

A COMPARISON OF SMOOTH SPECIMEN  
AND ANALYTICAL SIMULATION TECHNIQUES  
FOR NOTCHED MEMBERS AT ELEVATED TEMPERATURES

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Experimental strain measurements have been made at the highly strained regions on notched plate specimens that were made of Hastelloy X. Tests were performed at temperatures up to 1,600°F. Variable load patterns were chosen so as to produce plastic and creep strains. Where appropriate, notch root stresses were experimentally estimated by subjecting a smooth specimen to the measured notch root strains. The results of three analysis techniques are presented and compared to the experimental data. The most accurate results were obtained from an analysis procedure that used a smooth specimen and the Neuber relation to simulate the notch root stress-strain response. When a generalized constitutive relation was used with the Neuber relation, good results were also obtained, however, these results were not as accurate as those obtained when the smooth specimen was used directly. Finally, a general finite element program, ANSYS, was used which resulted in acceptable solutions, but, these were the least accurate predictions.

### INTRODUCTION

A variety of methods are available for the analysis of structures and components that are subjected to variable loads that result in inelastic strains. These analysis techniques usually can accommodate inelastic strains produced by both time-dependent creep and time-independent plastic strain. The finite element method is perhaps the most popular technique for these problems. However, other methods exist that show potential for much more economical analyses.

Regardless of the analysis procedure employed, an accurate set of constitutive relationships are required. If the uniaxial stress-strain response of the material is not adequately described, an analysis of any notch geometry will not be successful. Also, with all the possible variables associated with elevated temperature cyclic behavior, it is necessary to experimentally verify any analysis technique on simple notched laboratory specimens before attempting to analyze a complicated component such as a gas turbine engine.

Experimental data on smooth specimens and center notched plates have been generated. Smooth specimen data were generated at 70°, 1,200° and 1,600°F. Notched specimen data include temperatures of 70°, 1,200° and 1,550°F. All of

these data were generated for the purpose of establishing experimental evaluation criteria for constitutive models of time-dependent cyclic plasticity.

For comparison purposes, three analysis techniques were compared to some of these data. References 1-3 describe the analysis techniques and experimental procedures in detail. This paper presents a short summary of the three techniques and several examples of experimental versus analysis predictions.

#### MATERIAL, SPECIMENS AND EXPERIMENTAL TECHNIQUE

All tests were performed on specimens machined from Hastelloy X, a nickel-based superalloy used in components requiring oxidation resistance. Smooth specimens used for room temperature testing were machined with straight gage sections; specimens tested at elevated temperatures contained hour glass shaped gaged sections. For these high temperatures tests, diametral strain and axial stress were converted to axial strain. Elevated temperatures were produced with an induction furnace.

Notch specimen data that are presented in this paper were produced on thin plates with a notch located at the center of the plate. This notch was a circular hole with a theoretical stress concentration factor of 2.37 based on net section nominal stress. Notch root strains were determined with an interferometric technique that is described in Ref. 1 and 4. With this technique, normal strains were measured over very short gage lengths. The physical part of the gage consisted of indentations on the flat surface of the specimen. These indentations were pyramidal in shape with inclined sides tilted 45° to a normal of the surface. The indentations used for this experiment were placed 100 microns apart and were 25 microns square. They were placed 50 microns from the edge of the notch.

A He-Ne laser was used to simultaneously illuminate both indentations. Due to the coherent and monochromatic nature of this light, two interference fringe patterns resulted that were 90° relative to each other and 45° relative to the laser light. Movement of the indentations resulted in proportional movement of the fringe patterns. Averaging the movement of both fringe patterns eliminated rigid body motion. By monitoring the motion of these fringe patterns, strain could be determined. The fringe patterns were electronically sensed and the analog signal of relative light intensity was relayed to a minicomputer system. Final output of this system was an analog equivalent of strain that ranged from 0 to 10 volts.

