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Spiral Bevel and Circular Arc Helical Gears: Tooth Contact Analysis and the Effect of Misalignment on Circular Arc Helical Gears

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SPIRAL BEVEL AND CIRCULAR ARC HELICAL GEARS: TOOTH CONTACT ANALYSIS AND THE EFFECT OF MISALIGNMENT ON CIRCULAR ARC HELICAL GEARS

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SUMMARY

A computer aided method for tooth contact analysis was developed and applied. Optimal machine-tool settings for spiral bevel gears are proposed and when applied indicated that kinematic errors can be minimized while maintaining a desirable bearing contact. The effect of misalignment for circular arc helical gears was investigated and the results indicted that directed pinion refinishing can compensate the kinematic error due to misalignment.

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INTRODUCTION

A computer aided tooth contact analysis (TCA) is currently the most important topic in the gearing area. The purpose of TCA is the simulation of meshing directed at the determination of kinematic errors and the bearing contact. The kinematic errors are due to manufacturing and assembly errors. A TCA program for spiral bevel and hypoid gears was developed by Gleason Works in the sixties (ref. 1). Litvin had proposed the basic principles of TCA in 1962 and Litvin and Go Kai applied this method for the analysis and the improvement of conditions of meshing for the Gleason's spiral bevel gears (refs. 2 to 4).

Also a TCA program for hypoid gear drives was developed by Litvin and Gutman (ref. 7). Methods for the generation of conjugate gear tooth surfaces are very important in theoretical and practical aspects. These surfaces will have a higher load capacity and reduce the amount of mesh generated noise. The contents of this paper cover the generation of (1) spiral bevel gears with almost zero kinematic errors, and (2) helical gears with circular arc teeth. Adjustment of helical gears with circular arc teeth to the misalignment is also considered.

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NOMENCLATURE

c_f, c_p	centers of tooth circular arcs
M_1, M_2	points of tangency for circular arc pinion and gear
m_{12}	gear ratio, $m_{12} = \frac{\omega^{(1)}}{\omega^{(2)}}$
N_1, N_2	pinion and gear tooth numbers
$\underset{\sim}{n}_f^{(1)}, \underset{\sim}{n}_f^{(2)}$	pinion and gear tooth surface unit normals
$\underset{\sim}{r}_f^{(1)}, \underset{\sim}{r}_f^{(2)}$	pinion and gear tooth surface position vectors
r_1, r_2	pinion and gear pitch circle radii, in
Δc	change of center distance, in
$\Delta\gamma$	misalignment of gear rotation axes, deg
$\Delta\phi_2(\phi_1)$	kinematical error function
λ	helical gear lead angle, deg
Σ_F, Σ_P	pinion and gear generating surface
Σ_1, Σ_2	pinion and gear tooth surface
ϕ_F, ϕ_P	generating pinion and gear rotation angle, deg
ϕ_1, ϕ_2	pinion and gear rotation angle, deg
Ψ	pressure angle, deg
$\omega^{(1)}, \omega^{(2)}$	pinion and gear angular velocity, rad/sec

DISCUSSION

Tooth Contact Analysis

Figure 1 shows two surfaces being in contact at point M. The gear tooth surfaces must be in continuous tangency at every instant. This requirement can be satisfied if the following equations are observed (ref. 6).

$$\underset{\sim}{r}_f^{(1)} = \underset{\sim}{r}_f^{(2)}, \quad \underset{\sim}{n}_f^{(1)} = \underset{\sim}{n}_f^{(2)} \tag{1}$$

where $\underset{\sim}{r}_f^{(i)}$ and $\underset{\sim}{n}_f^{(i)}$ $(i = 1,2)$ are the position vector and the surface unit normal, respectively. The TCA program provides the numerical solution for equation (1). Using this solution we may determine the kinematic errors by the function

$$\Delta\phi_2(\phi_1) = \phi_2(\phi_1) - \frac{N_1}{N_2}\phi_1 \qquad (2)$$

Here ϕ_2 and ϕ_1 are the angles of gear rotation, function $\phi_2(\phi_1)$ is determined by the computer aided solution for equation (1); and N_i $(i = 1,2)$ are the numbers of gear teeth.

