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> ( $\mathrm{MASA}-\mathrm{TM}-86511$ ) EDBBLE SPACE TELESCORE:
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# HUBBLE SPACE TELESCOPE - POINTING ERROR EFFECTS OF NONLINEAR BALL JOINTS 

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# HUBBLE SPACE TELESCOPE - POINTING ERROR EFFECTS OF NONLINEAR BALL JOINTS 

## INTRODUCTION AND OBJECTIVES

The extreme pointing accuracy of the Hubble Space Telescope (HST) requires the consideration of effects not normally included when performing a Pointing Control System (PCS) analysis. For this reason, an effort has been made to model those mechanisms which could possibly cause very small limit cycles, on the order of 1 milli-arc-sec. The method chosen to mount the Scientific Instrument (SI) Optical Benches on-voard the HST provide one such mechanism.

While the primary concern was for the HST, the program used for this analysis was written in modular form and can, therefore, be applied to other spacecraft. As shown in Figure 1, the mathematical model for the Optical Bench receives only the vehicle angular body rates and the spacecraft to inertial frame transformation. These variables are processed and returned to the main program as reactive torques affecting the attitude control system performance. It is assumed for purposes of this study that the spacecraft is operating under drag free conditions. However, an input for external forces (such as atmospheric resistance) has been provided to satisfy the more general situation of a vehicle experiencing and reacting to various external disturbances.


Figure 1. Optical Bench/Spacecraft simplified diagram.
There were three primary objectives to be accomplished by this study:

1) Define the nonlinear dynamics associated with the Sil Optical Benches mounted on fre? ball joints.
2) Develop a computer program describing these dynamics.
3) Combine this program with an existing HST simulation to determine the effects on vehicle pointing accuracy.

## EQUATIONS OF MOTION

The optical benches were assumed to be mounted as shown in Figure 2.


Figure 2. Optical Bench mounting arrangement.
The variables $\Delta C G, R_{1}, R_{2}, R_{V C G}$ are defined in the body-fixed frame $\left(V_{1}, V_{2}, V_{3}\right)$ to be

$$
\begin{align*}
& \Delta \mathrm{CG}=\left[\begin{array}{l}
\Delta \mathrm{CG}_{\mathrm{V} 1} \\
\Delta \mathrm{CG}_{\mathrm{V} 2} \\
\Delta \mathrm{CG}_{\mathrm{V} 3}
\end{array}\right] ; \quad \mathrm{R}_{\mathrm{VCG}}=\left[\begin{array}{l}
\mathrm{R}_{\mathrm{CGV} 1} \\
\mathrm{R}_{\mathrm{CGV} 2} \\
\mathrm{R}_{\mathrm{CGV} 3}
\end{array}\right]  \tag{1}\\
& \mathrm{R}_{1}=\left[\begin{array}{l}
\mathrm{R}_{1 \mathrm{~V} 1} \\
\mathrm{R}_{1 \mathrm{~V} 2} \\
\mathrm{R}_{1 \mathrm{~V} 3}
\end{array}\right] \quad ; \quad \mathrm{R}_{2}=\left[\begin{array}{l}
\mathrm{R}_{2 \mathrm{~V} 1} \\
\mathrm{R}_{2 \mathrm{~V} 2} \\
\mathrm{R}_{2 \mathrm{~V} 3}
\end{array}\right]
\end{align*}
$$

The vectors $\mathbf{R}_{1}$ and $\mathbf{R}_{2}$ represent the distance to the geometric center of the aft and forward cavities (sockets), respectively. The distance from the vehicle center of mass (CM) to each cavity center and the optical bench CM is depicted in Figure 3 and is expressed by equation (2).

$$
\begin{equation*}
\mathbf{R}_{\mathbf{F}}=\mathrm{R}_{2}-\Delta \mathrm{CG} \text { (Forward Cavity) } \tag{2}
\end{equation*}
$$

$$
\begin{align*}
& \mathrm{R}_{\mathrm{A}}=\mathrm{R}_{1}-\Delta \mathrm{CG} \text { (Aft Cavity) }  \tag{2}\\
& \mathrm{R}_{\mathrm{CG}}=\mathrm{R}_{\mathrm{VCG}}-\Delta \mathrm{CG} \text { (Optical Bench CM) }
\end{align*}
$$