#### SMOOTH SPECIMEN SIMULATION

The most direct approach to determine uniaxial constitutive behavior, is to directly control a smooth specimen so as to produce the required stress-strain combination that is dictated by a mechanics analysis of the notch geometry. The Neuber relation is the result of such an analysis that has been extensively used for room temperature fatigue life predictions. For cyclic loading this relation is written as:

$$(\Delta\sigma)(\Delta\varepsilon) = (K'_t)^2(\Delta S)(\Delta e) \quad (1)$$

where:  $\Delta\sigma$  and  $\Delta\epsilon$  are the notch root stress and strain ranges, respectively;  $\Delta S$  and  $\Delta e$  are the remote stress and strain ranges, respectively; and  $K_t$  is the experimentally determined elastic stress concentration factor (the  $\hat{\phantom{x}}$  indicates an experimental value as opposed to a calculated value,  $K_t$ ).

Equation (1) by itself is indeterminate. Knowing the remote stress or strain range leaves three unknowns. For this study it was assumed that smooth specimens could be used to supply the needed stress-strain (constitutive behavior) at both remote and local regions. Notched specimens were subjected to controlled loading rates and peaks. Remote strains were measured with the ISG. Smooth specimens were subjected to the same strain patterns that were recorded from the ISG (the same strain rate was also maintained). The remote stress and strain versus time plots were multiplied by  $(k_t)^2$  so that  $(K_t)^2(\Delta S)(\Delta e)$ , which is the right side of Eqn. (1) can be determined as a function of time. A smooth specimen was then controlled so that the product of stress and strain,  $(\Delta\sigma)(\Delta\epsilon)$ , would follow the pattern predicted by the Neuber relation, Eqn. 1. Figures 1 and 2 show the stress-strain behavior as predicted by the Neuber relation versus the experimentally determined notch root stress-strain simulation. Notch root stresses were simulated by subjecting a smooth specimen to the same strains as measured with the ISG.

Room temperature results are shown in Fig. 1. Four load levels were used for this part of the program.

<u>Level #</u>	<u>Load (KN)</u>
1	±14.0
2	±14.5
3	±15.5
4	±16.0

All these data were generated with the material in the stable condition. As can be seen, the agreement is excellent. Similar elevated temperature data at 1,200°F are shown in Fig. 2. Two load levels are shown.

<u>Level #</u>	<u>Load (KN)</u>
1	±10.5
2	±11.3

A 100 sec. hold time in both tension and compression was used for the elevated temperature tests. At both load levels the direct ISG-stress simulation data did not show stress relaxation from the hold periods, whereas the Neuber prediction showed a pronounced effect. The actual difference in the general trend of the stress-strain response would result in significant errors in the peak values, which are often used for damage analyses.

## MODEL OF UNIAXIAL BEHAVIOR

The direct use of smooth specimens for determining constitutive behavior is not practical for most design applications. An accurate mathematical model of a materials behavior that can be used with mechanics analyses would be beneficial. In an attempt to satisfy this need, a new constitutive modeling technique was developed that is capable of predicting typical uniaxial materials behavior at room and elevated temperatures. Simulation of the time-independent phenomena of cyclic hardening or softening, cyclic relaxation of mean stress and history dependent memory, and the time-dependent behavior of creep and stress relaxation was accomplished. This constitutive model is based on a generalized analysis of any configuration of classical rheological model elements and special purpose elements that were developed specifically for this constitutive modeling technique.

The modeling technique provides for the use of classical elements such as elastic springs, viscous dampers and frictional sliders. Special elements to simulate cyclic hardening and relaxation of mean stress were also added. All these elements could be readily arranged in any manner to predict the stress-strain response of materials under complex loading. The theory supporting this technique is based on the ability to formulate matrix representations of the model parameters so as to provide a set of equations that may be solved numerically to determine the model response. For the analysis of notched members, a numerical technique was created to expand the Neuber relation with the constitutive model to include time-dependent phenomena. This technique was used to form a specific constitutive model that was constructed from the material properties of Hastelloy X.

