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Energy CONSERVATION

EVALUATION OF HALF WAVE INDUCTION MOTOR DRIVE FOR USE IN PASSENGER VEHICLES

Final Report

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March 21, 1985

Work Performed Under Contract No. AI01-78CS54209

University of Missouri-Columbia
Columbia, Missouri

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1.0 ABSTRACT

This report describes research performed at the University of Missouri-Columbia to devise and design a lower cost inverter-induction motor drive for electrical propulsion of passenger vehicles. A two-phase inverter-motor system is recommended. It is predicted to provide comparable vehicle performance, improved reliability and nearly a 10% cost advantage for a high production vehicle because of the reduction in total parts count, decreased total rating of the power semiconductor switches and somewhat simpler control hardware compared to the conventional three-phase bridge inverter-motor drive system. The major disadvantages of the two-phase inverter-motor drive are that the two-phase motor is larger and more expensive than a three-phase machine, the design of snubbers for the power switches is difficult because motor lead and bifilar winding leakage inductances produce higher transient voltages, and the torque pulsations are relatively large because of the necessity to limit the inverter switching frequency to achieve high efficiency.

The conclusions of this research are based on analyses and simulations. An actual model of the two-phase system must be constructed and evaluated to prove the advantages claimed for the proposed system. The most challenging engineering design task will be to design the inverter, motor and snubber circuits to minimize transient voltages with high system efficiency.

2.0 EXECUTIVE SUMMARY

This report describes research performed at the University of Missouri-Columbia to devise a lower cost induction motor drive for use in electrical propulsion of passenger vehicles. The work was done during the period from January through August of 1984 in two phases.

Phase I - Concept and Feasibility Study
(January-May)

Phase II - Detailed Design and Performance Analysis
(June-August)

As a result of Phase I, three preferred candidate systems were identified.

A. Half Wave Three-Phase Inverter-Motor (three power switches, three pairs of bifilar wound stator windings)

B. Four Switch Three-Phase Inverter-Motor (four power switches - two legs of conventional three-phase inverter, conventional three-phase machine, motor line returned to center-tap of dc source)

C. Two-Phase Inverter-Motor (four power switches, two pairs of bifilar wound stator windings)

After a detailed analysis of the candidate systems at the beginning of Phase II, the two phase inverter-motor was selected as the lowest cost approach to meet the electrical passenger vehicle system requirements.

Detailed analyses and digital computer simulations were carried out to design the inverter circuit and the two-phase induction motor required. The two-phase system has the following advantages over the conventional three-phase inverter-motor drive:

1. The number of power semiconductor switching devices is reduced from six to four.
2. The current rating of each power switch is 65% of that required for each switch in the equivalent three-phase system.
3. The total number of components is estimated to be about 70% of the number required for the three-phase circuit. This also implies higher reliability and lower manufacturing asembly cost.
4. The motor transient temperature rise is reduced due to the increased volume of copper.
5. Somewhat simpler control hardware is possible because of the common emitter circuit so that isolation is not required in the base drive circuits.
6. The total system cost is estimated to be about 90% of that for the three-phase system.

The overall efficiency is almost the same for the two-phase system and the three-phase system - approximately 91% at the 12kw, 80% speed operating condition.

The major disadvantages of the two-phase approach are increased motor size and cost, more difficult snubber design for transient voltage suppression, and greater torque pulsations.

For a high production electric passenger vehicle, the two-phase inverter-motor system developed on this research should provide significant cost advantages for the electric vehicle application when compared to the conventional three-phase bridge inverter-induction motor drive. It is important to note that the conclusions of this research are based on analysis and simulations. An actual model of the two-phase system must be constructed and evaluated to prove the advantages claimed for the proposed inverter-motor system. The most challenging engineering task will be to design the inverter, motor and snubber circuits to minimize transient voltages while retaining high system efficiency.

3.0 TWO-PHASE BIFILAR INDUCTION MOTOR/INVERTER SYSTEM

3.1 Specifications

The inverter-motor drive system requirements are defined by the contractual motor specifications in Appendix A. These system requirements are as follows:

Inverter input:

240V battery

Motor output:

a. Shaft power output

   1) 30kW for 1 hour with 20% duty cycle at maximum speed

   2) 60kW for 3 minutes with 30/% duty cycle at 80% of maximum speed

   3) 16kW for 3 minutes with 30% duty cycle at 10% of maximum speed

b. Maximum motor speed between 8,000 and 16,000 rpm

c. Two or four pole motor, totally enclosed, convection cooled with 40°C maximum ambient air and 4000 hours operating life at 80% of maximum speed with 12kW output

Fig. 1 is a graphical representation of the motor requirements. It illustrates one method of satisfying the specifications. However, later in this section, an alternate motor design is described which results in a reduction in motor size and weight below that implied by

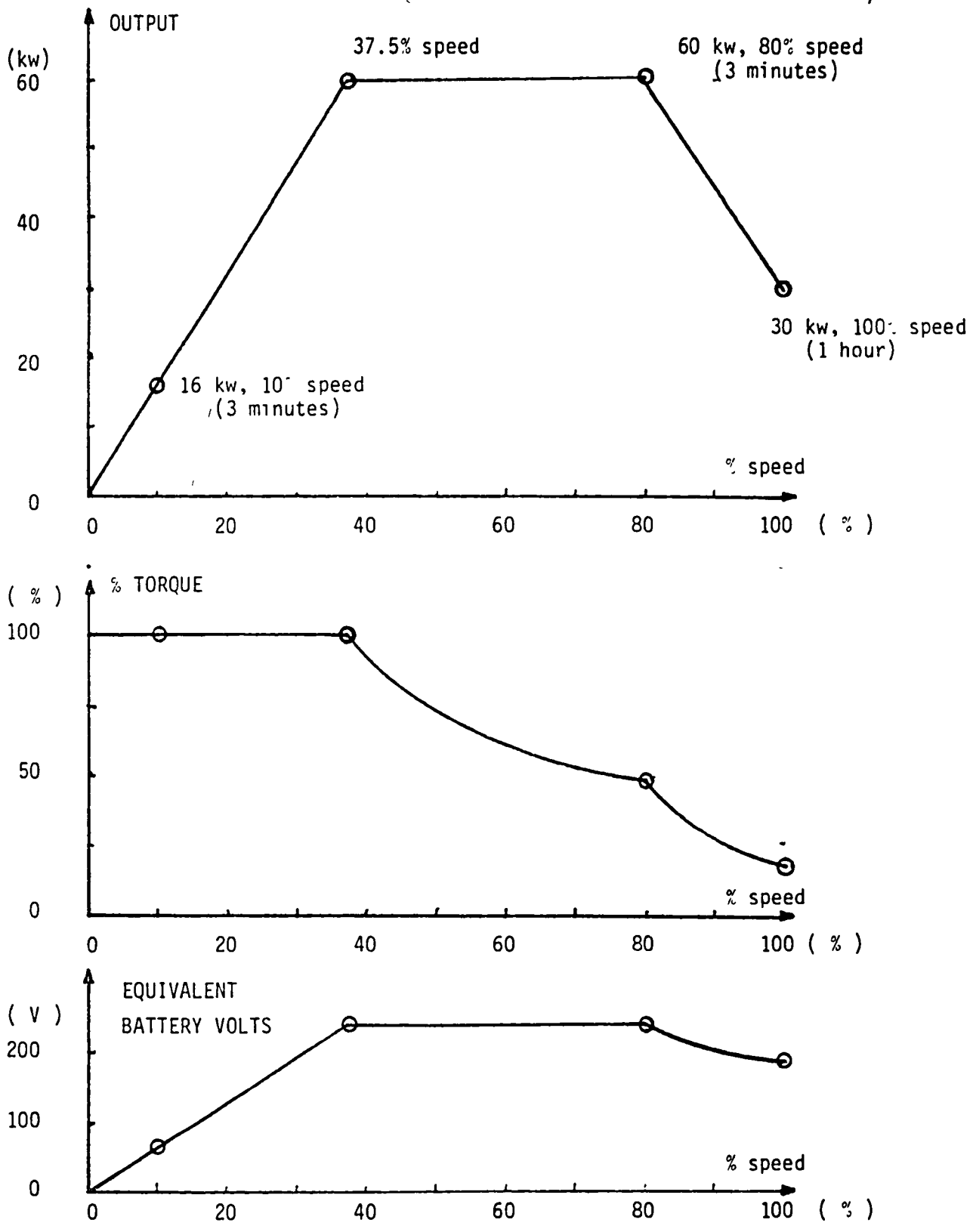

Fig. 1 Graphical Representation of Motor Requirements

Fig. 1. For Fig. 1, it is assumed that a constant volts/Hz is applied to the motor from zero speed to 37.5% speed. In this range the air-gap flux is essentially constant, implying a constant torque capability and a linearly increasing output power with speed. At 37.5% speed, the maximum voltage available is required, and so the motor voltage is assumed constant from 37.5% speed to 80% speed. It should be noted that whenever the equivalent battery voltage is less than 240V, it is assumed that PWM control of converter output voltage is used to provide this voltage reduction. Since the output power requirement is only 30kW at 100% speed, it is possible to reduce the motor applied voltage from 80% to 100% speed. The capability implied by Fig. 1 would meet the specification requirements since the three specified operating points lie on the power vs. speed curve, which defines the power capability at each speed.

The inverter-motor system also is to be designed to minimize the function

$$F = \text{Cost} + 10 \times \text{Weight} - 120 \times \text{Efficiency} \qquad (1)$$

where each of the parameters on the right side of this relationship is for the total inverter and motor system expressed as follows:

Cost = retail cost in dollars

Weight = total weight in kilograms

Efficiency = % efficiency at 80% of maximum motor speed with 12kW output

As a result of this research, a two-phase inverter/motor drive is proposed. The remainder of this report describes the system design to meet the specifications. In addition, the report explains why the system proposed is considered the preferred induction motor approach for this application.

3.2 System Description

3.2.1 Circuit Configuration and Operation

During the first phase of the research, several alternative inverter-induction motor combinations were considered. The following preferred candidate systems were selected for more detailed analyses:

A. Half-wave inverter driven three-phase bifilar wound induction motor (Appendix B, Fig. B-1).

B. Four switch inverter driven three-phase induction motor (Appendix B, Fig. B-2).

C. Two-phase inverter driven induction motor with two pairs of bifilar wound stator windings as shown in Fig. 2.

As a result of an evaluation of each of these systems, including simulations to determine the peak current in the power switches at the 60kW - 80% speed operating point, the two-phase approach shown in Fig. 2 was selected. It has two significant advantages compared to the alternative candidate systems: (1) the lowest total power switch rating (the power switch peak current times its peak voltage times the

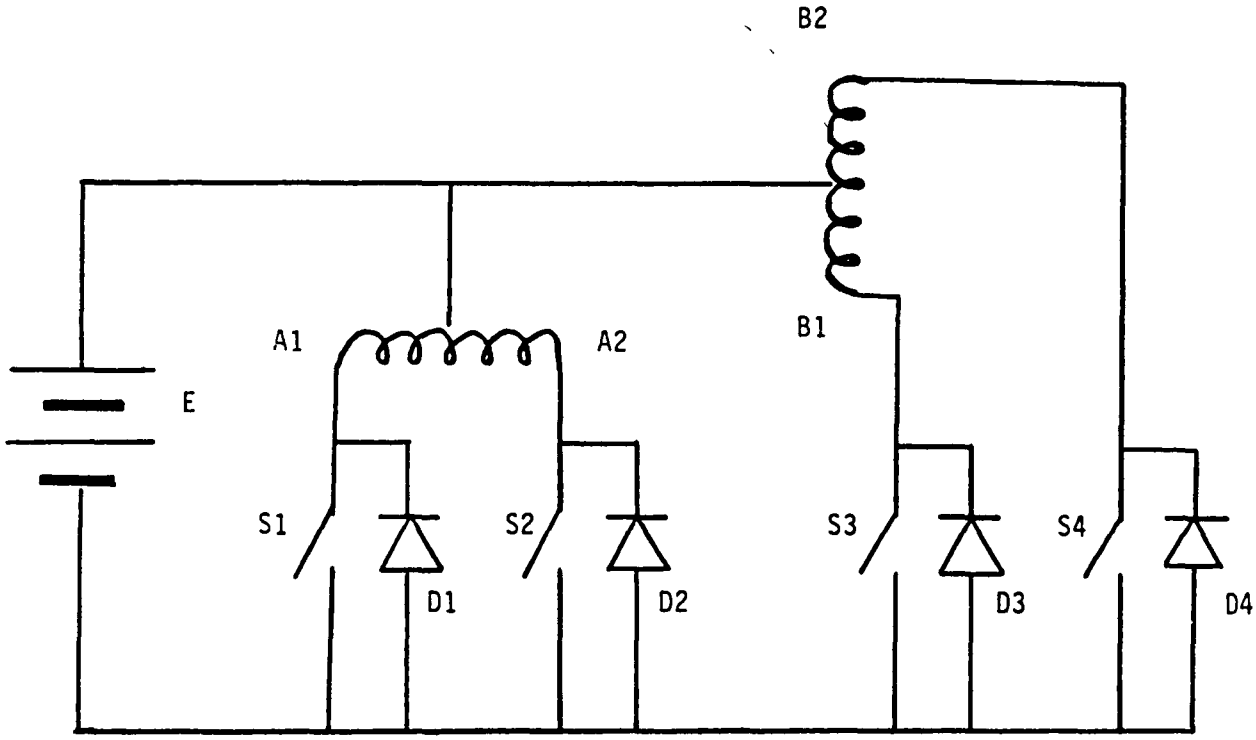

Fig. 2 Two-phase Bifilar Induction Motor/Inverter

number of switches required); (2) it is one of the two alternatives (Fig. B-1-b is the other) which may be arranged to have a common emitter connection with transistor switches or common cathodes if thyristors are used. This second advantage makes it possible to simplify the control hardware since gate or base drive isolation is not required. The primary disadvantages of the scheme in Fig. 2 are that the motor size and weight is increased by 10%-20% compared with the three-phase bridge system (Fig. B-3), the power switch snubber design is more difficult since line and bifilar winding leakage inductances produce additional transient voltages, and there are relatively large torque pulsations because of the necessity to limit the inverter switching frequency to achieve high efficiency. Appendix C contains an idealized comparative analysis of the two-phase and three-phase inverter arrangements.

The two-phase bifilar induction motor has only two stator phases, each of which includes a pair of tightly magnetically coupled coils. Fig. 2 shows the circuit configuration. Fig. 3 is an alternate circuit. However, Fig. 2 is the preferred approach since it simplifies the control hardware as mentioned previously. With transistor switches, all emitters are common, and all cathodes are common with thyristor switches. Thus, isolation is not required for the base drive or gate drive circuits.

Four switching devices and four free-wheeling diodes are required in the circuit of Fig. 2. The center-taps of

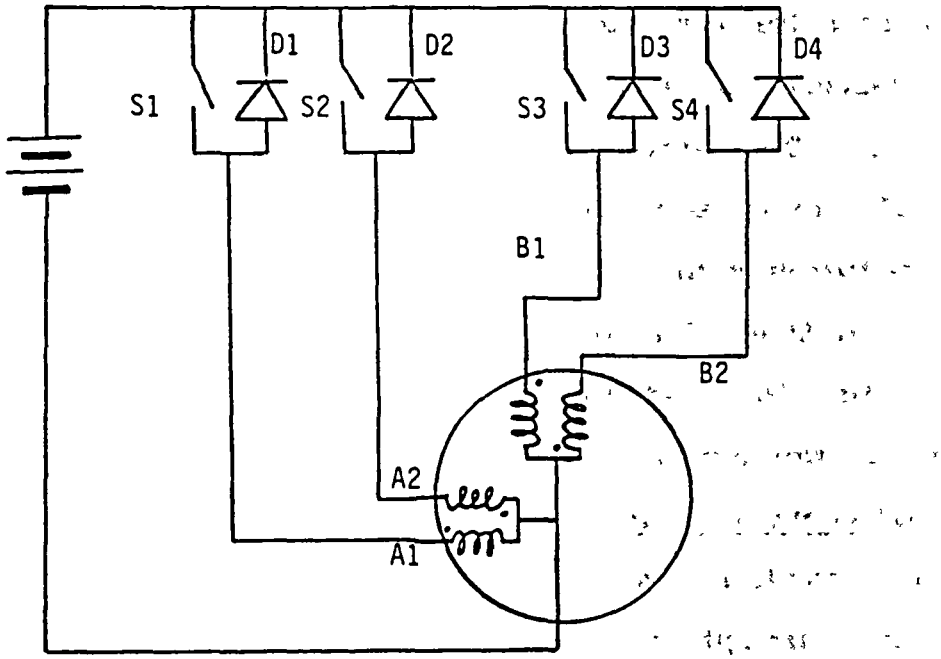

Fig. 3 Alternate configuration for Two-phase Bifilar Induction Motor/Inverter

the stator windings are connected to the positive side of the battery. The operation of the inverter is as follows. Suppose switch S1 is closed, and current is flowing from the battery through stator winding A1. When S1 is opened and S2 is closed, the current through stator winding A1 is transferred to the bifilar winding A2. As a result, free-wheeling diode D2 conducts, and the stored energy in the winding returns to the battery. After this diode current reduces to zero, the current starts to flow through switch S2. The current in stator windings B1 and B2 is controlled in a similar manner. However, the conduction intervals for switches S3 and S4 are displaced 90 electrical degrees from those for S1 and S2. Controlling the two-phase currents in this fashion, a rotating magnetic flux is produced in the machine.

3.2.2 PWM Control of Inverter

PWM control is employed in the inverter system so that the current through the stator windings can be shaped to the optimum waveform. In the light load case, the current is shaped to approach a sinusoidal waveform as shown in Fig. 4-a. The switching pattern is controlled such that the stator current always stays within a given band from the sinusoidal function. With heavy load, a current waveform approaching a square wave is delivered to the motor to provide maximum power with minimum peak motor current as shown in Fig. 4-b.

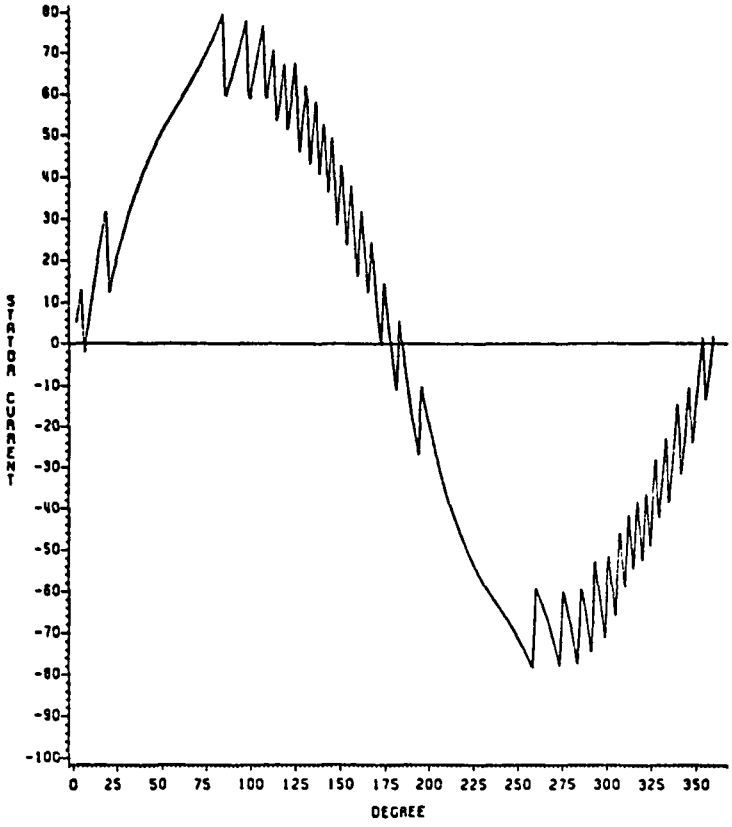

Fig. 4-a Current waveform for light load

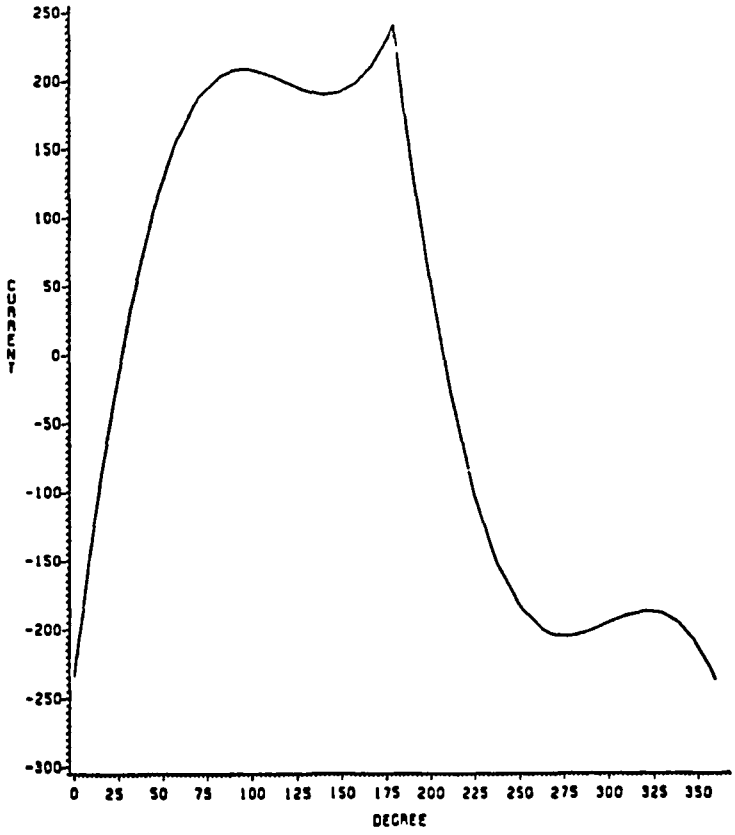

Fig. 4-b Current waveform for heavy load

With a microprocessor controller, it is also possible to control the PWM voltage waveform and the applied frequency to maximize efficiency at the 80% speed, 12 kW operating point. This might be accomplished by storing the required control program in memory, or possibly by an on-line efficiency optimization technique.

3.3 Motor Design

3.3.1 Operating Points

As indicated in section 3.1, three specific operating points are required.

16kW output at 10% speed (3 minutes, 30% duty)

60kW output at 80% speed (3 minutes, 30% duty)

30 kW output at 100% speed (1 hour, 20% duty)

The two-phase bifilar induction motor proposed must satisfy the operating conditions at each of these points.

3.3.2 Nature of Induction Motor

The polyphase induction motor maximum power output as a function of inverter frequency is given by

$$P_{max} = C_1 \left(\frac{V}{f}\right)^2 f \qquad (2)$$

where

C_1 $\triangleq$ constant $= \frac{3P}{8\pi L_{1r}}$

P $\triangleq$ number of poles

L_{1r} $\triangleq$ rotor leakage inductance

V $\triangleq$ rms value of stator voltage

f $\triangleq$ source frequency

When an induction motor is connected to an inverter, a similar maximum output is achieved, neglecting the effects of harmonics. Since the output voltage and frequency of the inverter may be adjusted, the maximum output power given by (2) can be changed, depending on the speed control strategy. If V/f is maintained constant over the low frequency range, the maximum power is proportional to the frequency (or rotor speed). After maximum stator voltage V is reached, the maximum power decreases proportional to 1/f since V is constant and f increases. In Fig. 5, generalized curves for the maximum power P_{max}, rated power P_R, stator current I_p, stator voltage V, and slip frequency f_s are plotted as a function of rotor speed.