The TCA program determines the line of action which represents the set of contact points in the fixed coordinate system and the working line which represents the set of contact points on the gear tooth surface. Due to the elasticity of gear tooth surfaces the contact of gear tooth surfaces is spread over an elliptical area. The bearing contact is formed by the set of contact ellipses whose centers are located at the points of the working line. We may determine the orientation of the instant contact ellipse and its dimension if the principal curvatures and directions for the contacting surfaces and their elastic approach are known.

Generation of Spiral Bevel Gears with Improved Criteria

The Gleason machining method for the generation of spiral bevel gears may provide improved conditions of meshing and bearing contact if certain machine-tool settings are used for pinion generation.

The generation of Gleason's spiral bevel gears is based on the following principles: (1) Figure 2 shows the head-cutter used for the gear generation. The shapes of the blades of the head-cutter are straight lines which generate two cones while the head-cutter rotates about axis C-C. The angular velocity of rotation about axis C-C does not depend on the generation motion but on the desired velocity of cutting only. Two head cutters are used for the pinion generation, they are provided with one-sided blades and cut the respective tooth sides separately. The so-called cradle of the cutting machine carries the head-cutter and rotates about $X_m^{(i)}$-axis $(i = 1,2)$ while the generated gear (pinion) rotates about z_1-axis (figs. 3 and 4).

Figure 3 shows the generation of the gear. The axis of the cradle rotation, $X_m^{(2)}$, is perpendicular to the gear root cone and intersects axis z_2 of gear rotation; γ_2 is the pitch angle and Δ_2 is the gear dedendum angle; ϕ_p is the angle of the cradle rotation.

Figure 4 shows the pinion generation. Unlike the generation of gear 2, the axes of rotation $X_m^{(1)}$ and z_1 do not intersect each other, rather they cross, and ΔE_1 and ΔL_1 are the sought-for corrections of the machine-tool settings with which the meshing of gears 1 and 2 is to be improved.

The new approach to the generation of Gleason's spiral bevel gears is based on the following principles: (1) Two generating surfaces, Σ_F and Σ_P are used for the generation of the pinion tooth surface, Σ_1, and the gear tooth surface, Σ_2 respectively. Both sides of the gear tooth are cut simultaneously (duplex method) but each side of the pinion tooth are cut separately (single method); (2) Four surfaces, Σ_F, Σ_P, Σ_1, and Σ_2 are in contact at the main contact point. The proposed machine-tool settings provide that:

(a) the gear ratio function $m_{12}(\phi_1) = \frac{\omega^{(1)}}{\omega^{(2)}}$ is of the required value at the main point of contact, P; (b) the derivative $dm_{12}/d\phi_1$ is equal to zero at P; (c) the normal to the gear tooth surfaces does not change its direction within the neighborhood of P; and (d) the contact ellipse moves along the surface (from heel to the toe) which provides better conditions for lubrication.

The main advantages of applying the calculated machine-tool settings for Gleason's spiral bevel gears are: (a) almost zero kinematical errors; (b) a higher contact ratio; and (c) the gears can be cut using the existing Gleason's equipment and tools.

The derivation of the proposed machine-tool settings is based on the following considerations: (1) The axis of rotation of the generating gear is perpendicular to the root cone; (2) The cutting ratio for the gear generation provides the coincidence of the instantaneous axis of rotation by cutting with the gear pitch line; the cutting ratio for the pinion generation is determined with the equation of meshing by cutting which must be observed for the main contact point; (3) The blade angle for the pinion satisfies the requirement that the generating surfaces, Σ_F and Σ_P, have a common normal at the main contact point; and (4) The corrections of machine-tool settings, ΔE_1 and ΔL_1, do not depend on the cutting ratio and require the derivative $dm_{12}/d\phi_1$ is zero at the main contact point.

A computer aided method (ref. 6) for the tooth contact analysis of spiral bevel gears has been developed. Figure 5 shows the calculated bearing contact for a gear mesh with the parameters as shown in table I.

The kinematic error as a function of pinion position is shown in figure 6. The maximum kinematic error found by the TCA as one pitch of rotation is simulated for the gear is less than 0.5 arc sec.

Generation of Helical Gears with Circular Arc Teeth

Helical gears with circular arc teeth have been proposed by Wildehaber-Novikov. Many contributions have been made to investigate this type of gearing (ref. 8). The advantages associated with the use of Wildhaber-Novikov gears are: (1) reduced contact stresses and, (2) improved conditions of lubrication due to the motion of the contact ellipse along the gear tooth surface. However, circular arc gears are sensitive to the change in center distance and misalignment of the axes of rotation. The analysis performed and contained in this paper includes: (1) the method for gear generation, (2) the principles of the TCA program, and (3) the adjustment of the gears to the misalignment.