For comparison purposes this model was used to predict the response of a uniaxial specimen that was subjected to a complicated strain history at 1,600°F. Figure 3 shows this comparison. The maximum discrepancy between the two responses is about 5 ksi or 9% of the total stress range experienced. The major differences occur during the times of stress relaxation.

This constitutive model was combined with the Neuber relation to predict the notch root strain response of a circular notched specimen tested at 1,200°F. This specimen was subjected to completely reversed constant rate, cyclic loads with hold times at both the tension and compression peaks. Comparisons of the experimental and model prediction for this test is shown in Fig. 4. The results of the comparison are relatively good. The general form of the response was very close to the measured output. The model strain values were within 18% of the experimental values at all times.

## FINITE ELEMENT ANALYSIS

The previous two analysis techniques employed the Neuber relation to relate remote and local behavior. Although these analysis techniques are relatively economical, their ability to deal with complicated geometries, without any experimental data on the stress concentrations, is limited. The most popular and versatile method of stress analysis is the finite element method. This method was used in a straight forward manner to calculate the notch root strains for two notch geometries.

The finite element analysis of the experimental data that were generated in this study used a large scale general purpose program, ANSYS. This program was utilized on a Prime 750 computer system that is linked to Tektronix interactive graphics terminals.

For this study, 2-D models were created for elliptical and circular notched specimens. Because of symmetry, the models were reduced to quarter sections. These models consisted of approximately 100 elements.

ANSYS uses the initial stress method for plasticity effects. Yielding is governed by the von Mises yield criterion and multiaxial effects are based on Prandtl-Reuss flow equations. Plastic solutions are restricted to isotropic behavior. Bilinear kinematic hardening was found to best fit the available experimental data.

All materials behavior that were required for this program were obtained from uniaxial data. Only cyclic stable behavior was simulated under isothermal conditions. For the creep portion of the program, only secondary creep was accounted for even though ANSYS does allow for primary creep.

For notched members made of Hastelloy X, good correlation was obtained between the analysis and experimental data at room temperature. Figure 5 shows the notch root strains on an elliptical center notched plate as predicted by ANSYS and as measured by the ISG. This plate was subjected to a completely reversed, symmetric load pattern. This agreement is extremely good considering that the stress-strain behavior is simulated by only two straight line segments.

Figure 6 shows experimental data and predictions for a circular notch. This test was performed at 1,200°F. The load pattern was symmetric with hold times in both tension and compression. Correlation of experimental ISG results and analytical predictions are very poor compared to the room temperature results. At least a portion of this inaccuracy can be attributed to not including primary creep in the program.

## CONCLUSIONS

The results presented in this paper represent only a small fraction of the experimental data that are available and of the analyses that were performed. In general, all three analysis techniques produced reasonably accurate predictions for both smooth specimen stress-strain behavior and notch root response for center notched plates made of Hastelloy X. The easiest experimental data to simulate were those generated at both temperature extremes, 70° and 1,600°F, where creep either dominated the strain response or had relatively little effect.

## REFERENCES

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4. Sharpe, W.N., Jr., "Interferometric Surface Strain Measurement," International Journal of Nondestructive Testing, Vol. 3, 1981, pp. 59-76.

