NEW CALORIMETRIC ALL-PARTICLE ENERGY SPECTRUM

John Linsley
Department of Physics and Astronomy
University of New Mexico, Albuquerque, NM 87131
USA

ABSTRACT

Both the maximum size N_m and the sea level muon size N_μ have been used separately to find the all-particle energy spectrum in the air shower domain. However the conversion required, whether from N_m to E or from N_μ to E, has customarily been carried out by means of calculations based on an assumed cascade model. It is shown here that by combining present data on N_m and N_μ spectra with data on 1) the energy spectrum of air shower muons and 2) the average width of the electron profile, one can obtain empirical values of the N_m to E and N_μ to E conversion factors, and an empirical calorimetric all-particle spectrum, in the energy range $2 \cdot 10^6 < E < 2 \cdot 10^9$ GeV.

1. Introduction. The great majority of shower particles are electrons, so it is natural that in the earliest air shower experiments the energy estimates were based on the number of electrons at the observation level. The first estimates (Auger 1939) were too low by about a factor of 4, because 1) the correction for longitudinal development was too conservative, and 2) the energy given to muons, neutrinos and low energy hadrons ($E_{\mu\nu h}$) was ignored. (At the energies in question $E_{\mu\nu h}$ amounts to some 35% of the whole.) The first difficulty stems from the average electron energy being comparatively low ($\sim E_c$, so that electrons are continually absorbed and regenerated. In order to estimate the energy deposited in the atmosphere one must use an integral signal such as the yield of atmospheric Cerenkov or fluorescent light, or else face the problem of accurately evaluating a correction factor that may be as large as a factor 20. By observing showers near maximum development (which generally means at a very high altitude) one can reduce the uncertainty in E_{EM} by minimizing the correction factor. Following this approach, one finds the all-particle energy spectrum by combining measurements of the N_m spectrum with estimates of the conversion factor E/N_m.

The alternative is to use shielded counters, which respond only to muons, and measure the N_μ spectrum, where $N_\mu(>E_\mu)$ is the 'muon size', the number of muons with enough energy to penetrate the shielding. This was done on a very large scale in the SUGAR experiment (Horton *et al.* 1983) and more recently in experiments at Chacaltaya, Tien Shan and Akeno (Kakimoto *et al.* 1981, Kirov *et al.* 1981, Hara *et al.* 1983). The difficulty with this method is that calculations relating N_μ to primary energy are relatively complex and model-dependent (see for example McComb *et al.* 1977 and Hillas 1981). Calculations of E/N_m are less affected by these difficulties, but they also require estimating the energy given to muons.

My purpose here is to show that by treating the experimental N_m and N_μ spectra *simultaneously*, using also experimental data on 1) the energy spectrum of air shower muons and 2) the width of the electron profile, one can obtain conversion factors which are almost entirely empirically based, and a new result on the all-particle energy spectrum which is almost entirely model independent.

2. Relation between maximum size and electronic energy. The energy dissipated by electrons is given by the track length integral

$$E_{EM} = (E_c/x_o)\int N(x)\,dx \ , \qquad (1)$$

where $N(x)$ is the number of electrons at depth x g/cm^2, E_c is the critical energy (=81 MeV in air according to Dovzhenko and Pomanskii 1964), and x_o is the radiation length (= 37.1 g/cm^2 in air according to conference paper HE4.4-4). Writing $E_{EM} = K(E_c/x_o)\sigma N_m$, where N_m is the height and σ is the width (standard deviation) of the average shower profile, what can be said about the value of K? Using a Gaussian distribution for N (surely quite a crude approximation), $K = \sqrt{2\pi} = 2.51$. Using a gamma distribution, $N = N_o\xi^q\exp(-q\xi)$ where $\xi=x/x_m$, which can be adjusted to fit very well (see conference paper HE4.4-5), the value of K ranges from 2.35 for q=6 (small showers) to 2.42 for q=12 (large showers). Thus it hardly varies at all, so adopting an average value for K, and substituting for (E_c/x_o), I obtain

$$E_{EM} = (\sigma/192)\ N_m \ , \qquad (2)$$

where σ is in g/cm^2 and E_{EM} is in GeV, accurate to 1-2%.* The profile width has been measured in the Yakutsk and Utah experiments (Grigoriev *et al.* 1983, Baltrusaitis *et al.* 1985), but only for $E \sim 10^9$ GeV. The energy dependence is expected on theoretical grounds to take the form $\sigma^2 = A + BD_{10}\log E$, where D_{10} is the elongation rate per decade, ≅ 65 g/cm^2 (Linsley and Watson 1981) and B is a characteristic length of order 60-70 g/cm^2 (conference paper HE4.4-5). Using the Yakutsk-Utah data to fix the value of A, one obtains $\sigma^2 = 1.1\cdot10^4 + 4.2\cdot10^3\log E$ (Linsley 1983),† and finally, by substitution in (2),