The rated output power as a function of slip frequency f_{SR}, which may change at different source frequencies, is

$$P_R = C_2 (\frac{V}{f})^2 f \; f_{SR} \qquad (3)$$

where

$C_2 \triangleq$ constant $= \frac{3P}{2R_2}$

$R_2 \triangleq$ rotor resistance

$f_{SR} \triangleq$ rated slip frequency at different source frequencies

3.3.3 Motor Design Procedure

For these discussions, the following variables are defined:

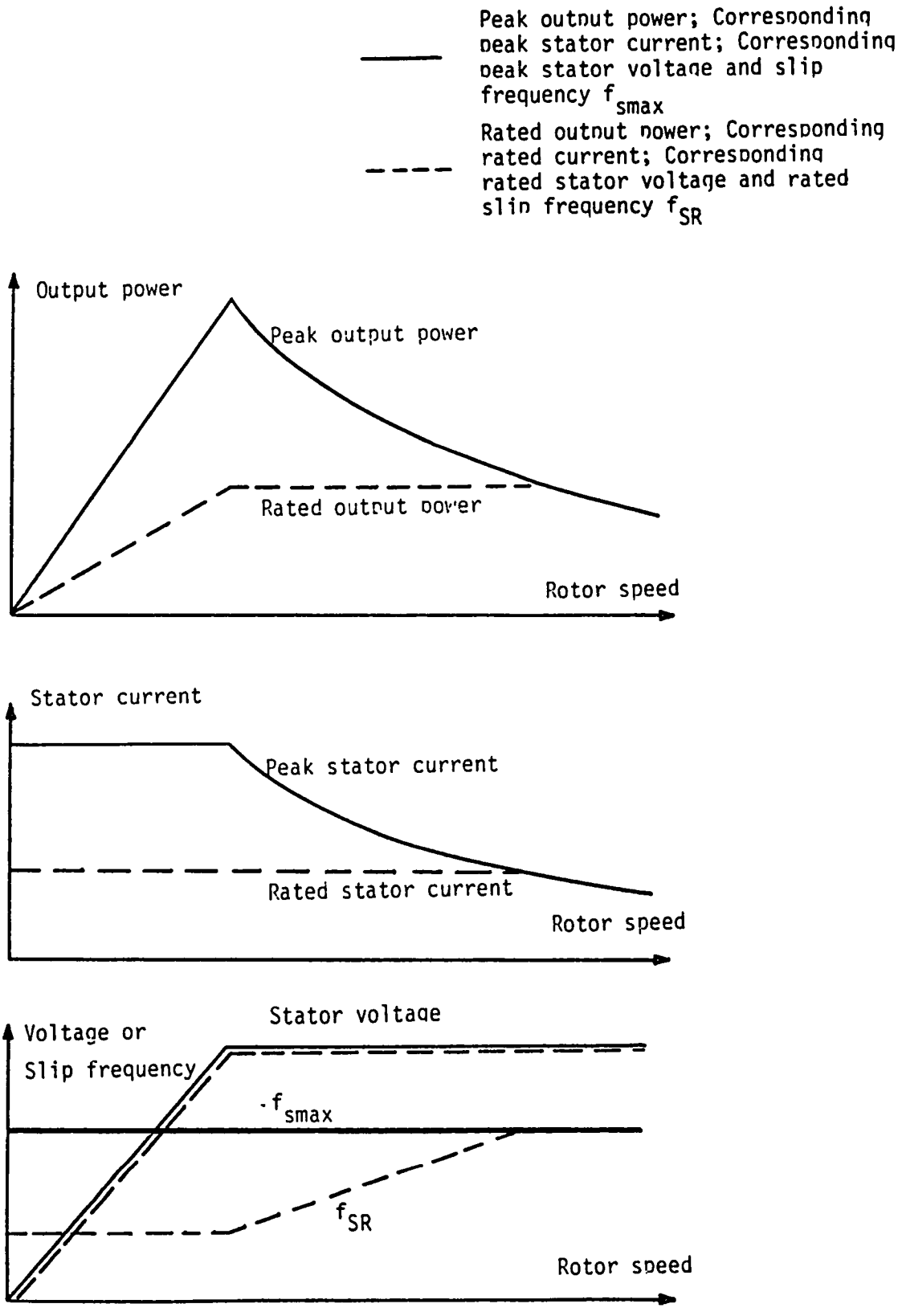

Fig. 5 Generalized motor characteristics

Base frequency f_B $\triangleq$ the design frequency for the motor (the frequency when V reaches a maximum)

Base output P_B $\triangleq$ the rated output at the base frequency

Base current I_B $\triangleq$ the current which produces rated output at the base frequency

Base voltage V_B $\triangleq$ the motor voltage at base frequency

Peak output power P_P $\triangleq$ the maximum output power at f_B

Peak current I_P $\triangleq$ the current which achieves P_P

There is a simple relation between these variables:

$$\left.\frac{P_P}{P_B} = \frac{I_P}{I_B} = \frac{f_{smax}}{f_{SR}}\right| \quad \text{at } f_B \qquad (4)$$

where f_{smax} is the maximum slip frequency which produces P_P at f_B

Fig. 6 shows three possible motor designs. One possible design to satisfy the specifications is shown by the B curves in Fig. 6, where the maximum power output is 16kW at 10% speed and 60kW at 80% speed. However, this is not the optimum motor design, because the peak current is

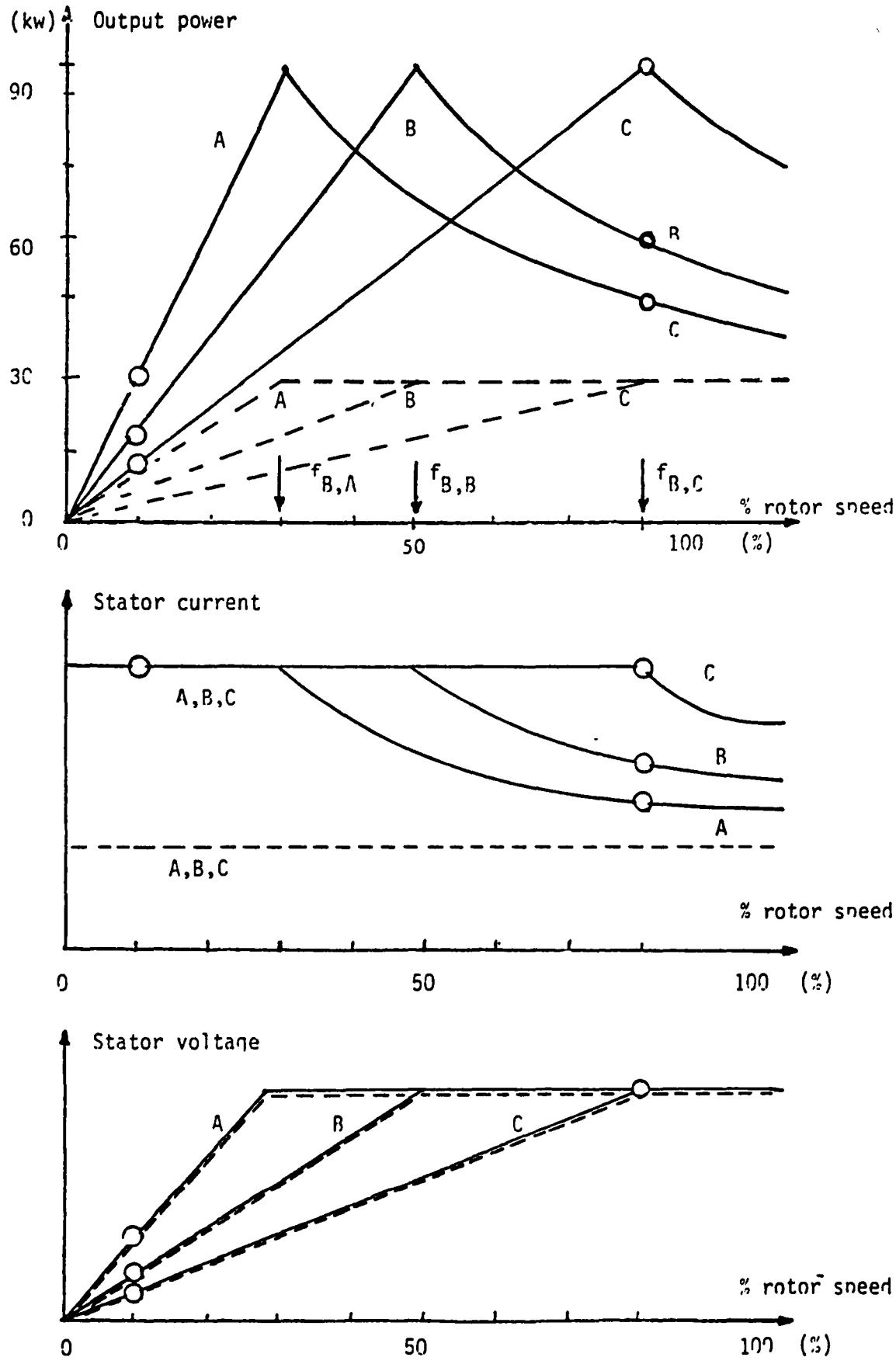

Fig. 6 Three-phase possible machine designs

excessive at low speed and the motor weight is not a minimum. It is very important to have minimum peak current since the power switch device ratings are determined by the maximum current through the devices. The peak current can be reduced for a given output power by lowering the base frequency, but this increases the size of the machine. Therefore, the base frequency should be increased, resulting in a smaller machine. Then, the voltage should be increased at low frequency to saturate or overexcite the machine. This permits higher power output with reduced peak current and minimum motor size and weight. The optimum machine design should have nearly equal peak currents at 10% speed.

Constant V/f operation means nearly constant air-gap flux. Thus, by increasing the V/f ratio, oversaturated air-gap flux can be achieved. Since output power is proportional to the product of air-gap flux and rotor current, larger air-gap flux helps to decrease the rotor current, and as a result, lower stator current can deliver the same output power. This mechanism is shown in Fig. 7. Case I on this figure corresponds to case B on Fig. 6. In the digital simulation program, this saturation effect can be easily checked by sensing the magnetizing current. Thus, by a trial and error method, oversaturation of air-gap flux is simulated, and stator current can be reduced until the allowable saturation flux is reached.

Also, when the motor is operated at the overloaded output, the temperature rise due to the greater loss is a

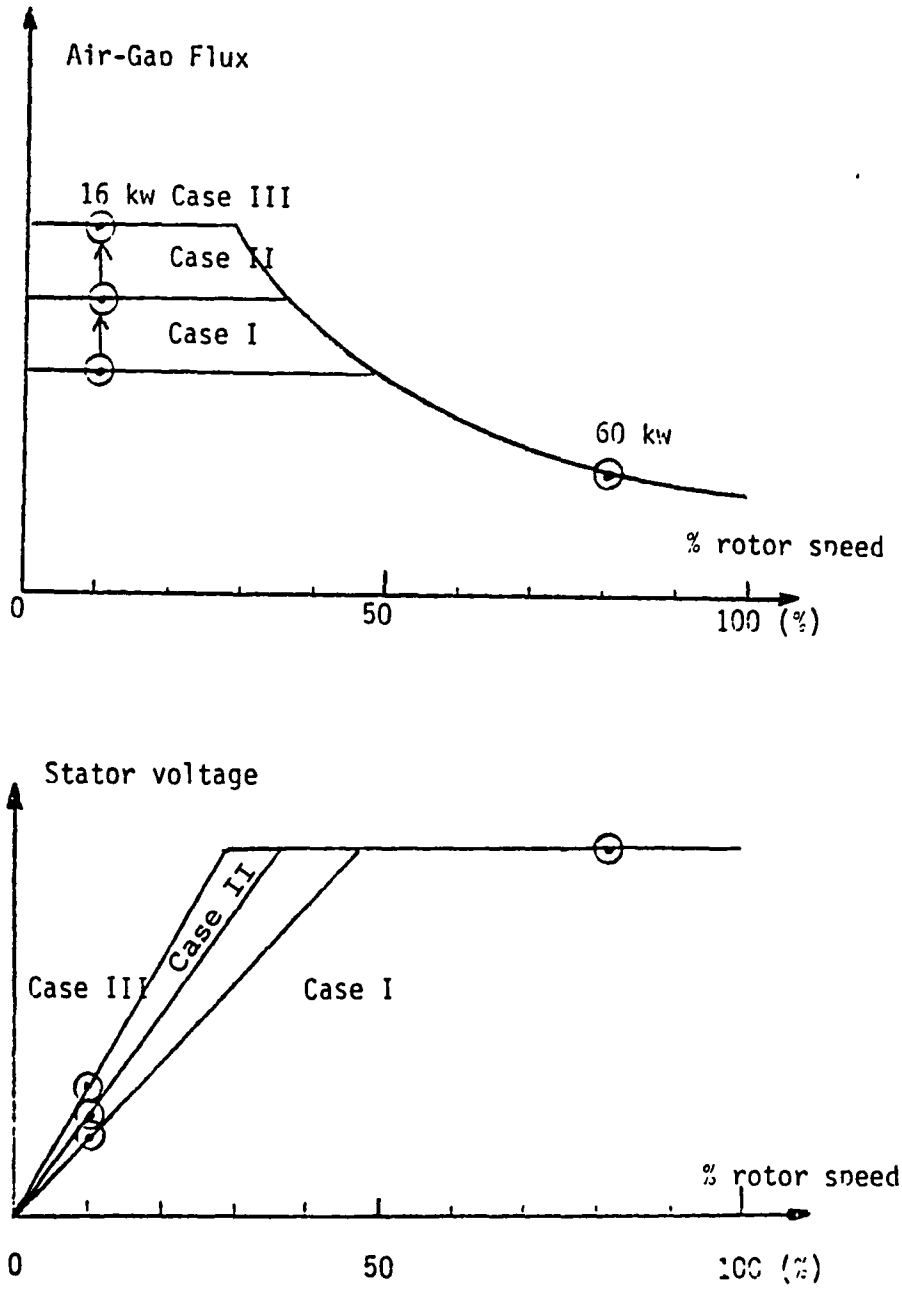

Fig. 7 Mechanism of air-gap flux saturation

very important factor for machine design. In summary, the induction motor design is optimized when the following conditions are satisfied:

1. 60kW output is designed to be maximum output power at 80% speed.
2. 16kW output at 10% speed is achieved by over-saturation of air-gap flux so that the peak current is almost the same as that for 60kW at 80% speed.
3. 30kW output is equal to the rated output at 100% speed.
4. The temperature rise at the overload condition is within an allowable range.
5. The weight of the motor is minimized.

3.3.4 Computer-Aided Design

The design of the two-phase induction motor involves four steps. A flow chart is given in Fig. 8.

Step 1: Choose the Base Operating Point

Since a machine can be almost uniquely designed at a base operating point (base frequency, base voltage and base output power), the base point should be selected carefully so that the designed machine is the optimum one satisfying all required operating conditions. Usually one must come back to Step 1 after checking the given conditions in Step 3.

Maximum frequency and pole number

For a given power rating, the size and the weight of an ac machine decrease as the rated frequency increases. On

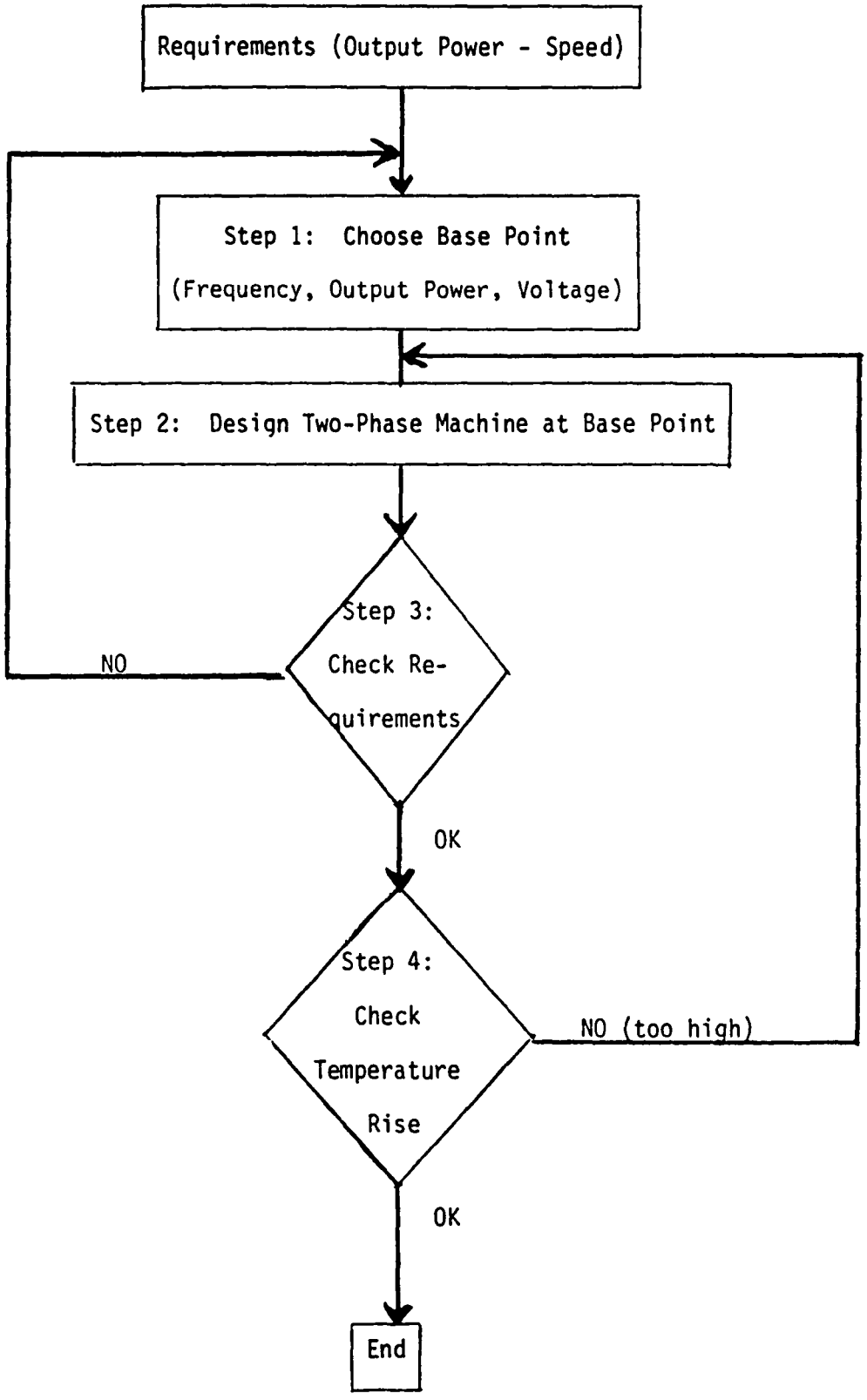

Fig. 8 Design flow chart

the contrary, the iron loss increases at high frequency, and the mechanical bearings become very expensive at high rotor speeds. A rotor maximum speed of 12,000 rpm is chosen because this is believed to provide a reasonable practical trade-off between losses, weight and bearing cost. For a given power rating and maximum speed, as the pole number increases, the maximum ac source frequency increases, which causes greater iron loss and switching loss. Thus, a two-pole machine is chosen, which sets the maximum ac source frequency at 200Hz.

Base voltage, base frequency and base output power

The base operating condition to establish the motor design is chosen as the 30kW output, 40% speed point shown in Fig. 9. The base voltage is selected as follows. With the PWM technique, the peak value of the fundamental component of the output voltage can change from zero to a maximum of $(4/\pi)(V_{DC})$, where V_{DC} is the battery voltage of 240V. The motor is designed such that the applied voltage as a function of speed changes as shown in Fig. 10. The V/Hz (Volts/ Hertz) is constant from zero to 40% speed. From 40% to 80% speed, the V/Hz is reduced with speed as the voltage is linearly increased to a maximum of 306V peak at 80% speed. Then the voltage is fixed at the maximum value from 80% speed to 100% speed. A base voltage of 241V peak at 40% speed is chosen by an iterative technique. Since, roughly speaking, the induction motor output power is proportional to the square of the applied voltage and inversely proportional to the applied frequency $(P \propto V^2/f)$,

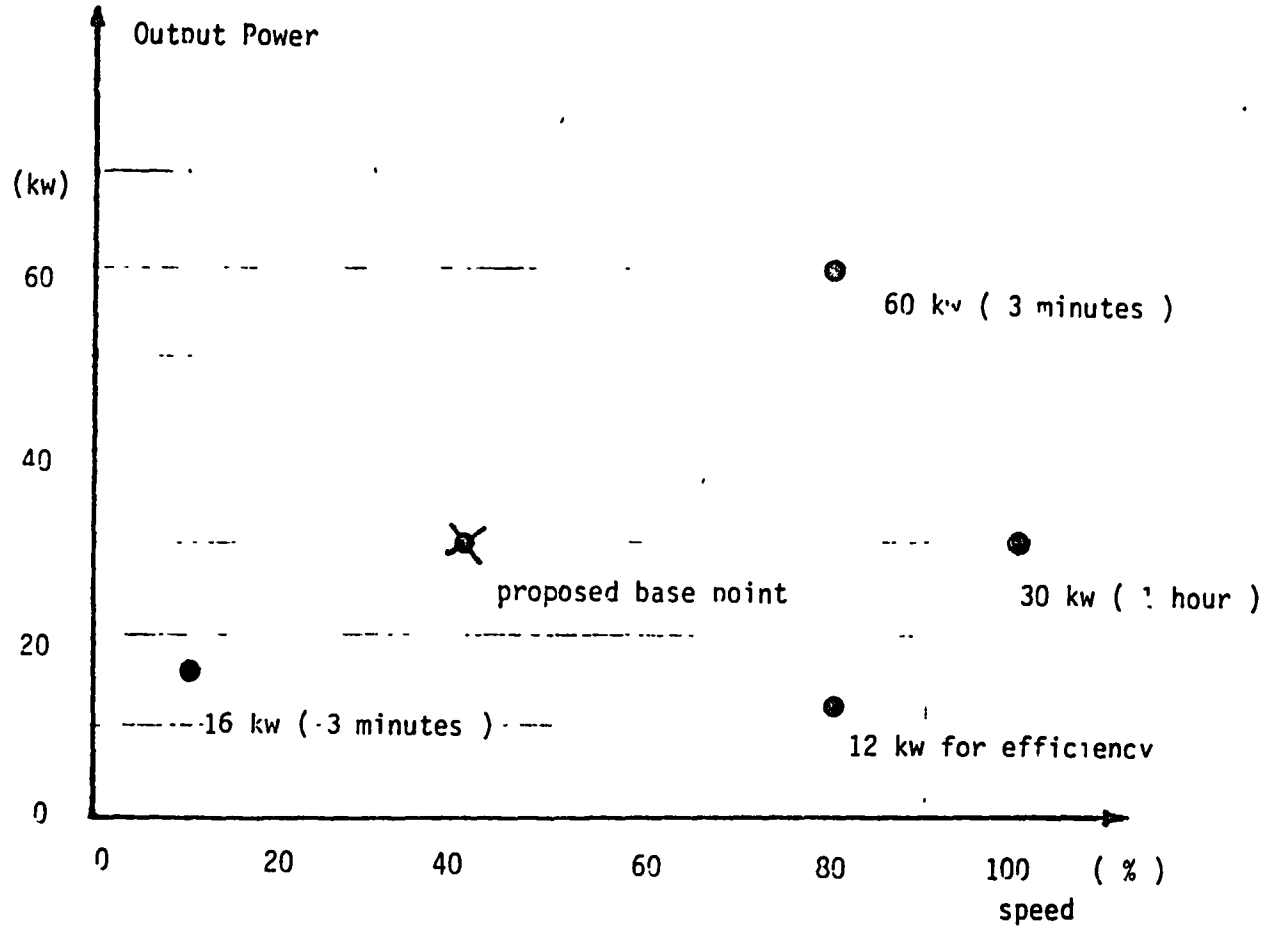

Fig. 9 Output power-speed requirement

Note: ● are specification operating points.