## ACKNOWLEDGEMENTS

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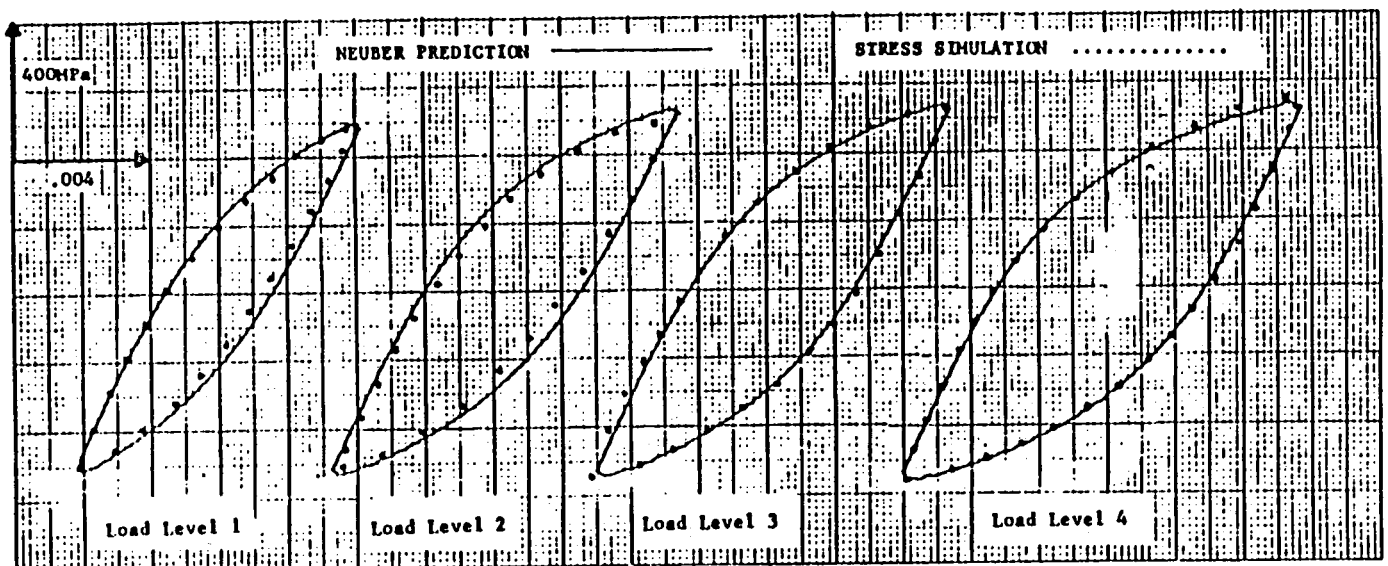
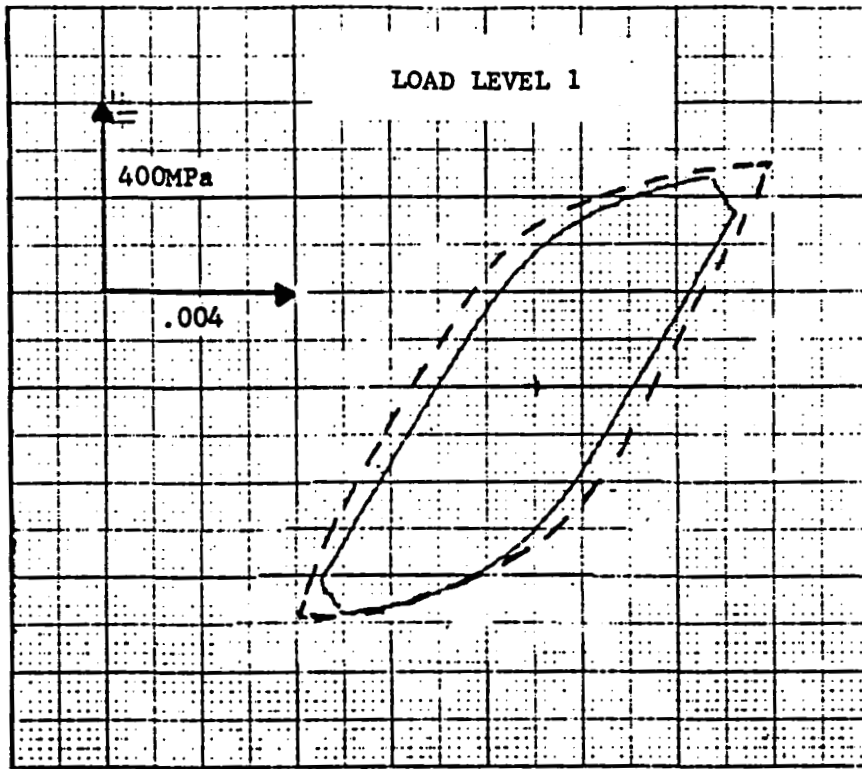


Figure 1. - Neuber prediction and stress simulation of stabilized local behavior.



