MODELING TAPER CHARGE WITH A NON-LINEAR EQUATION

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Introduction

This is a report of work which has been in progress for six months aimed at modeling the charge voltage and current characteristics of nickel-cadmium cells subject to taper charge. Work reported at previous NASA Battery Workshops has shown that the voltage of cells subject to constant current charge and discharge can be modeled very accurately with the equation:

$$\text{Voltage} = A + \frac{B}{C - X} + De^{-Ex}$$

where A, B, D, and E are fit parameters and x is amp-hrs of charge removed during discharge or returned during charge. In a constant current regime, x is also equivalent to time on charge or discharge.

The present study is aimed at deriving equations for fitting the charge taper portion of the charge curve since the previously derived equations are not appropriate for this application. Figure 1 shows a typical current vs time plot for a cell in constant current discharge with a constant current charge to some predetermined voltage limit (VL). The current is held at -16 amps for 30 minutes during the discharge portion and then jumps to +16 amps for approximately 20 minutes of charge. At this point, the voltage hits the limit, and the current drops rapidly over the next 10 minutes flattening out to around 2 amps by the time the charge terminates at approximately 90 minutes. As we shall see in this report, the shape of the taper current will vary widely depending on the test conditions. The slope at which the current drops initially is quite variable as is the current when the charge curve flattens out at the end of the charge.

Test Matrix

Data for this study is taken from a test of 50 amp-hr cells being conducted at the Goddard Space Flight Center. This test is unique in the sense that the same group of cells in the cell pack are being tested under various conditions of charge, discharge, temperature and voltage limit. The cells are cycled under one set of conditions (8 cycles) to reach equilibrium, then returned to baseline cycling regime before being subjected to a new set of environmental test conditions. The data for the study is taken from the eighth-cycle at each test condition.

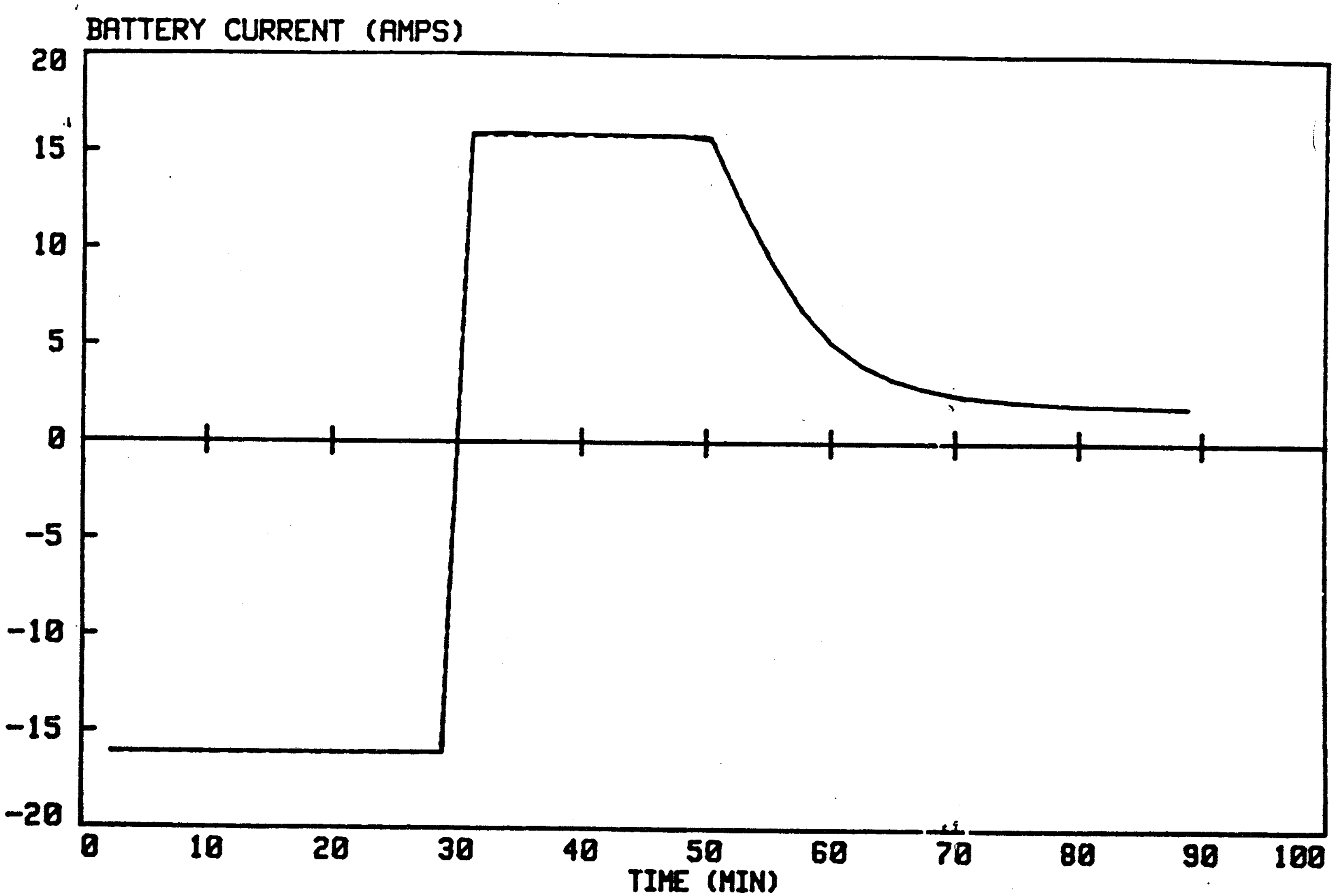

Figure 1. Typical Current Profile

Figure 2 shows the test matrix for this study. There are essentially four conditions of discharge (.1C, .2C, .5C, and .8C), four conditions of charge (.2C, .5C, .6C and .8C), three temperatures (0°C, 10°C, and 20°C), and three voltage limits (VL 3, VL 5, and VL 7). The box in the figure shows the voltage at which the cell goes into taper for the three voltage limits at the three different test temperatures. Depth of discharge (DoD) could also be considered a test variable which is totally dependent on discharge current since discharge time is always 30 minutes. DoD ranges from 5% at the .1C discharge rate to 40% for the .8C discharge rate.

The 4 x 4 matrix in Figure 2 shows all of the combinations of charge and discharge which were employed in the test. Note, however, that three of the combinations were not tested (see boxes which are crossed out) because of extreme conditions. The charge rates were not sufficient to return the amount of charge required at the 25% and 40% DoD levels.

Each box in the 4 x 4 matrix represents an additional set of nine test conditions which are the various combinations of the temperature (three levels) and voltage limit (three levels). All told, there are 117 possible combinations of these variables in the testing scheme.

Time to Voltage Limit

The various combinations of charge and discharge rates had a significant impact on the time in which the cell remained in constant current charge before hitting the voltage limit. Figure 3 shows the time to voltage limit for the various combinations of charge and discharge rate. The very short times (less than 2 minutes) are seen to occur at the high charge rates for those cells which were subjected to lower discharge rate, and, therefore, lower depths of discharge (5% and 10%).

The longer times to voltage limit occur for various combinations of charge rate and discharge rate along a diagonal from the lower left hand side of each matrix to the upper right hand side. A maximum of 30 minutes, one half of the total time in discharge, is observed at VL 7 for cells at .8C discharge and .6C charge.