The method for the gear generation is based on application of two generating surfaces Σ_F and Σ_P which are rigidly connected to each other (proposed by Litvin and Davidov, see ref. 8). We may imagine that Σ_F and Σ_P are the surfaces of two rigidly connected rack cutters and are in tangency along the straight line a-a (fig. 7(a)). The normal sections of the rack cutters are two circular arcs. While the rack cutters translate with velocity $\underset{\sim}{v}$, the gears rotate with angular velocities $\omega_{(1)}$ and $\omega_{(2)}$, respectively. Cylinders of radii $r_1 = v \div \omega^{(1)}$ and $r_2 = v \div \omega^{(2)}$ are the gear axodes (the pitch cylinders) and plane Π, which is tangent to the cylinders, is the axode (pitch plane) of the rack cutters (fig. 7(b)). The line of tangency of the axodes, I-I, is the instantaneous axis of rotation (Axis I-I passes through point I shown in fig. 7(b) and is parallel to the gear axes). Consider that the rack cutter surface Σ_F generates gear 1 tooth surface Σ_1 and Σ_P generates gear 2 tooth surface Σ_2. Surfaces Σ_F and Σ_1, and correspondingly Σ_P and Σ_2, are in line contact, but Σ_1 and Σ_2 are in point contact.

Two hobs and two grinding wheels may also be used instead of two rack cutters for the generation of gears. The design of these tools is based on the idea of application of two rack cutters. The shape of these mating tools depends on the gear pitch only and the same tools can be used for the generation of mating gears with different combination of teeth. The normal sections of two mating gears are shown in figure 8.

The TCA program developed by the authors is based on the principles presented in this paper. This program determines the bearing contact and the kinematic errors exerted by the misalignment of the gears. For instance, figure 9 shows the location of the contact ellipses for the gears with the unchanged (fig. 9(a)) and changed (fig. 9(b)) center distance.

The adjustment of the gears to the misalignment is based on the refinishing of one of the mating gears (usually the pinion) with the calculated change of the gear parameters (ref. 8) Here are some examples which illustrate this approach.

The change of the gear center distance, ΔC, brings in the dislocation of the bearing contact which moves to the bottom or to the top of the tooth according to the sign of ΔC (fig. 9). That dislocation may be avoided just by the change of the vertical setting of the generating surface Σ_F which generates the pinion.

Now, consider that due to the misalignment of the axes of gear rotation they become not parallel but skewed. An example of how misalignment induced kinematic errors could be almost eliminated is shown in table II. The induced kinematic error may be represented by an approximate linear function; the maximum value of the kinematic error is about 60 arc sec. Changing the lead angle from 75° to 75.10° we could compensate in full for the kinematic error.

CONCLUDING REMARKS

Basic principles of the computer aided tooth contact analysis have been proposed and applied for the simulation of meshing and bearing contact of spiral bevel gears and helical gears with circular teeth. Optimal machine-tool settings for the generation of spiral bevel gears with almost zero kinematic errors have been proposed. Methods for the adjustment of helical gears with circular arc teeth to the misalignment have been suggested.

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TABLE I.

Pinion (N_1) 16 teeth
Gear (N_2) 20 teeth
Pressure Angle = 25°
Diametrical pitch = 6.4 in
Mean pitch cone distance = 1.701 in
Point diameter for the gear head cutter = 3.5 in
Spiral angle = 35°
Machine correction settings = (Calculated by computer algorithm)
Work offset (ΔE_1) = 0.0487 in
Machine center to back (ΔL_1) = -0.0389 in

TABLE II.