NEUBER PREDICTION —————

STRESS SIMULATION - - - - -

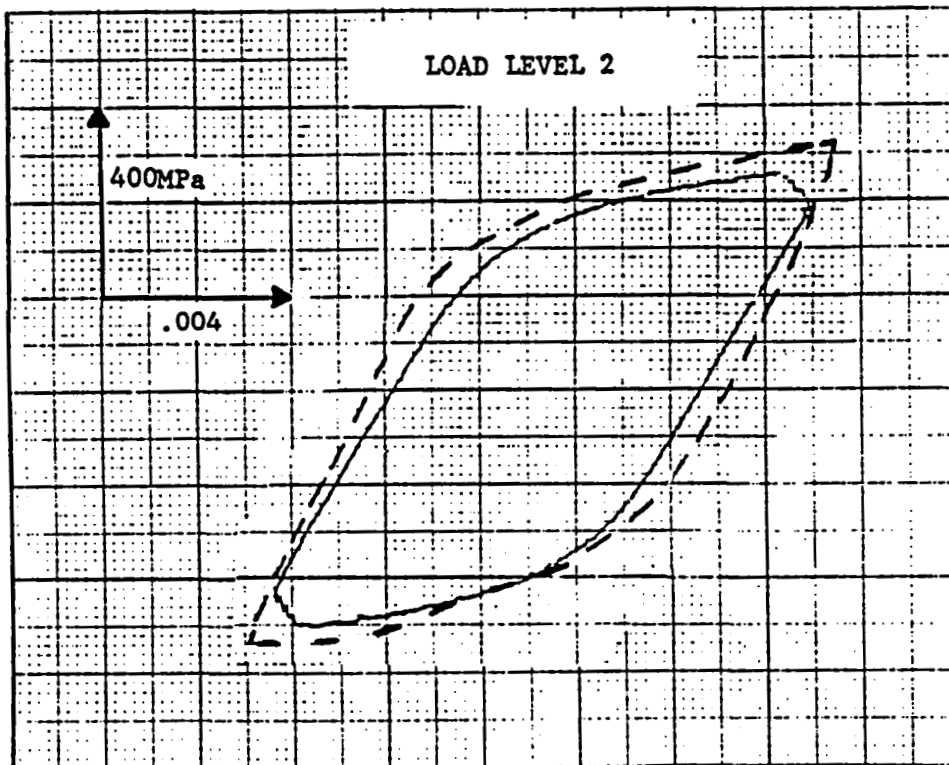


Figure 2. - Neuber prediction and stress simulation at 1200 °F.