Let the vehicle angular rates be represented by

$$
\omega_{\mathrm{V}}=\left[\begin{array}{c}
\omega_{\mathrm{V} 1}  \tag{3}\\
\omega_{\mathrm{V} 2} \\
\omega_{\mathrm{V} 3}
\end{array}\right] \quad(\mathrm{RAD} / \mathrm{SEC})
$$



Figure 3. Mass center and cavity center parameters.
In one computational period (specified by $\Delta \mathrm{tsec}$ ) these rates cause the center point of each cavity to sweep out an arc of length

$$
\begin{align*}
& \Delta S_{F}=R_{F} \omega_{V} \Delta t  \tag{4}\\
& \Delta S_{A}=R_{A} \omega_{V} \Delta t
\end{align*}
$$

Expressing equation (4) in matrix notation produces

$$
\begin{align*}
& \Delta S_{F}=\left[-\widetilde{R}_{F}\right]\left[\omega_{V}\right] \Delta t  \tag{5}\\
& \Delta S_{A}=\left[-\widetilde{R}_{A}\right]\left[\omega_{V}\right] \Delta t
\end{align*}
$$

where the tilde operation for any vector

$$
A=\left[\begin{array}{l}
\mathrm{X}  \tag{6}\\
\mathrm{Y} \\
\mathrm{Z}
\end{array}\right]
$$

is defined to be

$$
[\mathrm{A}]=\left[\begin{array}{ccc}
0 & -Z & \mathrm{Y}  \tag{7}\\
\mathrm{Z} & 0 & -X \\
-Y & \mathrm{~V} & 0
\end{array}\right]
$$

Applying the definition [equation (7)] to equation (5) gives

$$
\begin{align*}
& \Delta S_{F}=\left[\begin{array}{lll}
0 & R_{F V 3} & -R_{F V 2} \\
-R_{F V 3} & 0 & R_{F V 1} \\
R_{F V 2} & -R_{F V 1} & 0
\end{array}\right]\left[\begin{array}{c}
\omega_{V 1} \\
\omega_{V 2} \\
\omega_{V 3}
\end{array}\right] \Delta t  \tag{8}\\
& \Delta S_{A}=\left[\begin{array}{lll}
0 & R_{A V 3} & -R_{A V 2} \\
-R_{A V 3} & 0 & R_{A V 1} \\
R_{A V 2} & -R_{A V 1} & 0
\end{array}\right]\left[\begin{array}{l}
\omega_{V 1} \\
\omega_{V 2} \\
\omega_{V 3}
\end{array}\right] \Delta t
\end{align*}
$$

Let [EV] be the vehicle to inertial frame coordinate transformation. Then the arc lengths expressed in inertial space are

$$
\begin{align*}
& \Delta S_{F}=[E V]\left[-\widetilde{R}_{F}\right]\left[\omega_{V}\right] \Delta t \\
& \Delta S_{A}=[E V]\left[-\widetilde{R}_{A}\right]\left[\omega_{V}\right] \Delta t \tag{9}
\end{align*}
$$

The sum of the external and internal forces acting on the spacecraft are represented by

$$
F=\left[\begin{array}{l}
F_{V 1}  \tag{10}\\
F_{V 1} \\
F_{V 3}
\end{array}\right]
$$

The body fixed forces given by equation (10) produce vehicle accelerations in the inertial coordinate frame given by

$$
\begin{equation*}
A_{I}=[E V] \frac{F}{M_{V}} \tag{11}
\end{equation*}
$$

where $\mathrm{M}_{\mathrm{V}}$ is the weight of the vehicle expressed in kilograms. Therefore, the average inertial translational velocity of each cavity center during a single computational period is

$$
\begin{equation*}
V_{A V G}=V+\frac{A_{I}}{2} \Delta t \tag{12}
\end{equation*}
$$

and the inertial distance traversed in one time period is given by

$$
\begin{equation*}
\Delta \mathrm{D}=\mathrm{V}_{\mathrm{AVG}} \Delta \mathrm{t} \tag{13}
\end{equation*}
$$