$$E_{EM} \cong 0.71\ N_m^{1.025} \ . \qquad (3)$$

3. Relation between muon size and $E_{\mu\nu h}$. There is good agreement among independent measurements of the energy spectrum of air shower muons (Atrashkevich *et al.* 1983 and references therein). This spectrum is quite hard; almost half of the observed energy is given to particles with individual energies above 30 GeV. Over the range of shower energies where it has been studied ($3\cdot10^5$-10^8 GeV), the shape of this spectrum is invariant; hence the total energy of the observed muons is proportional to N_μ(>1GeV), the number of muons (at sea level) with energy > 1 GeV, where the proportionality constant equals 10.0±.5 GeV. To obtain the energy given to neutrinos the observed muons are propagated backward to a production spectrum. In the air shower region it is found that $E_\nu \sim 0.4\ E_{\mu,obs}$, where E_ν includes both ν_μ and ν_e. This result checks with a forward propagation calculation by Hillas (1981). Experiment based estimates of E_h, the energy deposited by low energy hadrons, range from $0.8\ E_{\mu,obs}$ (Greisen 1956) to $0.3\ E_{\mu,obs}$. Adopting $E_h \sim 0.4\ E_{\mu,obs}$ as a conservative estimate, the total non-electronic contribution is obtained:

$$E_{\mu\nu h} = (18\ {}^{+3.5}_{-1.5}\ \text{GeV})\cdot N_\mu(\text{>1GeV})_{\text{sea level}} \ . \qquad (4)$$

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* An alternative form which may sometimes be more convenient is $E_{EM} = (x_{hm}/428)\ N_m$, where x_{hm} is the full width at half maximum (Linsley 1981).
† In the energy range of interest here, the simpler formula $\sigma = 130 + 10.2\log E$ is equivalent.

Table 1. Muon size for a given intensity from various experiments.

integral intensity $(m^2 sr\ s)^{-1}$	N_μ (>1GeV)	muon threshold (GeV)	Ref.
10^{-6}	2.3×10^4	10.0	VK*
10^{-7}	6.5 "	"	"
10^{-8}	1.6×10^5	"	"
"	1.6 "	1.0	Ha@
10^{-9}	3.8 "	10.0	VK*
"	4.0 "	1.0	Ha@
10^{-10}	1.0×10^6	"	"
10^{-11}	2.5 "	0.22	L*@
"	2.0 "	0.70	Dm*
"	3.5 "	0.75	Ho*
"	2.4 "	1.0	Dx
"	2.6 "	"	Ha@
10^{-12}	5.7 "	0.70	Dm*
"	9.1 "	0.75	Ho*
10^{-13}	1.6×10^7	0.70	Dm*
"	2.4 "	0.75	Ho*
10^{-14}	6.4 "	"	"
10^{-15}	1.7×10^8	"	"
10^{-16}	4.5 "	"	"
10^{-17}	1.2×10^9	"	"

* adjusted to 1 GeV threshold
@ adjusted to sea level

Neither this result nor the one expressed by (3) depends on any assumption about the primary composition; they are properties of cosmic rays *as they occur*, in this energy range, at the solar system. As an experimental result, (4) applies to the energy range given above, $3 \cdot 10^5$-10^8 GeV. Extrapolation up to 10^{11} GeV is justified unless there occurs some radical change affecting the production of *very high energy* muons and neutrinos.

4. Calorimetric all-particle energy spectrum. Data on the N_μ and N_m spectra are summarized in Tables 1 and 2, in inverse form (N_μ and N_m as functions of integral intensity). The (inverse) all-particle energy spectrum is obtained by adding together E_{EM} from (3) and $E_{\mu\nu h}$ from (4), using tabulated values of N_μ and N_m for the same intensity, in the range 10^{-6}-$10^{-12}/m^2 sr\ s$ where there are reliable data for both N_μ and N_m. The result, changed to differential form, is shown in Fig. 1. In case of Table 1 the various N_μ values for a given intensity were averaged; in case of Table 2, the later results at Chacaltaya were used. There is good agreement with the Yakutsk energy spectrum in this range (Efimov and Sokurov 1983), and with Haverah Park results (Cunningham *et al.* 1980). The present result supports a rather high location, nearly 10^7 GeV, for the transition region where the change of slope (knee) occurs.

5. Other results: conversion factors. As I showed previously, this derivation of the energy spectrum yields as by-products factors for converting N_m and N_μ separately to primary energy, and also yields the fraction of primary energy given to electrons, vs energy (Linsley 1983). These results are shown in Fig. 2. The low value found for E/N_m, 1.3±.2, confirms an important prediction by Hillas (1983). An apparent conflict between results from Chacaltaya and from lower elevations is resolved. The earlier values of N_m (La Pointe *et al.* 1968) were somewhat too low, but energies were about correct because the conversion factor was somewhat too high. The later values of N_m are more nearly correct, but the energies were too high because the conversion factor was much too high. Although here the conversions, N_m to E and N_μ to E, have been treated symmetrically, the energy dependence of the N_μ-E 'conversion factor' makes this inconvenient in practice. Convenient formulae for representing the

N_μ data in Fig. 2 and Table 1 are:

$$N_\mu(>1\text{GeV})_{s.l.} = E(\text{GeV})^{0.835}/6.8, \quad (5)$$

$$J[>N_\mu(>1\text{GeV})]_{s.l.} = 3\cdot10^4 N_\mu^{-2.4}, \quad (6)$$

where J is in $m^{-2}sr^{-1}s^{-1}$.