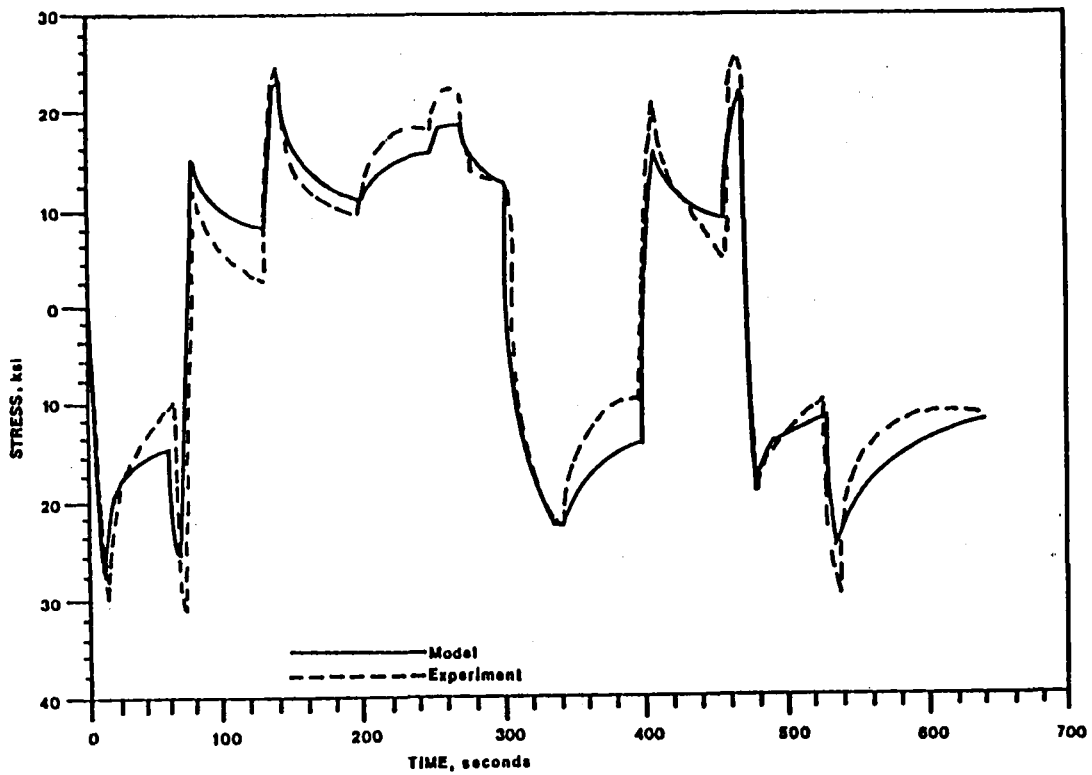


Figure 3. - Comparison of model and experimental response for 1600 °F test.