The total displacement of each cavity center can now be expressed as

$$
\begin{align*}
& C_{F}=\sum_{i=1}^{t_{f} / \Delta t}\left(\Delta S_{F}+\Delta D\right)_{i}  \tag{14}\\
& C_{A}=\sum_{i=1}^{t_{i} / \Delta t}\left(\Delta S_{A}+\Delta D\right)_{i}
\end{align*}
$$

where $t_{f}$ is the total time period being considered.
Now consider the inertial ball displacements $\mathrm{B}_{\mathrm{F}}$ and $\mathrm{B}_{\mathrm{A}}$. Iet the inertial frame three-axis angular rates of the Optical Bench be defined by $\left[\dot{\beta}_{I}\right]$, Referring to Figure 4 the arc lengths traversed by each ball in one time period are defined by


Figure 4. Translational and rotational impedance.

$$
\begin{align*}
& \Delta K_{F}=\left[-\widetilde{L}_{F}\right]\left[\dot{\beta}_{I}\right] \Delta t \\
& \Delta K_{A}=\left[-\widetilde{L}_{A}\right]\left[\dot{\beta}_{I}\right] \Delta t \tag{15}
\end{align*}
$$

Also, let the inertial translation of the Optical Rench center-of-mass in one time period be defined by the symbol $\Delta \mathrm{D}_{\mathrm{T}}$. The total displacement of each ball can now be described as

$$
\begin{align*}
& B_{F}=\sum_{i=1}^{t_{f} / \Delta t}\left(\Delta K_{F}+\Delta D_{T}\right)_{i} \\
& B_{A}=\sum_{i=1}^{t_{f} / \Delta t}\left(\Delta K_{A}+\Delta D_{T}\right)_{i} \tag{16}
\end{align*}
$$

The magnitude of the force acting on each ball due to cavity wall contact is specified by

$$
\begin{align*}
& \left|F_{F}\right|=K_{1}\left[\left|C_{F}-B_{F}\right|-\mathrm{TOL}_{F}\right] \\
& \left|F_{A}\right|=K_{2}\left[\left|C_{A}-B_{A}\right|-\mathrm{TOL}_{A}\right] \tag{17}
\end{align*}
$$

where
$\mathrm{K}_{1}=$ spring constant for forwar' hall. This constant represents the restoring force which acts when the forward ball ،tacts the cavity wall.
$K_{2}=$ spring constant for aft ball.
$\mathrm{TOL}_{\mathrm{F}}=$ forw ard ball dead zone. This constant represents the free space (or tolerance) between the forward ball and the cavity wall.
$\mathrm{TOL}_{\mathrm{A}}=$ aft ball dead zone.

The quantity within the brackets of equation (17) represents the magnitude of the compression (extension is not possible) of the respective snrings, i.e., this is the distance the ball "sinks" into the cavity wall before the generated force can restore the ball to the free space. Equation (17) acts o.aly after cavity wall contact has been made and becomes zero after contact has been lost according to equation (18)

$$
\begin{array}{lll}
\left|\mathrm{F}_{\mathrm{F}}\right|=0 & & \left|\mathrm{C}_{\mathrm{F}}-\mathrm{B}_{\mathrm{F}}\right|<\mathrm{TOL}_{\mathrm{F}} \\
& \text { for } &  \tag{18}\\
\left|\mathrm{F}_{\mathrm{A}}\right|=0 & & \left|\mathrm{C}_{\mathrm{A}}-\mathrm{B}_{\mathrm{A}}\right|<\mathrm{TOL}_{A}
\end{array}
$$

A simplifying assumption has been made that no tangential forces can be exerted on the balls due to wall contact. Therefore, the forces are always directed from each ball toward the center of the cavity. The unit vectors expressing the direction of the forces $\mathrm{F}_{\mathrm{F}}$ and $\mathrm{F}_{\mathrm{A}}$ are given by

$$
\begin{align*}
& \mathrm{U}_{\mathrm{F}}=\frac{\left(\mathrm{C}_{\mathrm{F}}-\mathrm{B}_{\mathrm{F}}\right)}{\left|\mathrm{C}_{\mathrm{F}}-\mathrm{B}_{\mathrm{F}}\right|} \\
& \mathrm{U}_{\mathrm{A}}=\frac{\left(\mathrm{C}_{\mathrm{A}}-\mathrm{B}_{\mathrm{A}}\right)}{\left|\mathrm{C}_{\mathrm{A}}-\mathrm{B}_{\mathrm{A}}\right|} \tag{19}
\end{align*}
$$