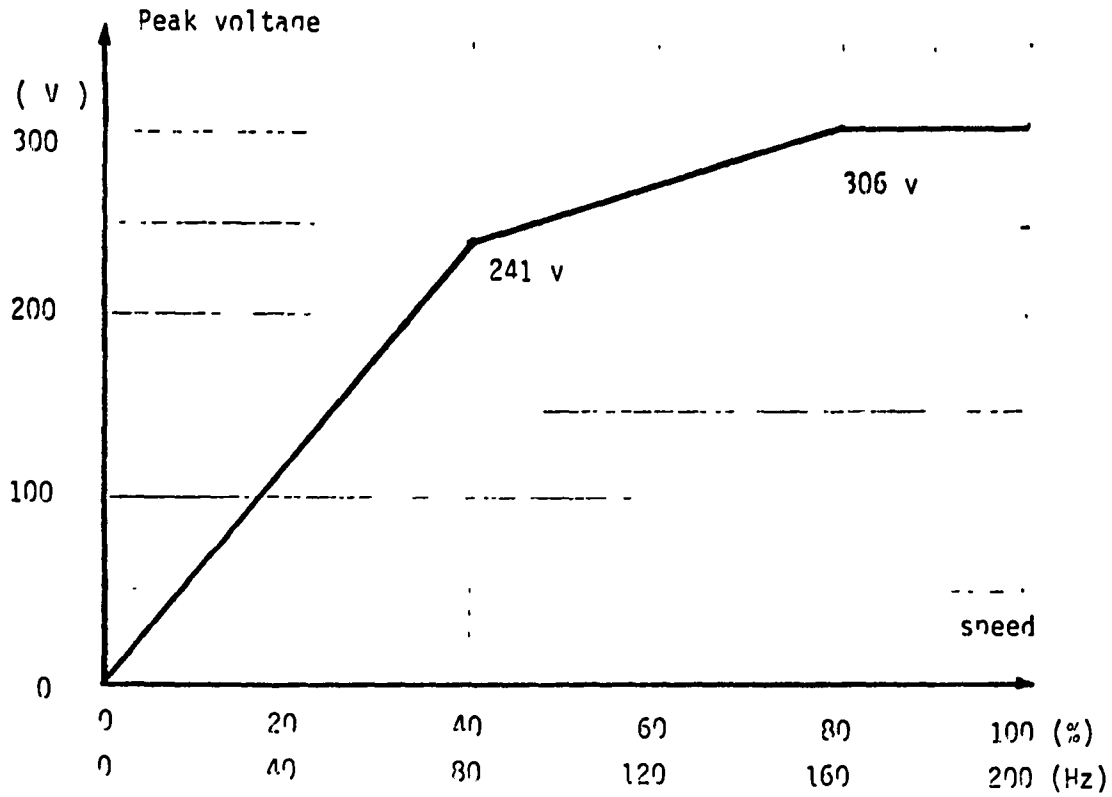

Fig. 10 Motor fundamental peak voltage vs. speed or frequency for proposed base point

the proposed voltage variation in Fig. 10 implies that the rated output power at different speeds is

7.5kW at 10% speed

30kW at 40% speed

38kW at 80% speed

30kW at 100% speed

The 16kW, 10% speed point requires about twice the rated output power that would be delivered with the low speed range constant V/Hz of 241V/80Hz. (This implies approximately 60V at 10% speed, or 20Hz.) Thus, the motor voltage must be increased above 60V at 10% speed to provide 16kW. When the voltage is increased at a given frequency, the air-gap flux increases, which drives the machine further into saturation. This may result in excessive motor current due to the reduction of the magnetizing reactance. Thus, the motor voltage required for maximum power at 10% speed must be high enough to drive the machine into saturation without excessive current. However, if the V/Hz is too high at 10% speed, it will be too low at 80% speed because the maximum voltage available with a 240V source will be reached at a lower speed. A low V/Hz at 80% speed will mean that the 60kW output cannot be achieved.

Step 2: Design the Machine at the Base Operating Point

The detailed design of the induction motor, both in mechanical and electrical characteristics, is included in Appendix D. When the base operating point is given, the

machine is essentially uniquely defined. The final design is determined in Appendix D after several iterative trials. The resulting machine parameters are as follows:

External diameter	D_o	11.5 inches
Internal diameter	D	5.75 inches
Rotor length	L	6.3 inches
Weight	W	≃ 190 lb (≃ 86kg)
Stator resistance	R_s	0.0274 Ω
Rotor resistance	R_r	0.0213 Ω
Magnetizing inductance	L_m	13.86mH
Stator leakage inductance	L_{ls}	0.25mH
Rotor leakage inductance	L_{lr}	0.27mH

Step 3: Check the Output Power by Simulations, Considering Saturation Effects

Since the machine is designed for the base operating point, the maximum output power should be checked by digital simulations, including the saturation effect of the air-gap flux. Appendix G describes the detailed simulations.

Fig. 11 shows the simulation results for the output power-speed relations at 10%, 40%, 80% and 100% speed of the finally designed motor, when the stator current is limited to 245A peak. All of the specified operating points are satisfied. At the maximum power output of each curve, the current is almost a square wave since this current waveform results in more power output than with sinusoidal current for the same peak current. At the 16kW, 10% speed point, the greatest saturation of the magnetic core occurs. The

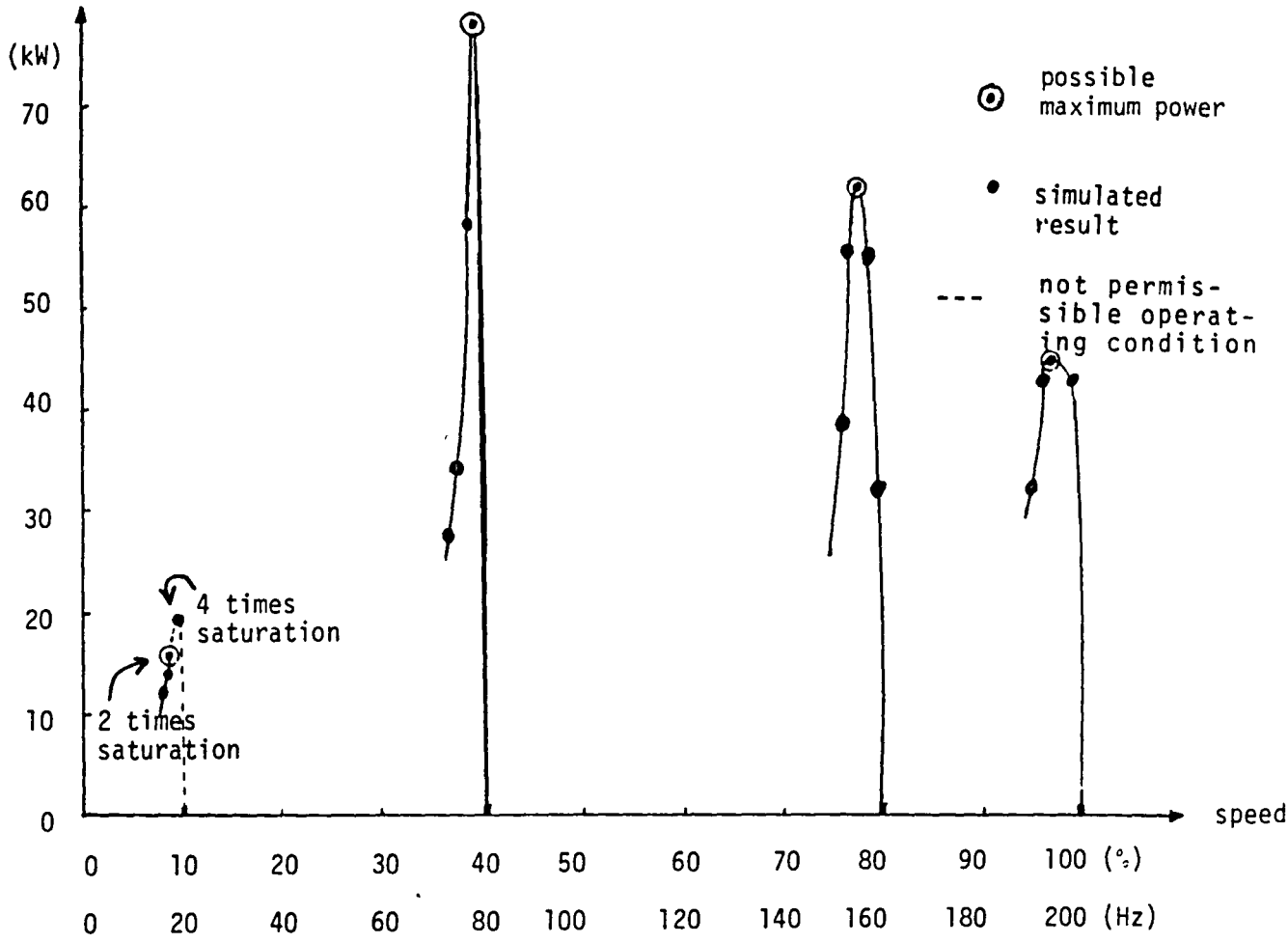

Fig. 11 Output power-speed curves of two-phase motor from simulations.

(Peak current limited to 245 A)

saturation effect of the core is implemented in the simulation study by changing the magnetizing and leakage inductances of the machine. If the magnetizing current exceeds the allowable limit, then the magnetizing and leakage inductances are replaced with adjusted values, which are calculated from the B-H curve. This modification is easily implemented in Step 2. If the magnetizing current still exceeds the allowable limit, which occurs for very low slip frequency, then this is not considered a permissible operating condition.

At the 60kW output, 80% speed operating point, under-saturation of the core occurs because the voltage is not high enough to produce rated flux. In this case, when the magnetizing current in the simulation is below the allowable limit, the magnetizing and leakage inductances are recalculated in Step 2 using the B-H curve.

Step 4: Check the Temperature Rise

Most of the heat generated in the machine comes from the copper loss, which is proportional to stator resistance times the square of the current. According to the simulation results, the rms currents at the given operating points are as follows:

1) 16kW at 10% speed (3 mins., 30% duty) $I_{1rms} = 200A$

2) 60kW at 80% speed (3 mins., 30% duty) $I_{1rms} = 175A$

3) 30kW at 100% speed (1 hour) $I_{1rms} = 115A$

Assuming a conventional rotor fan type construction, the air flow is reduced at lower rotor speeds so that the worst-case cooling condition is the 10% speed point. If the

iron and mechanical losses are considered negligible, then the highest transient temperature rise occurs for the 16kW, 10% speed operating point. Since this condition continues for three minutes with a 30% duty cycle, the load pattern is assumed to be 16kW for three minutes, followed by no output for seven minutes, repeated several times. The temperature rise is estimated by solving the heat balance differential equations (Appendix E). The estimated temperature rise is 50 °C. The bifilar windings help to reduce the temperature rise because of the increased volume of copper. The temperature rise is higher if the same heat loss is generated in a smaller volume of copper.

In the calculation of the temperature rise, the most pessimistic conditions are considered. First, static air is assumed for the heat dissipation. Next, the heat transfer to the core and frame is neglected. Thus, the actual temperature rise should be somewhat less than 50°C.

3.3.5 Efficiency

In accordance with the JPL specifications, the most important operating point from the standpoint of efficiency is the 12kW, 80% speed point. The theoretical approach to optimize the efficiency of an induction machine implies that the magnetizing current should be equal to the rotor current.* But, since the V/hz is reduced at the 80% speed

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* T. W. Jian, N. L. Schmitz, D. W. Novotny, "Characteristic Induction Motor Slip Values for Variable Voltage Part Load Performance Optimization," IEEE PAS, Jan. 1983, pp. 38-46.

point, the magnetizing current does not reach the optimized value because of the undersaturation of the air-gap flux. Thus, the best that is possible is to make the magnetizing current its maximum value and to control the current to get 12kW output at 80% speed. Simulation results, performed by an iterative method changing the slip frequency and the amplitude of the sinusoidal motor current, give 98.6% efficieny at a slip frequency of 1.16Hz (0.725% slip). The current and output power are shown in Appendix G. The pulsating power is ±65%. These simulations neglect the iron loss, stray loss, and mechanical losses. If these losses are included, the efficiency is expected to be about 95%.

3.4 Inverter Design

3.4.1 Switching Device Requirements

In Step 3 of the design procedure in the previous section, three given output power-speed operating points were checked by simulations. The maximum peak current through the stator winding is 245A when either 16kW at 10% speed or 60kW at 80% speed is required. Since the forward blocking voltage of the device is two times 240V, the total VI rating of the devices for the two-phase inverter will be (240 X 2 X 245) x 4 = 470kVA.

From the viewpoint of the power dissipation in the switching devices, 16kW output at 10% speed produces the maximum switching loss. From the simulations, the switching frequency is 3.4kHz at 16kW, 10% speed, and 160Hz at 60kW, 80% speed.

In summary, the switching device requirements are

Peak current = 245 A

RMS current = 200.5A

Maximum switching frequency = 3.4kHz

Peak voltage (not including transients) = 480V

3.4.2 Power Circuit

As a result of a study of the available transistor and thyristor devices to meet the switching requirements for this application, MG300M1FK1 Toshiba power transistors are recommended. These transistors have 1000V blocking capability, 300A dc rating and 1.6kW maximum power dissipation capability. Thus, this Toshiba transistor provides a nominal dc voltage safety factor of 1000/480 = 2.08 and an rms current safety factor of 300 X $\sqrt{2}$ = 2.12. (Note: This does not include the transient voltages across the transistors. The transient voltages are discussed in the next section.)

The two-phase inverter requires four power switch modules, each of which includes a Darlington power transistor, a freewheeling diode, a snubber circuit and a base drive. However, only one heat sink is required since the MG300M1FK1 transistor is an isolated package. Also, no base drive isolation is necessary for the circuit of Fig. 2 with common emitters.

The two-phase and three-phase inverter circuits both can deliver power to the motor load or take power from the source. This is accomplished by the proper control of the inverter switching instants relative to the ac voltage generated by the load machine, which produces a negative

slip. Appendix G.5 gives simulation results for regenerative operation.

The detailed inverter design is described in Appendix F.

3.4.3 Transient Voltage

The two-phase system does present a more difficult snubber design problem because of the leakage inductances that will be present in the bifilar windings and because of the inductances in the motor leads from the inverter to the motor. It will be necessary to use special techniques to minimize the motor lead inductance, such as using parallel closely spaced flat conductors between the inverter and the motor or packaging the inverter essentially on the motor housing.

The amplitude of the voltage spike primarily depends on the equivalent leakage inductance of the bifilar winding and the lead inductance between the inverter and motor. The energy stored in these inductances must be dissipated in the snubbers. Based on estimated practical values of total leakage and lead inductance, the voltage spike phenomena can be simulated by the digital computer, as shown in Appendix F. However, a precise design of the snubbers required can be accomplished only through experiment with an actual model of the inverter.

As shown in Appendix F, the peak transistor voltage is about 900V with 4 μH total lead and leakage inductance. However, if the inductance is reduced to 1μH, the peak transistor voltage is approximately 600V. This would reduce the

transistor voltage safety factor from 2.08 (as given in the previous section) to about 1.6, which is still a quite reasonable voltage safety factor.

3.4.4 Control Circuit

A microprocessor controller will be used, which consists of an MPU, ROM, RAM, timer, I/O port and current sensors which can provide a variety of speed and torque controls. In either case using PWM, the speed or torque controller generates the frequency and amplitude commands for the stator current. This current is regulated so as to stay within a hysteresis band of the reference command by PWM control of the inverter. Also, the switching frequency is monitored and limited to less than 3.4kHz. The maximum fundamental inverter frequency command is 200Hz.

The use of the microprocessor permits the implementation of very sophisticated controls to provide optimal efficiency at any operating point and protection of the drive system.

4.0 FINAL DESIGN AND EXPECTED PERFORMANCE OF A TWO-PHASE INVERTER/BIFILAR INDUCTION MOTOR SYSTEM

4.1 Machine Data

External diameter	D_o = 11.5 inches
Internal diameter	D = 5.75 inches
Rotor length	L = 6.3 inches
Weight	W ≈ 190 lb (≈ 86kg)
Output rating	30kW
Number of phases	2
Base frequency	80Hz

Maximum speed	12,000 RPM
Base voltage (rms)	170V
Base current (rms)	110A
Stator resistance	R_s = 0.0274 Ω
Rotor resistance	R_r = 0.0213 Ω
Magnetizing inductance	L_m = 13.86mH
Stator leakage inductance	L_{ls} = 0.25mH
Rotor leakage inductance	L_{lr} = 0.27mH

4.2 Inverter Data

Power switching devices	(4) Toshiba MG300M1FK1 Transistors
Input voltage	240V
Output voltage	240V (peak or rms)
Output current	245A peak; 200A rms
kVA rating	96kVA
kW rating	60kW
Maximum output frequency	200Hz
Maximum switching frequency	3.4kHz
Estimated efficiency at 12kW, 80% speed operating point	95.4%

4.3 Inverter/Motor Performance

The digital computer simulations described in Appendix G provide the following performance data at the specified operating points:

16.4kW output at 10% speed

Slip frequency = 1.7Hz (slip = 8.50%)

Switching frequency = 3.4kHz

Stator peak current= 245A*

Stator rms current = 200A

Pulsating torque = ±20%

60.5kW output at 80% speed

Slip frequency = 3.2Hz (slip = 2%)

Switching frequency = 160Hz

Stator peak current = 245A**

Stator rms current = 175A

Pulsating torque = ±30%

13.1kW output at 80% speed

Slip frequency = 1.16 Hz (slip = 0.725%)

Switching frequency = 1.6 kHz

Stator peak current = 103A***

Stator rms current = 53.2A

Pulsating torque = ±65%

---

*The current is a PWM controlled wave form with a square wave envelope 205±40A.

**The inverter voltage is a square wave producing an unmodulated current waveform.

***The current is a PWM controlled sinusoidal waveform with an envelope of ± 38A.

Motor efficiency (only with copper loss) = 98.6%

Motor efficiency (with 3.4% iron and stray mechanical loss) = 95.2%

Inverter efficiency (with snubber, on-state and switching loss) = 95.4%

Total efficiency = 91%

4.4 System Schematic Diagram

The complete schematic diagram of the inverter/motor drive system is shown in Fig. 12.

As mentioned previously, transient voltages are produced by lead inductances and leakage inductances due to imperfect coupling between the bifilar motor windings. With the input capacitor connected across the inverter dc bus very close to the inverter transistors and motor windings, the dc lead inductance is minimized. Snubber circuits are included to suppress voltage transients due to the imperfect coupling between bifilar windings and the inductance of the leads between the inverter transistors and motor windings. This also implies that the inverter transistors must be mounted very close to the motor.

The design of the snubbers is discussed in Appendix F. However, the precise snubber configuration can be determined only through experimentation with an actual model of the inverter. A final optimum arrangement may include varistors and additional capacitors.

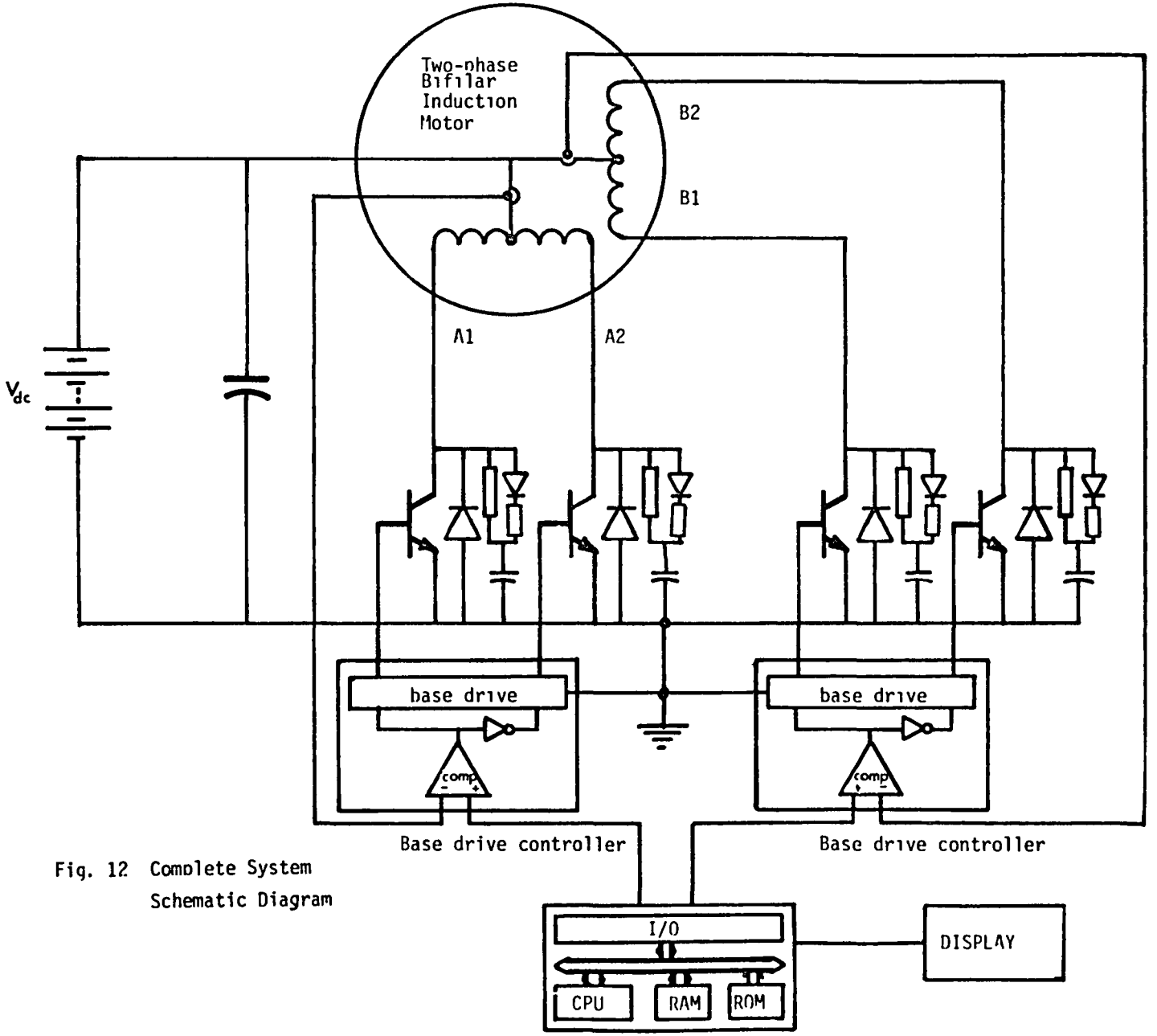


Fig. 12 Complete System Schematic Diagram

The base drive circuits are critical for reliable power transistor operation with minimum switching loss. Current sensors are required to detect each motor winding current for the PWM controlled current operations as well as for overcurrent protection. Sophisticated control strategies can be implemented in the microprocessor software.

5.0 COMPARISON OF TWO-PHASE BIFILAR AND THREE-PHASE BRIDGE INVERTER MOTOR SYSTEMS

5.1 Advantages and Disadvantages of Two-Phase System

Advantages

A. The number of power switching devices is reduced from six to four.

B. The current rating of each power switch is 65% of the value required for the three-phase bridge inverter (Fig. B-3). This has been verified by simulation as shown in Table II of the next section.

C. The total number of components is estimated to be about 70% of the number required for the three-phase system. Since four transistor switches are used instead of six as in the thre-phase bridge inverter, this would imply about two-thirds as many components. However, some circuits are not repeated for each switch and the snubbers are more complex for the two-phase inverter. The component count reduction also implies higher reliability and lower manufacturing assembly cost.

D. The motor transient temperature rise is reduced due to the increased volume of copper.

E. The control hardware is somewhat simpler due to the common emitter arrangements so that isolation is not required in the base drive circuits.

F. The total component cost for the two-phase system is estimated to be about 90% of that for the three-phase system. This cost reduction cannot be proven until prototype equipments are manufactured. The cost will be reduced because of the reduced total number of components, the elimination of the need for base drive circuit isolation, and the reduced peak current in the transistor switches. However, the cost will be greater because of the higher peak transistor voltages required, the need for very effective snubbers to minimize transient voltages, the requirement for a somewhat larger motor than for three-phase, and the necessity to minimize motor bifilar winding leakage inductances and lead inductances in the inverter current switching paths. It is presently estimated that these tradeoffs will result in reducing the two-phase system cost to about 90% of that for a three-phase inverter/motor drive.

Disadvantages

A. The size, weight and cost of the two-phase motor is greater than for the three-phase machine. It

is estimated that an equivalent three-phase motor would have 85% of the weight and 90% of the cost of the two-phase bifilar machine.

B. The design of the snubbers for the power switches is more difficult since motor lead inductances and bifilar winding leakage inductances produce additional transient voltages.

C. The torque pulsations are relatively large because of the necessity to limit the inverter switching frequency to achieve high efficiency.

5.2 Simulation Results

Digital simulations to compare the two-phase and three-phase systems are presented in Appendix G. Table I lists the parameters used for the simulations. The results of these simulations are summarized in Tables I and II.

Fig. 13-a shows the stator voltage and current per phase, and Fig. 13-b shows the instantaneous output power for the two-phase machine. Fig. 13-c and Fig. 13-d show the corresponding variables for the three-phase machine.

