interface conditions but which makes δW_{FL} arbitrarily small in magnitude?

In general, there is not. Recall from the discussion of the extended energy principle following equation (9) that the correction vector to $\vec{\xi}$ is assumed to be of order ε and localized to a layer of width ε. Its contribution to δW_F is then of order ε. But in the present case, if $\vec{\xi}\cdot\hat{n}$ is of order 1 at the interface, and ξ is again localized to a layer of order ε, δW_{FL} will be order ε^{-1}. We cannot always make δW_{FL} negligibly small.

The case $\xi \cdot \hat{n} = 0$ is an exception. In this case, ξ can be zero on the boundary and of order ε in a layer of width ε. For example, take $y = 0$ to be the surface; $y \to -\infty$ with depth. Take the Lagrangian pressure and magnetic field perturbations $\Delta p_s(x,z)$, $\Delta \vec{B}_s(x,z)$ to be prescribed by the displacement of the upper atmosphere. Then, for $y \leq 0$, let

$$\xi_x(x,y,z) = -\frac{\varepsilon}{B_y}\left(\Delta B_{xs} - \frac{B_x \Delta p_s}{\gamma P}\right)e^{y/\varepsilon}(1-e^{y/\varepsilon})$$

$$\xi_y(x,y,z) = \frac{\varepsilon \Delta p_s}{\gamma P}\, e^{y/\varepsilon}(1-e^{y/\varepsilon})$$

$$\xi_z(x,y,z) = -\frac{\varepsilon}{B_y}\left(\Delta B_{zs} - \frac{B_z \Delta p_s}{\gamma P}\right)e^{y/\varepsilon}(1-e^{y/\varepsilon}),$$

where the functions multiplying the exponentials are evaluated at $y = 0$. This choice of ξ gives δW_F of order ε for $y < 0$.

These arguments have the following implications for MHD stability analysis of the solar corona. If we restrict ourselves to displacements with $\xi \cdot \hat{n} = 0$, then δW_S and δW_{FL} can both be made zero, and it is both necessary and sufficient for the stability of the "isolated" coronal modes that $\delta W_{FC} > 0$. If we consider displacements with $\xi \cdot \hat{n} \neq 0$, the positive definite term δW_S (cf. equation 12) must be added to δW_{FC}. In addition, δW_{FL} must also be minimized. This requires an explicit model of the lower atmosphere, but none of the presently available coronal equilibrium models include the lower atmosphere. Thus, the stability of modes with $\xi \cdot \hat{n} \neq 0$ is indeterminate. In summary, it is sufficient but not necessary for the stability of the isolated ($\xi \cdot \hat{n} = 0$) displacements that δW_{FC} is positive when minimized with unrestricted $\vec{\xi} \cdot \hat{n}$. Modes with $\vec{\xi} \cdot \hat{n} \neq 0$ cannot properly be tested for stability without a model of the lower atmosphere.

III. EFFECT OF SURFACE TERMS ON CORONAL STABILITY

In this section, we discuss the relevance of the term δW_S and δW_{FC} derived in Section II to a number of studies of coronal MHD stability in the literature in which different lower boundary conditions were assumed. We first consider the conditions originally used by Einaudi and Van Hoven (1981) and then discuss the conditions used by Schindler et al. (1983). Finally, we treat the work of Hood (1984a,b), for which the necessary analysis is somewhat more involved. In all the papers we will treat, the components of $\vec{\xi}$ perpendicular to the magnetic field are assumed to vanish at the base of the atmosphere. The physical motivation for this is that the fieldlines are assumed to be fixed in dense, infinitely conducting photospheric gas.

a) Line Tying with Flow at Lower Boundary

Einaudi and Van Hoven (1981) studied the stability of coronal loops idealized as cylinders of finite length with twisted magnetic fields. They did not include gravitational stratification, an approximation which applies when the thermal scale height much exceeds the size of the system. the conditions they imposed at the ends of the cylinder are

$$\xi_\perp = 0 \text{ at } z = \pm L$$

$$\xi_\parallel(-L) = \xi_\parallel(L) \tag{13}$$

$$\frac{d}{dz}\xi_z(-L) = \frac{d}{dz}\xi_z(L)$$

where z is along the axis of the cylinder and $\xi_\parallel$ and $\xi_\perp$ are the components of $\vec{\xi}$ parallel and perpendicular to B. These conditions have also been used by Migliuolo et al. (1984), Einaudi and Van Hoven, (1983) and references therein. Since $\vec{\xi} \cdot \hat{n} \neq 0$ in their model, they can derive sufficient but not necessary stability conditions for the isolated modes.