Each ball force (expressed in the inertial frame) can now be represented as

$$
\begin{align*}
& \mathrm{F}_{\mathrm{F}}=\left|\mathrm{F}_{\mathrm{F}}\right| \mathrm{U}_{\mathrm{F}}  \tag{20}\\
& \mathrm{~F}_{\mathrm{A}}=\left|\mathrm{F}_{\mathrm{A}}\right| \mathrm{U}_{\mathrm{A}}
\end{align*}
$$

The syste: block diagram shown in Figure 5 gives a more complete definition of this logic.
OF POOR QUAL:TY

Figure 5. System block diagram.

## CABLE CONNECTIONS

Each Optical Bench is connected to the spacecraft through electrical cables used for power and scientific data transmission. These cables will have an effect on the mution of the Optical seach sinze they have properties which produce translational and rotational spring and damping coefficients. The cable forces and torques are assumed to act through the Optical Bench center of mass as depicted in Figure 4. Let the mass and inertia tensor be defiaed by $\mathrm{M}_{1}$ and

$$
J_{1}=\left[\begin{array}{ccc}
\mathrm{J}_{\mathrm{V} 1} & 0 & 0  \tag{21}\\
0 & \mathrm{~J}_{\mathrm{V} 2} & 0 \\
0 & 0 & \mathrm{~J}_{\mathrm{V} 3}
\end{array}\right]
$$

Establish the $\mathrm{x}, \mathrm{y}, \mathrm{z}$ coordinate frame with origin at the Optical B.nch center of mass and with axes iritially parallel to the vehicle and inertial frames. The lever arms $\mathrm{L}_{\mathrm{A}}$ and $\mathrm{L}_{\mathrm{F}}$ are (referring to Figs. 1 and 4)

$$
\begin{align*}
& \mathrm{L}_{\mathrm{A}}=\mathrm{R}_{1}-\mathrm{R}_{\mathrm{VCG}} \\
& \mathrm{~L}_{\mathrm{F}}=\mathrm{R}_{2}-\mathrm{R}_{\mathrm{V}^{\prime} \mathrm{CG}}
\end{align*}
$$

The cable dynamics are shown in Figure 4 as an impedance ( $\mathbf{z}_{\mathbf{x}}, \mathbf{z}_{\mathbf{y}}, \mathbf{z}_{\mathbf{z}}$ ) in each axis connecting the Optical Bench center of mass and the spacecraft.

Consider fast the translational dynamics. A mechanica! representation used for equation derivation is shown in Figure 6 and has beer simplified (for ease of drawing) such that the attach point lever


Figure 6. Translational model.
arm lies only along the $\mathrm{V}_{3}$ axis. Consider the vehicle to have an angular rate ( $\omega_{\mathrm{V} 2}$ ) about the $\mathrm{V}_{2}$ axis aild a translation ( D ) along the $\mathrm{V}_{1}$ axis. Under these conditions, the motion of the attach point is given by the sum of $D$ and $D_{R T}$ where

$$
\begin{equation*}
\mathrm{D}_{\mathrm{RT}}=\int \mathrm{V}_{\mathrm{RT}} \mathrm{dt}=-\int \mathrm{R}_{\mathrm{CG}} \omega_{\mathrm{V} 2} \mathrm{dt} \tag{23}
\end{equation*}
$$

Expressing this in the more general three-dimensional case and in matrix notation gives

$$
\begin{equation*}
\dot{\mathrm{D}}_{\mathrm{RT}}=\mathrm{V}_{\mathrm{RT}}=\left[-\widetilde{\mathrm{R}}_{\mathrm{CG}}\right] \omega_{\mathrm{V}} . \tag{24}
\end{equation*}
$$

Writing the transfer function from the input force $F$ to the attach point produces,

$$
\begin{equation*}
\mathrm{F}=\mathrm{M}_{1} \ddot{\mathrm{D}}_{\mathrm{T}}+\mathrm{d}_{\mathrm{T}}\left(\dot{\mathrm{D}}_{\mathrm{T}}-\dot{\mathrm{D}}_{\mathrm{RT}}-\dot{\mathrm{D}}\right)+\mathrm{K}_{\mathrm{T}}\left(\mathrm{D}_{\mathrm{T}}-\mathrm{D}_{\mathrm{RT}}-\mathrm{D}\right) \tag{25}
\end{equation*}
$$