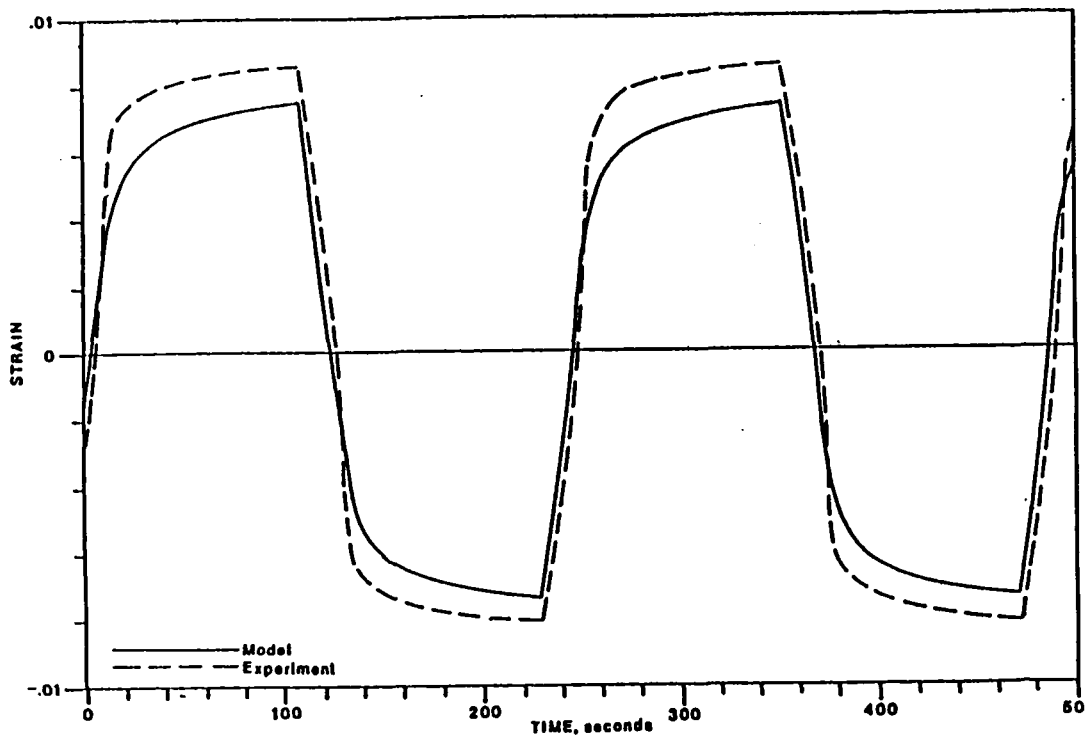


Figure 4. - Comparison of model and experimental notch root response for 1200 °F test.



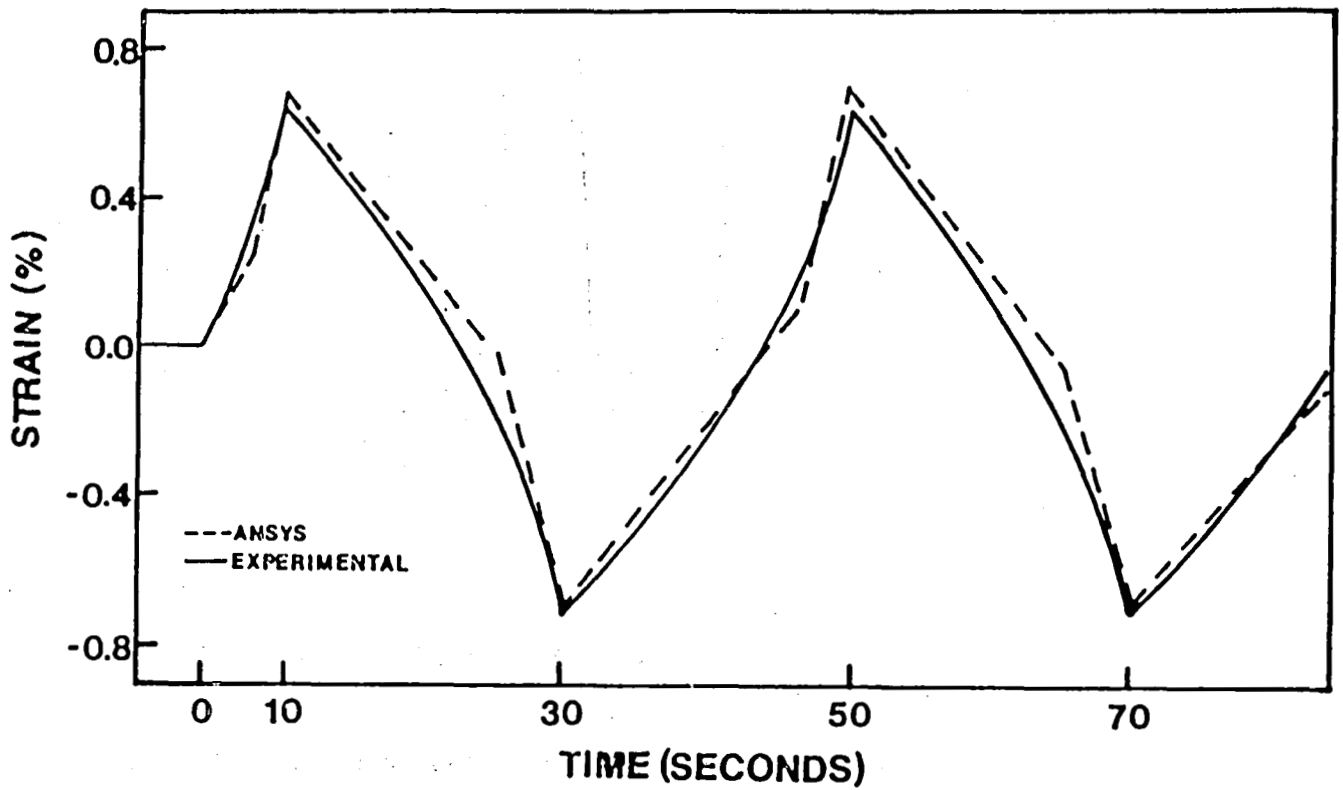


Figure 5. - Elliptical notch strain versus time at room temperature.