Pinion (N_1) = 12 teeth (right handed)
Gear (N_2) = 94 teeth
Nominal lead angle = 75°
Nominal pressure angle = 30°
Misalignment of the pinion axis ($\Delta\alpha$) = 0.1° (this is measured clockwise from the gear axis)
Changed lead angle = 75.10°

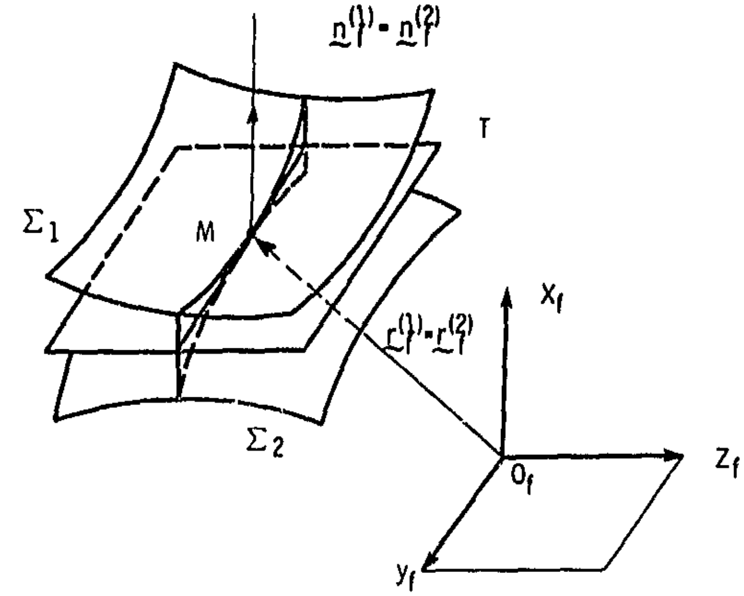


Figure 1. - Gear tooth surfaces in contact.

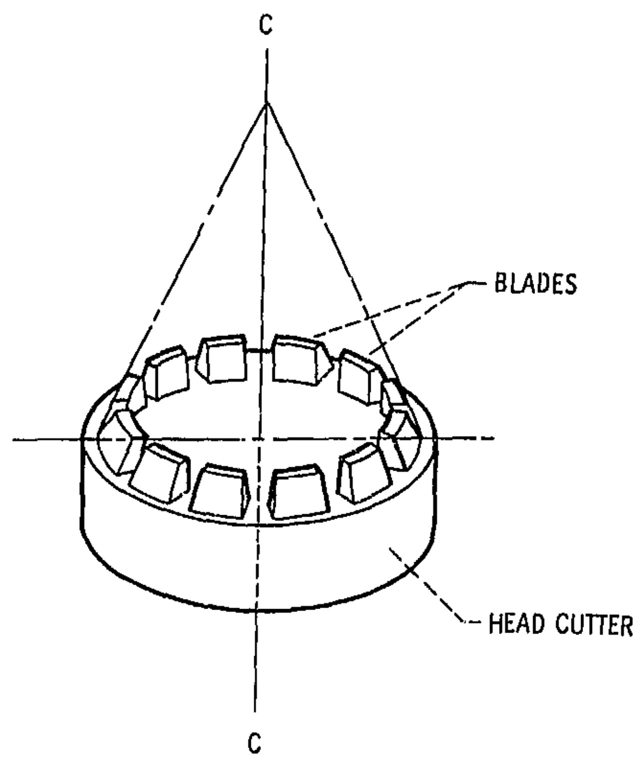
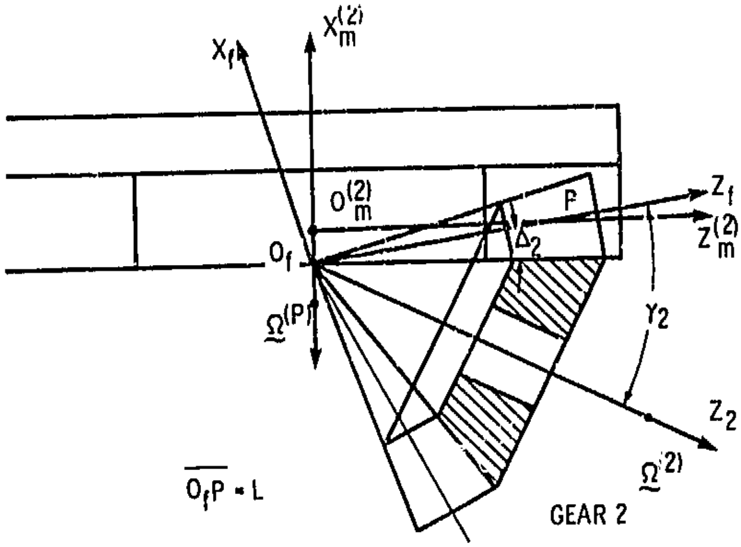
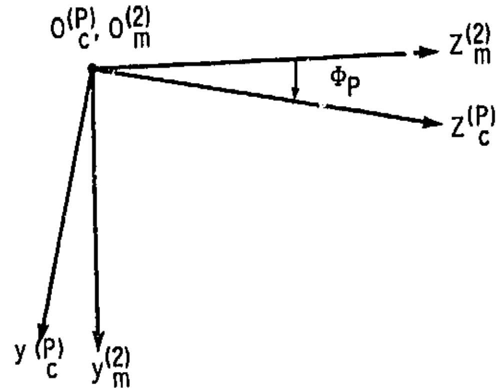