Rearranging equation (25) and introducing LaPlace notation gives

$$
\begin{equation*}
F=\left(M_{1} S^{2}+d_{T} S+K_{T}\right) D_{T}-\left(d_{T} S+K_{T}\right) D_{R T}-\left(\mathrm{G}_{\mathrm{T}} S+K_{T}\right) D \tag{26}
\end{equation*}
$$

If it is assumed that the vanables $\mathrm{D}_{\mathrm{RT}}$ and D are zero (as would be the case for a stationary vehicle) then equation (26) simplifies to

$$
\begin{equation*}
\frac{D_{T}}{F}=\frac{1 / M_{1}}{s^{2}+d_{T} / M_{1} S+K_{T} / M_{1}}=\frac{1 / M_{1}}{s^{2}+2 \zeta} \frac{\omega_{N} S+\omega_{N}^{2}}{} \tag{27}
\end{equation*}
$$

For the purpose of this report it was assumed that the spring constant $\left(\mathrm{K}_{\mathrm{T}}\right)$ and damping ratio $(\zeta)$ were

$$
\mathrm{K}_{\mathrm{T}}=1 \mathrm{lb} / \mathrm{in} .=175.118 \mathrm{~N} / \mathrm{m} \text { and } \quad \zeta=0.005 \text { (unitless). }
$$

With these assumptions the natural frequency and damping coefficient are calculated to be

$$
\mathrm{f}=0.174 \mathrm{~Hz} \quad \text { and } \quad \mathrm{d}_{\mathrm{T}}=1.6 \mathrm{~N}-\mathrm{S} / \mathrm{m}
$$

Since the Optical Bench mass is invariant all of the transiational coefficients will be the same for all vehicle axes

A simplified vie of the manchal model used for rotational equation derivation is shown in Figure 7.


Figure 7. Rotational model.
The torque applied to the Optical Bench inertia [equation (21)] due to wall contact is

$$
\begin{equation*}
\mathrm{T}=\left[\tilde{\mathrm{L}}_{\mathrm{F}}\right]\left[\mathrm{F}_{\mathrm{F}}\right]+\left[\tilde{\mathrm{L}}_{\mathbf{A}}\right]\left[\mathrm{F}_{\mathrm{A}}\right] \tag{28}
\end{equation*}
$$

where $L_{A}$ and $L_{F}$ are defined in Figure 4.
Summation of the torques on each inertia produces the following equation,

$$
\begin{equation*}
\mathrm{T}=\mathrm{J}_{1} \ddot{\beta}_{\mathrm{Z}}+\mathrm{d}_{\mathrm{R}}\left(\dot{\beta}_{\mathrm{Z}}-\omega_{\mathrm{V}}\right)+\mathrm{K}_{\mathrm{R}}(\beta-\theta) \tag{29}
\end{equation*}
$$

where

$$
\beta_{\mathrm{I}}=\left[\begin{array}{l}
\beta_{\mathrm{V} 1}  \tag{30}\\
\beta_{\mathrm{V} 2} \\
\beta_{\mathrm{V} 3}
\end{array}\right] ; \quad \theta=\left[\begin{array}{l}
\theta_{\mathrm{V} 1} \\
\theta_{\mathrm{V} 2} \\
\theta_{\mathrm{V} 3}
\end{array}\right],
$$

are the rotation angles of the Optical Bench and the vehicle, respectively.
Again introducing LaPlace notation and letting $\omega_{\mathrm{V}}=\boldsymbol{\theta}=0$ produces

$$
\begin{equation*}
\frac{\beta}{\mathrm{T}}=\frac{\mathrm{J}_{1}^{-1}}{\mathrm{~s}^{2}+\mathrm{d}_{\mathrm{R}} / \mathrm{J}_{1} \mathrm{~S}+\mathrm{K}_{\mathrm{R}} / \mathrm{J}_{1}}=\frac{\mathrm{J}_{1}^{-1}}{\mathrm{~s}^{2}+25 \omega_{\mathrm{N}} \mathrm{~S}+\omega_{N}^{2}} \tag{31}
\end{equation*}
$$