The output power or torque pulsations are greatest for the two-phase system at the 60kW operating point since a square wave voltage is supplied from the two-phase inverter. However, at the 12kW operating point, with sinusoidal current control, the torque pulsations of the two-phase system are nearly as large because the inverter switching frequency must be limited to achieve high efficiency.

Table I Machine Parameters

Bifilar Two-Phase Induction Motor (at 160Hz)	Three-Phase Induction Motor (at 320Hz)
R_s=0.0274	R_s =0.00578
R_r'=0.0213	R_r'=0.0038
$L_{ls}=2.94\times10^{-4}$H	$L_{ls}=5.95\times10^{-5}$H
$L_{lr}=3.23\times10^{-4}$H	$L_{lr}=7.0\times10^{-5}$H
$L_m=26.8\times10^{-3}$H	$L_m=3.32\times10^{-3}$H
Weight=86kg	Weight=73kg

Note: A four pole three-phase motor was used for this comparison because more commercial data was available to us on this type of machine. However, we believe a two pole three-phase machine is somewhat preferable.

Table II Comparison of Two-Phase and Three-Phase Inverter-Motor System

Case I: 80% speed, 60kW output (for peak current comparison)

Two-Phase Induction Motor	Three-Phase Induction Motor
Peak current I_p=245A	Peak current I_p=375A
rms current I_{rms}=175.8A	rms current I_{rms}=243A
slip = 2.0%	slip = 1.25%
P_{out} = 60.5kW	P_{out} = 60.2kW
frequency = 160.0Hz	frequency = 329.8Hz

Case II: 80% speed, 12kW output (for efficiency comparison)

Two-Phase Induction Motor	Three-Phase Induction Motor
Peak current I_p= 103A	Peak current I_p=105A
rms current I_{rms}= 53.2A	rms current I_{rms}=66A
slip = 0.725%	slip = 0.8%
P_{out} = 13.1kW	P_{out} = 12.5kW
frequency = 160Hz	frequency = 329.8Hz
efficiency (only copper loss)=98.6%	efficiency (only copper loss)=98.9%

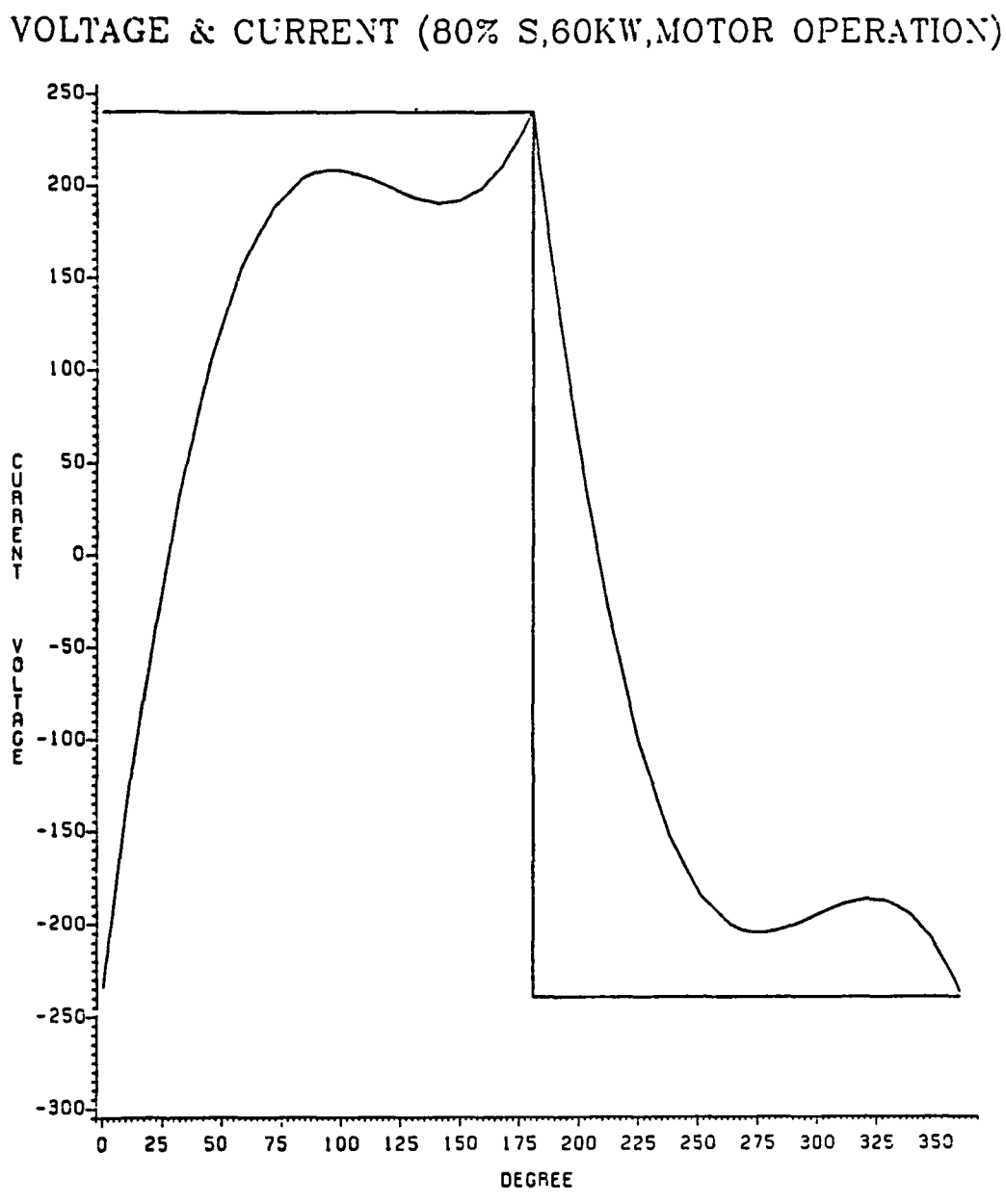


Fig. 13-a Two-phase

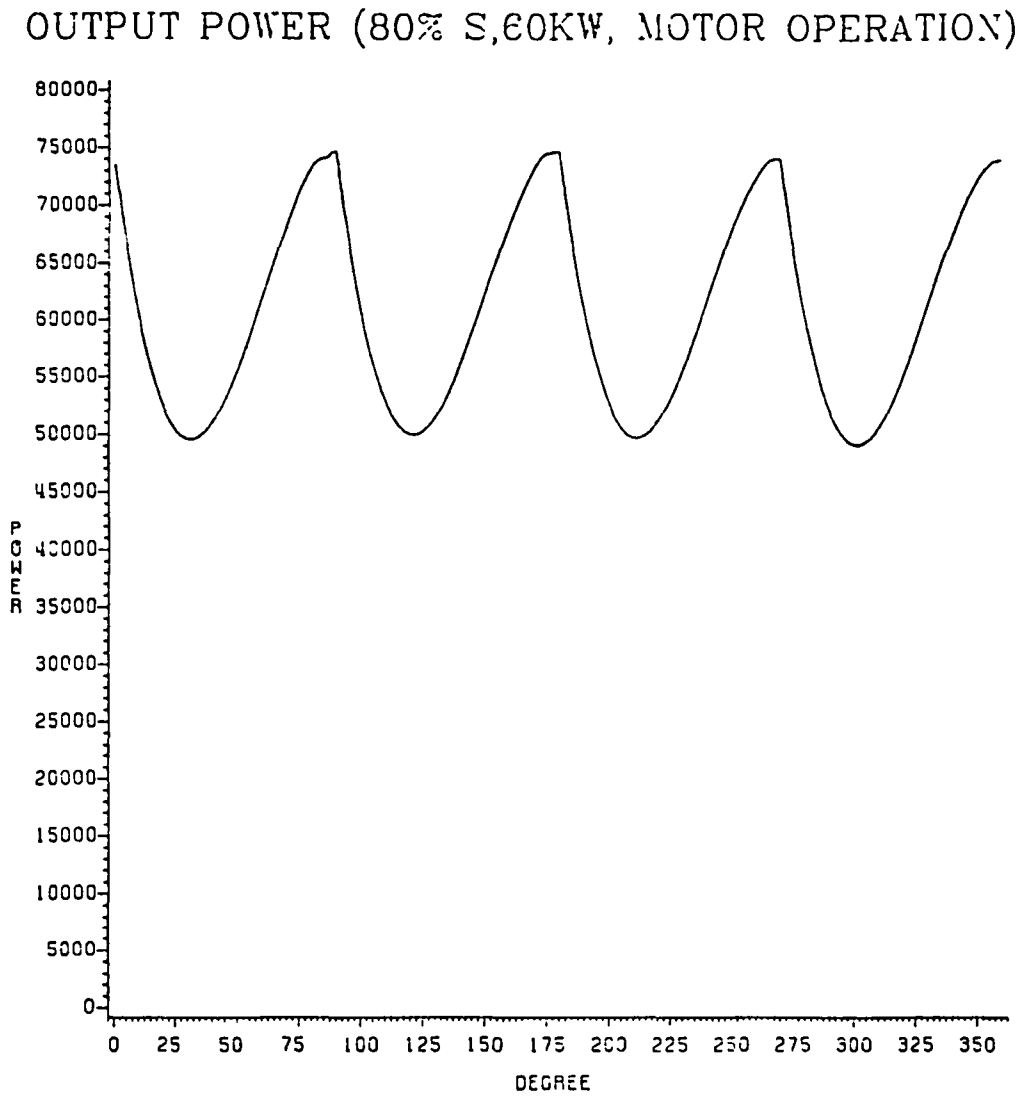


Fig. 13-b Two-phase

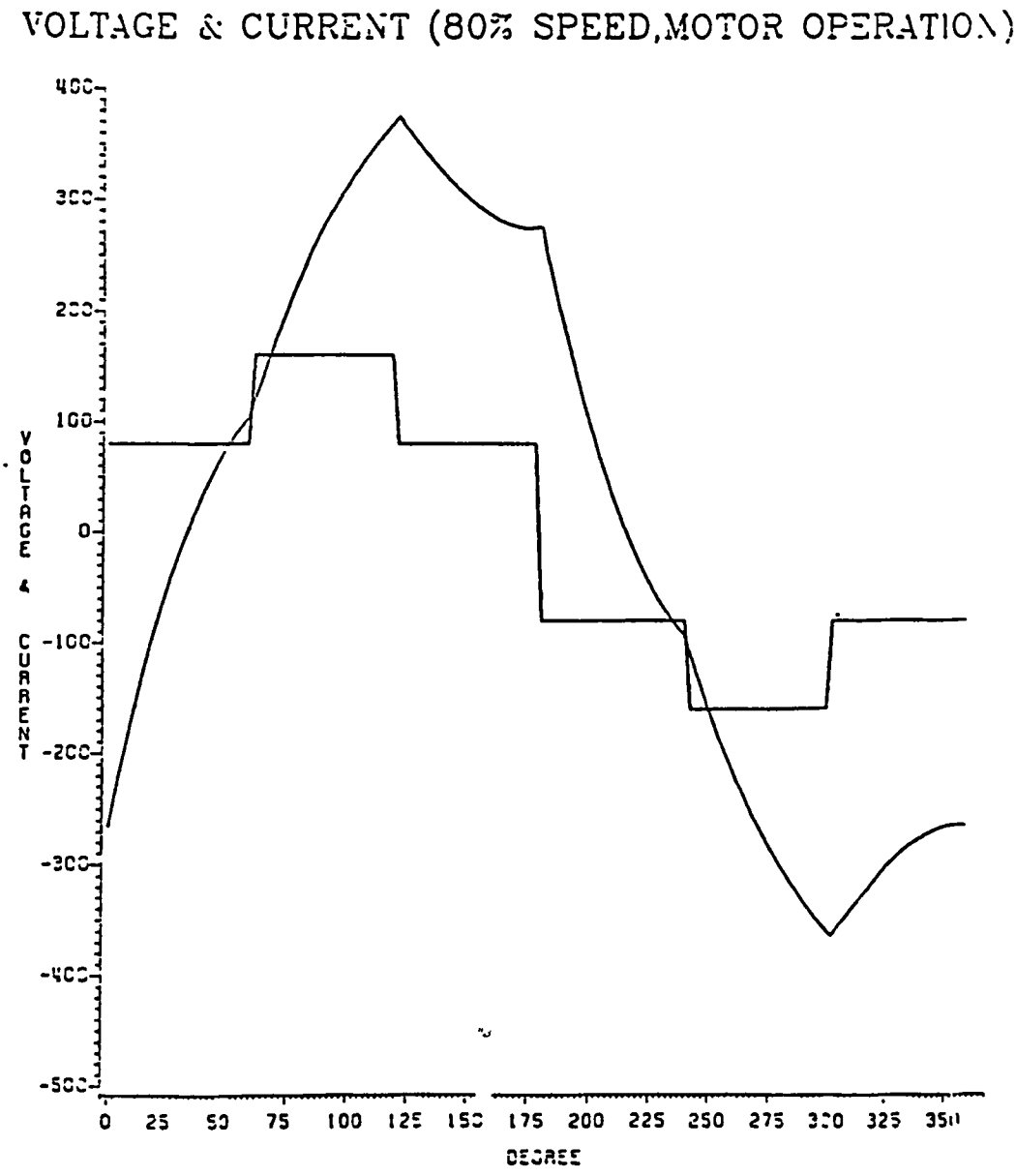


Fig. 13-c Three-phase

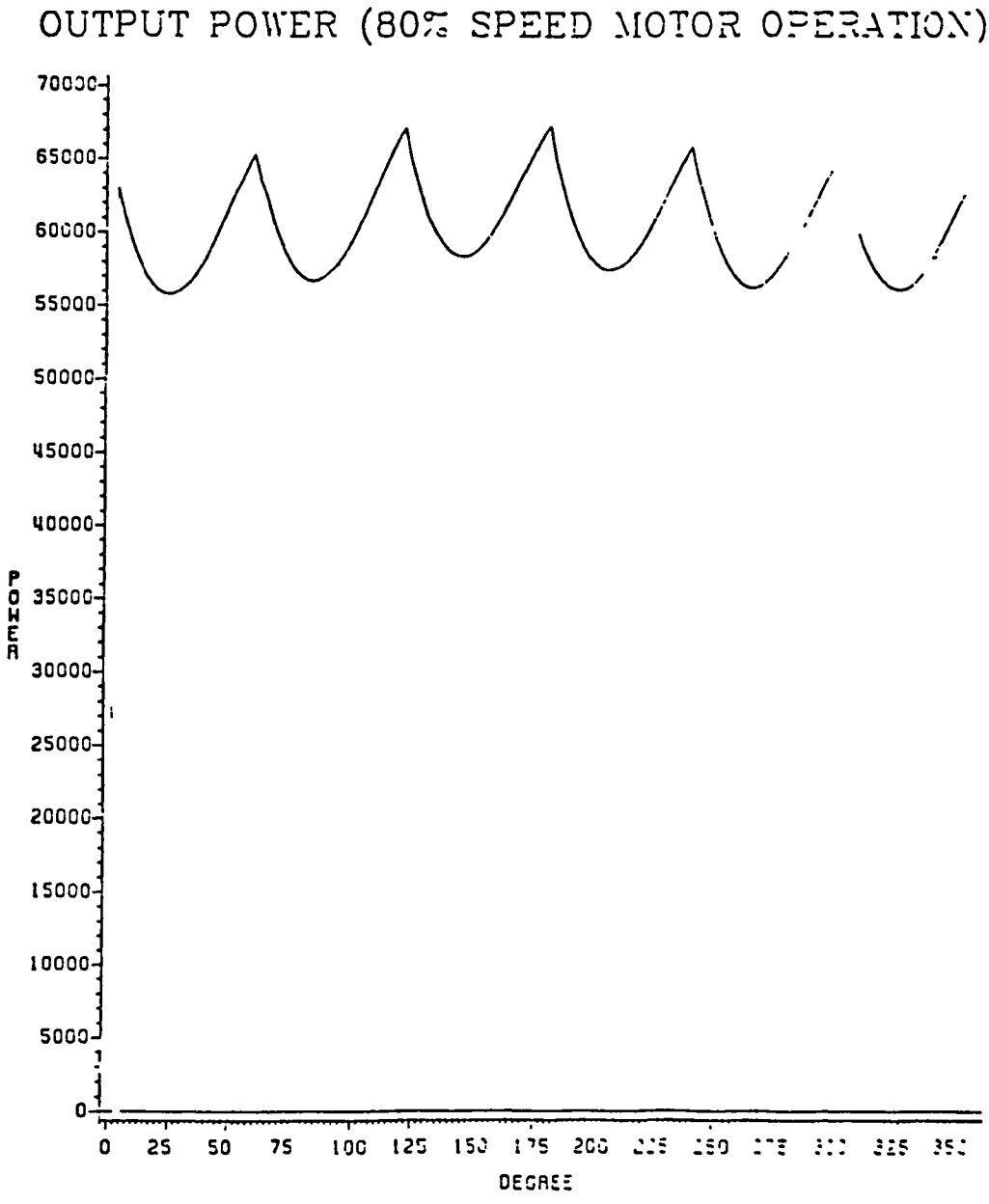


Fig. 13-d Three-phase

5.3 Performance Function Comparison

Machine: The following data are used

1)	Cost of two-phase machine	$360*
	Cost of three-phase machine	$325*
2)	Weight of two-phase machine	86kg
	Weight of three-phase machine	73kg
3)	Efficiency at 12kW, 80% speed	
	Two-phase machine	95%
	Three-phase machine	95%
4)	Performance function F (F=C + 10W - 120E)	
	Two-phase machine	-10.1K
	Three-phase machine	-10.3K

Inverter: The following data are used

1)	Estimated manufacturing cost of two-phase inverter	$1000
	Estimated manufacturing cost of three-phase inverter	$1200
2)	The weight and efficiency of the two-phase and three-phase inverters are estimated to be essentially the same.	
3)	Performance function (neglecting the weight and efficiency terms)	
	Two-phase inverter	$1000
	Three-phase inverter	$1200

---

*These are best judgment estimates assuming at least 10,000 units/year.

Inverter-Motor System: (Including only the parameters in the Performance Function which are different.)

Two-phase system $360+$1000+$10X86=$2220

Three-phase system $325+$1200+$10X73=$2255

Thus, the two-phase system has only a very slight advantage over the three-phase system based on the Performance Function.

6.0 CONCLUSIONS

The proposed two-phase inverter-motor drive has the following advantages over the traditional three-phase bridge inverter-motor system:

1. The number of power semiconductor switching devices is reduced from six to four.
2. The current rating of each power switch is 65% of that required for each switch in the equivalent three-phase system.
3. The total number of components is estimated to be about 70% of the number required for the three-phase system. This also implies higher reliability and lower manufacturing assembly cost.
4. The motor transient temperature rise is reduced due to the increased volume of copper.
5. Somewhat simpler control hardware is possible because of the common emitter circuit so that isolation is not required in the base drive circuits.

6. The total component cost is estimated to be about 90% of that for the three-phase system.

The overall efficiency of the two systems is about the same and the Performance Function in the JPL Specifications is also nearly equal for the two-phase and three-phase systems.

The major disadvantages of the two-phase system are:

1. The weight and cost of the two-phase machine are greater. It is estimated that the three-phase machine would have 85% of the weight and 90% of the cost of the two-phase bifilar machine.
2. The snubber requirements for the inverter power transistors are much more severe for the two-phase inverter since motor lead inductances and bifilar winding leakage inductance produce additional transient voltages.
3. The torque pulsations are relatively large because of the necessity to limit the inverter switching frequency to achieve high efficiency.

For a high production electric passenger vehicle, the two-phase inverter-motor system developed on this research should provide significant cost advantages for the electric vehicle application when compared to the conventional three-phase bridge inverter-induction motor drive. It is important to note that the conclusions of this research are based on analysis and simulations. An actual model of the two-phase system must be constructed and evaluated to prove the advantages claimed for the proposed inverter-motor system. The most challenging engineering task will be to design the inverter, motor and snubber circuits to minimize transient voltages while retaining high system efficiency.

APPENDIX A

MOTOR SPECIFICATIONS PROVIDED BY JPL

EXHIBIT I

Induction Motor Specification

The following is a specification for an ac induction motor to be used in conjunction with a Pulse Width Modulated (PWM) Inverter and a 240 volt battery for propulsion of electric vehicles using a fixed ratio gear reduction.

I. BACKGROUND

Numerous traction drives have been developed using inverter driven induction motors. In all cases the inverter converts dc battery power into three-phase power of controlled frequency and voltage. While control algorithms vary, with all schemes, the electrical frequency is maintained close to the mechanical frequency and the voltage to frequency ratio is maintained below a critical maximum (lest magnetic saturation occur).

Compared with dc systems, such ac drives are advantageous in that inherent four quadrant performance is achieved, motor cost is greatly reduced, motor weight and losses are approximately halved while motor maintenance is virtually eliminated. Until recently, however, these advantages were offset by cost, weight, and inefficiency problems of the inverter.

II. SYSTEM DESIGN PHILOSOPHY

With traction drives, where PWM inverters provide the ac power, the usual constraints of voltage and frequency are nonexistent. Accordingly, the requirement for producing torque under high slip conditions is deleted, and thus the

need for compromise between "starting characteristics" versus "running efficiency" is eliminated. Accordingly, traction type machines are designed for maximum efficiency with minimal rotor resistance.

For each torque-speed point of operation, a range of excitation voltage-slip frequency values may be used; however, only one value of voltage and slip frequency produces optimal efficiency. With modern inverter technology it is possible to control both voltage and frequency such that optimal system efficiencies prevail over the entire torque speed plane. From experience it has been found that the optimal solution corresponds to maintenance of a constant slip frequency with the excitation voltage per Hertz varying roughly as the square root of the torque. Since selection of the inverter control parameters is flexible, it follows that the appropriate optimization sequence is:

1. Design optimized motor.
2. Characterize this motor.
3. Design appropriate inverter control algorithm.

The above philosophy is embodied in the attached specification.

With traction systems, cost, weight, and efficiency parameters are of key importance. In order to meaningfully optimize these parameters, relationships must be established between these parameters. From JPL studies it has been found that for passenger vehicles the "cost of weight" is

approximately \$10/kg and the value of efficiency is approximately \$120/%. Accordingly, for both the motor and the inverter, the total cost, C^1, to be minimized is:

$$C^1 = C + 10W - 120E \quad (1)$$

where C is the retail dollar cost

W is the weight in kg

E is the efficiency in %

Since efficiency is primarily important in terms of range and since maximum range is under freeway conditions, E is defined under the speed and power conditions associated with freeway driving (55 mph, 12 kW shaft output).

A totally enclosed convective cooled design is recommended since:

1. Expected heat loss is low as motor efficiency will be in the range of 92% to 95%.
2. Duration of operation is limited by the battery storage capability. (With 150 km of driving, heat produced by motor will be less than 1.2 kWh.)
3. With open design, dirt particles may cause stator winding damage.

III. DISCUSSION OF SPECIFICATIONS

1. Voltage

The nominal battery voltage is 240 V dc. This corresponds to a maximum inverter output voltage of 165 V

rms (sine modulation) and 185 V rms (six-step mode). The inverter will control to lower voltages in accordance with the control algorithm.

2. Power Output

Since the bidder shall determine actual speeds and torques, specifications are presented in terms of shaft output power and fraction of maximum speed. (The gear reduction ratio and inverter control constants will be determined after establishing the motor.)

2a. Power at Maximum Speed

This specification establishes one hour thermal and torque breakdown capability of handling the rated power at full speed. Since efficiency and breakdown torque both improve with decreasing speed, it follows that this power level can be maintained for all speeds above base speed.

2b. Power at 10% Maximum Speed

This specification establishes the torque breakdown capability below base speed and corresponds to vehicle acceleration.

2c. Power at 80% Maximum Speed

This specification establishes the torque breakdown capability at 0.8 N_{max} (55 mph) and corresponds to vehicle acceleration at this speed.

3. Maximum Speed

As maximum speed is increased, cost and weight decrease. Accordingly, moderately high speeds are desirable - consistent with mechanical and windage limits. With previous efforts, maximum speeds between 9,000 and 20,000 rpm have been used. The actual maximum speed selected will be on the basis of Equation 1.

4. Number of Poles

As the number of poles is increased, the conductor losses generally drop while the magnetic losses increase. (The inverter switching losses are proportional to frequency and, therefore, proportional to pole number; this impact is, however, small and will be neglected.) The optimum pole number will be determined on the basis of Equation 1.