The shape of the current taper, as will be seen later in the report, is roughly correlated to the amount of time that the cell stays in constant current charge before hitting the taper limit. The drop-off in current is much sharper for the cells which have a short time to voltage limit, especially those at less than 2 minutes. The parameters of the fit equations for this portion of the charge curve are highly dependent on this initial rate of drop in the current after hitting the voltage limit.

Percent of Charge Returned During the Charge Cycle

Figure 4 shows the percentage of charge returned to the cells during the charge portion of the cycle. As with the time to voltage limit, there is a significant variation in the percent return based on the

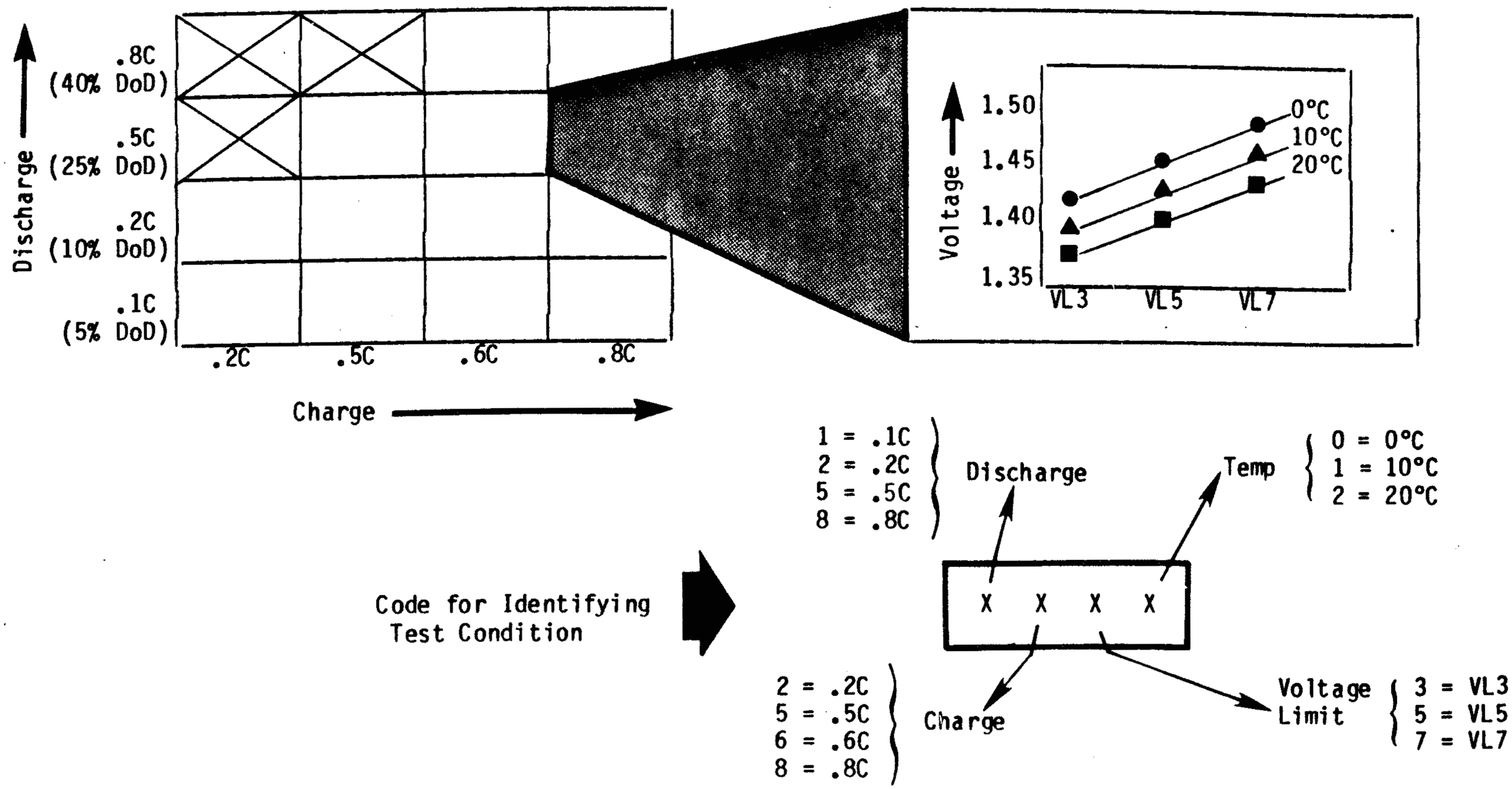

Figure 2. Text Matrix

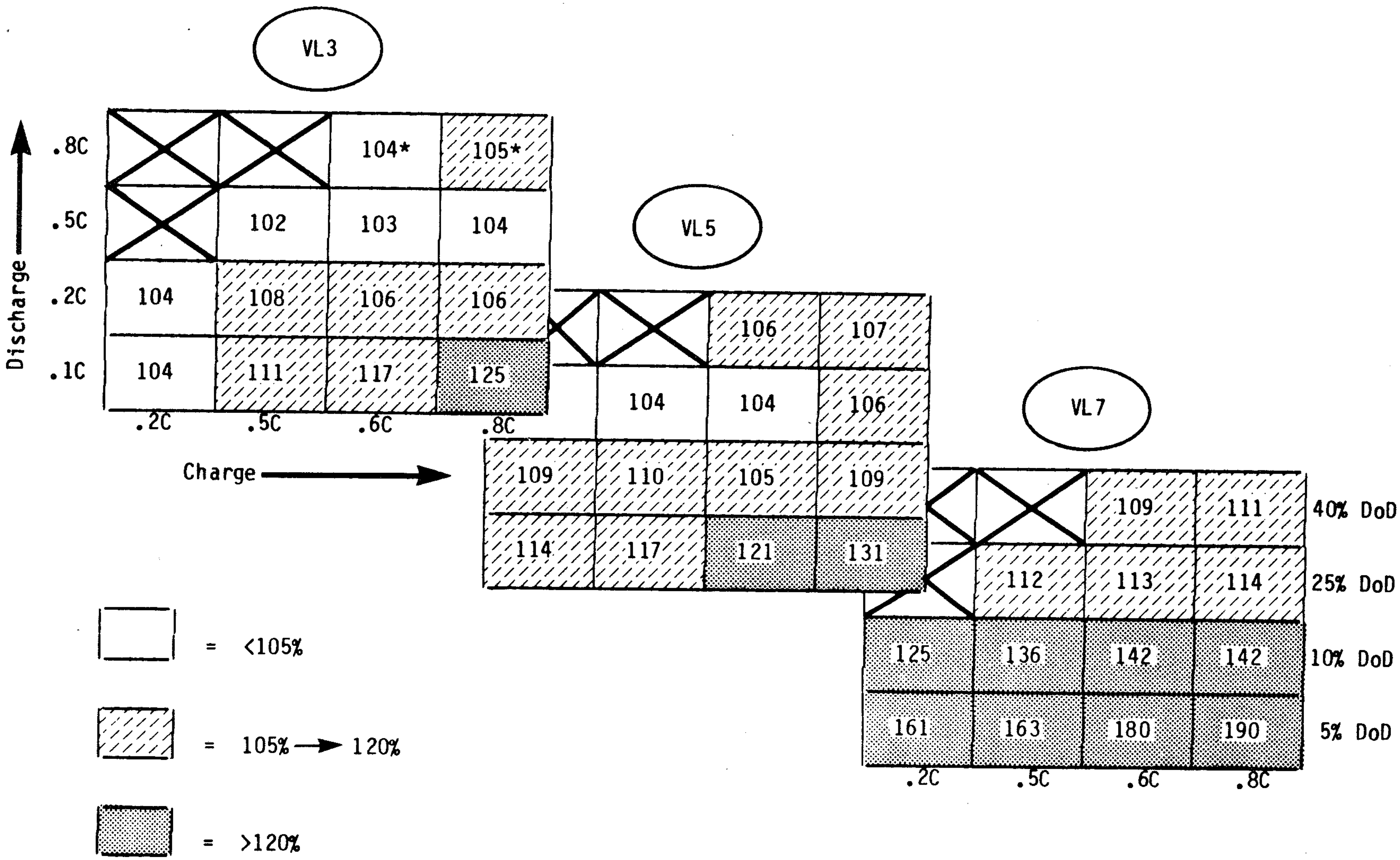

Figure 3. Percent Returned During Charge (Advantage of All Temperatures)