With the assumption that

$$
\mathrm{K}_{\mathrm{R}}=1.3558 \frac{\mathrm{~N} \cdot \mathrm{~m}}{\mathrm{RAD}}
$$

$\zeta=0.005$ unitless
the following constants are calculated

$$
\mathrm{d}_{\mathrm{R}}=\left[\begin{array}{l}
0.0877  \tag{Hz}\\
0.1431 \\
0.1431
\end{array}\right](\mathrm{N} \cdot \mathrm{~m} \cdot \mathrm{~S}) \quad ; \quad \mathrm{f}_{\mathrm{N}}=\left[\begin{array}{l}
0.0246 \\
0.0151 \\
0.0151
\end{array}\right]
$$

The forces $\mathrm{F}_{\mathrm{A}}$ and $\mathrm{F}_{\mathrm{F}}$ are calculated in the inertial frame; however, the translational and rotational spring and damping constants only have meaning in the vehicle reference axis system. Consequently, the forces and torques generated as a result of motional differences between the vehicle and Optical Bench must be converted into the vehicle frame and back to the inertial frame. Equation (25) can be written as

$$
\begin{equation*}
\ddot{\mathrm{D}}_{\mathrm{T}}=\frac{1}{\mathrm{M}_{1}}\left[\mathrm{~F}+\mathrm{F}_{\mathrm{DAMP}}+\mathrm{F}_{\mathrm{S}}\right] \tag{32}
\end{equation*}
$$

where

$$
\begin{align*}
& \mathrm{F}_{\mathrm{DAMP}}=\mathrm{d}_{\mathrm{T}}\left(\dot{\mathrm{D}}+\dot{\mathrm{D}}_{\mathrm{RT}}-\dot{\mathrm{D}}_{\mathrm{T}}\right)=\mathrm{d}_{\mathrm{T}}\left(\mathrm{~V}-\mathrm{V}_{\mathrm{T}}+\mathrm{V}_{\mathrm{RT}}\right)  \tag{33}\\
& \mathrm{F}_{\mathrm{S}}=\mathrm{K}_{\mathrm{T}}\left(\mathrm{D}-\mathrm{D}_{\mathrm{T}}+\mathrm{D}_{\mathrm{RT}}\right)
\end{align*}
$$

Since the spring and damping factors can be applied only in the vehicle frame, equation (33) must be rewritten as

$$
\begin{aligned}
& \left.\mathrm{F}_{\mathrm{DAMP}}=[\mathrm{EV}]\left\{\left[\mathrm{d}_{\mathrm{T}}\right] \times(\mathrm{IV}]\left[\mathrm{V}-\mathrm{V}_{\mathrm{T}}\right]+\mathrm{V}_{\mathrm{RT}}\right)\right\} \\
& \mathrm{F}_{\mathrm{S}}=[\mathrm{EV}]\left\{\left[\mathrm{K}_{\mathrm{T}}\right] \times\left([\mathrm{VE}]\left[\mathrm{D}-\mathrm{D}_{\mathrm{T}}\right]+\mathrm{D}_{\mathrm{RT}}\right)\right\}
\end{aligned}
$$

where $[\mathrm{VE}]=[\mathrm{EV}]^{\mathrm{T}}=$ vehicle to inertial frame transformation.
For the rotational dynamics, equation (29) is expressed as
$\ddot{\beta}=\mathrm{J}_{1}{ }^{-1}\left[\mathrm{~T}+\mathrm{T}_{\text {DAMP }}+\mathrm{T}_{\mathrm{S}}\right]$
where

$$
\begin{align*}
& \mathrm{T}_{\text {DAMP }}=\mathrm{d}_{\mathrm{R}}\left[\omega_{\mathrm{V}}-\dot{\beta}\right] \\
& \mathrm{T}_{\mathrm{S}}=\mathrm{K}_{\mathrm{R}}[\theta-\beta] \tag{36}
\end{align*}
$$