5. Weighting Function (See Equation 1)

This function relates cost and efficiency to the retail price of the motor. The motor is to be designed such that the value of this function is minimized. The efficiency is defined at 0.8 N_{max} and 12 kW shaft output. Excitation for this point of operation will be the optimal efficiency voltage (probably less than 185 V rms). The corresponding slip frequency will be noted.

6. Efficiency and Power Factor

Efficiency and power factor are defined under "optimal conditions" where the excitation voltage is such that the

resulting slip frequency matches that associated with optimal efficiency at 0.8 N_{max}, 12 kW. Since excitation voltage is limited to 185 V rms, slip frequencies higher than optimal will prevail for some combinations of high torque and high speed. Efficiency and power factor are to be specified for at least five torque and five rpm values.

IV. EXPECTED RANGES OF PARAMETER VALUES

Based on JPL studies, the following ranges of parameters are expected for an optimized machine:

1.	Weight	65 to 95 kg
2.	Frame Size	10" to 12"
3.	Pole Number	2 or 4
4.	Maximum Speed (N_{max})	8,000 to 16,000 rpm
5.	Base Speed	0.35 to 0.45 N_{max}
6.	Optimal Slip Frequency	1 to2 Hz
7.	Efficiency at 0.8 N_{max}, 12 kW	92% to 95%
8.	Power Factor for 7.	85% to 90%

Induction Motor Specification

1. Input line voltage, maximum rms: 185 V
   (will be varied as required for speed and torque control)
2. Power output:

1 hr. (20% duty) at maximum speed (N_{max})	30 kW
3 min. (30% duty) at 0.8 N_{max}	60 kW
3 min. (30% duty) at 0.1 N_{max}	16 kW

3. Speed, N_{max}, between 8,000-16,000 rpm
   (to be selected by contractor)
4. Number of poles: two or four
   (to be selected by contractor)
5. Weighting function F to be minimum:

   $F = \text{cost} + 10(\text{weight}) - 120(\text{efficiency})$

   where

   cost is retail cost in $

   weight is in kg

   efficiency is in % measured at 0.8 N_{max}, 12 kW shaft power with optimal voltage excitation
6. Construction: totally enclosed.
7. Cooling: convection air, maximum ambient 40 C.
8. Life: 4000 hours at full load.

In presenting the quotation the bidder shall:

Describe the selected design.

Specify the size and weight.

Specify the torque breakdown profile over the whole range of speed (either graphically or in tabular form).

Specify efficiency and power factor versus speed and torque for optimal excitation.

T. W. Macie

9/23/83

APPENDIX B

ALTERNATIVE INVERTER/INDUCTION MOTOR CONFIGURATIONS

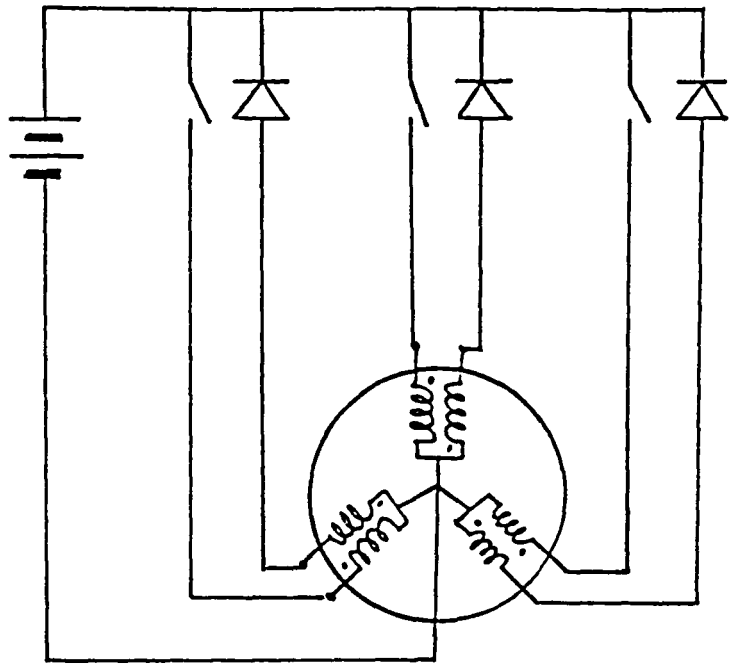


Fig. B-1-a Half-wave inverter driven three-phase bifilar induction motor

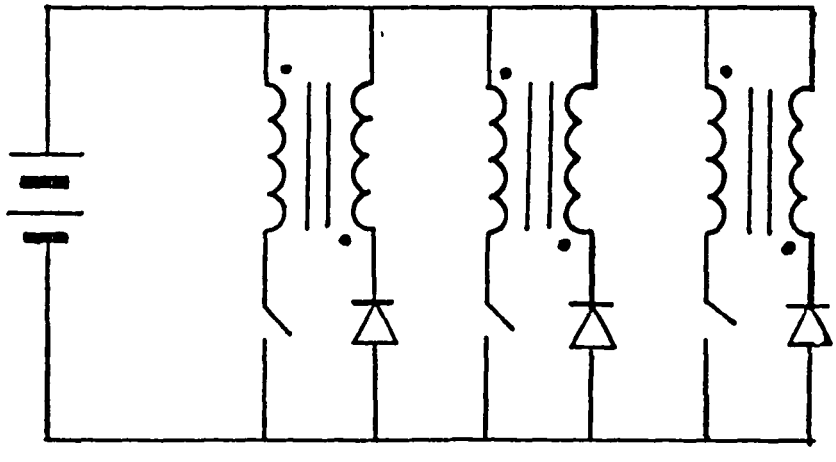


Fig. B-1-b Alternative arrangement for half-wave inverter driven three-phase motor

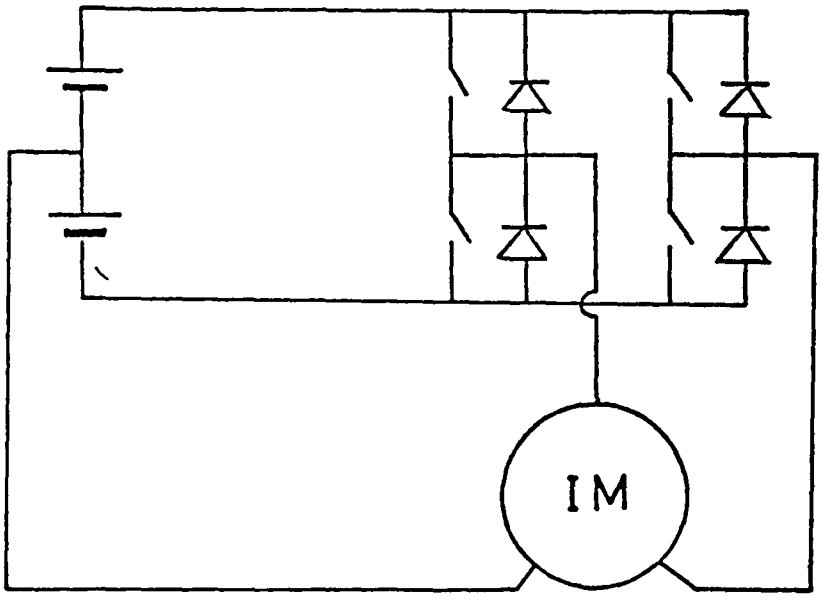


Fig. B-2 Four-switch inverter driven three-phase induction motor

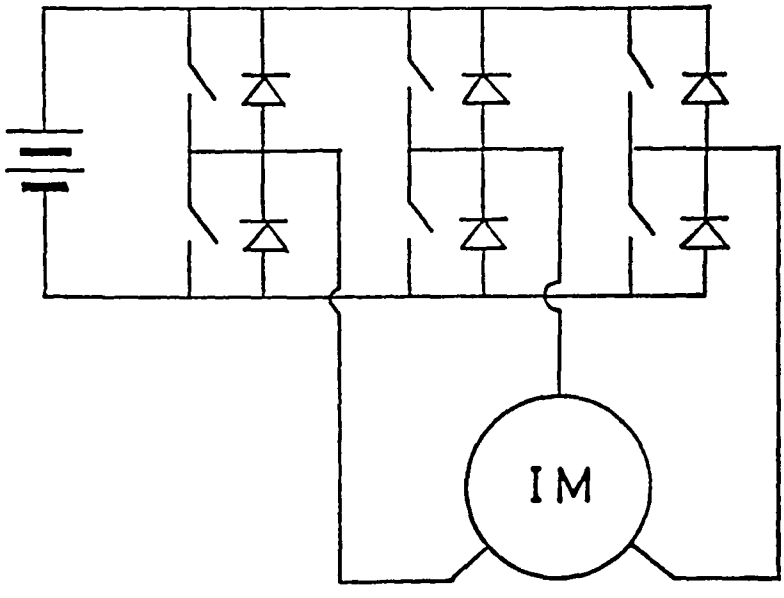


Fig. B-3 Three-phase bridge inverter driven induction motor

APPENDIX C

IDEALIZED COMPARISONS OF TWO-PHASE AND THREE-PHASE INVERTERS

The two-phase inverter circuit shown in Fig. 2 and the three-phase arrangement in Fig. B-3 are used for these comparisons.

Assuming Equal DC Power to a Resistive Load

Two-phase

For this situation, both the load current and load voltage are assumed to be square waves as shown in Fig. C1. This would be the case if the motor center-tapped windings in Fig. 2 were the primary windings of two ideal transformers with a purely resistive load on each secondary.

Let

$V_{2\phi} \triangleq$ rms volts/phase on each half of center-tapped winding

$= V_{DC}$

and

$I_{2\phi} \triangleq$ rms current/phase through the resistive load

Assuming that the ideal transformers each have unity turns ratio from half of the primary winding to the secondary,

$$I_{2\phi} = I_{DC}/2$$

The current I_{DC} is the dc source current. Since the two pairs of switches in Fig. 2 are operated with 90° phase displacement, the total dc source current is always equal to twice the current in one phase. Basically, I_{DC} divides equally at all instants, with $I_{DC}/2$ flowing through one half of each transformer primary.

Thus

$V_{T,P} \triangleq$ switch peak voltage

$= 2V_{DC}$

and

$I_{T,P} \triangleq$ switch peak current

$= I_{DC}/2$

and let

$TR_{2\phi} \triangleq$ total switch rating $V_{T,P}I_{T,P}$ for two-phase

$= (4)\ (2V_{DC})\ (I_{DC}/2)$

$= 4V_{DC}\ I_{DC}$

Three-phase

In this case, the circuit of Fig. B-3 is considered, except with the motor winding replaced with a wye connected balanced set of load resistors. With a current I_{DC} flowing from the dc source in Fig. B-3 the line-to-neutral load voltage and current are as shown in Fig. C-1. In this case

$V_{T,P} = V_{DC}$

$I_{T,P} = I_{DC}$

and let

$TR_{3\phi} \triangleq$ total switch rating $V_{T,P}\ I_{T,P}$ for three-phase

$= (6)\ (V_{DC})\ (I_{DC})$

$= 6V_{DC}\ I_{DC}$

Therefore, the total switch rating for two-phase is 2/3 of that for three-phase.

Note: This can be checked in another very simple manner as follows. If the power delivered to the resistive loads is the same for the two-phase and the three-phase circuits, then the power taken from the dc supply must be the same in each case.

$$P_{DC} = V_{DC}\ I_{DC}$$

Also

$$I_{T,P,2\phi} = I_{DC}/2$$

and

$$I_{T,P,3\phi} = I_{DC}$$

Therefore

$$TR_{2\phi} = (4)\ (2V_{DC})\ (I_{DC}/2) = 4V_{DC}\ I_{DC}$$

$$TR_{3\phi} = (6)\ (V_{DC})\ (I_{DC}) = 6V_{DC}\ I_{DC}$$

and

$$TR_{2\phi} = 2/3\ TR_{3\phi}$$

Assuming Equal Fundamental Power to a Resistive Load

Two-phase

Let

$V_{1,2\phi} \triangleq$ amplitude of fundamental voltage/phase

$$= \frac{1}{\pi} \int_0^{2\pi} v_{2\phi} \sin \omega t \, d(\omega t)$$

$$= \frac{2V_{DC}}{\pi} \int_0^{\pi} \sin \omega t \, d(\omega t)$$

$$= \frac{4V_{DC}}{\pi}$$

and similarly for the current

$$I_{1,2\phi} = \frac{4I_{T,P,2\phi}}{\pi}$$

Then

$P_{1,2\phi} \triangleq$ total fundamental power to load

$$= (2) \left(\frac{4V_{DC}}{\pi\sqrt{2}}\right) \left(\frac{4I_{T,P,2\phi}}{\pi\sqrt{2}}\right)$$

$$= \frac{16}{\pi^2} V_{DC} I_{T,P,2\phi}$$

Three-phase

Let

$V_{1,3\phi} \triangleq$ amplitude of fundamental voltage/phase

$$= \frac{1}{\pi} \int_0^{2\pi} v_{3\phi} \sin \omega t \, d(\omega t)$$

$$= \frac{2V_{DC}}{\pi} \left[\int_0^{\pi/3} \frac{1}{3} \sin \omega t \, d(\omega t) + \int_{\pi/3}^{2\pi/3} \frac{2}{3} \sin \omega t \, d(\omega t) \right.$$

$$\left. + \int_{2\pi/3}^{\pi} \frac{1}{3} \sin \omega t \, d(\omega t) \right]$$

$$= \frac{2V_{DC}}{\pi}$$

and for the current

$$I_{1,3\phi} \triangleq \frac{1}{\pi}\int_0^{2\pi} i_{3\phi} \sin \omega t \, d(\omega t)$$

$$= \frac{2I_{PT,3\phi}}{\pi}\left[\int_0^{\pi/3} \tfrac{1}{2}\sin \omega t \, d(\omega t) + \int_{\pi/3}^{2\pi/3} \sin \omega t \, d(\omega t) + \int_{2\pi/3}^{\pi} \tfrac{1}{2}\sin \omega t \, d(\omega t)\right]$$

$$= \frac{3I_{T,P,3\phi}}{\pi}$$

Then

$P_{1,3\phi} \triangleq$ total fundamental load power

$$= (3)\left(\frac{2V_{DC}}{\pi\sqrt{2}}\right)\left(\frac{3I_{T,P,3\phi}}{\pi\sqrt{2}}\right)$$

$$= \frac{9V_{DC}I_{T,P,3\phi}}{\pi^2}$$

For equal load power

$$P_{1,2\phi} = P_{1,3\phi}$$

$$\frac{16}{\pi^2} V_{DC} I_{T,P,2\phi} = \frac{9V_{DC}I_{T,P,3\phi}}{\pi^2}$$

or

$$I_{T,P,2\phi} = \frac{9}{16} I_{T,P,3\phi}$$

Finally, the total switch ratings $TR_{2\phi}$ and $TR_{3\phi}$ are

$$TR_{2\phi} = (4)\ (2V_{DC})\ (I_{T,P,2\phi})$$

$$TR_{3\phi} = (6)\ (V_{DC})\ (I_{T,P,3\phi})$$

Then

$$TR_{2\phi} = (4)\ (2V_{DC})\ (\frac{9}{16}\ I_{T,P,3\phi})$$

$$= \frac{9}{2}\ V_{DC}\ I_{T,P,3\phi}$$

and

$$TR_{3\phi} = 6V_{DC}\ I_{T,P,3\phi}$$

Thus

$$TR_{2\phi} = \frac{3}{4}\ TR_{3\phi}$$

Two-phase load voltage and current

Three-phase line-to-neutral load voltage and current

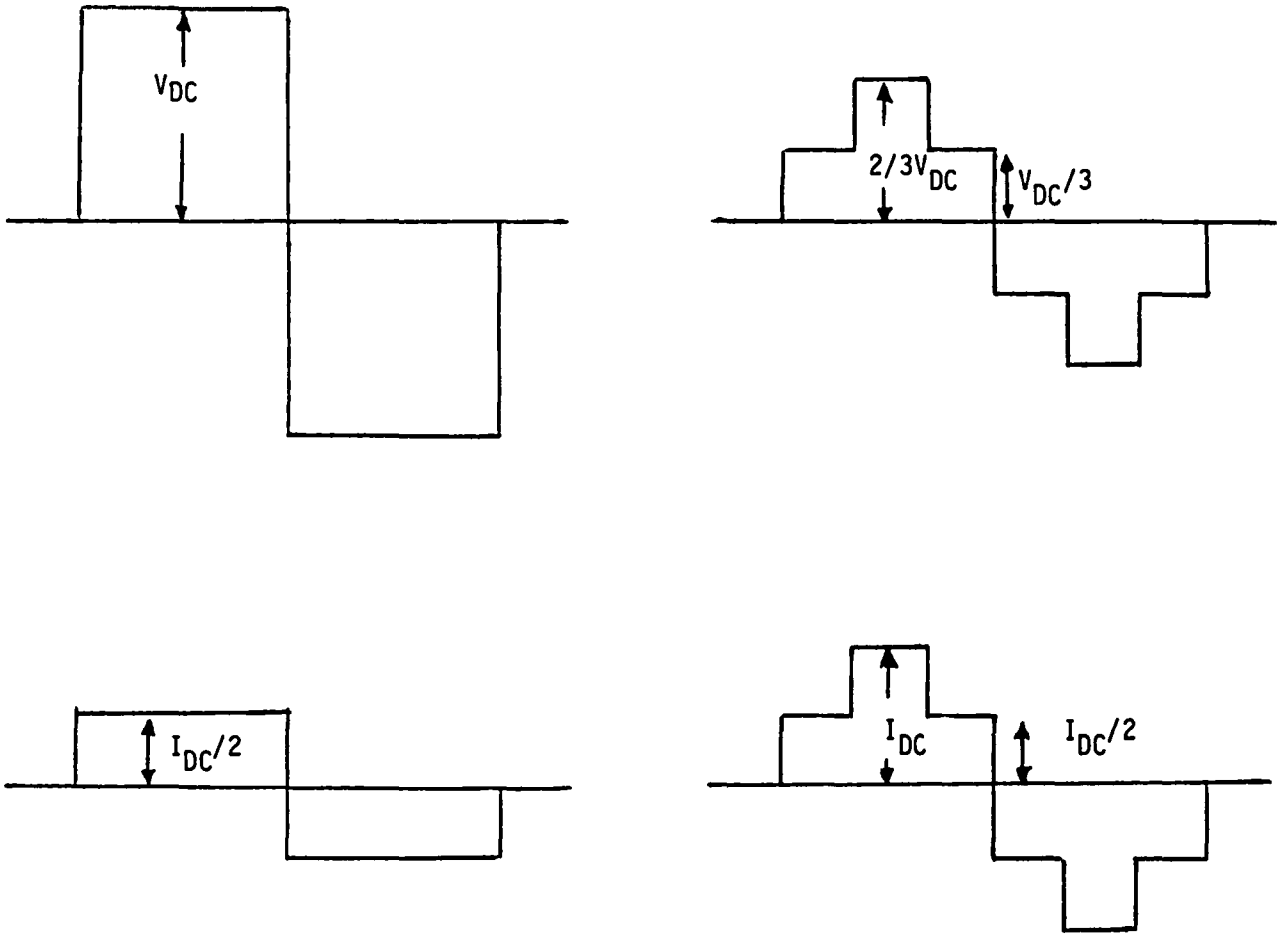


Fig. C-1 Load voltages and currents for Fig. 2 and Fig. B-3 respectively with motors replaced by resistive load

Note: Two ideal transformers are used in Fig. 2, with unity turns ratio from each half of their primaries to their secondaries connected to resistive loads. A wye-connected set of balanced load resistors replaces the motor in Fig. B-3.

APPENDIX D

DETAILED DESIGN OF TWO-PHASE BIFILAR INDUCTION MACHINE

MAIN DATA OF UMC TWO-PHASE BIFILAR INDUCTION MOTOR

1.	Output	P_m = 30 kW
2.	Number of phases	m = 2
3.	Frequency	f = 80 Hz
4.	Equivalent voltage of battery	V_D = 189 V
5.	Phase current	I_{ph} = 110.2 A
6.	Efficiency	η = 0.91
7.	Power factor	$\cos \phi$ = 0.88
8.	Number of poles (p $\triangleq$ pole pairs)	2p = 2
9.	External diameter of stator	D_o = 29.21 cm (11.5 in)
10.	Internal diameter of stator	D = 14.6 cm (5.75 in)
11.	Length of stack	L = 16 cm (6.3 in)
12.	Length of air gap	l_g = 0.03 cm (0.0118 in)
13.	Number of slots of stator	Z_1 = 24
14.	Number of slots of rotor	Z_2 = 20
15.	Turns per coil	Z_{S1} = 11
16.	Number of coils	n_c = 12
17.	Series turns per phase	T_{ph} = 33
18.	Number of branches	a_p = 2
19.	Resistance of stator	R_s = 0.0274Ω
20.	Resistance of rotor	R'_r = 0.0213Ω
21.	Leakage inductance of stator	L_{ls} = 2.5 X 10^{-4} H
22.	Leakage inductance of rotor	L_{lr} = 2.7 X 10^{-4} H
23.	Magnetizing inductance	L_m = 13.86 X 10^{-3} H

UMC TWO-PHASE BIFILAR MOTOR

1. Given data

   P_m = 30 kW

   m = 2 (number of phases)

   f = 80 Hz (basic frequency)

   P = 1 (pair of poles)

   V_D = 189 V (equivalent base battery voltage)

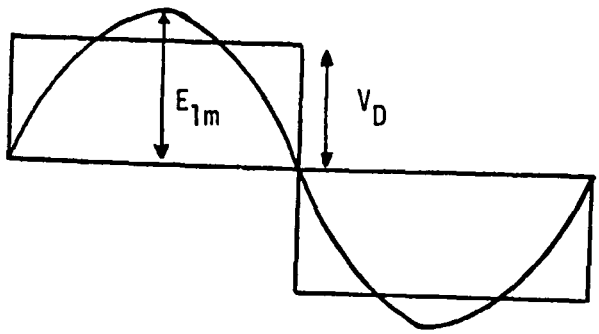


Fig. D-1

$$E_{1m} = \frac{4}{\pi} V_D = \frac{4}{\pi} \times 189 \text{ V} = 241 \text{ V}$$

$$E_{1(rms)} = \frac{4}{\sqrt{2}\,\pi} \times 189 \text{ V} = 170 \text{ V}$$

2. External diameter of stator D_o

   choose D_o = 29.21 cm = 11.5 in

3. Internal diameter of stator D

   let D = 14.6 cm = 5.75 in

4. Specific magnetic loading $\overline{B} = \frac{2P\phi_m}{\pi DL}$

   assumed $\overline{B}$ = 0.45 wb/m^2

   $\overline{B}$ = 0.3 — 0.6 wb/m^2 for common motor

5. Slots of stator $Z_1 = 24$

6. Slots of rotor Z_2

$$Z_2 = Z_1 \pm 4P = 20$$

7. Assumed Efficiency $\eta = 0.91$

Power factor $\cos \phi = 0.88$

8. The kVA rating S

$$S = \frac{P_m}{\cos \phi} = \frac{30}{0.91 \times 0.88} = 37.46 \text{ kVA}$$

9. The motor constant G

$$G = \frac{S}{D^2 Ln} = 11 \text{ kw } \bar{B} \text{ X ac X } 10^{-3} = 11 \text{ X } 0.9029 \text{ X } 0.45 \text{ X } 30{,}000 \text{ X } 10^{-3} = 134$$

where L is length of stack

kw is winding factor

ac is specific electric loading

n is synchronous speed (rps)

kw = kp x kd kp = 1 (single layer winding)

$$kd = \frac{\sin (\pi/2m)}{n' \sin (\pi/2mn')} = \frac{\sin (\pi/2x2)}{6 \times \sin (\pi/2x2x6)} = 0.9029$$

$$n = \frac{Z_1}{2pm} = \frac{24}{2x2} = 6 \text{ (number of slots per phase per pole)}$$

$$n = \frac{f}{p} = \frac{80}{1} = 80 \text{ rps}$$

$$ac = 2m\, T_{ph}\, I_{ph}/\, D = 5000 \quad 45{,}000 \text{ Amp-cond/m}$$

Let ac = 30,000

10. $D^2L = \frac{S}{Gn} = \frac{37.46}{134 \times 80} = 3.4944 \times 10^{-3}\ m^3$

$$L = \frac{D^2L}{D^2} = \frac{3.4944 \times 10^{-3}}{0.146^2} = 0.164\ m$$

Let L = 0.16 m = 16 cm = 6.3 in

11. The flux per pole ϕ_m

$$\phi_m = \bar{B} \times \frac{\pi D}{2P} \times L = 0.45 \times \frac{\pi \times 0.146}{2} \times 0.16 = 0.0165\ wb$$

12. The turns per phase (in series) T_{ph}

$$T_{ph} = \frac{E_1}{4.44\ kw\ f\ _m} = \frac{170}{4.44 \times 0.9029 \times 80 \times 0.0165} = 32.1$$

choose $T_{ph} = 33$

13. The real flux per pole ϕ_m

$$\phi_m = \frac{E_1}{4.44\ kw\ f\ T_{ph}} = \frac{170}{4.44 \times 0.9029 \times 80 \times 33} = 0.016\ wb$$

14. The number of coils n_c

$$n_c = \frac{Z_1}{2} = \frac{24}{2} = 12 \text{ (single layer)}$$

The number of branches a_p

$a_p = 2$

15. The coils per phase in series n_{ph}

$$n_{ph} = \frac{n_c}{ap \times m} = \frac{12}{2 \times 2} = 3$$

16. The turns of coil Z_{s1}

$$Z_{s1} = \frac{T_{ph}}{n_{ph}} = \frac{33}{3} = 11$$

17. $$\bar{B} = \frac{\phi_m}{\frac{\pi DL}{2P}} = \frac{0.016}{\frac{x0.146x0.16}{2}} = 0.436 \text{ wb/m}^2$$

18. Phase current I_{ph}

$$I_{ph} = \frac{S \times 10^3}{m \times E_1} = \frac{37.46 \times 10^3}{2 \times 170} = 110.2 \text{ A}$$

$ac = 2m\ T_{ph} \times I_{ph}/\pi D = 2 \times 2 \times 33 \times 110.2/\pi \times 0.146 = 31714$ Amp-cond/m

19. The density of current of stator winding δ_1

choose $\delta_1 = 5 \text{ A/mm}^2$

20. The section of coil S_c

$$S_c = \frac{I_{ph}}{a_{p\ 1}} = \frac{110.2}{2 \times 5} = 11.02 \text{ mm}^2$$

choose 3 x #13 wire and 2 x #15 wire

#13 wire d = 1.83 mm $A_d = 2.63 \text{ mm}^2$

insulated d' = 1.83 + 0.18 = 2.01 mm

#15 wire d = 1.45 mm $A_d = 1.65 \text{ mm}^2$

d' = 1.63 mm

$S_c = 3 \times 2.63 + 2 \times 1.65 = 11.19 \text{ mm}^2$

21. The number of wires in parallel n_p

$n_p = 3 + 2 = 5$

22. The total number of wires in a slot n_s

$n_s = 2 \times n_p \times Z_{s1} = 2 \times 5 \times 11 = 110$

The first number 2 on the right side represents the bifilar winding.