b) Rigid Boundary Condition

Schindler et al. (1983) used the condition $\vec{\xi} = 0$ on the lower boundary, which corresponds to treating the photosphere as a rigid, perfectly conducting wall. It is clear in this case that δW_S and δW_{FC} vanish. Thus, none of the stability results arising from their minimization of δW_F are affected by addition of the corresponding δW_S or δW_{FC}. In a sense, the $\vec{\xi} = 0$ boundary condition is a limiting case of the two fluid analysis for $\rho_\ell \to \infty$. If $\rho_\ell \to \infty$, δW_S becomes large unless $\xi_n \to 0$. One must also impose $\xi_\perp = 0$.

c) Line Tying with Unrestricted $\xi_\parallel$

Hood (1983a) derived a form of δW including gravitational stratification. He used the lower boundary condition $\vec{\xi} \times \vec{B} = 0$, although only in the approximate sense described below, and did not restrict $\xi_\parallel$. Although his formulation (as does that of Schindler et al.) extends to systems in which the magnetic fields have three spatial components, the analysis here is restricted to systems in which the fieldlines lie in parallel, vertical planes, such that

$$\vec{B} = + \hat{x} \frac{\partial A}{\partial y} - \hat{y} \frac{\partial A}{\partial x} \tag{14}$$

All the equilibrium quantities are functions of x and y only, and the magnetic fieldlines are assumed to form loops which are symmetric in x. These systems resemble solar magnetic arcades. Defining A_1 as $\vec{\xi} \cdot \nabla A$ and using (14), Hood's boundary condition becomes $A_1 = 0$ on the boundary. Using only this boundary condition, he writes δW_{FC} as

$$2\delta W_{FC} = \int d^3x \left(\frac{\partial A_1}{\partial s}\right)^2 + \left[\nabla A \cdot \nabla\left(\frac{J}{B^2}\right) - \frac{2J^2}{B^2} - \frac{\partial J}{\partial A}\right]A_1^{\,2} + \left(B\cdot\nabla\xi_z\right)^2 +$$

$$\left[B\frac{\partial \xi_z}{\partial z} - \frac{\nabla A\cdot\nabla A_1 + JA}{B}\right]^2 + p\left[(\gamma-1)(\nabla\cdot\vec{\xi})^2 + e^{2y/H}(\nabla\cdot\vec{\xi}e^{-y/H})^2\right] \qquad (15)$$

Here, J is the current density, H is the thermal scale height, and $\partial/\partial s = 1/B\ \vec{B}\cdot\nabla$ is the derivative along a fieldline.

To derive stability criteria, Hood assumes that the perturbations are isothermal ($\gamma = 1$) and minimizes with respect to $\partial\xi_z/\partial z$ and $\nabla\cdot(\vec{\xi}e^{-y/H})$. This results in the conditions

$$\nabla\cdot\vec{\xi} = \frac{\xi_y}{H} \qquad (16)$$

$$\frac{\partial \xi_z}{\partial z} = \left(\frac{\nabla A\cdot\nabla A_1 + JA_1}{B^2}\right) \qquad (17)$$

Equation (16) is a generalization of the incompressibility condition that results from minimizing δW in plasmas that are not gravitationally stratified (e.g. Freidberg, 1982). Equations (16) and (17), taken together, imply that the total (gas plus magnetic) pressure perturbation is zero, and thus eliminates the stabilizing restoring force due to a pressure perturbation.