Again converting to the vehicle frame and back to the inertial reference gives

$$
\begin{align*}
& \mathrm{T}_{\mathrm{DAMP}}=[\mathrm{EV}]\left\{\mathrm{d}_{\mathrm{R}} \times\left(\omega_{\mathrm{V}}-[\mathrm{VE}]\left[\dot{\beta}_{\mathrm{I}}\right]\right)\right\} \\
& \mathrm{T}_{\mathrm{S}}=[\mathrm{EV}]\left\{\mathrm{K}_{\mathrm{R}} \times\left(\theta_{\mathrm{V}}-[\mathrm{VE}]\left[\beta_{\mathrm{I}}\right]\right)\right\} \tag{37}
\end{align*}
$$

Equations (34) and (37) represent the dynamics as implemented and as shown in the block diagram (Fig. 5).

Referring to equation (11) the vehicle acceleration can now be expressed in more detail by

$$
\begin{equation*}
A_{I}=\frac{[E V][F]}{M_{V}}=-\frac{\left[F_{A}+F_{F}+F_{D A M P}+F_{S}-(E X T E R N A L \text { FORCES })\right]}{M_{V}} \tag{38}
\end{equation*}
$$

It is possible for the Optical Bench to transmit reactive torques to the vehicle through three different paths:

1) Forward and aft wall contact forces acting through their respective lever arms.
2) Optical Bench rotational damping and spring torques.
3) Translational spring and damping forces acting at the Optical Bench attach point.

The total reactive torque (in vehicle coordinates) can now be expressed as

$$
\begin{equation*}
T_{R}=-[V E]\left\{\left[\widetilde{R}_{F}\right]\left[F_{F}\right]+\left[\tilde{R}_{A}\right]\left[F_{A}\right]+\left[\widetilde{R}_{C G}\right]\left[F_{D A M P}+F_{S}\right]+T_{D A M P}+T_{S}\right\} \tag{39}
\end{equation*}
$$

Equation (39) represents the reactive torques that drive the spacecraft attitude control system.

## SIMPLIFYING ASSUMPTIONS

All the necessary equations have been derived and the loop can now be closed through equation (39) as shown in Figure 5. Certain simplifying assumptions have been made in the equation derivation either because their effects were considered to be negligible or because the data for their implementation was not available.

1) The arc lengths $\Delta \mathrm{S}, \Delta \mathrm{D}_{\mathrm{T}}$, and $\Delta \mathrm{K}$ were considered to be staight lines instead of curved paths.
2) No tangential forces are exerted on the Optical Bench balls by cavity wall contact. Contact forces are always directed along a line through the contact point and the cavity center.
3) Cable damping and spring constants for rotation and translation act through the Optical Bench center of mass.
4) The Optical Bench inertia tensor was considered to be diagonal (i.e., all coupling terms were zero).
5) The ball and socket dead zones (specified by $\mathrm{TOL}_{\mathrm{F}}$ and $\mathrm{TOL}_{\mathbf{A}}$ ) are small enough to allow the Optical Bench reference frame to re considered congruent with the vehicle frame.
6) The total torque on the Optical Bench is expressed by
$T_{T}=J_{1} \ddot{\beta}+(\dot{\beta} \times H)$
where H is the angular momentum. Assuming a diagonal inertia tensor and small angular rates, the term ( $\dot{\beta} \times \mathrm{H}$ ) can be ignored.
7) The ball contact spring stiffness was assumed to be constant.
8) Thc Optical Bench was assumed to be rigid.

## INPIJT DATA AND PROGRAM RESULTS

The Faint Object Camera (FOC) is the only Scientific Instrument on bcard the HST which has the ball and socket mounting arrangement at each end. Referring to Figure 2 the variables shown for the FOC are given by (all units are in meters)

$$
\Delta \mathrm{CG}=\left[\begin{array}{c}
6.617 \\
0.005 \\
-0.145
\end{array}\right] ; \quad \mathrm{R}_{\mathrm{VCG}}=\left[\begin{array}{c}
4.461 \\
-0.297 \\
-0.325
\end{array}\right] ; \quad \mathrm{R}_{1}=\left[\begin{array}{c}
3.1 \\
-0.102 \\
-0.102
\end{array}\right] ; \quad \mathrm{R}_{2}=\left[\begin{array}{c}
5.125 \\
-0.7 \\
-0.7
\end{array}\right]
$$

