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WEIBULL DISTRIBUTION BASED ON MAXIMUM LIKELIHOOD WITH INTERVAL INSPECTION DATA

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This technical note determines the two Weibull parameters based upon the method of maximum likelihood as presented in memorandum, "Oxidizer Turbine (HPOTP) First Stage Blade Reliability Analysis," dated July 10, 1985, by the author. The test data used were failures observed at inspection intervals. The application was the reliability analysis of the SSME oxidizer turbine blades.

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INTRODUCTION

This technical note determines the two Weibull parameters based upon the method of maximum likelihood as presented in memorandum, "Oxidizer Turbine (HPOTP) First Stage Blade Reliability Analysis," dated July 10, 1985, by the author. The test data used were failures observed at inspection intervals. The application was the reliability analysis of the SSME oxidizer turbine blades.

DISCUSSION

This memo presents the results of a reliability analysis of the HPOTP first stage blades. The failure mode investigated here is a shank cracking of the blades on the downstream pressure side. It was assumed that the time distribution of this failure mode follows the two-parameter Weibull distribution whose probability density function is given by

$$(1) \quad f(t) = (\beta/\eta^{\beta}) t^{\beta-1} \, \mathrm{EXP}[-(t/\eta)^{\beta}].$$

The parameter β is called the shape parameter and is positive. The parameter η is called scale parameter and is also positive. Because the scale parameter η always represents the 63.2 percent point, it is also called the "characteristic life". It has the same units as t, namely, time, whereas β is a dimensionless number.

The Weibull reliability function (survivorship function) is

$$(2) \qquad R(t) = EXP - [(t/\eta)^{\beta}].$$

This is the probability of a blade surviving beyond time t.

Two sets of test data were analyzed, one set for unchamfered blades (Table 1) and one set for chamfered/blended blades (Table 2).

Estimation of the two Weibull parameters β and η was based upon the method of maximum likelihood. This method was selected for two reasons. First, it is capable of using all the statistical information contained in the test data. Second, the method utilizes the intuitively appealing principle to determine the two parameters in such a way that the sequence of observations that actually occurred is the one having maximum probability. Usually the observations are a random sample of independent observations from the same distribution. Most reliability analyses assume that each failure time of a component is known exactly. However, for the present test data only the intervals in which failures occurred are known. The likelihood function can be set up to properly account for this condition.

Suppose that R blades have been found to fail in the interval $t_{1_i} < t_i < t_{2_i}$ (i=1, 2,...,R) and S blades have been found to survive beyond time t_{3_j} (j = 1, 2, ...S). Then the sample likelihood function is given by

$$(3) \qquad L = \prod^{R} [EXP-(t_{1_i}/\eta)^{\beta} - EXP - (t_{2_i}/\eta)^{\beta}] \times \prod^{S} EXP - (t_{3_j}/\eta)^{\beta}.$$

where the first product $\prod^{R}$ is the probability of R blades failing within their respective test intervals and the second product $\prod^{R}$ is the probability of S blades surviving their respective test runs.

For the following analysis, it is convenient to work with the logarithm of the above likelihood function, which is

$$(4) \qquad \ell n\, L = \sum^{R} \ell n\, [EXP-(t_{1_i}/\eta)^{\beta} - EXP - (t_{2_i}/\eta)^{\beta}] - (1/\eta)^{\beta} \sum^{S} t_{3_j}^{3}.$$

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The given test data represent a special case in which the upper failure interval limit is equal to the total test time, i.e., $t_{2_i} = t_{3_i}$. Moreover, there are M=78 blades per disk and it is observed at the end of the i-th test that R_i blades have failed since the inspection time t_{1_i} and that $(M-R_i)$ blades have survived the test. Supposing that there are a total of N test runs, then eq (4) can be modified accordingly to yield

$$(5) \qquad \ell n\, L = \sum^{N} R_i\, \ell n[\mathrm{EXP}-(t_{1_i}/\eta)^{\beta} - \mathrm{EXP}-(t_{2_i}/\eta)^{\beta}] - (1/\eta)^{\beta} \sum^{N} (M-R_i) t_{2_i}^{\beta}\ .$$

Several numerical methods exist to maximize the log-likelihood function of eq (5). The results of this analysis were obtained by the Newton-Raphson Method which finds the maximum of the log-likelihood function by solving the set of equations

$$(6) \qquad \frac{\partial\, \ell n\, L}{\partial \eta} = 0\ , \qquad \frac{\partial\, \ell n\, L}{\partial \beta} = 0\ .$$

This was done on a programmable hand calculator. The method works well if the starting parameters are close to the maximum likelihood estimators. With some experience, it is not very difficult to find good starting values.

