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"Planetary Dynamos"

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FOREWORD

The discovery of the magnetic fields of the planets and the detailed measurement of the geomagnetic field by space probes has introduced a new era in the development of dynamo theory.

Only a decade ago, it seemed rather unlikely that the problem of the origin of geomagnetism could ever be solved with sufficient accuracy to permit quantitative comparison with the observations. The uncertainty about the energy source and the general lack of knowledge about the physical conditions in the earth's core tended to prohibit a unique determination of the mechanism of the geodynamo. The discovery of the magnetic fields of other planets and the recognition of their general similarity with the geomagnetic field have changed this unsatisfactory state of the theory. With respect to some parameters relevant to the dynamo process, the cores of the major planets are actually better known than the earth's core. And the constraints offered by the various realizations of planetary magnetism further restrict the possibilities of different dynamo models. Thus the theory of geomagnetism has become a part of the general theory of planetary magnetism. The MAGSAT-program has added significantly to our knowledge of planetary magnetism. The accuracy of observations has been improved such that a reliable extrapolation of the magnetic field to the core surface is now much more feasible than it has been before, and the prospect of further MAGSAT missions raises the expectation that the time dependence of the geomagnetic field will be known with similar accuracy in the future. In the research support under NASA Grant NSG 7484 it has been attempted to develop dynamo theory with these applications in mind.

Because of the three-dimensional nonlinear magnetohydrodynamic equations which must be solved in spherical geometries and because of the complex feedback provided by the Lorentz force in the strongly nonlinear regime,

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the development of realistic models of planetary dynamos is anything but straightforward. Much of the recent work had to be devoted towards an exploration of the parameter space in order to distinguish between important and less important influences and to gain the experience necessary for the construction of more flexible and more cost effective dynamo computer codes. But the progress made in the past year and the promising results obtained so far have encouraged us to continue these efforts. In the following we discuss those areas where we have made progress during the past few years.

1. Generation of magnetic fields by convection in a rotating sphere

A major step in understanding planetary dynamos depends on the analysis of magnetic field generation by convective motions in rotating spheres and spherical shells. This problem has been attacked in collaboration with Dr. Cuong and more recently with Dr. Bolton.

Major results presented in the paper by Cuong and Busse (1981) are: (i) For small amplitudes of convection the magnetic field has quadrupolar symmetry. For larger amplitudes axisymmetric circulations and differential rotation become important and a field of dipolar symmetry becomes preferred. The close competition of fields of dipolar and quadrupolar symmetry is in agreement with the analysis of apherical α^2-dynamos by Proctor (1977). (ii) Since convection is most vigorous outside the cylindrical surface touching the equator of an inner spherical boundary, the poloidal field strength is lower at the poles and higher at the equator than a purely dipolar field. (iii) A sufficiently high azimuthal wave number ($m \geq 6$) of the convection motion is required for dynamo action. Dynamos appear to function best with a clear separation between large and small scale components of the magnetic field. (iv) The increase of magnetic field strength with increasing Rayleigh number suggests a stable equilibrium state for the parameter range that has been investigated.

The work of Cuong and Busse (1981) has been extended, at least as far as the convection part is concerned, in the Ph.D. thesis of Bolton (1985). The axisymmetric interaction approximation of the earlier work has been dropped, since larger computer memories have become available. Accordingly, the higher harmonics in the azimuthal direction have been taken into account. The comparison with the earlier work indicates that the axisymmetric interaction approximation works well at low amplitudes of convection (as is to be expected) and at higher values of the dominant azimuthal wavenumber m.

Unfortunately the limited time available for the completion of the thesis has not allowed Dr. Bolton to complete the solution of the nonlinear dynamo problem. The attempts to generate equilibrium solutions for the complete nonlinear magnetohydrodynamic problem of the generation of magnetic fields by convection have failed so far. The forward integration in time, which has also been attempted, has indicated a divergence of the axisymmetric component of the magnetic field. Further testing of the magnetic part of the computer code of Bolton appears to be necessary before it can be decided whether a programming error or numerical instabilities or physical instability of the equilibrium solution are the cause of convergence failure. This work is currently in progress.

2. Dynamo oscillations

Compared to the problem of thermal convection, the problem of the magnetohydrodynamic dynamo driven by convection offers even more possibilities for time dependent states because of the additional time derivative in the equation of induction. It is thus not surprising that stable steady equilibrium states are exceptional in dynamo theory even though they are most accessible to calculations.

To elucidate some typical features of nonlinear dynamo oscillations the stability of two cases of magnetic equilibrium states has been studied. The investigation of the model of magnetic field generation by convection rolls in the presence of a shear (Busse, 1973) revealed an instability which is caused by the reduction of the convection amplitude with increasing magnetic field strength in combination with the property that the efficiency of dynamo action decreases for high magnetic Reynolds numbers because of flux expulsion. Thus, an incidental increase of the magnetic field strength reduces the amplitude of convection which increases the efficiency of the dynamo process and leads to a further growth of the magnetic field strength. The numerical integration of the time-dependent equations for the problem exhibits nonlinear relaxation oscillations with short-lived peak amplitudes of the magnetic field, sharp decays, and long periods of slowly growing magnetic energy in the neighborhood of the equilibrium value (Busse, 1982). It is not unlikely that reversals could occur if small additional disturbances are introduced.