Combining the data in Figure 2 according to previously developed equations produces the variables of Figure 3

$$
\mathrm{R}_{\mathrm{A}}=\left[\begin{array}{c}
-3.517 \\
-0 . \mathrm{i} 07 \\
0.043
\end{array}\right] ; \quad \mathrm{R}_{\mathrm{CG}}=\left[\begin{array}{c}
-2.156 \\
-0.302 \\
-0.18
\end{array}\right] ; \quad \mathrm{R}_{\mathrm{F}}=\left[\begin{array}{c}
-1.492 \\
-0.705 \\
-0.555
\end{array}\right]
$$

and Figure 4

$$
\mathrm{L}_{\mathrm{A}}=\left[\begin{array}{c}
-1.361 \\
0.195 \\
0.233
\end{array}\right] ; \quad \mathrm{L}_{\mathrm{F}}=\left[\begin{array}{c}
0.664 \\
-0.403 \\
-0.375
\end{array}\right]
$$

The FOC mass and inertia terms are defined by

$$
M_{1}=146 \mathrm{Kg}
$$

and

$$
\mathrm{J}_{1}=\left[\begin{array}{ccc}
7.256 & 0 & 0 \\
0 & 39.207 & 0 \\
0 & 0 & 39.207
\end{array}\right] \mathrm{Kg} \mathrm{~m}^{2}
$$

where $\mathrm{J}_{1}$ is given about the FOC center of mass.

As a test case the vehicle was maneuvered 0.05 deg about the positive $\mathrm{V}_{2}$ axis. This rotation was sufficient to cause an aft cavity wall collision. The system performance was examined with and without fexible body dynamics. The rotation about the $\mathrm{V}_{2}$ (maneuver axis) produced very similar results in each case as shown in Figures 8 and 9. Each plot shows a rotation to 0.05 deg ( 180 arosec ) but the scale is such that the small variations due to body bending are not evident. Plotting the angular error in the maneuver axis produces Figures 10 and 11 . The maneuver takes approximately 80 sec to complete. The small variations in each plot are less than 1 milli-arc-sec and are principally due to the translational and rotational dynamics of the suspended mass. Flexible body effects are much more pronounced in Figures 12 through 15 which represent the angular rotations for the $V_{1}$ and $V_{3}$ axes. Collisions between the aft ball and the cavity wall are most evident in Figures 14 and 15 which indicate six collisions of the aft ball at $14,22,26,54,62$, and 67 sec . After the maneuver has been completed, the flexible body effects are once again very small and the variations can be attributed primarily to mass translation.

## CONCLUSIONS

The simulation results indicate that there is no strong coupling from the wall/ball collisions into the vehicle elastic body. Flexible body "ringing" soon damps out after the vehicle rotation has been accomplished and the last collision has occurred. Pointing axis deviations as determined from the RMS value of the $\mathrm{V}_{2}$ and $\mathrm{V}_{3}$ axis rotations are less than $1 \times 10^{-3}$ arc-sec. The principal cause of the damped vibration seen beginning at 80 sec is the translational effects of the Optical Bench acting through the cable connections. Studies will continue to determine if uncertainties in flexible body and translational mode frequencies could cause an instability or limit cycle condition to exist.


Figure 8. Rotation about $\mathrm{V}_{2}$ axis with rigid body dynamics.


Figure 9. Rotation about $\mathbf{V}_{2}$ axis with flexible body dynamics.


Figure 10. $\mathrm{V}_{2}$ axis error with rigid body dynamics (difference between commanded and actual vehicle angle).


Figure 11. $\mathrm{V}_{2}$ axis error with flexible body dynamics (difference between commanded and actual vehicle angle).


Figure 12. Rotation about $\mathrm{V}_{1}$ axis with rigid body dynamics.


Figure 13. Rotation about $\mathrm{V}_{1}$ axis with flexible body dynamics.


Figure 14. Rotation about $\mathrm{V}_{3}$ axis with rigid body dynamics.


Figure 15. Rotation about $\mathrm{V}_{3}$ axis with flexible body dynamics.

## APPROVAL

## HUBBLE SPACE TELESCOPE - POINTING ERROR EFFECTS OF

 NONLINEAR BALL .hINTSBy John E. Farmer and Floyd R. Grissett

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified

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