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An Extended Two-Target Differential Game Model for Medium-Range Air Combat Game Analysis

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ABSTRACT

This interim report summarizes the first phase of an investigation of a two-target game, representing an air combat with boresight limited all-aspect guided missiles. The results, obtained by using a line of sight coordinate system, are compared to a similar recently published work. The comparison indicates that improved insight, gained by using line of sight coordinates, allows to discover important new features of the game solution.

Detailed results will be presented in a verbal briefing at NASA Ames Research Center, Moffett Field, Cal. in August 1985.

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1. Introduction

The research activity in the first six months of the investigation was oriented to evaluate the usefullness of a line of sight coordinate system and to develop a computer program which generates barrier trajectories in these coordinates. As a first example, a simple pursuit-evasion game (the game of two identical cars) with a circular target set solved. This solution was presented at the 26 Israel Annual Conference on Aviation and Astronautics (Feb. 1985), and demonstrated the advantages of using a line of sight coordinate system. The copy of the paper (Ref. 1) is enclosed with this report. The convenient coordinate system allowed to discover a new type of singular surface, which was overlooked in a previous investigation of the same game.

Guided by the successful results of the above step the next phase was oriented to use the line of sight coordinate system in a two-target game analysis. The example selected for this phase represents an air combat between two aircraft with boresight limited all aspect guided missiles. One of the reasons for this selection was the information obtained from Prof. J.V. Breakwell at the beginning of the investigation, that a comparable work (Ref. 2) had been submitted for publication. It seemed that testing the validity of the new approach, used in the present investigation, by an appropriate comparison is of a major importance.

In this interim report the main findings of a "first-firing" two-target game analysis, including the results of the comparison with Ref. 2, are summarized. The detailed solution will be presented in a verbal briefing at NASA Ames Research Center in Auqust 1985, as well as in a scientific paper, which is now in preparation. (The paper has been invited for a special issue of Computer and Mathematics with Applications on Pursuit-Evasion Differential Games by the Guest Editor Dr. Y. Yavin.)

2. Problem Formulation

The dynamic model of the "first-firing" two-target game to be investigated is of the game of two identical cars, used in Ref.1 and expressed by Eqs.(12)-(14) there. The target sets of the game are line segments aligned with the respective velocity vectors and limited by the minimum and maximum effective ranges of the guided weapon system. The variables of the game and the target sets are depicted in a fixed (inertial) coordinate system in Fig. 1. In Fig. 2 the usable parts of the target sets are shown in a line of sight coordinate system. As it is shown in the figure, a part of the minimum range boundary is determined by the rate of turn constraint,

$$R_i = \sin \phi_j > \ell_{min} \quad i \neq j \quad i,j=1,2.$$

The objective of the two-target game analysis is to determine the winning zones of each player (1,2) as well as the regions of draw and eventual mutual kill. The first phase of such an analysis is the solution of the respective qualitative pursuit-evasion games (1 vs. 2 and 2 vs. 1).

3. Pursuit-Evasion Game Solution (1 pursuing 2)

The starting point is to determine the optimal strategies at the boundary of the usable parts of the target set.

It turns out that on the "maximum-range boundary" ($R = \ell_{max}$) only a single point A_1 ($\phi_1 = 0$, $\phi_2 = \pm \pi$) can serve as a terminal point for optimal strategies. It indicates that all barrier trajectories terminating at maximum range do it in a "tail-chase".

On the "minimum range boundary" every point can serve as an end point of an optimal barrier trajectory. Moreover, optimal strategies can terminate on both sides of the zero-thickness target set.

Based on the optimal strategies, determined along the boundary of the usable parts of the target sets, optimal trajectories can be integrated backwards in time. In the line of sight coordinate system the backward integration can be performed analytically yielding a closed form solution, as outlined in Section 3.4 of Ref. 1 (see Eqs.(45)-(71) there).

The only end point (A_1) on the "maximum-range" boundary is a junction of <u>4 universal lines</u> (2 for each player). These universal lines are also optimal trajectories and can be obtained by backward integration. Moreover, at any point of a universal line two optimal trajectories coming from opposite directions meet. These regular barrier trajectories are also obtained in a closed form by similar backward integration. A part of these trajectory families, ending on the 4 universal lines, intersect each other along dispersal lines and generate the "maximum-range barrier section".

The barrier trajectories, emanating backwards from both sides of the "minimum-range boundary" generate two symmetrical surface sections (one for $\phi_1 > 0$ and the other for $\phi_1 < 0$) and the majority of them itersect with trajectories coming from the "maximum-range barrier". These two surface sections can be called the "minimum-range barrier section."

The maximum-range and minimum-range barrier section do not form yet a closed barrier surface. Near to the "tail-chase" zone ($|\phi_2| \to \pi$) the "minimum-range barrier section" ends before reaching the "maximum-range barrier section". This gap is closed by a third type of barrier section, baptized as the "interconnecting barrier section". The origin of this third barrier section is a strategy change (switching line) of the pursuer. The optimal trajectories generating this "interconnecting barrier section" emanate from an evader dispersal line and continue towards a "tail-chase" due to the above mentioned "strategy switch" of the pursuer.