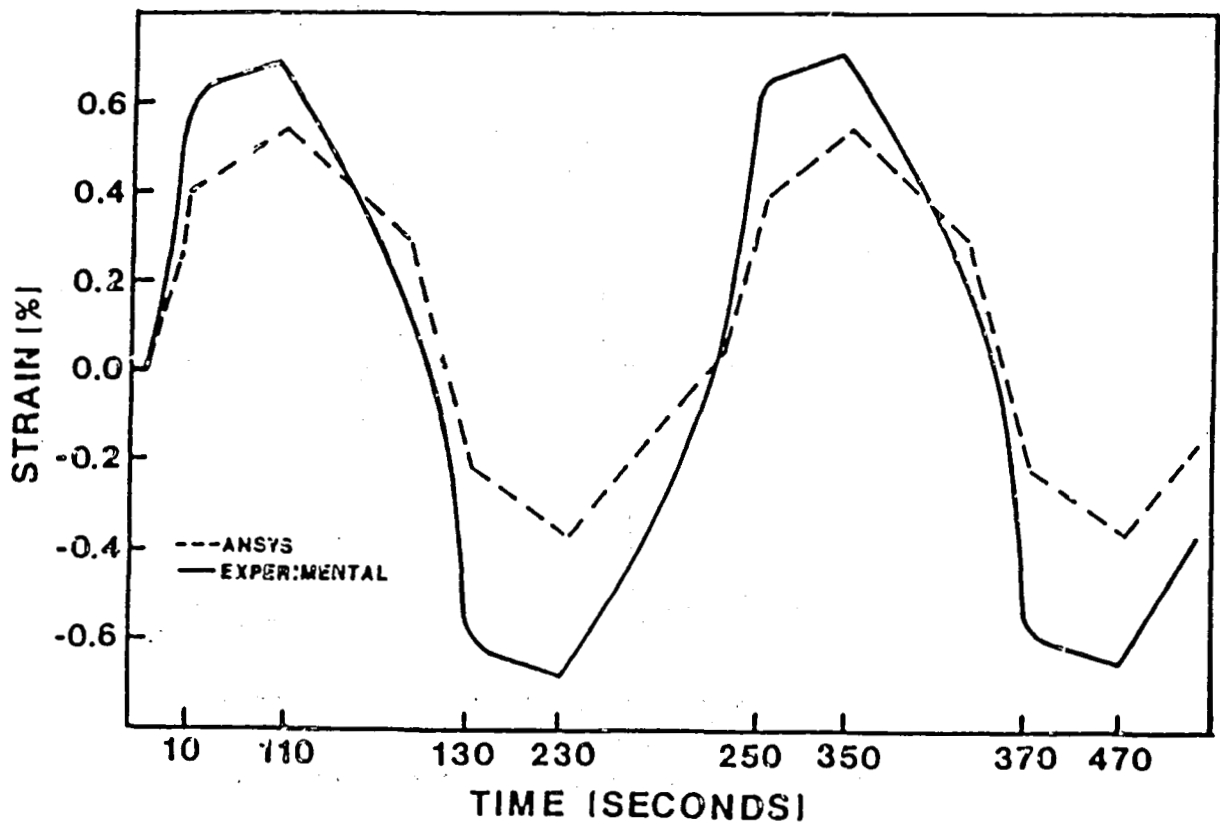


Figure 6. - Strain versus time of circular notched specimen at 1200 °F.