23. The real density of current δ_1

$$\delta_1 = \frac{I_{ph}}{a_p s_c} = \frac{110.2}{2 \times 11.19} = 4.92 \text{ A/mm}^2$$

24. Length of mean conductor

$L_{mc} = L + 1.1Y + 3$ (cm)

$= 16 + 1.1Y + 3 = 16 + 1.1 \times 22.93 + 3 = 44.2$ cm

where $Y = \frac{\pi D}{2P} = \frac{\pi \times 14.6}{2} = 22.93$ cm

We use concentric coils: 1-8

1-10 coil spans

1-12

25. Length per phase L_{ph}

$L_{ph} = T_{ph} \times 2 \times L_{mc} = 33 \times 2 \times 0.442 = 29.17$ m

26. Resistance per phase R_s (75°C)

$R_s = P_{75} \times L_{ph}/S_c \times a_p = 0.021 \times 29.17/11.19 \times 2 = 0.0274\Omega$

27. Reduced rotor current I_2'

$I_2' \simeq I_{ph} \cos\phi = 110.2 \times 0.88 = 97$ A

28. The tooth pitch of rotor τ_{s2}

$$\tau_{s2} = \frac{\pi D_2}{Z_2} = \frac{\pi \times 14.54}{20} = 2.284 \text{ cm}$$

where diameter of rotor $D_2 = D - 2 \times l_g = 14.6 - 2 \times 0.03 = 14.54$ cm

$l_g = 0.3$ mm (length of air gap)

29. Number of slots per pole S_2 (rotor)

$$S_2 = \frac{Z_2}{2p} = \frac{20}{2} = 10$$

30. The current of the bar I_b

$$I_b = \frac{2m\ k_w\ T_{ph}}{Z_2}\ I_2' = \frac{2 \times 2 \times 0.9029 \times 33}{20} \times 97 = 578\ A$$

31. The current of the end ring I_c

$$I_c = \frac{I_b S_2}{\pi} = 578 \times 10/\pi = 1840\ A$$

32. Casting aluminum alloy for rotor cage

P_{Al} (75°C) = 0.04Ω mm^2/m

(The value can be in negotiation with manufacturer)

33. Let $\delta_{2b} = 3.2\ A/mm^2$

34. The section of each bar a_2

$$a_2 = \frac{I_b}{\delta_{2b}} = \frac{578}{3.2} = 180.6\ mm^2$$

35. The resistance per bar λ_b

$$\lambda_b = P_{Al} \times L/a_2 = 0.04 \times 0.16/180.6 = 3.54 \times 10^{-5}\ \Omega$$

36. The section of end ring a_{2c}

Let $\delta_{2c} = 2.8\ A/mm^2$

$$a_{2c} = \frac{I_c}{\delta_{2c}} = \frac{1840}{2.8} = 657\ mm^2$$

37. The resistance of ring r_c

$$r_c = \rho_{Al} \times \pi \times D_r/a_{2c} = 0.04 \times \pi \times 0.126/657 = 2.41 \times 10^{-5}\ \Omega$$

where $D_r = 0.126$ m (the mean diameter of ring)

38. The reduced resistance of rotor r_2'

$$r_2' = \frac{Z_2(r_b + 2r_c S_2^2/\pi^2 Z_2) I_b^2}{m\, I_2'^2} = \frac{20\ (3.54 + 2 \times 2.41 \times 10^2/\pi^2 \times 20) \times 578^2 \times 10^{-5}}{2 \times 97^2}$$

$$= 0.02124\Omega \simeq 0.0213\Omega$$

39. Carter's factor k_c

$$k_c = k_{c1} \cdot k_{c2}$$

$$k_{c1} = \frac{\tau_{S1}}{\tau_{S1} - b_o + 2 l_g \times {}^{l}_{n}\ [1 + \frac{b_o}{2 \times l_g}]}$$

$$= \frac{19.1}{19.1 - 3.5 + 2 \times 0.3 \times {}^{l}_{n}[1 + \frac{3.5}{2 \times 0.3}]} = 1.14$$

where $\tau_{S1} = \frac{\pi D}{Z_1} = \frac{\pi \times 146}{24} = 19.1$ mm

(tooth pitch of the stator)

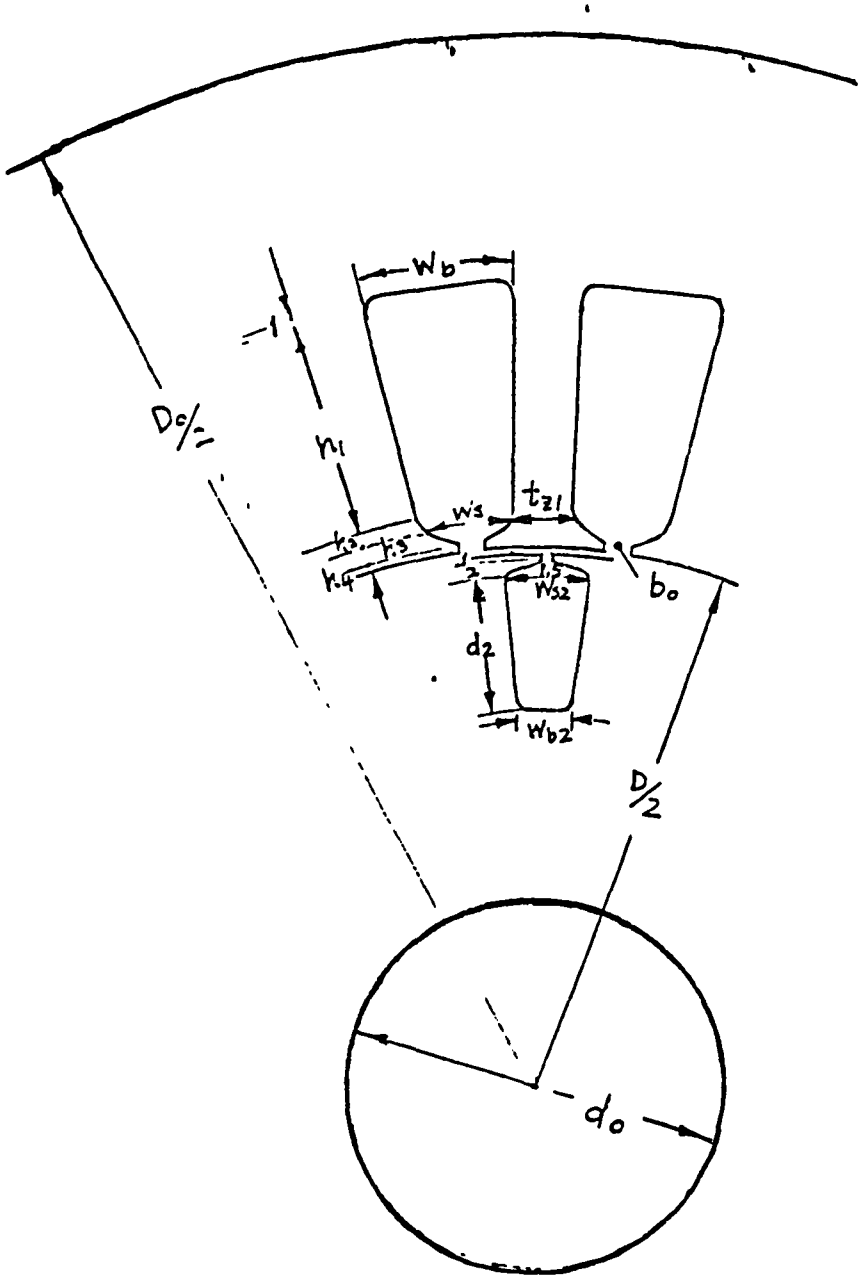
$b_o = 3.5$ mm (width of slot opening - refer to Fig. 1)

$$k_{c2} = \frac{\tau_{S2}}{\tau_{s2} - b_{o2}\ 2\ l_g \times {}^{l}_{n}\ [1 + \frac{b_{02}}{2\ l_g}]}$$

$$= \frac{2.284\ X\ 10}{22.84 - 1.5 + 2 \times 0.3 \times {}^{l}_{n}\ [1 + \frac{1.5}{2 \times 0.3}]} = 1.034$$

$$k_c = k_{c1} \cdot k_{c2} = 1.14 \times 1.034 = 1.18$$

Effective length of air gap l_g'



h_1 = 31 mm
h_2 = 2 mm
h_3 = 2 mm
h_4 = 1 mm
b_o = 3.5 mm
t_{Z1} = 7.8 mm
t_{Z2} = 9.2 mm

$$W_S + t_{Z1} = \frac{[D+2(h_3+h_4)]\,\pi}{Z_1}$$

$$= \frac{[146 + 2(2+1)]}{24}\,\pi$$

$$= 19.9 \text{ mm}$$

$$W_S = 19.9 - 7.8 = 12.1 \text{ mm}$$

$$W_b = \frac{[D+2(h_1+h_2+h_3+h_4)+2]\,\pi}{Z_1} - t_{Z1} = \frac{(146+2x36+2)\pi}{24} - 7.8 = 21 \text{ mm}$$

$$W_{S2} = \frac{[D-21g-2x3]\,\pi}{Z_2} - t_{Z2} = \frac{(146-2x0.3-6)\pi}{20} - 9.2 = 12.7 \text{ mm}$$

d_2 = 17 mm

$$W_{b2} = \frac{(D-21g-2x3-2xd_2)\,\pi}{Z_2} - t_{Z2} = \frac{(146-2x0.3-6-2x17)\,\pi}{20} - 9.2 = 7.356 \text{ mm}$$

Fig. D-2

$$l_g' = k_c\, l_g = 1.18 \times 0.3 = 0.354 \text{ mm}$$

40. The area of air gap A_g

$$A_g = \frac{\pi D}{2p} L = \frac{2 \times 0.146}{2} \times 0.16 = 0.0367 \text{ m}^2$$

41. Modified density of flux of air gap B_g

$$B_g = 1.36 \times \phi_m / A_g = 1.36 \times 0.016/0/0367 = 0.593 \text{ wb/m}^2$$

where 1.36 is for considering saturation in both teeth of stator and rotor

42. Area of teeth per pole of stator A_{Z1}

$$A_{Z1} = t_{z1} \times L \times 0.92 \times S_1 = 0.78 \times 16 \times 0.92 \times 12$$

$$= 138 \text{ cm}^2 = 0.0138 \text{ m}^2$$

where $S_1 = \frac{Z_1}{2p} = \frac{24}{2} = 12$

43. Modified density of flux in teeth B_{Z1}

$$B_{Z1} = 1.36 \times \phi_m / A_{Z1} = 1.36 \times 0.016/0.0138 = 1.577 \text{ wb/m}^2$$

44. Length of tooth l_{Z1}

$$l_{Z1} = k_1 + k_2 = 31 + 2 = 33 \text{ mm} = 0.033 \text{ m}$$

45. Height of core h_{c1} (stator)

$$h_{c1} = \frac{D_o - [D + 2(h_1 + h_2 + h_3 + h_4)]}{2}$$

$$= 29.21 - \frac{[14.6 + 2\,(3.1 + 0.2 + 0.2 + 0.1)]}{2} = 3.705 \text{ cm}$$

$$= 0.03705 \text{ m}$$

46. Area of core A_{c1} (stator)

$$A_{c1} = h_{c1} \times L \times 0.92 = 0.03705 \times 0.16 \times 0.92 = 0.005454 m^2$$

47. Maximum density of flux through the core B_{c1}

$$B_{c1} = \frac{\phi_m}{2A_{c1}} = \frac{0.016}{2 \times 0.005454} = 1.4668 \text{ wb/m}^2$$

48. Length of core per half pole l_{c1}

$$l_{c1} = \frac{(D_o - h_{c1})\pi}{4p} = \frac{(29.21 - 3.705)\pi}{4} = 20.03 \text{ cm}$$

49. Effective length of core l'_{c1}

$$l'_{c1} = \zeta l_{c1} = 0.38 \times 20.03 = 7.6 \text{ cm} = 0.076 \text{ m}$$

50. $\zeta = 0.38$ considering the densities of flux along the length of core are not equal

51. The fill coefficient of slot k_f

$$k_f = \frac{n_s d'^2}{\frac{W_s + W_b}{2} \times (h_1 + h_2)} = \frac{2 \times 11 \times (3 \times 2.01^2 + 2 \times 1.63^2)}{\frac{12.1 + 21}{2} \times (31 + 2)} = 0.702$$

$k_f = 0.702 < 0.75$ (This means that bifilar winding can be inserted in slots without difficulty.)

52. Area of rotor teeth per pole A_{Z2}

$$A_{Z2} = t_{Z2} \times L \times 0.92 \times S_2 = 0.92 \times 16 \times 0.92 \times 10 = 135.4 \text{ cm} = 0.01354 \text{ m}^2$$

53. Modified density of flux in rotor teeth B_{Z2}

$$B_{Z2} = \frac{1.36\phi_m}{A_{Z2}} = \frac{1.36 \times 0.016}{0.01354} = 1.607 \text{ wb/m}^2$$

54. Length of rotor tooth l_{Z2}

$$l_{Z2} \simeq d_2 = 17 \text{ mm} = 0.017 \text{ m}$$

55. The height of rotor core h_{c2}

$$h_{c2} = \frac{D - 2l_g - 2d_2 - 2 \times 2 - d_o}{2} + \frac{1}{6} \times d_o$$

$$= \frac{146 - 2 \times 0.3 - 2 \times 17 - 4 - 45}{2} + \frac{1}{6} \times 45$$

$$= 38.7 \text{ mm}$$

where $d_o = 45$ mm (d_o is diameter of the shaft)

56. The maximum density of flux through the rotor core B_{c2}

$$B_{c2} = \frac{\phi_m}{2A_{c2}} = \frac{0.016}{2 \times 0.0057} = 1.404 \text{ wb/m}^2$$

57. Area of rotor core A_{c2}

$$A_{c2} = h_{c2} \times L \times 0.92 = 0.387 \times 0.16 \times 0.92 = 0.0057\text{m}^2$$

According to the B-H curve on Fig.D-3(Sec. 4-264, Fig. 4-31, Standard Handbook for Electrical Engineers, Tenth Ed., Donald G. Fink and John M. Carroll, McGraw-Hill), we get

$B_{Z1} = 1.577 \text{ wb/m}^2 = 15770$ gauss

$H_{Z1} = 32$ oe $= 32 \times 79.577 = 2546$ A/m

$B_{c1} = 1.4668 \text{ wb/m}^2 = 14668$ gauss

$H_{c1} = 17$ oe $= 17 \times 79.577 = 1352$ A/m

$B_{Z2} = 1.607 \text{ wb/m}^2 = 16070$ gauss

$H_{Z2} = 40$ oe $= 40 \times 79.577 = 3183$ A/m

$B_{c2} = 1.404 \text{ wb/m}^2 = 14040$ gauss

$H_{c2} = 13$ oe $= 13 \times 79.577 = 1034$ A/m

MAGNETIC MATERIALS

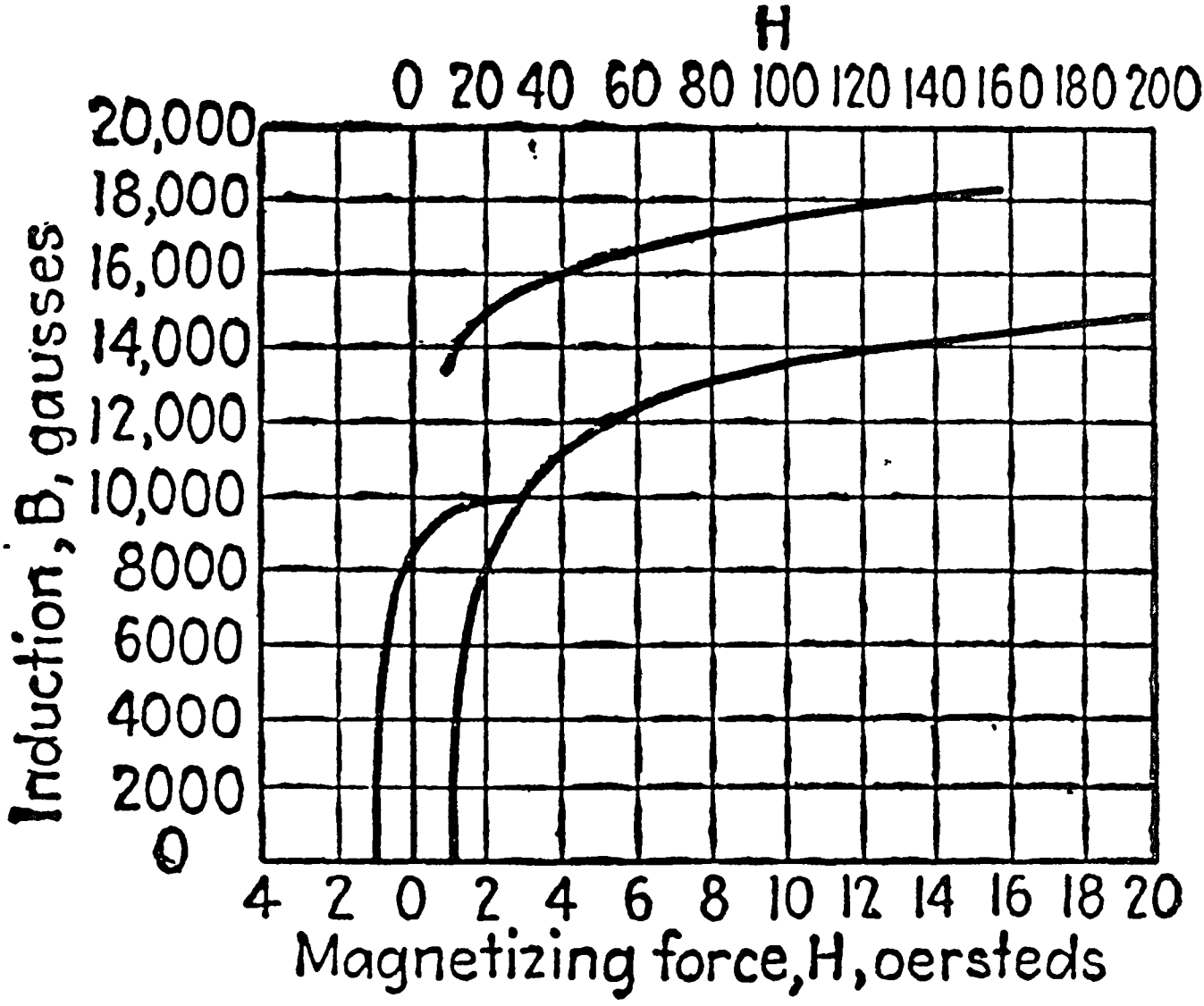


Fig. D_3 Normal-induction curve and hysteresis loop of electrical sheet, No. 22 gage.