Figure 2. - Head cutter used for spiral bevel gear generation.



(a)



(b)

Figure 3. - Generation of spiral bevel gear.

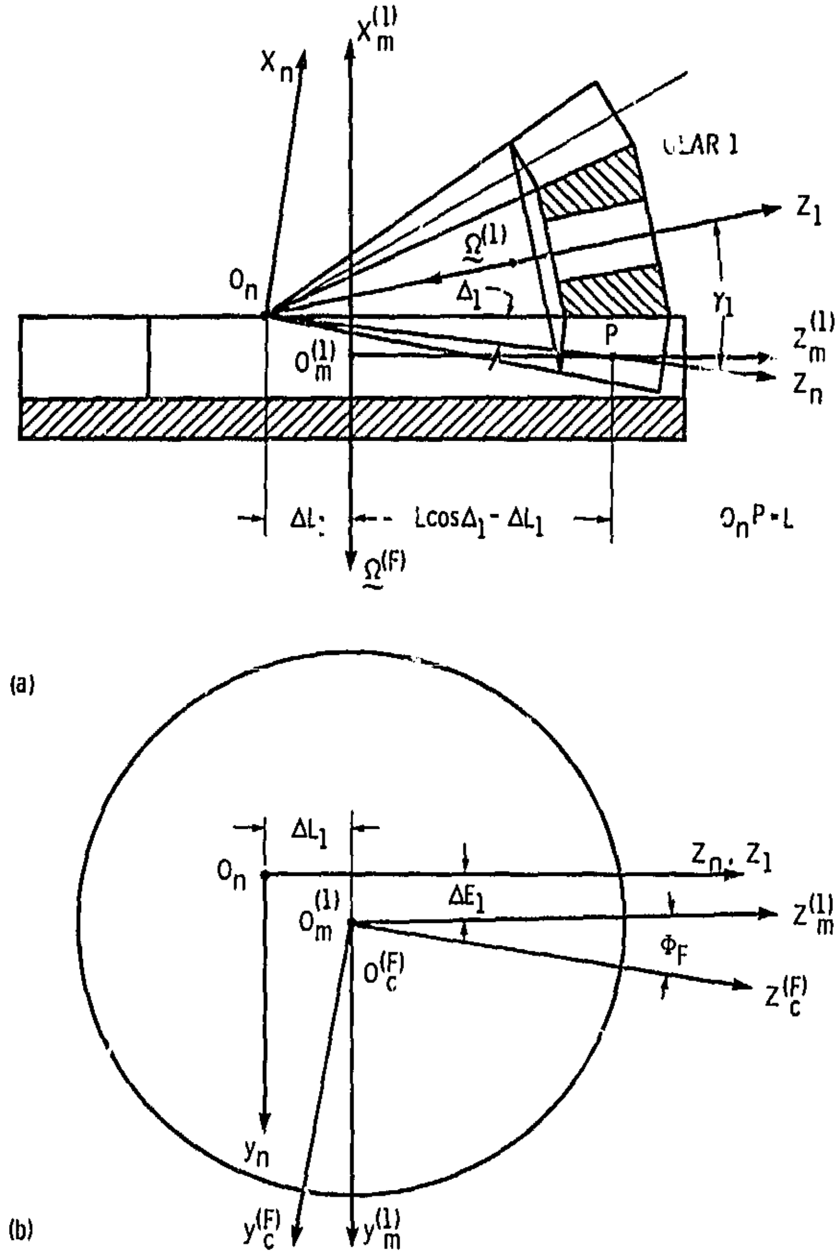


Figure 4. - Generation of spiral bevel pinion.