It can be thus concluded that the barrier of the 1 against 2 pursuit-evasion game is a closed surface generated by three different types of sections. It encloses the "winning zone" of the pursuer and separates it from the other part of the state space.

The barrier of the second pursuit-evasion game (2 pursuing 1) is identical if the roles of ϕ_1 and ϕ_2 are interchanged.

4. Two-Target Game Solution

The respective barriers of the two pursuit-evasion games intersect along lines in the planes of symmetry $\Phi_1 = \Phi_2$ and $\Phi_1 = -\Phi_2$. These planes are in fact semipermeable surfaces of the "first-firing" two-target game and intersect along the R axis ($\Phi_1 = \Phi_1 = 0$). The consequence of barrier intersection is that a part of the respective "winning zones", being enclosed by the opponents' barrier, disappears. The remaining regions have to be considered in the two-target game analysis as only candidates for an eventual winning. A part of each "candidate winning zone" may turn out to belong to the "region of mutual kill".

Simultaneous mutual kill occurs at any point along the target set intersection ($\Phi_1 = \Phi_2 = 0$, $\ell_{min} \leq R \leq \ell_{max}$). Optimal trajectories leading to this "mutual kill target set" generate a barrier separating the "region of mutual kill" from the remaining "winning zones". At the maximum range point "P" ($R = \ell_{max}$, $\phi_1 = \phi_2 = 0$) four singular trajectories, characterized by "pure-pursuit" strategies $\phi_i = \dot{\phi}_i = 0$ $(i = 1,2)$ meet.

The singular trajectories serve also as <u>universal lines</u> for one of the players. At any point along the universal lines two regular trajectories meet. These trajectories form a surface which intersects the "maximum-range barrier sections" of the original pursuit-evasion games along new <u>dispersal lines</u>. This pyramid type surface, generated by all the trajectories ending at the point "P" separate the "region of mutual kill" from the respective "winning zones" and can be called as the "internal mutual kill barrier sections."

A second surface, section that separates the "region of mutual kill" from the "zone of draw", consists of parts of "maximum-range barrier sections" of the pursuit-evasion games. It also has a pyramid shape and can be called as the "outer mutual kill barrier section". At any point on this barrier section one of the players can select either to be an evader - and in this case the game terminates with a marginally successful tail chase escape - or to act agressively and then the trajectory enters to the "region of mutual kill". The aggressive strategy in the "region of mutual kill" is leading towards a "head-on" ($\phi_1 = \phi_2 = 0$) encounter at $R = \ell_{max}$. Inside this region the players are committed to a mutual kill and any of them who tries to evade will be killed by the opponent. A mutually agreed disengagement leading to a draw does not seem to be likely, because in this case any of the players can deceive his opponent at any time and drive the trajectory closer to his own "winning zone".

It can be summarized that the "region of mutual kill" is enclosed by two pyramid shape barrier sections. The remaining "winning zones" of the two target game are substantially reduced compared to the pursuit-evasion game solution. They are rather limited to initial conditions of clear directional advantage with respect to the opponent.

5. Comparison to Ref. 2

The results of the above outlined two-target game analysis were compared to a recently published paper (Ref. 2). In that paper a similar problem is investigated, but without any restriction on the minimum firing range, allowing therefore collision type mutual kills. In all other respects the game models are identical.

Most of the results of the two independent investigations confirm each other. There are, however, two elements discovered in the present analysis which could not be found in Ref. 2. It has been verified that these elements are not connected to the difference in the minimum range definition.

The first element is the existence of a strategy switch of the pursuer in the pursuit-evasion game. It leads to generate the "interconnecting barrier section" and enlarges the "winning zone" of the pursuer compared to the results of Ref. 2.

The second element relates to the "mutual kill barrier". The "pure pursuit" strategy found to be optimal along the universal lines of the internal "mutual kill barrier section" is not mentioned in Ref. 2. This strategy leads to reduce the "region of mutual kill" and consequently enlarges the remaining "winning zones".

6. Conclusions.

In the first six months of the investigation two major steps were accomplished. The methodology of game analysis in a line of sight coordinate system was developed and tested by comparing results with similar game models analyzed in previous studies. The new methodology allows to generate barrier trajectories in a closed form and to identify dispersal lines and other types of singularities. It provides an improved insight for game analysis as it was demonstrated by discovering features which had been overlooked in previous works.

The validated methodology can be thus considered as a ready tool for the forthcoming phases of the three years research program outlined in the original research proporsal.

References

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2. Merz, A.W.: "To pursue or to evade? - That is the Question". J. of Guidance and Control, Vol. 8, No. 2, 1985, pp.161-166.