The other example of an unstable equilibrium state occurs in the annulus model of a convection driven dynamo if the enhancement of dynamo action owing to a particular component of the Lorentz force is taken into account (Busse, 1977). For a particular parameter range two unstable steady magnetic states exist which gives rise to nonlinear oscillations between these states (Busse, 1982).

3. A model of nonlinear convection in rapidly rotating spheres

Numerical computations of convection in rotating spheres are limited to relative low values of the Taylor number which is the dimensionless parameter incorporating the rotation rate. Because of the small scale in the direction perpendicular to the axis of rotation assumed by the convection columns it is difficult to obtain sufficiently high resolution as the Taylor number is increased. To overcome this limitation analytical expressions for convection columns have been developed recently (Busse, 1983a,b) using an expansion of the basic equations in powers of the Prandtl number. This expansion is particularly appropriate for planetary cores where the Prandtl number is certainly less than unity and probably small compared to unity.

In the present form the analysis has first been applied to the atmospheres of the major planets with the goal of explaining the structure of the observed mean zonal flows. The success of the model in describing the major features of zonal flows on Jupiter and Saturn suggests that it is likely to give a good description of convection in planetary cores if magnetic effects are taken into account. This is the basic idea of a new approach to model dynamos in rapidly rotating planetary cores which is discussed in more detail in the following section.

4. A semi-analytical model for planetary dynamos

The success of the analytical solution for convection in a rapidly rotating internally heated sphere (Busse, 1983a,b, which is referred to in the following by B83) has motivated us to develop a dynamo model on the basis of that solution. This model describes explicitly in the numerical scheme only the large scale magnetic field and the large scale motions, while analytical expressions are used for the small scale convection velocity field and the small scale magnetic field. Expressions for the latter have been obtained from the velocity field derived in B83 since boundary conditions for the small scale magnetic field can be neglected. At the same time the small scale component of the Lorentz force acting on the conv-velocity field has been taken into account as a perturbation in analogy to the buoyancy and viscous forces treated in B83.

The objective of the semianalytical model is to complement the fully numerical model discussed. Because of the limited parameter space accessible to the latter and its high cost of computations, the semi-analytical model offers considerable advantages in exploring the parameter domain relevant for planetary dynamos. On the other hand the assumptions underlying the analytical part of the model can be tested by comparison with the fully numerical model. Such a test will be of special importance when the two scales are not widely separated and an influence of boundary conditions for the small scale magnetic field may become noticeable.

In its kinematic aspects the semi-analytical model corresponds to the method of mean field electrodynamics developed by Steenbeck and Krause and Rädler (1966). But instead of introducing assumptions about the small scale velocity field, the latter is obtained directly as a solution of the equations of motion. This method permits the introduction of the Lorentz force without additional complications. The large scale component of the Lorentz force may even exceed the large scale Coriolis force. Only on the small scale the Lorentz force is taken into account as perturbation which is realistic in terms of planetary applications. Because of the analytical expression for the small scale velocity field, the small scale Lorentz force becomes a parameter of the problem. The main advantage of the new formulation is that the α_{ik} tensor, which is assumed to be isotropic in the earlier work, can be calculated from first principles.

In other respects the new model can be regarded as an extension of the analytical annulus model of Busse (1975). Owing to the replacement of the annulus geometry by the realistic configuration of a spherical shell the magnetohydrodynamic part of the problem becomes much more interesting. The large scale Lorentz force is no longer automatically balanced by the pressure gradient and the consequences of Taylor's (1963) constraint must be faced directly. Taylor has pointed out that the component of large scale flow that corresponds to a differential rotation $\omega(s)\ \underset{\sim}{k} \underset{\sim}{\times} \underset{\sim}{r}$ is driven most effectively by the Lorentz force. There $\underset{\sim}{k}$ denotes the unit vector in the direction of the axis of rotation and $\omega(s)$ is the angular velocity which depends only on the distance from the axis. The Lorentz force can be balanced only by viscous forces or Reynolds stresses for this particular component, while the Coriolis force is available for other components of large scale flow. While the problem pointed out by Taylor (1963) is usually regarded as a severe difficulty of nonlinear dynamo theory (see, for example, Malkus and Proctor, 1975; Ierley, 1982), the Taylor shear has turned out to be essential in our new dynamo model. Because the α_{ik} tensor is highly anisotropic in the asymptotic case of large rotation rates, no steady dynamos have been found in spherical shells (Busse and Miin, 1979). By taking into account the Taylor shear, however, Busse and Zhang (1985) have found steady dynamos. Presently the parameter dependence of these solutions is investigated and attempts are being made to derive a realistic spatial dependence of the α_{ik} tensor. This new type of dynamo has been called $\alpha^2\omega$-dynamo.

5. Papers resulting from research supported under NASA Grant NSG 7484

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E.W. Bolton	Ph.D. thesis - "Problems in Nonlinear Convection in Planar and Spherical Geometries," UCLA, 1985.