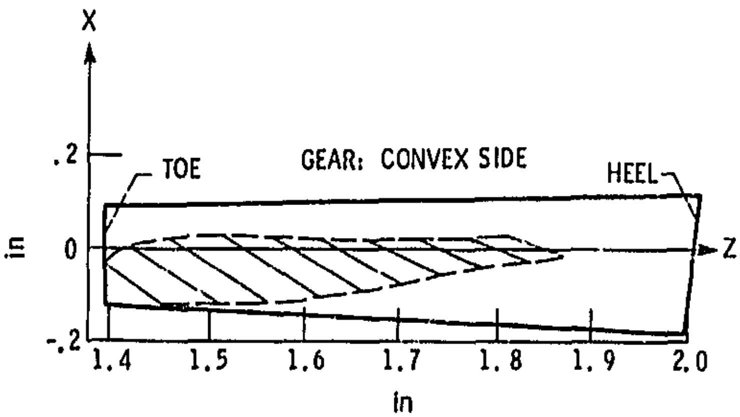


Figure 5. - Bearing contact on gear as it moves through the mesh.

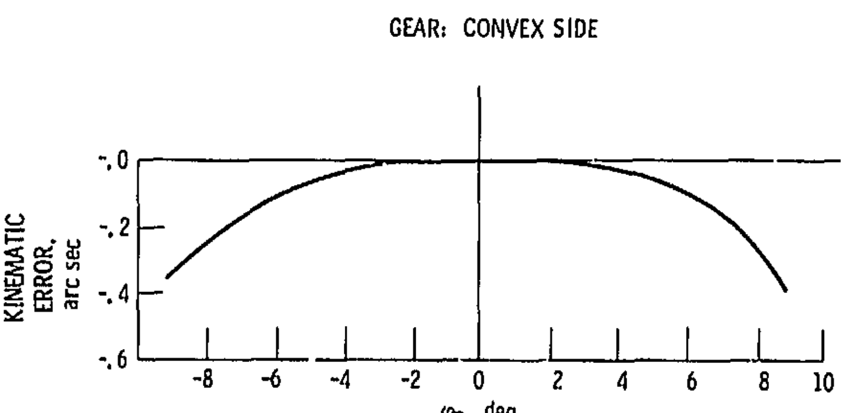


Figure 6. - Kinematic error as a function of pinion rotation.

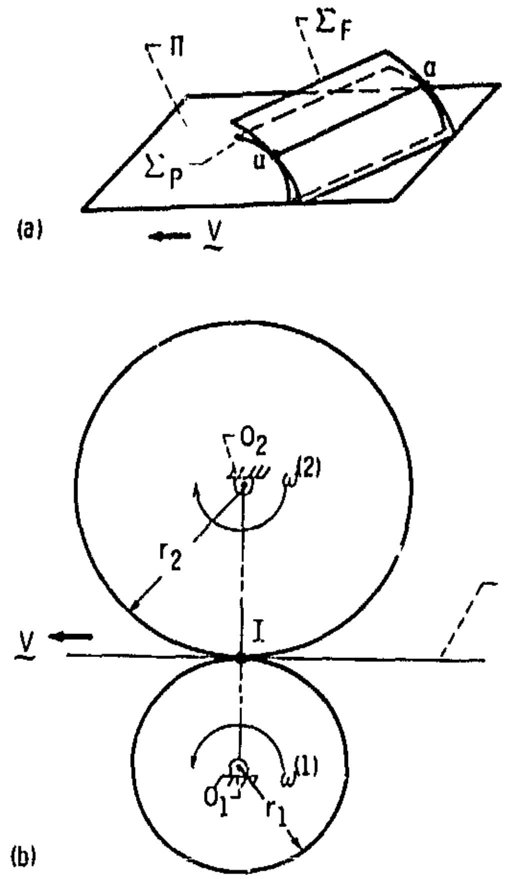


Figure 7. - Generation of helical circular arc gears.