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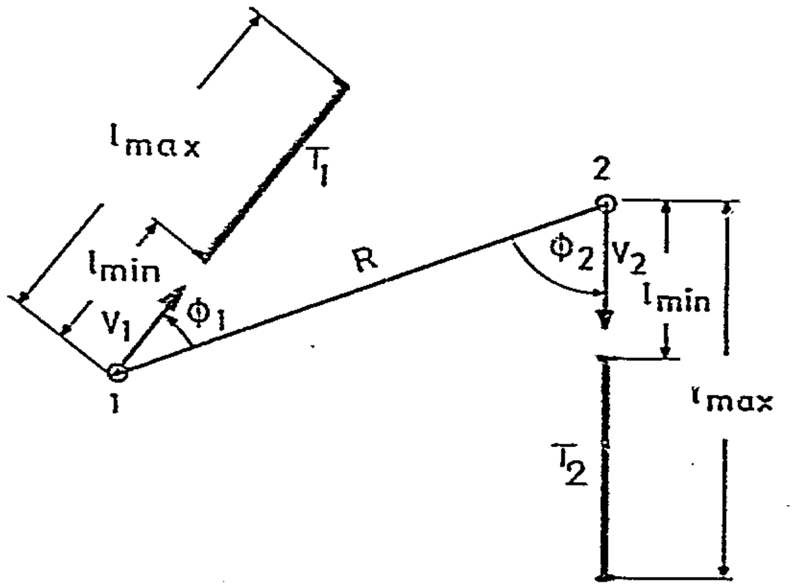

Fig. 1. Two-target game geometry in a fixed coordinate system.

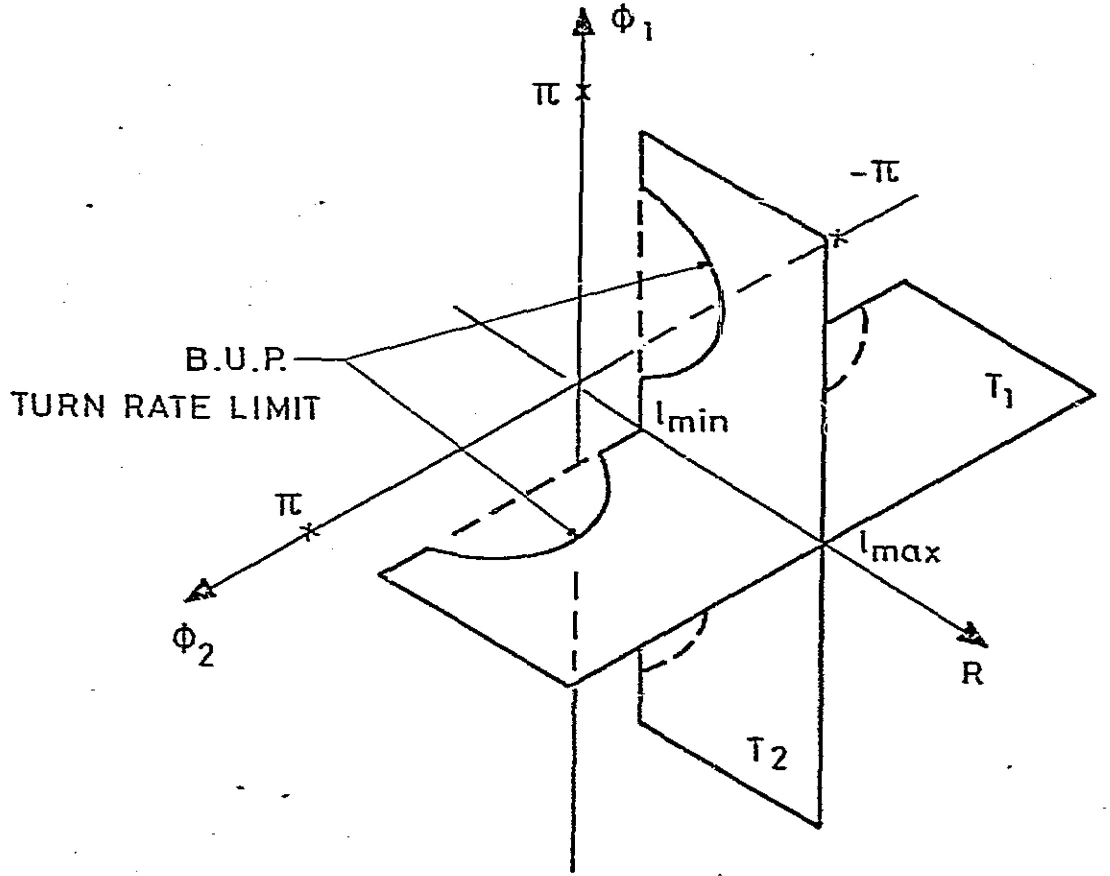

Fig. 2. Target sets in the line of sight coordinate system.