58. Magnetizing force AT

$$AT = AT_g + AT_{Z1} + AT_{c1} + AT_{Z2} + AT_{c2}$$

59. Magnetizing force of air gap AT_g

$$AT_g = 8 \times 10^5 \times l'_g \times B_g = 8 \times 10^5 \times 0.000354 \times 0.593 = 168 \text{ A}$$

60. Magnetizing force of stator tooth AT_{Z1}

$$AT_{Z1} = H_{Z1} \times l_{Z1} = 2546 \times 0.033 = 84 \text{ A}$$

61. Magnetizing force of stator core AT_{c1}

$$AT_{c1} = H_{c1} \times l'_{c1} = 1352 \times 0.076 = 102.7 \text{ A}$$

62. Magnetizing force of rotor tooth AT_{Z2}

$$AT_{Z2} = H_{Z2} \times l_{Z2} = 3183 \times 0.017 = 54 \text{ A}$$

63. Magnetizing force of rotor core AT_{c2}

$$AT_{c2} = H_{c2} l'_{c2} = 1034 \times 0.0152 = 16 \text{ A}$$

where $l'_{c2} = \zeta l_{c2} = 0.38 \times 4 = 1.52 \text{ cm} = 0.0152 \text{ m}$

$$l_{c2} = \frac{D - 2 l_g - 2 d_2 - 2 h'_{c2}}{4p} \simeq 4 \text{ cm}$$

$$AT = 168 + 84 + 102.7 + 54 + 16 = 424.7 \text{ A}$$

64. Magnetizing current I_{or}

$$I_{or} = {}^{p \times 2AT}/_{1.17 \times kw \times T_{ph}} = {}^{2 \times 424.7}/_{1.17 \times 0.9029 \times 33} = 24.4 \text{ A}$$

65. Magnetizing reactance X_m

$$X_m \simeq \frac{E_1}{I_{or}} = \frac{17.0}{24.4} = 6.967\Omega$$

66. Magnetizing inductance L_m

$$L_m = \frac{X_m}{2\pi f} = \frac{6.967}{2x\pi x80} = 13.86 \times 10^{-3} \quad H$$

67. The specific slot permeance of stator λ_{s1}

$$\lambda_{s1} = \frac{h_2}{W_2-W_s} \ln \left(\frac{W_2}{W_s}\right) + \frac{h_3}{W_s-b_o} \ln \left(\frac{W_s}{b_o}\right) + \frac{h_4}{b_o} + \frac{h_1+1}{W_b} \left[\frac{\beta^2 - \frac{\beta^4}{4} - \ln\beta - \frac{3}{4}}{(1-\beta)(1-\beta^2)^2}\right]$$

$$= \frac{2}{12.62-12.1} \ln \left(\frac{12.62}{12.1}\right) + \frac{2}{12.1-3.5} \ln \left(\frac{12.1}{3.5}\right) + \frac{1}{3.5} +$$

$$\frac{32}{21}\left[\frac{0.6^2 - \frac{0.6^4}{4} - \ln 0.6 - \frac{3}{4}}{(1-0.6)(1-0.6^2)^2}\right] = 0.162 + 0.288 + 0.286 + 0.822 = 1.558$$

where $W_2 = \frac{[146 + 2\ (h_4 + h_3 + h_2)]\ \pi}{24} - 7.8 = 12.62$ mm

$W_s = 12.1$ mm

$h_2 = 2$ mm

$h_3 = 2$ mm

$h_4 = 1$ mm

$b_o = 3.5$ mm

$\beta = \frac{W_2}{W_b} = \frac{12.62}{21} = 0.6$ (refer to Fig. 1)

68. The specific slot permeance of rotor λ_{s2}

$$\lambda_{s2} = \frac{h'_3}{W_{s2}-1.5} \ln\left(\frac{W_{s2}}{1.5}\right) + \frac{1}{1.5} + \frac{d_2}{W_{b2}}\left[\frac{\beta^2 - \frac{\beta^4}{4} - \ln\beta - \frac{3}{4}}{(1-\beta)(1-\beta^2)^2}\right] = \frac{2}{12.7-1.5} \times$$

$$\ln\left(\frac{12.7}{1.5}\right) + \frac{1}{1.5} + \frac{17}{7.356}\left[\frac{1.726^2 - \frac{1.726^4}{4} - \ln 1.726 - \frac{3}{4}}{(1-1.726)(1-1.726^2)^2}\right] = 1.49$$

where $W_{s2} = 12.7$ mm

$W_{b2} = 7.356$ mm

$d_2 = 17$ mm

$$\beta = \frac{W_{s2}}{W_{b2}} = 1.726$$

69. The reduced value λ'_{s2}

$$\lambda'_{s2} = \frac{kw^2 Z_1}{Z_2}\lambda_{s2} = \frac{0.9029^2 \times 24}{20} \times 1.49 = 1.46$$

70. The slot leakage reactance of stator χ_{s1}

$$\chi_{s1} = \frac{8f_m L T_{ph}^2}{10^7 Z_1}\lambda_{s1} = \frac{8 \times 80 \times 2 \times 6.3 \times 33^2}{10^7 \times 24} \times 1.558 = 0.057\ \Omega$$

where L is in inches

71. The slot leakage reactance of rotor χ_{s2}

$$\chi_{s2} = \frac{8fm L T_{ph}^2}{10^7 Z_1}\lambda'_{s2} = \frac{8 \times 80 \times 2 \times 6.3 \times 33^2}{10^7 \times 24} \times 1.46 = 0.535\ \Omega$$

72. The end leakage reactance X_e

$$X_e = \frac{4fm\ D\ T_{ph}^2}{10^7(2p)^2} = \frac{4 \times 80 \times 2 \times 5.75 \times 33^2}{10^7(2)^2} = 0.1\ \Omega$$

where D = 14.6 cm = 5.75 in

73. Differential leakage reactance (sum of the zigzag and belt leakage reactance)

$$X_d = \frac{5}{6} X_m \left(\frac{6K_{c1} - 1 + 5\sigma^2}{5\ S_1^2} + \frac{\sigma k_{c2} - 1}{5\ S_2^2}\right) = \frac{5}{6} \times 6.967 \times \left(\frac{6 \times 1.14 - 1 + 5(0.5)^2}{5 \times 12^2}\right.$$

$$\left. + \frac{0.5 \times 1.034 - 1}{5 \times 10^2}\right) = 0.0516$$

$\sigma = 0.5$ is the angle of rotor slot skew, as a fraction of one stator slot pitch

74. The leakage reactance of stator X_1

$$X_1 = X_{s1} + \frac{Z_2}{Z_1 + Z_2}(X_e + X_d) = 0.057 + \frac{20}{24 + 20}(0.1 + 0.0516) = 0.126\ \Omega$$

75. The reduced leakage reactance of rotor X'_2

$$X_2' = X_{s2} + \frac{Z_1}{Z_1 + Z_2}(X_e + X_d) = 0.0535 + \frac{24}{24 + 20}(0.1 + 0.0516) = 0.136\ \Omega$$

76. $$L_{1s} = \frac{X_1}{2\pi f} = \frac{0.126}{2 \times \pi \times 80} = 2.5 \times 10^{-4}\ H$$

$$L_{1r} = \frac{X_2'}{2\pi f} = \frac{0.136}{2 \times \pi \times 80} = 2.7 \times 10^{-4}\ H$$

f = 20 Hz V_D = 60 V	f = 80 Hz V_D = 189 V	f = 160 Hz V_D = 240 V
$R_s = 0.0274$	$R_s = 0.0274$	$R_s = 0.0274$
$R'_r = 0.0213$	$R'_r = 0.0213$	$R'_r = 0.0213$
$L_{lr} = 2.41 \times 10^{-4}$ H	$L_{lr} = 2.7 \times 10^{-4}$ H	$L_{lr} = 3.23 \times 10^{-4}$ H
$L_{ls} = 2.256 \times 10^{-4}$ H	$L_{ls} = 2.5 \times 10^{-4}$ H	$L_{ls} = 2.94 \times 10^{-4}$ H
$L_m = 6.51 \times 10^{-3}$ H	$L_m = 13.86 \times 10^{-3}$ H	$L_m = 26.8 \times 10^{-3}$ H
I_{or} = 66 A (rms)	I_{or} = 24.4 A (rms)	I_{or} = 8 A (rms)
$I_{or} = \sqrt{2} \times 66$ (peak) = 93 A	$I_{or} = \sqrt{2} \times 24$ (peak) ≃ 34 A	$I_{or} = \sqrt{2} \times 8$ (peak) ≃ 11 A

Modified Parameters of the Motor Operating at

$f = 20$ Hz $\quad V_D = 60$ V

(1) $E_{1(rms)} = \frac{4}{\sqrt{2} \times \pi} V_D = \frac{4}{\sqrt{2} \times \pi} \times 60 = 54$ V

(13) $\phi_m = \frac{E_1}{4.44\ kw\ f\ T_{ph}} = \frac{54}{4.44 \times 0.9029 \times 20 \times 33}$

$= 0.0204$

Neglect the change of resistance, as the operation frequency is changed.

(26) $R_s = 0.0274\ \Omega$

(38) $R_r' = 0.0213\ \Omega$

(41) $B_g = 1.36\ \phi_m/A_g = \frac{1.36 \times 0.0204}{0.0367}$

$= 0.756$

(43) $B_{Z1} = 1.36\ \phi_m/A_{Z1} = \frac{1.36 \times 0.0204}{0.0138}$

$= 2.01$

(47) $B_{c1} = \phi_m/2A_{c1} = \frac{0.0204}{2 \times 0.005454} = 1.87$

Modified Parameters of the Motor Operating at

$f = 160$ Hz $\quad V_D = 240$ V

$E_{1(rms)} = \frac{4}{\sqrt{2} \times \pi} \times 240 = 216$ V

$\phi_m = \frac{216}{4.44 \times 0.9029 \times 160 \times 33} = 0.0102$

$R_s = 0.0274\ \Omega$

$R_r' = 0.0213\ \Omega$

$B_g = \frac{1.36 \times 0.0102}{0.0367} = 0.378$

$B_{Z1} = \frac{1.36 \times 0.0102}{0.0138} = 1.005$

$B_{c1} = \frac{0.0102}{2 \times 0.005454} = 0.935$

(53) $B_{Z2} = \frac{1.36\phi_m}{A_{Z2}} = \frac{1.36 \times 0.0204}{0.01354} = 2.049$

$B_{Z2} = \frac{1.36 \times 0.0102}{0.01354} = 1.0245$

(56) $B_{c2} = \frac{\phi_m}{2 \times A_{c2}} = \frac{0.0204}{2 \times 0.0057} = 1.79$

$B_{c2} = \frac{0.0102}{2 \times 0.0057} = 0.895$

(57) $B_{Z1} = 2.01$

$B_{Z1} = 1.005$

$B'_{Z1} = 0.9 \times B_{Z1} = 1.809$

(When teeth are over saturated, part of flux passes through slots.)

$H_{Z1} = 130 \times 79.577 = 10345$

$H_{Z1} = 3 \times 79.577 = 239$

$B_{c1} = 1.87$

$B_{c1} = 0.935$

$B'_{c1} = 0.9 \times 1.87 = 1.683$

(Part of flux spreads to housing.)

$H_{c1} = 55 \times 79.577 = 4376$

$H_{c1} = 2.6 \times 79.577 = 207$

$B_{Z2} = 2.049$

$B_{Z2} = 1.0245$

$B'_{Z2} = 0.9 \times 13_{Z2} = 1.84$

$H_{Z2} = 160 \times 79.577 = 12732$

$B_{c2} = 1.79$

$B'_{c2} = 0.9 \times 1.79 = 1.61$

$H_{c2} = 40 \times 79.577 = 3183$

(59) $AT_g = 8 \times 10^5 \times l'_g \times B_g = 8 \times 10^5 \times 0.000354 \times 0.756$
$= 214$

(60) $AT_{Z1} = H_{Z1} l_{Z1} = 10345 \times 0.033 = 341$

(61) $AT_{c1} = H_{c1} l'_{c1} = 4376 \times 0.076 = 332$

(62) $AT_{Z2} = H_{Z2} l_{Z2} = 12732 \times 0.017 = 216$

(63) $AT_{c2} = H_{c2} l'_{c2} = 3183 \times 0.0152 = 48$

$AT = AT_g + AT_{z1} + AT_{c1} + AT_{z2} + AT_{c2}$

$= 214 + 341 + 332 + 216 + 48 = 1151$

(64) $I_{or} = \frac{P \times 2 \times AT}{1.17 \times kw \times T_{ph}} = \frac{2 \times 1151}{1.17 \times 0.9029 \times 33}$

$= 66$

(65) $\chi_m = \frac{E_1}{I_{or}} = \frac{54}{66} = 0.818$

$H_{Z2} = 3.2 \times 79.577 = 255$

$B_{c2} = 0.895$

$H_{c2} = 2.5 \times 79.577 = 199$

$AT_g = 8 \times 10^5 \times 0.000354 \times 0.378$
$= 107$

$AT_{Z1} = 239 \times 0.033 = 8$

$AT_{c1} = 207 \times 0.076 = 16$

$AT_{Z2} = 255 \times 0.017 = 5$

$AT_{c2} = 199 \times 0.0152 = 3$

$AT = 107 + 8 + 16 + 5 + 3 = 139$

$I_{or} = \frac{2 \times 139}{1.17 \times 0.9029 \times 33} \simeq 8$

$\chi_m = \frac{216}{8} = 27$

(66) $L_m = \frac{X_m}{2\pi f} = \frac{0.818}{2 \times \pi \times 20} = 6.51 \times 10^{-3}$ $\quad L_m = \frac{27}{2 \times \quad \times 160} = 26.8 \times 10^{-3}$

The overload current is around two times larger than rating, so slot leakage inductance still can be considered as a constant.

(70) $X_{s1} = \frac{8fm\, L\, T_{ph}^2}{10^7 Z_1} \lambda_{s1}$ $\quad X_{s1} = \frac{8 \times 160 \times 2 \times 6.3 \times 33^2}{10^7 \times 24} \times 1.558$

$= \frac{8 \times 20 \times 2 \times 6.3 \times 33^2}{10^7 \times 24} \times 1.558 = 0.01425$ $\quad = 0.114$

(71) $X_{s2} = \frac{8fm\, L\, T_{ph}^2}{10^7 \times Z_1} = \lambda'_{s2}$ $\quad X_{s2} = \frac{8 \times 160 \times 2 \times 6.3 \times 33^2}{10^7 \times 24} \times 1.46$

$= \frac{8 \times 20 \times 2 \times 6.3 \times 33^2}{10^7 \times 24} \times 1.46 = 0.1336$ $\quad = 0.107$

(72) $X_e = \frac{4fm\, D\, T_{ph}^2}{10^7 (2p)^2} = \frac{4 \times 20 \times 2 \times 5.75 \times 33^2}{10^7 (2)^2}$ $\quad X_e = \frac{4 \times 160 \times 2 \times 5.75 \times 33^2}{10^7 (2)^2}$

$= 0.025$ $\quad = 0.2$

(73) $X_d = \frac{5}{6} X_m \left(\frac{6k_{c1} - 1 - 5\sigma^2}{5S_1^2} + \frac{\sigma k_{c2} - 1}{5S_2^2}\right)$ $\quad X_d = \frac{5}{6} \times 27 \times \left(\frac{6 \times 1.14 - 1 + 5 \times 0.5^2}{5 \times 12^2} + \frac{0.5 \times 1.034 - 1}{5 \times 10^2}\right)$

$= \frac{5}{6} \times 0.818 \times \left(\frac{6 \times 1.14 - 1 - 5 \times 0.5^2}{5 \times 12^2} + \frac{0.5 \times 1.034 - 1}{5 \times 10^2}\right)$ $\quad = 0.2$

$= 0.00605$

(74) $X_1 = X_{s1} + \frac{Z_2}{Z_1+Z_2}(X_e + X_d) = 0.01425$

$+ \frac{20}{20+24}(0.025 + 0.00605) = 0.02835$

$X_1 = 0.114 + \frac{20}{20+24}(0.2 + 0.2)$

$= 0.296$

(75) $X_2' = X_{s2} + \frac{Z_1}{Z_1+Z_2}(X_e + X_d)$

$= 0.01336 + \frac{24}{24+20}(0.025 + 0.00605)$

$= 0.0303\ \Omega$

$X_2' = 0.107 + \frac{24}{24+20}(0.2 + 0.2)$

$= 0.325$

(76) $L_{1s} = \frac{X_1}{2\pi f} = \frac{0.02835}{2 \times \pi \times 20} = 2.256 \times 10^{-4}$

$L_{1s} = \frac{0.296}{2 \times \pi \times 160} = 2.94 \times 10^{-4}$

$L_{1r} = \frac{X_2'}{2\pi f} = \frac{0.0303}{2 \times \pi \times 20} = 2.41 \times 10^{-4}$

$L_{1r} = \frac{0.325}{2 \times \pi \times 160} = 3.23 \times 10^{-4}$

$R_s = 0.0274\ \Omega$

$R_s = 0.0274\ \Omega$

$R_r' = 0.0213\ \Omega$

$R_r' = 0.0213\ \Omega$

$L_m = 6.51 \times 10^{-3}$

$L_m = 26.8 \times 10^{-3}$

APPENDIX E

MOTOR TRANSIENT TEMPERATURE RISE

Peak Momentary Motor Temperature Rise

The specifications require three motor operating points.

1. 100 per cent speed, 30 kW output for 1 hour
2. 10 per cent speed, 16 kW output for 3 min.
3. 80 per cent speed, 60 kW output for 3 min.

The first one is obviously no problem.

$$I_{ph} = \frac{170}{216} \times 110.2 = 86.7 \text{ A}$$

$$\delta_1 = \frac{86.7}{2 \times 11.19} = 3.87 \text{ A/mm}^2 \quad \text{(See Appendix D)}$$

The copper losses are small due to the low current, and the motor operates under saturation, so the core losses are also small. Although the mechanical losses are greatest at top speed, high speed creates very good heat dissipation conditions. One hour operation is expected to produce nearly steady state temperature rise.

The second one is worse than the third case for temperature rise because at this operating point there are maximum losses and worst case heat dissipation conditions.

The heat balance differential (1) can be applied to evaluate the transient temperature rise.

$$\frac{d\theta}{dt} + \theta \frac{S\lambda}{GC_p} = \frac{P}{GC_p} \qquad (1)$$

where θ is temperature rise, °C

S is cooling surface of the hot body, m^2

G is weight of the hot body, kg

C_p is specific heat of the hot body, J/kg-°C

λ is specific heat dissipation or emissivity, J per second per m^2 of surface per °C difference between surface and ambient cooling medium

P is heat developed, J per second or W

During a short time we neglect the exchange between stator and rotor (the heat-resistance of air gap is very large), and even neglect the heat dissipation. This means that all the heat developed causes temperature rise, and of course, this is the most serious situation.

Then (1) becomes

$$\frac{d\theta}{dt} = \frac{P}{GC_p}$$

$$\theta = \int \frac{P}{GC_p}\, dt \qquad (2)$$

If we assume P and C_p are constant, then

$$\theta = \frac{P}{GC_p}\, t \qquad (3)$$

where t = 3 minutes = 180 s

$$G = m \times 2 \times L_{ph} \times S_c \times a_p \times 10^{-3} \times g_c$$

$$= 2 \times 2 \times 29.17 \times 11.19 \times 2 \times 10^{-3} \times 8.9 = 23.24$$

g_c = 8.9 kg/dm^2 (specific gravity of copper)

The total weight G of the two-phase bifilar winding is greater because each single winding conducts for only a half cycle. Thus, the bifilar windings have an advantage for short time transient temperature rise - not significant for steady state.

The specific heat C_p of copper is

$$C_p = 0.092 \text{ Btu/lbF}$$

$$= 0.092 \times \frac{1055}{0.454 \times \frac{5}{9}} = 384.8 \text{ J/kg}^\circ\text{C}$$

P is stator winding losses

$$P = m \times r_s \times I_{rms}^2 = 2 \times 0.0274 \times 200.4^2$$

$$= 2201$$

(As shown in Appendix G, I_{rms} is 200.4 A)

From (3), we get

$$\theta = \frac{P}{GC_p} t = \frac{2201}{23.24 \times 384.8} \times 180$$

$$= 44.3°C$$

We suggest the insulation class F. Specifications for the F class insulation are as follows:*

The hot-spot temperature is 155°C.

The maximum permitting temperature rise for totally enclosed fan-cooled motors is 105°C, based on ambient temperature of 40°C, 3330 ft. altitude, and determined by the resistance method.

If we directly apply (1) for 30% duty

$$\frac{d\theta}{dt} + \frac{s\lambda}{GC_p} \theta = \frac{P}{GC_p}$$

then
$$\theta = \frac{P/GC_p}{S\lambda/GC_p} - \frac{P/GC_p}{S\lambda/GC_p} e^{-\frac{S\lambda}{GC_p} t}$$

$$= \frac{P}{S\lambda} - \frac{P}{S\lambda} e^{-\frac{S\lambda}{GC_p} t} \qquad (4)$$

where $\lambda = 1/c = 1/0.04 = 25$ J/sec m^2°C

(assume no cooling air)

$$S = Z_1 \times L_{mc} \times L_{sp} = 24 \times 0.442 \times 0.091$$

$$= 1.05 \ m^2$$

L_{sp} is the perimeter of stator slot

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* Standard Handbook for Electrical Engineers, 11th Edition, McGraw-Hill, pp. 20-45.

$$L_{sp} = 2(h_1+h_2) + W_b + W_s$$

$$= 2\ (31 + 2) + 21 + 12.1 = 99.1 \text{ mm}$$

$$= 0.0991 \text{ m}$$

For Z_1, L_{mc}, h_1, h_2, W_b, and W_s refer to the design sheet of the motor in Appendix D.

From (4)

$$\theta = \frac{2201}{1.05 \times 25} - \frac{2201}{1.05 \times 25}\ e^{-\frac{1.05 \times 25}{23.24 \times 384.8}\ t}$$

$$= 83.85 - 83.85\ e^{-0.00294t}$$

30% duty 3 minutes operation
7 minutes off

For the first operation

$$t = 3 \times 60 = 180 \text{ sec}$$

$$\theta_1 = 83.85 - 83.85\ e^{-0.00294 \times 180}$$

$$= 34.5°C$$

During the 7 minute off interval

$$\theta'_1 = \theta_1\ e^{-0.00294 \times t}$$

$$= 34.5\ e^{-0.00294 \times 420} = 10°C$$

The second duty cycle

$$\theta_2 = \theta_1 + 10 = 44.5°C$$

$$\theta'_2 = 44.5\ e^{-0.00294 \times 420} = 13°C$$

The third duty cycle

$$\theta_3 = \theta_1 + 13 = 47.5°C$$

$$\theta_3' = 47.5 \times e^{-0.00294 \times 420} = 13.8°C$$

The fourth duty cycle

$$\theta_4 = \theta_1 + 13.8 = 48.3°C$$

$$\theta_4' = 48.3\ e^{-0.00294 \times 420} = 14.05°C$$

The fifth duty cycle

$$\theta_5 = \theta_1 + 14.05 = 48.55°C$$

$$\theta_5' = 48.55\ e^{-0.00294 \times 420} = 14.12°C$$

The sixth duty cycle

$$\theta_6 = \theta_1 + 14.12 = 48.62°C$$

$$\theta_6' = 48.62\ e^{-0.00294 \times 420} = 14.14°C$$

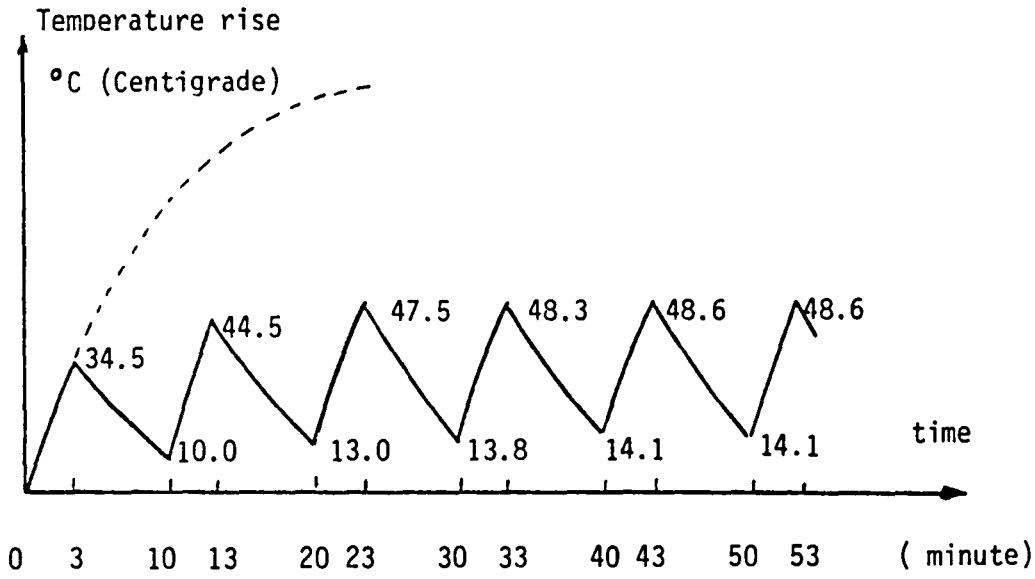


Fig. E-1 Transient Temperature Rise

APPENDIX F

DETAILED INVERTER DESIGN

F.1 INTRODUCTION

The complete system schematic diagram is shown in Fig. F-1. This is identical to Fig. 12 of section 4.4 in this report, but it is included again here for convenience. This appendix describes the detailed design of the two-phase inverter-bifilar induction motor system.