Hood then takes the limit of infinitely large wavenumber in the $\hat{z}$ direction, $k_z \to \infty$. Since condition (17) requires that the product $k_z\xi_z$ be finite, the magnetic tension term $(B\cdot\nabla\xi_z)^2$ in δW, which corresponds to bending the fieldlines out of their equilibrium plane, becomes negligible. (see Gilman 1970, Asséo et al.1980, Zweibel 1981). Thus, δW is reduced to the form

$$2\delta W = \int d^3x\left\{\left(\frac{\partial A_1}{\partial s}\right)^2+\left[\nabla A\cdot\nabla\left(\frac{J}{B^2}\right)-\frac{2J^2}{B^2}-\frac{\partial J}{\partial A}\right] A_1{}^2\right\}, \tag{18}$$

which is considerably simpler than equation (15), since only the perturbation variable A_1 and its derivative along a fieldline appear. When δW in the form (18) is minimized subject to the normalization condition $\int d^3x\ A_1{}^2 = 1$, the resulting Euler equation for A_1 is an eigenvalue equation. The solution of the eigenvalue problem is the basis of Hood's stability analysis (Hood 1984b) of the Zweibel and Hundhausen (1982) equilibrium solutions.

The minimizing conditions (16) and (17), together with the assumption that $\gamma = 1$ and $\xi_\perp = 0$ at the lower boundary, guarantee that δW_S as it appears in equation (11) vanishes. That is, Hood's displacements have a nonzero Lagrangian pressure perturbation on the lower boundary. As we showed in Section II, the appropriate form of δW_S is really Eq. (12), because of the Lagrangian force balance condition. Since $\xi_\parallel$ is unrestricted, δW_S will generally not vanish, and δW_{FL} cannot vanish either.

We should also note that when $\partial\xi_z/\partial z$ is given by equation (17), ξ_z will not vanish on the lower boundary (because ∇A_1 does not). Therefore, the condition $\xi_\perp = 0$ is technically violated. However, since ξ_z is small (see the discussion following equation (17)) it appears consistent to neglect ξ_z on the boundary when dropping $(B \cdot \nabla\xi_z)^2$ from δW_{FC}.

Suppose that when δW_F as given by equation (18) is minimized over A_1, the resulting $\vec{\xi}$ (which can be calculated from A_1 using conditions (16) and (17), as we do below) satisfies $\vec{\xi} \cdot \hat{n} = 0$. In this case, δW_{FC} itself will be a true minimum. On the other hand, if $\vec{\xi} \cdot \hat{n} \neq 0$, the minimization is unacceptable because δW_S and δW_{FL} must be included.

We now discuss the conditions under which the minimization of equation (18) will permit $\vec{\xi} \cdot \hat{n} = 0$. Since δW_{FC} in equation (18) only contains derivatives of A_1 along a fieldline, we can consider perturbations which are localized to a single flux-tube. To solve for $\vec{\xi} \cdot \hat{n}$ in terms of any given A_1, we eliminate $\partial\xi_z/\partial z$ between conditions (16) and (17) to give

$$(\vec{B} \cdot \nabla\vec{\xi}) \cdot \vec{B} - (\vec{B} \cdot \nabla\vec{B}) \cdot \vec{\xi} = \frac{B^2\xi_y}{H} . \tag{19}$$

Then, ξ_x may be written in terms of A_1 and ξ_y and eliminated from (19). The result is a first order differential equation for ξ_y as a function of A_1, with solution

$$\xi_y(s) = B_y(s)e^{y/H} \int_c^s \frac{ds'}{B} e^{-y'/H}\left(\frac{A_1}{BB_y}\frac{dB_x}{ds} - \frac{B_x}{B}\frac{d}{ds}\frac{A_1}{B_y}\right) \tag{20}$$

We assume here that B_x does not vanish anywhere on the field line, so s is a single valued function of x; the fieldlines Hood studied have these properties. Let the fieldline end at $\pm s_0$. Then, if ξ_y vanishes at the endpoints, equation (20) implies that

$$\int_{-s_0}^{s_0} \frac{ds'}{B} e^{-y'/H}\left(\frac{B_x}{B}\frac{d}{ds}\frac{A_1}{B_y} - \frac{A_1}{BB_y}\frac{dB_x}{ds}\right) = 0 \tag{21}$$

Since, in the geometry assumed, B_x and B_y are even and odd functions of x (or s), respectively, equation (21) will be satisfied only if A_1 is an odd function of x.