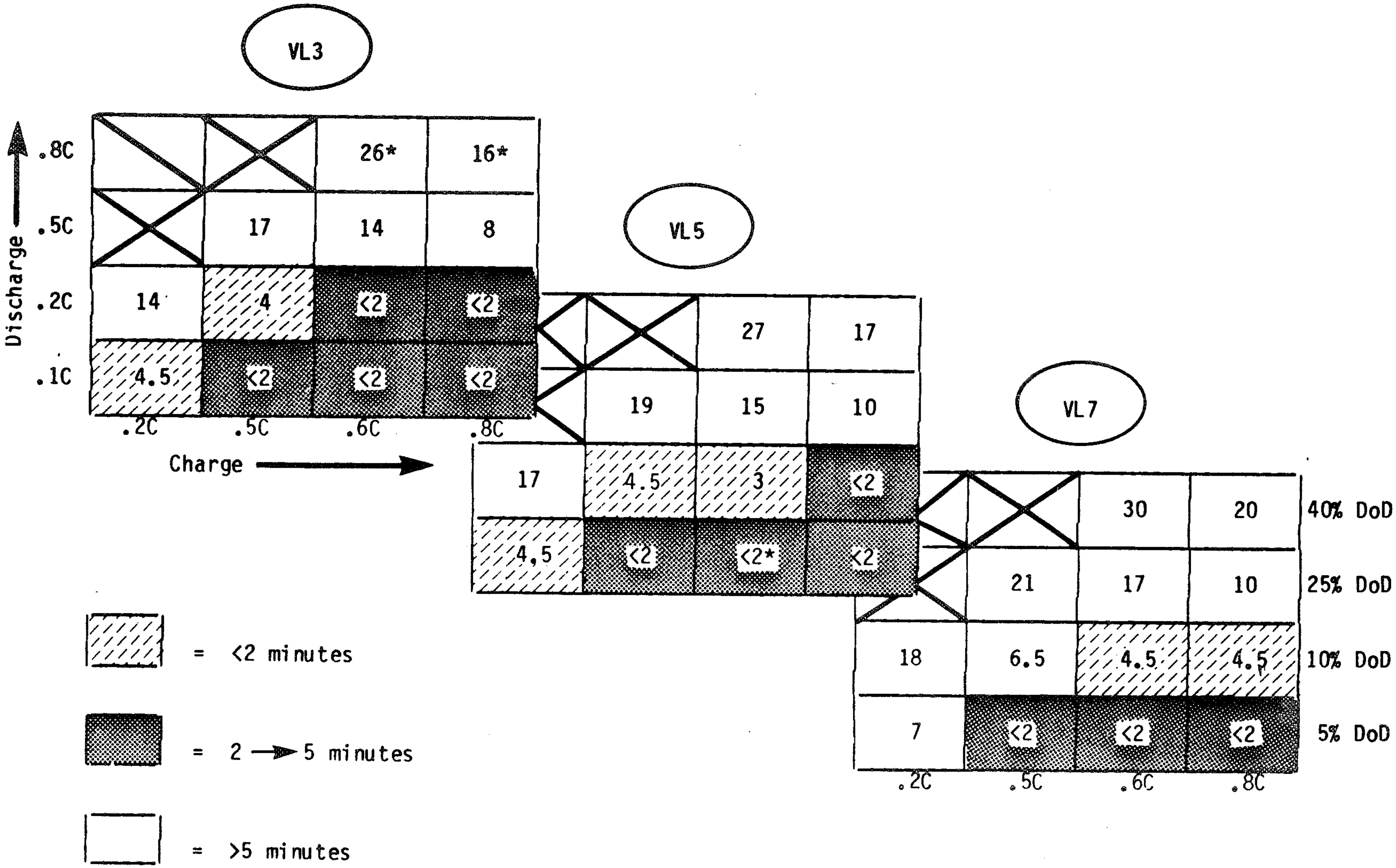

Figure 4. Time to Voltage Limit (Averages of All Temperatures)

differences in charge and discharge rates, and on the variation in voltage limit. For the matrices shown in Figure 4, the temperatures are averaged together for each specific test condition. At VL 3, the percent returned is less than 110% for most of the conditions. At VL 7, on the other hand, most of the test conditions show a percent return greater than 110%, with over half of the test conditions showing greater than 125%, at the lower DoDs. Again, there is a rough correlation between the time that the cells remain in charge before hitting the voltage limit and the percentage of charge returned to the cell. The cells at the high charge rates but modest DoDs show very short times to voltage limit, but, also, much higher percentages of return.

Fitting the Taper Portion of the Charge

It was noted by trial and error that the slope of the current in taper plotted against the current yielded a curve which was approximately linear. It was also noted that the linearity was improved if a constant term was subtracted from the current. The constant term in many cases was close to the current at the end of taper charge, suggesting that the constant was related to the value of current to which the experiment current was approaching as a limit. This observation suggested that the phenomenon could be fit by a first order differential equation of the form:

$$\frac{dI}{dt} = k\,(I - a)$$

where I is current and k and a are constants. The solution to this equation was then used in a non-linear regression fitting program to calculate fit parameters in the equation: $I = (1)\exp^{(2)t} + (3)$ where I is current and (1), (2), and (3) are the fit parameters.

This equation worked quite well in fitting data such as that shown in Figure 5a, the taper charge of the cell at .5C discharge, .5C charge, VL 7 and 0°C (Fit 5570 -- See Figure 1 for code). Parameter 3, as suggested above, is close to the point where the current is leveling off, and Parameter 3 plus Parameter 1 are approximately equal to 25 amps, the current for the cell as it hit the voltage limit.

The three parameter fit equation did not work so well, however, for curves where there was a sharp drop in current immediately after the cell hit the voltage limit. This is readily seen in Figure 6a which is a three parameter fit of the cell at .1C discharge, .5C charge, VL 3, and 20°C. As was mentioned earlier, many of the test conditions where the cell hits the voltage limit quickly demonstrated this sharp initial drop in current. It was also noted that a plot of dI/dt vs I for test conditions like those shown in Figure 6 would yield not one but two linear portions with different slopes. This suggested that there were two first order effects being demonstrated during taper charge which required a more complex equation in order to adequately fit the data.

Four and Five Parameter Fit Equations

In order to accommodate the two effects mentioned above, a four parameter equation with two exponential terms and a five parameter equation with two exponential terms and a constant were used with the non-linear regression program to fit the data. These equations are shown below:

$$\text{Four Parameter Equation:} \quad I = (1)\exp^{(2)t} + (3)\exp^{(4)t}$$

$$\text{Five Parameter Equation:} \quad I = (1)\exp^{(2)t} + (3)\exp^{(4)t} + (5)$$

where (1), (2), (3), (4), and (5) are the fit parameters.