As mentioned earlier, the reliability function of eq (2) represents the probability that a single blade will operate without a crack for at least t seconds. Of practical importance is also the reliability of a turbine disk, which for the HPOTP first stage contains n=78 blades. Considering the turbine disk to be a system composed of n identical and independent components the disk reliability is given by the cumulative binomial distribution as:

$$(7) \qquad R_D(k,\, t) = \sum_{0}^{k} \binom{n}{x} R^{n-x} (1 - R)^{x}.$$

where R is the reliability of the single blade. The reliability $R_D(k,t)$ is the probability of not more than k components failing within the time interval from 0 to t. This "system" reliability is, of course, always smaller than the reliability of a single blade.

Another statistical quantity of interest is the conditional reliability. This is the probability of a component (or system) to survive a specified operational time T after it has already been ir op ration for some known time T_o. For the Weibull distribution, it is given by

$$(8) \qquad R_C\,(T/T_o) = \frac{\text{EXP} - [(T + T_o)/\eta]^\beta}{\text{EXP} - (T_o/\eta)^\beta} \quad .$$

NUMERICAL RESULTS

Only the estimates of the Weibull parameters and the B1-life will be presented. The B1-life represents the operating time for which the reliability is 99% (1% unreliability). The shape parameter β may be used as an indicator of the underlying failure mechanism. If β < 1 the failure rate decreases with the age of the component. This is often referred to as infant mortality or as burn-in failure mode. For β=1, the Weibull distribution is identical to the well-known exponential distribution, which represents a random failure mode. In this mode, the failure rate is constant which means the component does not age. This condition is more appropriate for complex systems whose components have different ages due to previous failures and replacements. A shape parameter β>1 indicates a wear-out failure mechanism. However, it is important to understand that a more accurate assessment of the underlying failure mechanism has to be based upon the confidence intervals of the Weibull parameters. The characteristic life η is, in general, not of interest because it corresponds to a reliability of only 36.8%. When comparing different sets of Weibull parameters, it will be noticed that even though their values may differ substantially, the corresponding B1-lifes or B.1 lifes, that is high range reliabilities, will differ only slightly.

A. Unchamfered Blades (Table 1)

Scale parameter	η = 1,142,538 sec.
Shape parameter	β = 0.581
B1-life	t_1 = 413 sec.

B. Chamfered Blades (Table 2)

1. Blended blades only (exclusion of data set marked by asterisk).

Scale parameter	η = 69,324 sec.
Shape parameter	β = 1.806
B1-life	t_1 = 5,429 sec.

2. All chamfered blades (inclusion of data set marked by asterisk).

Scale parameter	η = 898,949 sec.
Shape parameter	β = 0.8208
B1-life	t_1 = 3,309 sec.

CONCLUSIONS

1) The given test data indicate a significant statistical difference between the chamfered and the unchamfered blades, the former showing an order of magnitude higher B1-life than the former.

2) There appears to be also a significant change when the single test run having unblended blades is included in the analysis. Especially worthy of notice here is the change of the shape parameter β from a wear-out failure mechanism ($\beta>1$) to an infant mortality condition. This could be indicative of a quality control problem. Whether this change can be attributed to the difference between blended and unblended blade design or to some other factor cannot be determined since only this single test run of unblended blades was available.

With the above given Weibull parameters it requires only a small computational effort to determine single blade and disk reliabilities for various time points of interest using the appropriate equations presented earlier.

TABLE 1. UNCHAMFERED BLADE TEST DATA

(No inspection time available.)

Number of Cracked Blades	Total Test Time (Sec)
2	10155
7	6474
1	6092
0	3100
1	4298
0	3000
5	3226
3	3152
13	3095
1	3048
0	2850
4	2406
2	2373
6	2180
0	1761
0	1615
0	1343
0	721

TABLE 2. CHAMFERED BLADE TEST DATA

A. TESTS WITH CRACKED BLADES

No. of Cracked Blades	Inspection Time (Sec)	Total Test Time (Sec)
3	4000	5996
1	2100	4252
1	1100	3441
1	1200	3360
1	1100	3281
3	0	3112
1	0	1263
13*	0*	1798*

*Represents unblended blades.

B. TESTS WITHOUT CRACKED BLADES (No inspection time necessary)

Total Test Time (Sec)	Total Test Time (Sec)
7840	2448
6332	1847
5982	1508.5
4653	1427
4391	1260
4135	1015
3491	895
3241	766
3155	756
3077	750
2946	666
2880	501
2868	371
2849	300
2810	

APPROVAL

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The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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