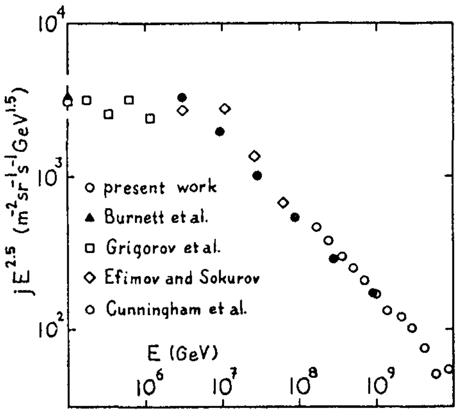

Fig. 1. All-particle Energy Spectrum.

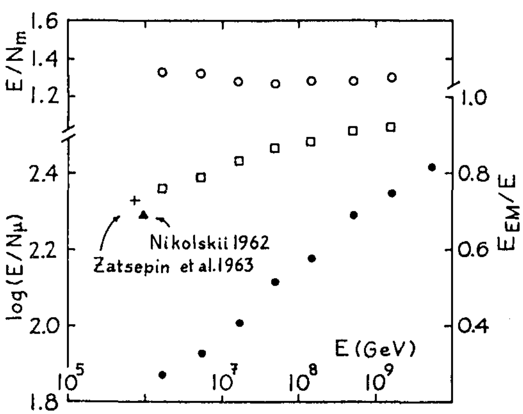

Fig. 2. Other Results. Open circles E/N_m, squares E_{EM}/E (r. h. scale), filled circles $\log(E/N_\mu)$.

Table 2. Maximum size for a given intensity from various experiments

integral intensity $(m^2 sr\ s)^{-1}$	N_m	obs. depth g/cm^2	Ref.
10^{-5}	3.5×10^5	210	AI
10^{-6}	1.3×10^6	"	A
"	1.3 "	540	LP
10^{-7}	4.2 "	"	"
10^{-8}	1.15×10^7	"	"
"	1.30 "	"	K
10^{-9}	3.6 "	"	LP
"	4.0 "	"	K
10^{-10}	1.05×10^8	"	LP
"	1.20 "	"	K
10^{-11}	3.2 "	"	LP
	4.0 "	"	K
10^{-12}	1.26×10^9	"	"
"	$5. \times 10^8$	835	L
10^{-13}	1.6×10^9	"	"
10^{-14}	$1. \times 10^{10}$	"	"

References. ANTONOV and IVANENKO 1975, Proc. 14th ICRC 8, 2708 (AI); ANTONOV *et al*. 1983, Proc. 18th ICRC 6, 19 (A); ATRASHKEVICH *et al*. 1983, Proc. 18th ICRC 11, 229; AUGER 1939, Rev. Mod. Phys. 11, 288; BALTRUSAITIS *et al*. 1985, Phys. Rev. Lett. 54, 1875; CUNNINGHAM *et al*. 1980, Ap. J. 236, L71; DIMINSTEIN *et al*. 1979, Proc. 16th ICRC 8, 122 (Dm); DIXON *et al*. 1974, J. Phys. A 7, 1010 (Dx); DOVZHENKO and POMANSKII 1964, Sov. Phys. JETP 18, 187; EFIMOV and SOKUROV 1983, Proc. 18th ICRC 2, 123; GREISEN 1956, *Progress in Cosmic Ray Physics* (North-Holland: Amsterdam); GRIGORIEV *et al*. 1983, Proc. 18th ICRC 6, 204; HARA *et al*. 1983, Proc. 18th ICRC 9, 198 (Ha); HILLAS 1981, *Proc. Paris Workshop on Cascade Simulations* (TCAST: Albuquerque) pp. 3, 13; 1983, *Proc. Cosmic Ray Workshop, Univ. Utah* (Bartol Res. Foundation: Newark) p. 1; HORTON *et al*. 1983, Proc. 18th ICRC 6, 124 (Ho); KAKIMOTO *et al*. 1981, Proc. 17th ICRC 11, 254 (K); KIROV *et al*. 1981, Proc. 17th ICRC 2, 109; LA POINTE *et al*. 1968, Can. J. Phys. 46, S68 (LP); LINSLEY and WATSON 1981, Phys. Rev. Lett. 46, 459; LINSLEY 1973, Proc. 13th ICRC 5, 3202 and 3205 (L); 1981, *Proc. Paris Workshop on Cascade Simulations* (TCAST, Albuquerque) p. 23; 1983, Proc. 18th ICRC 12, 135; McCOMB *et al*. 1977, J. Phys. G: Nucl. Phys. 5, 1613; VERNOV and KHRISTIANSEN 1968, Can. J. Phys. 46, S197 (VK).