These four and five parameter equations were able to fit both the sets of data shown in Figures 5 and 6 more accurately than the three parameter fit equation. Figures 5b and 6b are the fits obtained with the four parameter equation and Figures 5c and 6c are the fits obtained with the five parameter equation. The sum of squared residuals decreases (from 1.0 to 0.15 to 0.026) for fits 5a, 5b and 5c, suggesting that there is an increasing accuracy with the four and five parameter equations.

Tables 1 and 2 show parameters for the three fit and the five fit equations, for many of the test conditions in the test matrix. There are some trends evident as one reads from left to right (increasing VL) for any particular charge/discharge combination, or reads from top to bottom (increasing temperature). There is, however, no strong pattern as yet which suggests strong correlation with the environmental parameters.

Other Characteristics of the Taper Charge

Figure 7 shows an interesting correlation between the current at the end of charge and the percent of charge returned. When one plots one against the other, the data points cluster along straight lines, with the slope of the lines increasing the decreasing discharge rate or DoD. The currents at the end of charge are significantly higher for the cells with 40% DoD (4.0 to 5.0 amps) than for the cells at 5% DoD which range from 2.0 to 3.0 amps. What is more striking, however, are the differences in percent return. The 5% DoD cells range from 130% to over 200% charge return whereas the 40% DoD cells cluster around 110% charge return. It is interesting to note that the ln slope of the lines drawn through the data points when plotted against DoD (ln scale) yields a straight line as shown in the insert to Figure 7.

Another interesting effect is shown in Figure 8 where percent recharge is shown vs. temperature for each combination of charge and discharge. For the higher DoDs there is little or no temperature effect (40% and 25% DoD). The effect of temperature is more dramatic for the 5% and 10% DoD test conditions with increasing impact from left to right with increasing charge rate. The greatest difference, for example, is shown for the .1C discharge rate/.8C charge rate combination where percent recharge ranges from 140 to 200%.