We now consider Hood's study of the Zweibel and Hundhausen (1982) equilibria (Hood 1984b). These equilibria form a one parameter family in which the parameter measures the volume electric current, or distortion of the fieldlines from a potential field at the base of the atmosphere. The only previous stability analysis of these equilibria was a local analysis (Zweibel 1981) which showed that some portion of all the ZH equilibria were locally unstable. However, this analysis did not consider the stabilizing effect of magnetic tension. Hood found that instability along an entire fieldline only exists if the parameter $2\alpha H$ which measures the current exceeds a certain threshold. His solutions for A_1 are even, rather than odd, functions of x. According to the arguments above, the stability boundary for isolated coronal modes should be at a larger value of the current parameter than that found by Hood.[1] The stability boundary for these modes can be found by solving the Euler equation for the integral (18) with $A_1 = 0$ at the end and apex of a fieldline. Within the framework of the present analysis, Hood has found a sufficient, but not necessary, stability condition.

IV. DISCUSSION AND CONCLUSIONS

In this paper, we have considered the lower boundary condition for coronal MHD stability problems. These systems are characterized by magnetic fieldlines which connect the corona to a much denser, underlying atmosphere. Their ideal MHD stability has been studied using the BFKK energy principle.

The lower atmosphere has simply been modeled as a rigid, conducting wall in some previous treatments (e.g. Schindler et al. 1983) on which the fluid displacement $\vec{\xi}$ vanishes. Other studies have allowed a non-vanishing fluid displacement parallel to the magnetic field at the lower boundary, but required the perpendicular components $\xi_\perp$ to vanish.

In order to understand the effect of different boundary conditions, we considered the boundary as a contact surface between two ideal fluids of different temperatures and densities. We reviewed the boundary conditions which apply to a fluid-fluid interface and pointed out that these boundary conditions lead to surface integrals in the perturbed potential energy δW which represent PdV work done at the interface between the fluids. These terms (Eq. 12) can be written in a form which involves the density contrast between the two fluids, and is the same term one derives in an analysis of the Rayleigh Taylor

[1] After this paper was accepted for publication, we became aware of a recent study by Hood (1984c) which is consistent with this.

instability at an interface between unmagnetized fluids. Since the lower atmosphere is much denser than the upper atmosphere, the surface term is strongly stabilizing. It vanishes when the displacement normal to the boundary vanishes.

We also found that δW_{FC} cannot be ignored for displacements with $\vec{\xi} \cdot \hat{n} \neq 0$. Thus, there are two types of displacements; the isolated coronal modes, with $\vec{\xi} \cdot \hat{n} = 0$, for which δW_S and δW_{FC} can legitimately be set equal to zero, and the displacements with $\vec{\xi} \cdot \hat{n} \neq 0$. For the latter, the stability problem consists of jointly minimizing δW_{FC}, δW_{FL}, and δW_S. This is not only impractical, in view of the lack of available equilibrium models, but is counter to the concept of an isolated coronal MHD instability. Virtually all papers on the subject have been concerned with isolated coronal instabilities, whether or not they treated the lower boundary condition in a way consistent with that idea.

If the volume term δW_{FC} alone is minimized, as was done by Hood (1984a,b) the result can be used to give a sufficient, but not necessary, condition for stability of the isolated coronal modes. We showed that Hood's minimization of δW_{FC} for a particular set of equilibria (Hood 1984b) led to nonvanishing $\vec{\xi} \cdot \hat{n}$. The imposition of $\vec{\xi} \cdot n = 0$ on the boundary requires that Hood's trial functions A_1 have odd parity. We would argue, therefore, that some of the equilibria that Hood predicted are unstable are actually stable, according to the upper and lower-fluid model.

The rigid boundary condition with $\vec{\xi} = 0$ has a vanishing surface term. Assuming the rigid boundary condition with $\vec{\xi} = 0$ leads to a self-consistent problem in which δW_{FC} alone is minimized. This seems to be the simplest approach to treating the corona as an isolated system. The full problem, involving thermal exchange and dynamical forcing by motions of the fieldline endpoints, will have to be explored by other methods than the MHD energy principle.

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