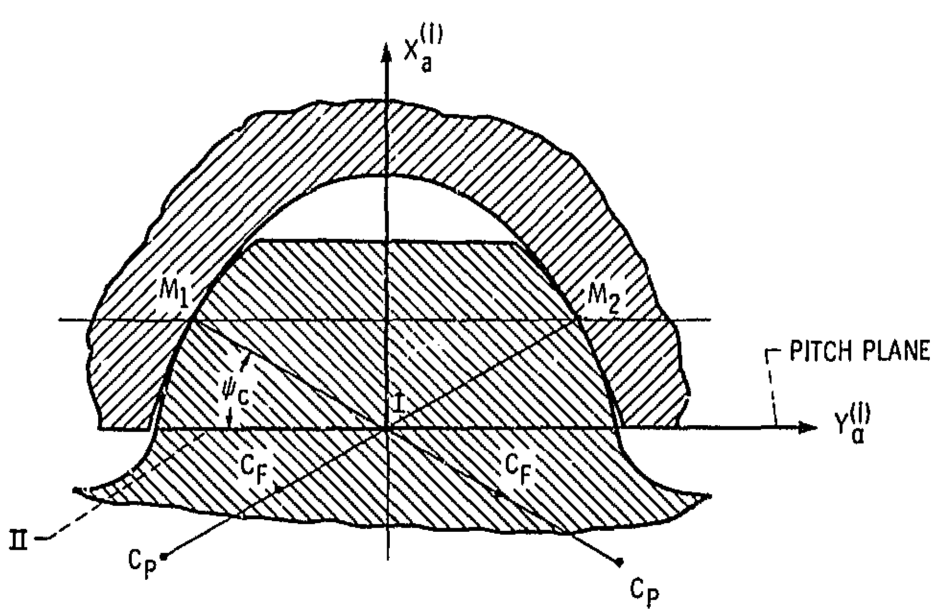
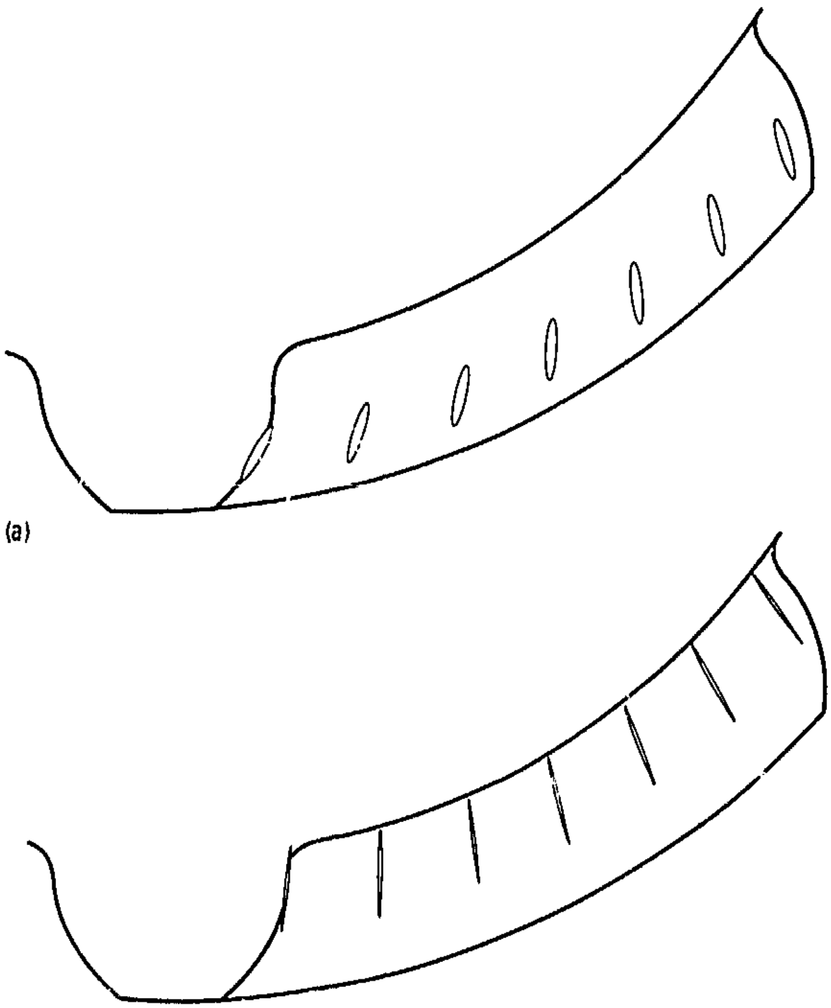


Figure 8. - Normal sections of two mating circular arc gears.



(a)

(b)

Figure 9. - Effect of center distance change on circular arc gear bearing contact.

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15. Supplementary Notes

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16. Abstract

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