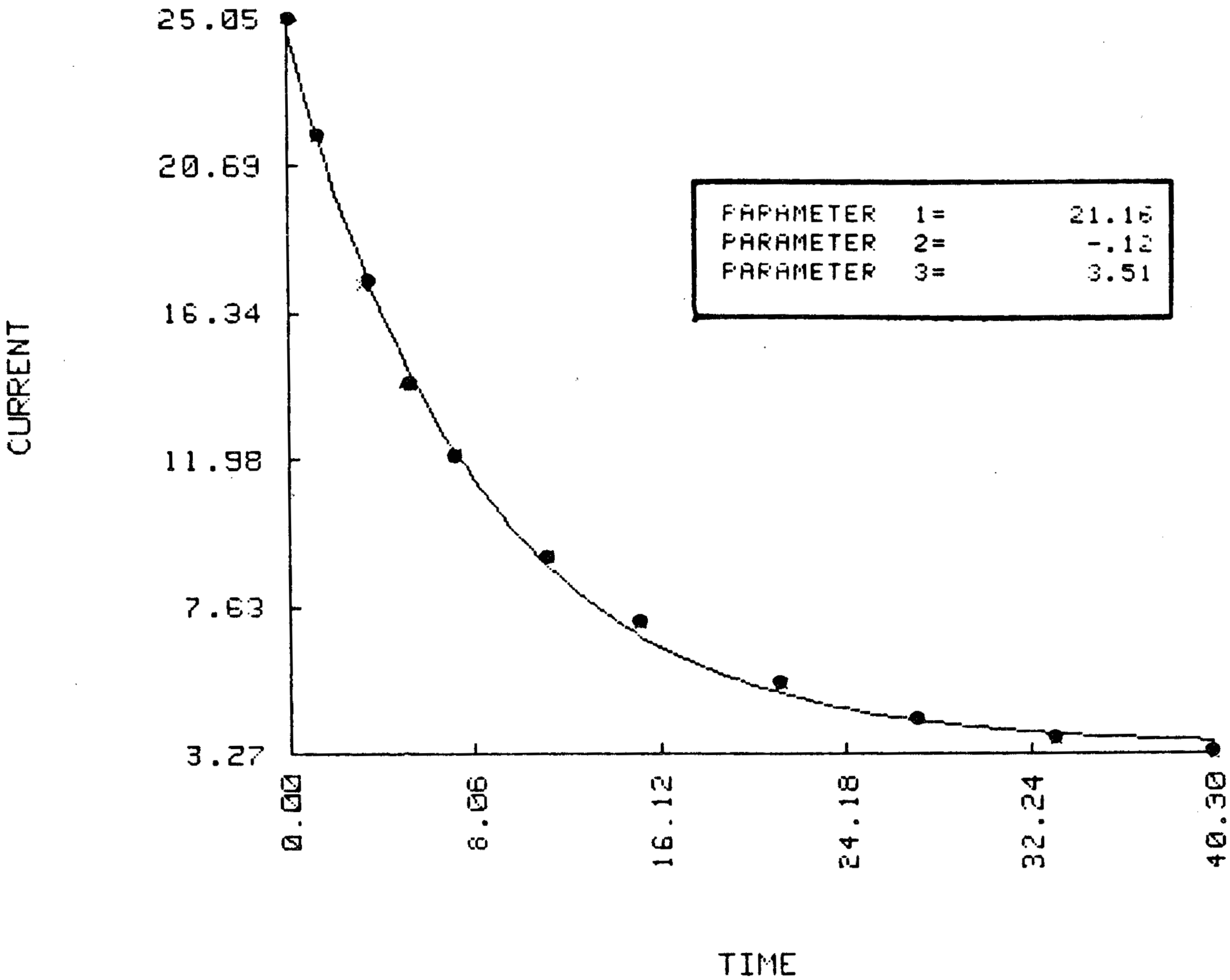

Figure 5a. Three Parameter Fit for .5C Discharge, .5C Charge, VL 7 and 0° C

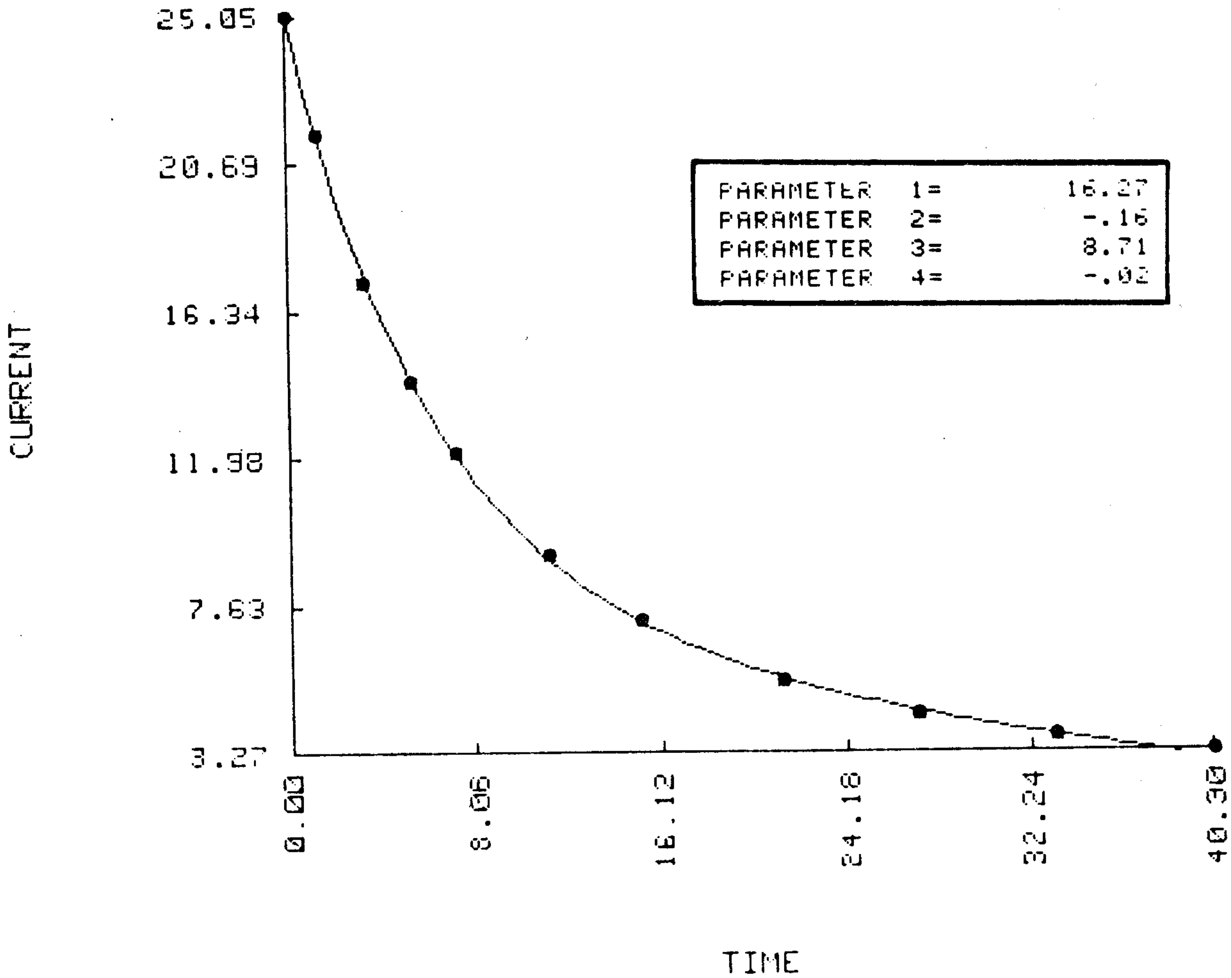

Figure 5b. Four Parameter Fit for .5C Discharge, .5C Charge, VL 7 and 0° C

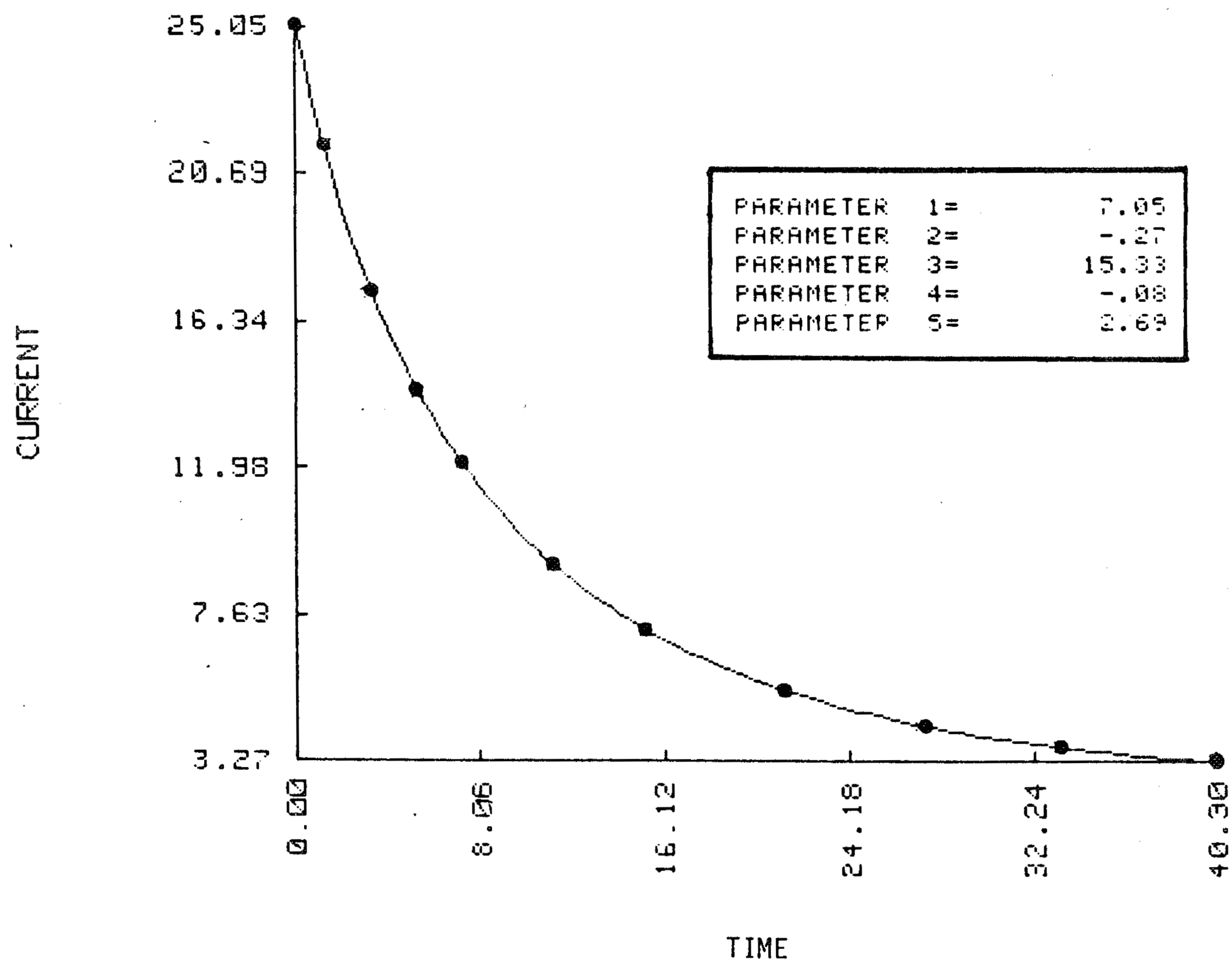

Figure 5c. Five Parameter Fit for .5C Discharge, .5C Charge, VL 7 and 0°C

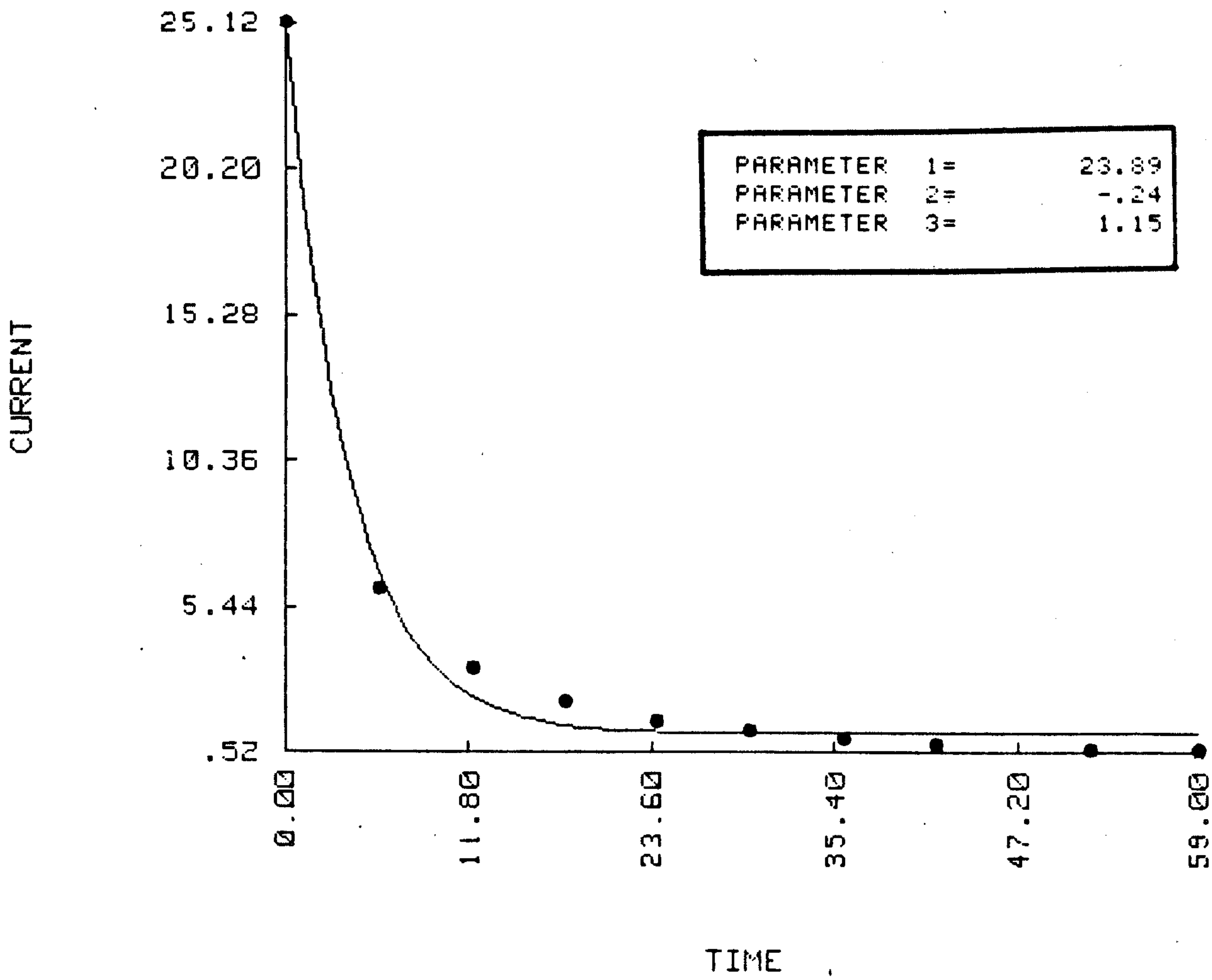

Figure 6a. Three Parameter Fit for .1C Discharge, .5C Charge, VL 3 and 20° C

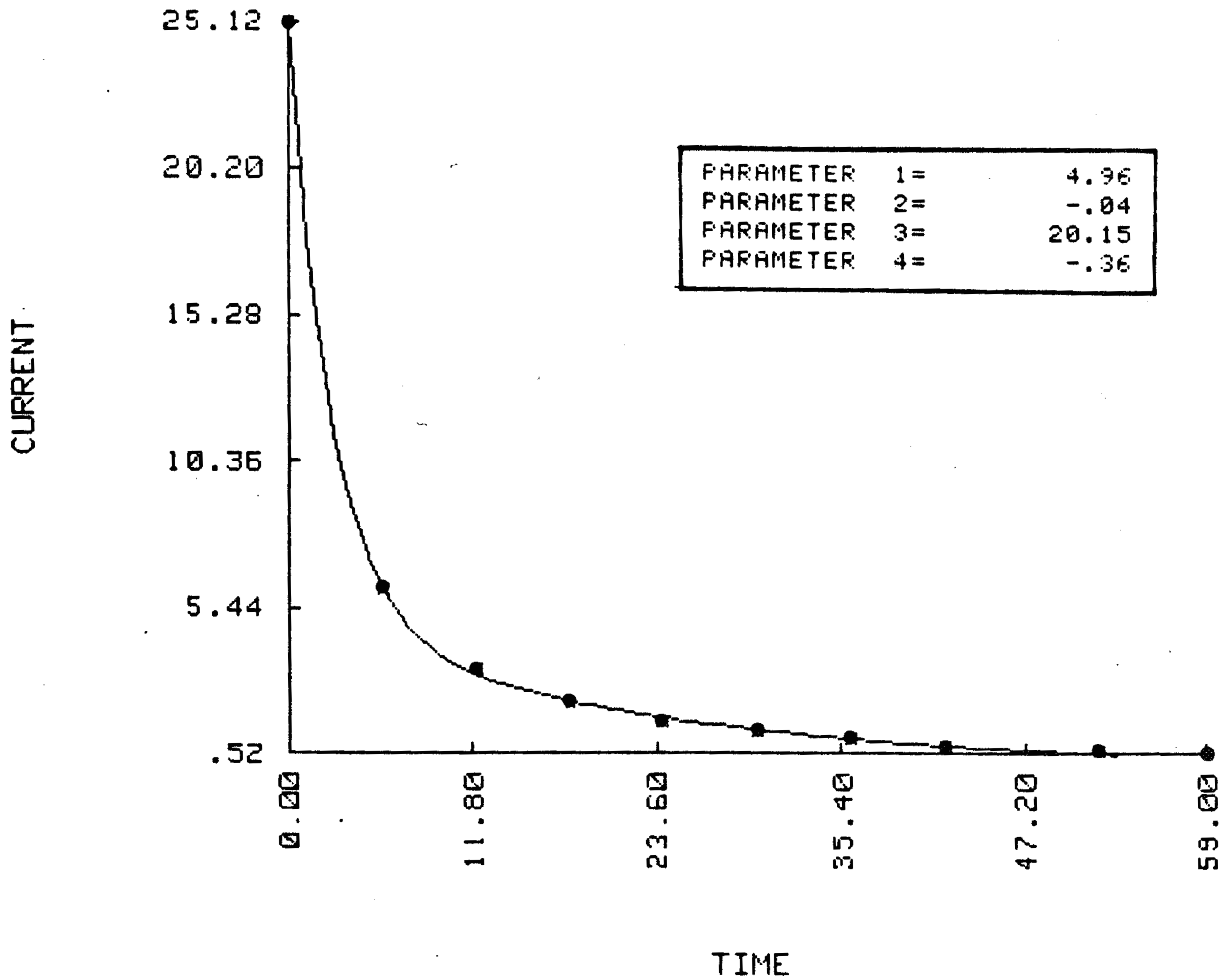

Figure 6b. Four Parameter Fit for .1C Discharge, .5C Charge, VL 3 and 20° C

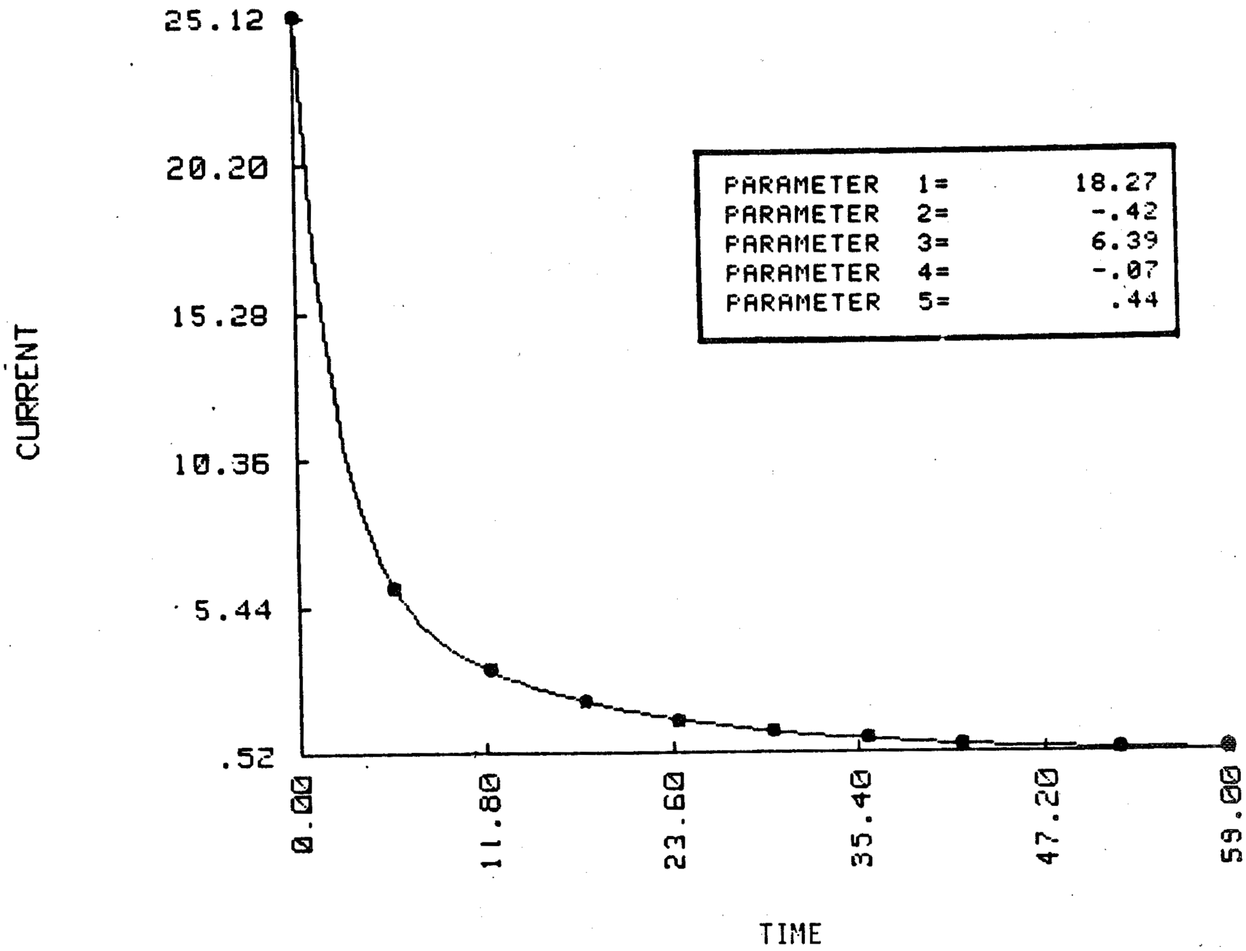

Figure 6c. Five Parameter Fit for .1C Discharge, .5C Charge, VL 3 and 20° C

Table 1. PARAMETERS FOR 3 PARAMETER FIT EQUATION

TEST CONDITION	1230	1250	1270	1830	1850	1870
Parameter 1	9.07	9.63	8.04	37.7	38.3	36.5
Parameter 2	-.137	-.103	.144	-.85	- .695	-.69
Parameter 3	1.117	.895	1.96	2.14	2.18	3.16
TEST CONDITION		1251	1271	1831	1851	1871
Parameter 1		9.087	7.66	37.9	37.08	36.36
Parameter 2		-.103	-.127	- .908	- .778	- .591
Parameter 3		.982	2.42	1.85	2.68	3.20
TEST CONDITION	1232		1272		1852	1872
Parameter 1	8.75		7.5		39.16	36.07
Parameter 2	-.134		-.173		- .489	- .41
Parameter 3	1.149		2.64		1.82	4.01
TEST CONDITION	5532	5552	5572	8830	8850	8870
Parameter 1	20.85	23.00	20.82	39.93	36.62	33.84
Parameter 2	- .096	- .116	- .132	- .066	- .088	- .12
Parameter 3	2.75	1.55	4.016	1.24	3.608	4.20
TEST CONDITION				8831	8851	8871
Parameter 1				38.68	36.7	39.09
Parameter 2				- .062	- .089	- .084
Parameter 3				1.03	3.67	3.51
TEST CONDITION				8832	8852	8872
Parameter 1				34.97	39.901	39.19
Parameter 2				- .068	- .066	- .085
Parameter 3				5.32	1.80	2.81

Table 2. PARAMETERS FOR 5 PARAMETER FIT EQUATION

TEST CONDITION	5530	5550	5570
Parameter 1	9.15	7.6	7.06
Parameter 2	- .02	- .27	- .27
Parameter 3	16.95	15.7	15.34
Parameter 4	- .17	- .056	- .08
Parameter 5	- .98	1.12	2.69
TEST CONDITION	**5531**	**5551**	**5571**
Parameter 1	9.7	6.72	7.38
Parameter 2	- .013	- .28	- .11
Parameter 3	18.1	17.7	17.14
Parameter 4	- .16	- .05	- .11
Parameter 5	-2.84	.651	3.47
TEST CONDITION	**5532**	**5552**	**5572**
Parameter 1	9.8	5.24	5.33
Parameter 2	- .04	- .062	- .14
Parameter 3	13.5	18.51	15.019
Parameter 4	- .14	- .134	- .148
Parameter 5	1.56	1.15	4.2

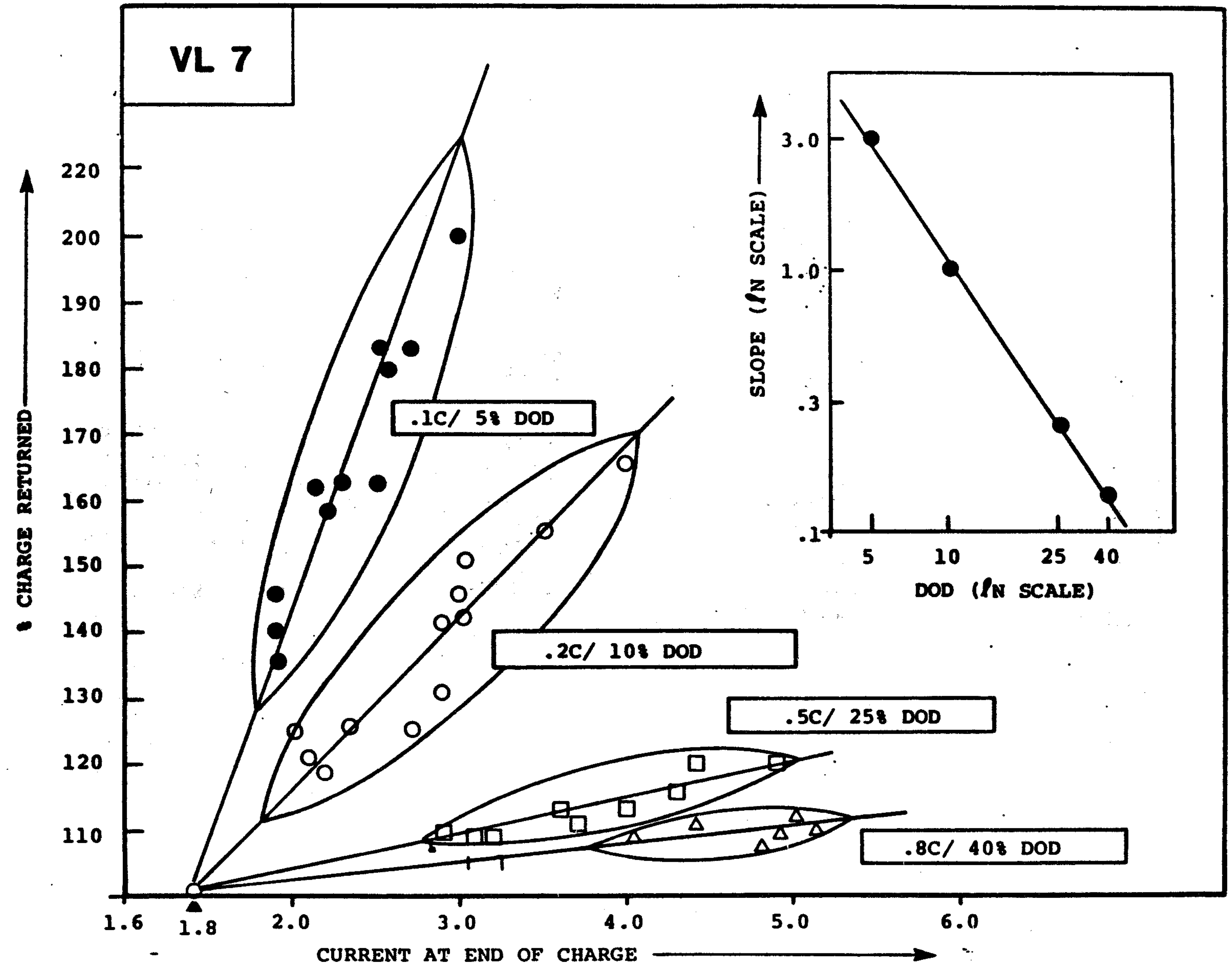

Figure 7. Correlation of Charge Returned with End of Charge Current

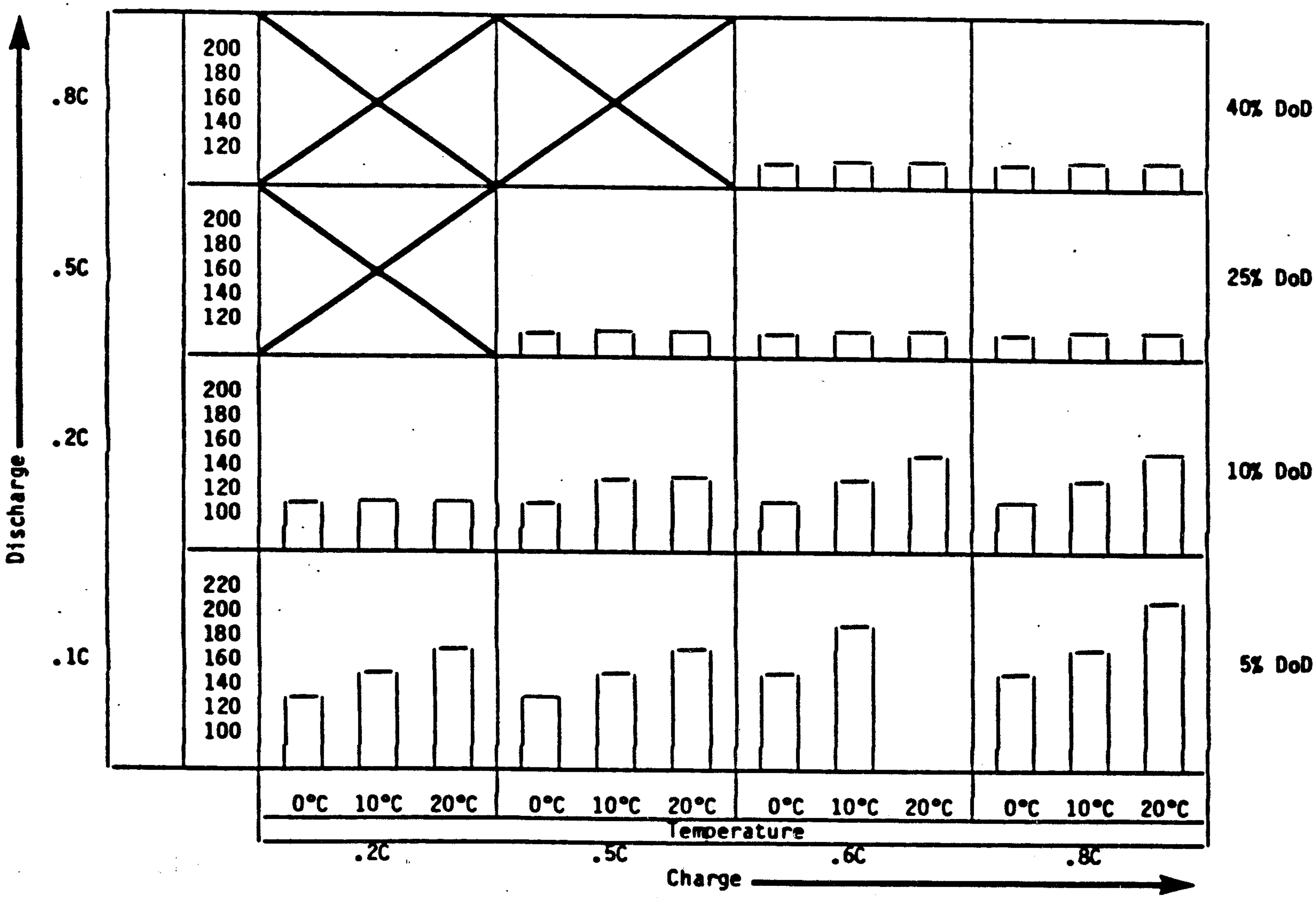

Figure 8. PERCENT RECHARGE VS TEMPERATURE FOR VL 7 TEST CONDITION

Summary and Conclusions

This study has shown that taper charge current profiles of all the environmental test conditions used in this test can be fit very accurately by a five parameter fit equation (Figures 5c and 6c). The study did not progress to the point where these parameters could then be correlated to the environmental test conditions. However, there were certain trends evident which could be pursued in future modeling efforts.

1. Percent charge returned and time to voltage limit show a distinct character which can be related to charge/discharge rate combinations and the level of voltage limit (Figures 3 and 4).

2. Cells with very modest DoDs (5% and 10%) and higher rates of charge generally showed a very short time to voltage limit, higher percentage of charge returned, a very steep drop in current after hitting the voltage limit and more modest currents toward the end of charge.

3. Cells with higher DoDs and charge rates showed, as might be anticipated, longer time to the voltage limit, lower percent of charge returned, a more modest rate of drop in current after VL, and greater currents at the end of charge.

4. Percent charge return, the relationship between DoD and current at the end of charge, can be easily modeled (Figure 7).

5. The effect of temperature is observed more at the lower discharge rates (and DoDs) and increases with increasing charge rate (Figure 8)

Future work should concentrate on a more comprehensive multiple correlation of test conditions (discharge rate (DoD), charge rate, voltage limit and temperature) with the effects of these conditions on battery performance (percent charge returned, charge efficiency, end of charge current, and current taper profile).