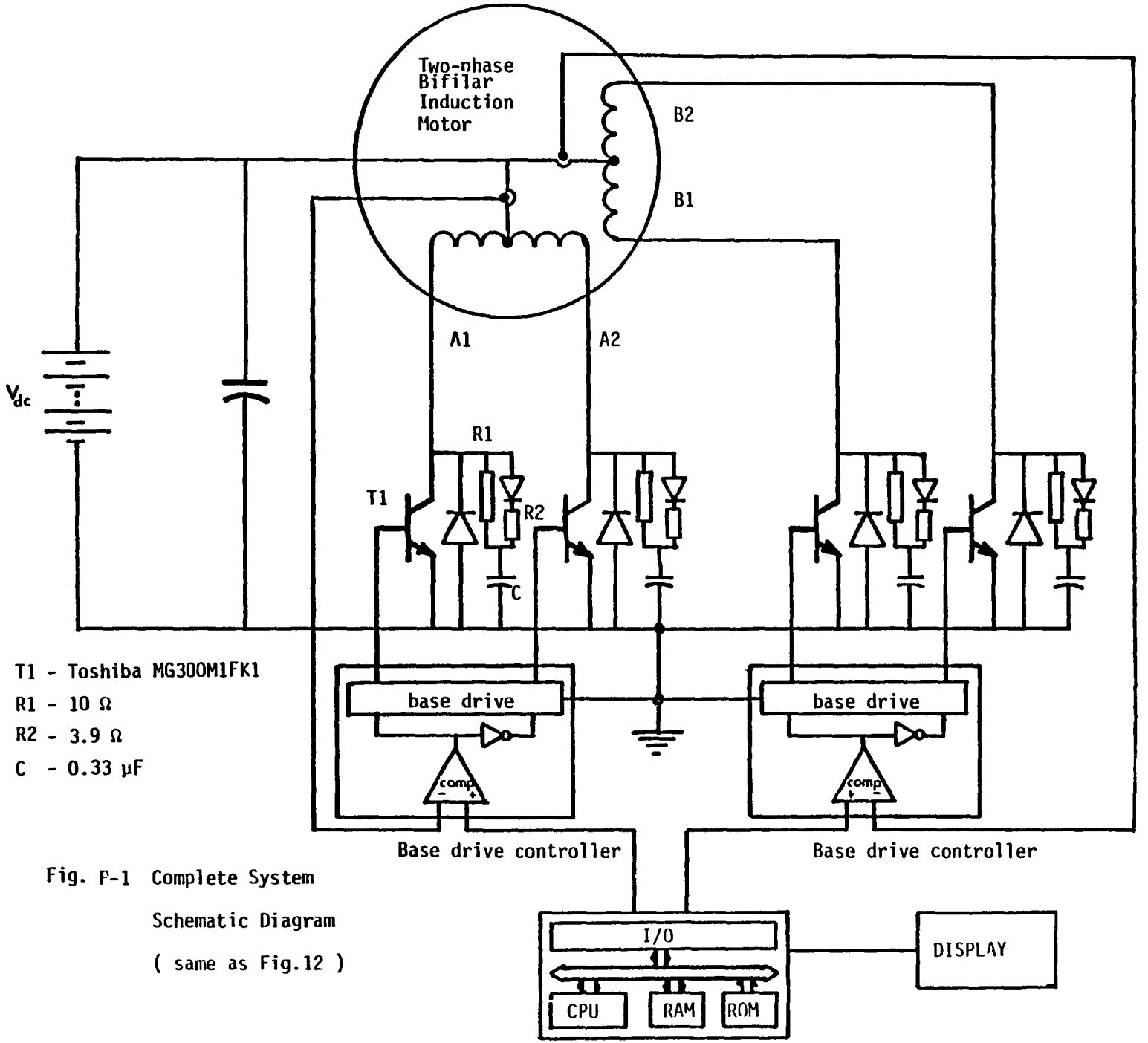
F.2 POWER CIRCUIT DESIGN

Fig. F-2 is the power circuit (same as Fig. 2). The most important aspect of the design of the power circuit is the determination of the rating requirements for the power switch devices. Analysis and simulation of the two-phase bifilar induction machine define the ideal switch voltage and current ratings to supply the power required at the specification operating points.

The power switch ratings (section 3.4.1),assuming ideal circuit conditions, are

Peak Voltage	V_p = 480V
Peak Current	I_p = 245A
RMS Current	I_{RMS} = 200A
Maximum Switching Frequency	$f_{S,MAX}$ = 3.4kHz at 245A peak

However, it is necessary to analyze the specific circuit configuration to determine the precise transient and steady state voltage and current requirements of the switching devices. One of the primary disadvantages of the circuit in Fig. F-2 is the existence of greater transient voltages. These are the result of leakage inductance due to imperfect coupling between the bifilar windings and motor lead inductance which is now in the commutation path where current switching occurs. The lead inductance does not produce voltage transients in the three-phase bridge system.



T1 - Toshiba MG300M1FK1
R1 - 10 Ω
R2 - 3.9 Ω
C - 0.33 μF

Fig. F-1 Complete System Schematic Diagram (same as Fig.12)

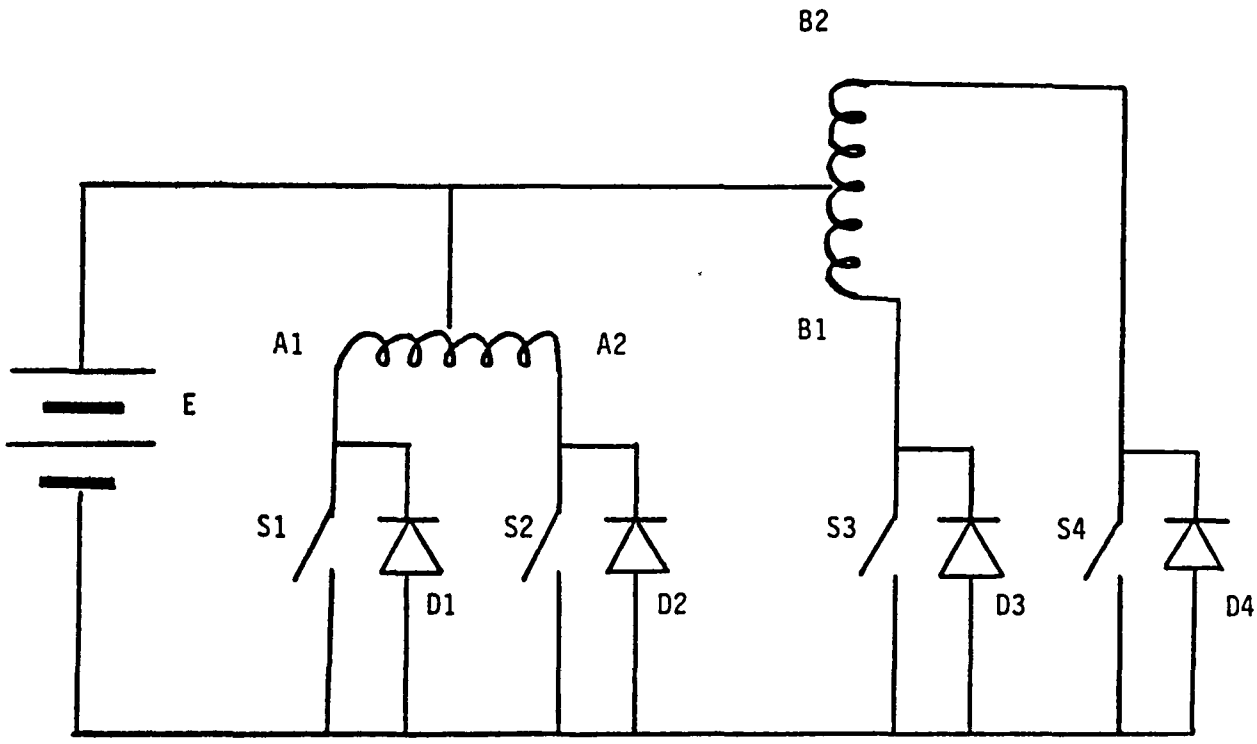


Fig. F-2 Two-phase Bifilar Induction Motor/Inverter

F.3 TRANSIENT VOLTAGE SPIKE ANALYSIS

Consider Fig. F-2 at the moment when switch S1 is opened to transfer its current to the S2-D2 branch. If the A1 and A2 bifilar windings are perfectly coupled, and if there is no lead inductance in the circuit including the dc source, motor leads and switch-diode branches, then the current commutation can occur instantly with no voltage transient. Unfortunately, the bifilar winding coupling is not perfect and lead inductances do exist. Therefore, voltage transients are generated proportional to the rate of change of current and the equivalent leakage inductances of the bifilar windings and the battery, motor and switch-diode wiring inductances. Thus, it is essential to minimize these inductances and determine accurate estimates of their maximum values.

Snubber Design

Fig. F-3 shows a simplified model of a single snubber circuit including a lumped inductance to represent bifilar winding leakage inductances and lead inductances.

The order of magnitude of the spike voltage is determined from assuming that all of the initial energy stored in the inductance is transferred to the capacitor, that is,

$$\frac{1}{2} L I_p^2 = \frac{1}{2} C V_p^2 \qquad (1)$$

where

$I_p \triangleq$ peak current at instant switch opens

$V_p \triangleq$ peak voltage on capacitor

$L \triangleq$ equivalent total leakage and lead inductance

$C \triangleq$ snubber capacitance

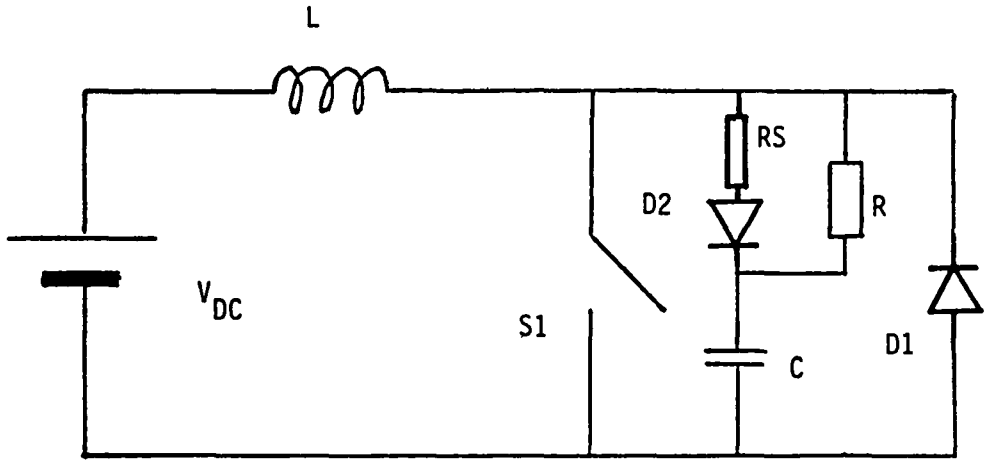


Fig. F-3 Simplified Switching Circuit

Thus

$$V_p = \sqrt{\frac{L}{C}}\ I_p \qquad (2)$$

First, it is necessary to determine the maximum value of L. This inductance is primarily the leakage inductance due to imperfect coupling in the bifilar windings. Normally, the lead inductances are small, particularly if the inverter is mounted close to the motor and if a high quality capacitor is connected directly from the motor winding center-tap to the negative dc bus, which minimizes the effect of battery lead inductance and the inductance in the winding center-tap lead.

The normal total leakage reactance for a 10-100hp induction motor is 0.23 per unit (pu). Of this total, the stator leakage is about 0.1 pu. This stator leakage can be reduced to about 0.05 pu for high torque motors. Finally, the equivalent leakage inductance between two tightly coupled windings in one motor phase should be less than .005 pu. This results in an estimated maximum value of L equal to 15μH. However, it is estimated that the leakage inductance between two bifilar motor windings could be reduced an order of magnitude below 15μH with special motor winding techniques. For the following calculations an L of 4μH is used. With I_p = 245A, from (2)

$$V_p = \sqrt{\frac{4\mu H}{C}}\ 245 = \frac{0.49}{\sqrt{C}} \qquad (3)$$

If C is chosen equal to 0.33μF,

$$V_p = \frac{0.49}{0.33\mu F} = 853V \qquad (4)$$

This is a reasonable limit for V_p.

The snubber R is necessary to limit the peak snubber capacitor discharge current at the moment of turning on the power switch. If R is chosen equal to 10 ohms, then

$$I_p(0) = \frac{2V_{DC}}{R} = 48A \qquad (5)$$

This is a reasonable $I_p(0)$ to allow for the device rating.

Voltage Spike Simulation

The snubber design carried out in the previous section is based on an approximate transient voltage estimation. The precise switching behavior can be determined by digital simulation including all non-linear effects, or by hardware experiments. Since no hardware construction was done, digital simulations were performed using SPICE to provide more accurate results than could be obtained from approximate analysis.

The switching transient is short relative to changes which occur in the rotor circuit of the induction motor. Thus, the steady state equivalent circuit model is used for the motor load, i.e., r_2/s where r_2 is rotor resistance and s is the slip. The equivalent stator circuit is derived using the three winding transformer input-output relations. Fig. F-4 shows the flux linkages of the transformer, which is equivalent to the bifilar wound induction motor with the assumptions that the switching transient is very fast and that all windings have equal turns. The #1 and #2 windings are the stator bifilar windings, and the #3 winding is the rotor winding. v_1, v_2, v_3 and i_1, i_2, i_3 are the terminal voltages and currents for each winding. ϕ_1, ϕ_2 and ϕ_3 are defined as the total flux of #1, #2 and #3 windings, respectively. $\phi_{\ell 1}$, $\phi_{\ell 2}$, $\phi_{\ell 3}$ are the corresponding leakage flux for the #1, #2 and #3 windings, respectively. ϕ_M is the mutual flux between either stator winding and the rotor winding, and ϕ_B is the mutual flux between the two stator windings. The following relations are obtained.

$$\phi_1 = \phi_{\ell 1} + \phi_B + \phi_M \qquad (6)$$

$$\phi_2 = \phi_{\ell 2} + \phi_B + \phi_M \qquad (7)$$

$$\phi_3 = \phi_{\ell 3} + \phi_M \qquad (8)$$

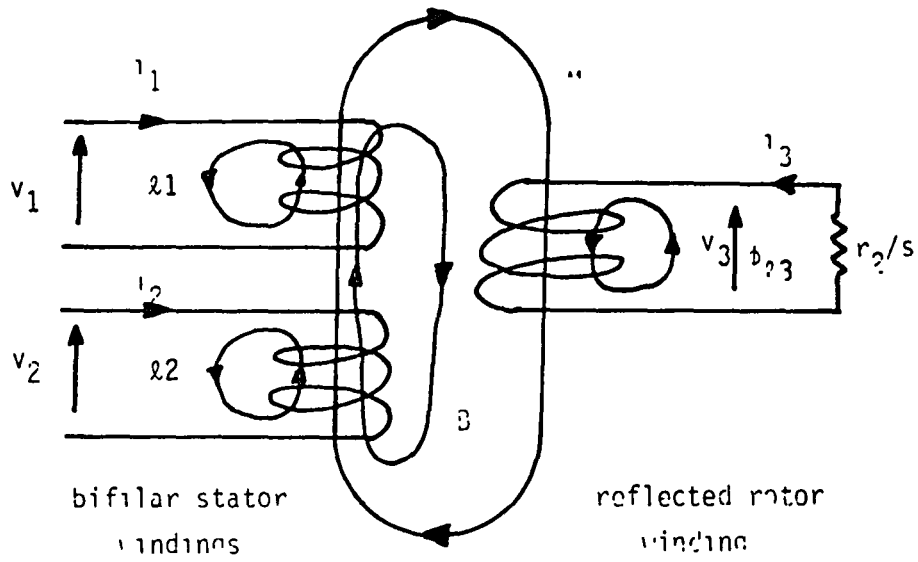


Fig. F-4 Flux linkage relations of three winding transformer

where $v_1 = \frac{d\phi_1}{dt}$, $v_2 = \frac{d\phi_2}{dt}$, and $v_3 = \frac{d\phi_3}{dt}$

Since the #1 and #2 windings are bifilar windings, they are very tightly coupled. Thus, all of the flux ϕ_M links the #3 winding as well as both the #1 and #2 windings.

Defining the mutual (or magnetizing) inductance L_M between the #1, #2 and #3 windings, the mutual flux ϕ_M is

$$\phi_M = L_M i_3 + L_M (i_1 + i_2) \tag{9}$$

Defining the mutual inductance L_{MB} between the #1 and #2 windings, the mutual flux ϕ_B is

$$\phi_B = L_{MB} i_1 + L_{MB} i_2 \tag{10}$$

Defining the leakage inductances $L_{\ell 1}$, $L_{\ell 2}$, and $L_{\ell 3}$ of the #1, #2 and #3 windings respectively, then $\phi_{\ell 1}$, $\phi_{\ell 2}$, and $\phi_{\ell 3}$ are

$$\phi_{\ell 1} = L_{\ell 1} i_1 \tag{11}$$

$$\phi_{\ell 2} = L_{\ell 2} i_2 \tag{12}$$

$$\phi_{\ell 3} = L_{\ell 3} i_3 \tag{13}$$

Combining (6) - (13)

$$\phi_1 = L_{\ell 1} i_1 + L_{MB}(i_1 + i_2) + L_M(i_1 + i_2 + i_3) \tag{14}$$

$$\phi_2 = L_{\ell 2} i_2 + L_{MB}(i_1 + i_2) + L_M(i_1 + i_2 + i_3) \tag{15}$$

$$\phi_3 = L_{\ell 3} i_3 + L_M (i_1 + i_2 + i_3) \tag{16}$$

These three equations provide the equivalent circuit shown in Fig. F-5, where the rotor load and stator resistance r_1 are added. v_1 and v_2 are controlled to be E, -E or 0. This equivalent circuit becomes the conventional equivalent circuit if one of the bifilar winding branches is removed.

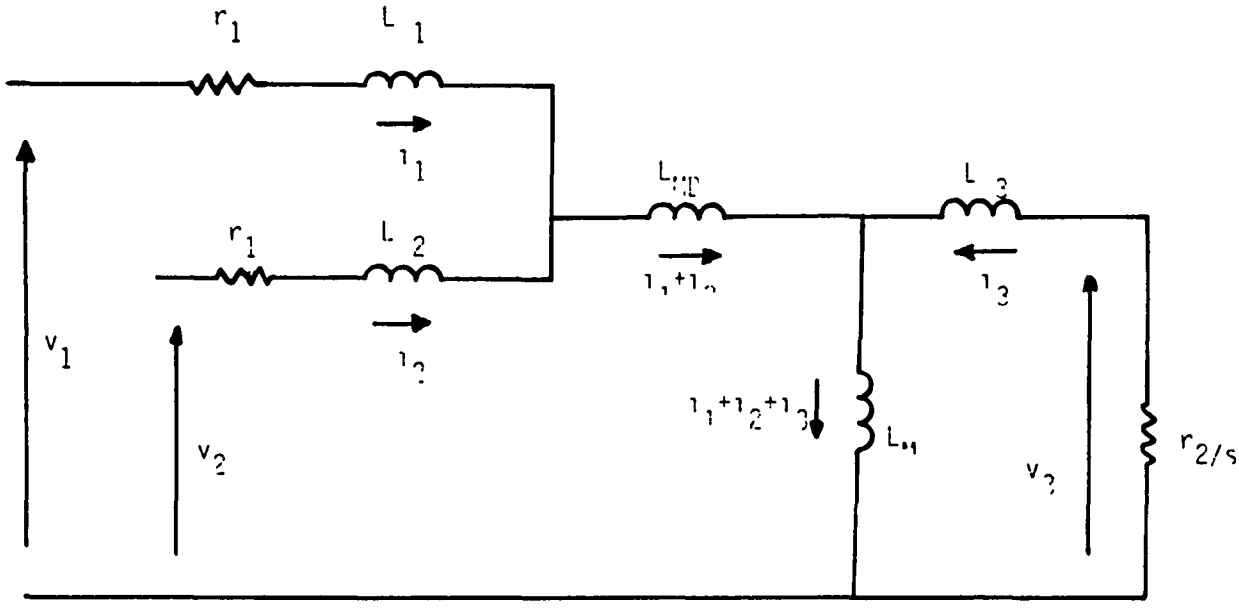


Fig. F-5 Equivalent Switching Circuit

SPICE is a general purpose circuit simulation program. Circuits may contain resistors, capacitors, inductors, mutual inductors, independent voltage and current sources, four types of dependent sources, transmission lines, and the most common semiconductor devices: diodes, BJTs, JFETs and MOSFETs.

Fig. F-6 shows the equivalent circuits used for simulation, including snubber circuits and base drive circuits. When one device is turned off, the base drive voltage source is changed linearly from 10 volts to zero for a 5 microsecond interval. After a 15μs delay, when the other is turned on, the base voltage is controlled linearly from zero to 10 volts for a 5μs interval. When one device is switched off, the current first flows through the feedback diode of the other device. Thus, the delay time in turn-on of the next device has no effect. The interval during which the base voltage source is linearly controlled is changed from 5μs to 2μs, but the transient voltage changes no more than 0.4% in its peak value. The lumped leakage inductances LLS1 and LLS2, shown in Fig. F-6, are chosen to be 4μH, while the sum of LLS1 and LMB is equal to the stator leakage inductance. LM and LLR are equal to the magnetizing and rotor leakage inductances respectively. RS1 and RS2 are the stator winding resistances, and RR2 is calculated from r_2/s where r_2 is the rotor resistance and s is the slip at the steady state operating point. The series resistances R11 and R22 help to suppress the transient voltage. By trial and error, 3.9 ohms are selected. A peak transient voltage of 943 volts across the transistors is obtained for the following operating conditions and system parameters:

10% speed, 16 kW output

Percent slip = 8.5%

Stator current = 245A

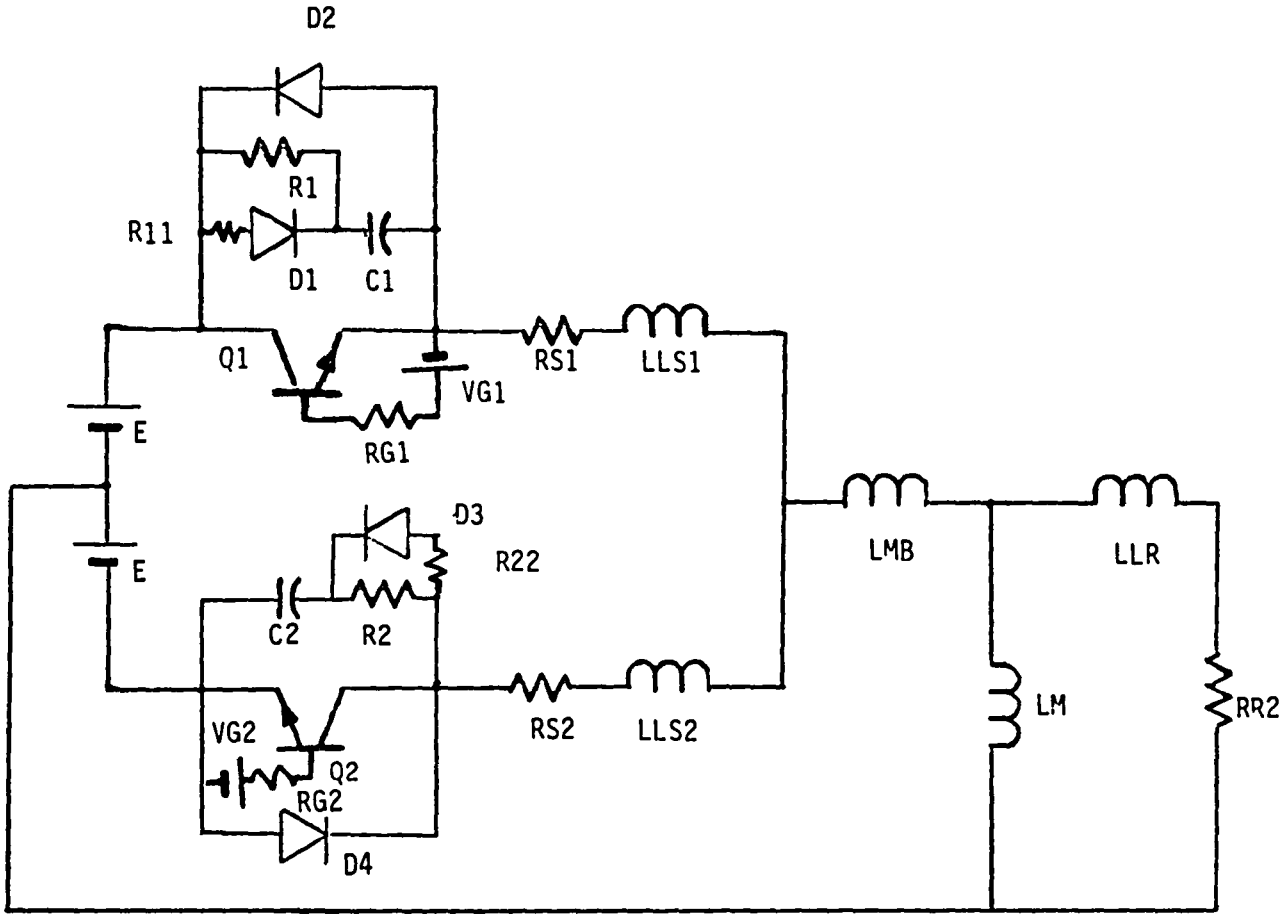


Fig. F-6 Equivalent circuit used for simulation

Note: LLS1 and LLS2 are equivalent leakage inductance.

Magnetizing current (through LM) = 93A

Rotor current = 142A

LM = 6.51mH; LLR = 241μH

LLS1 = LLS2 = 4μH; LMB = 221.6μH

RS1 = RS2 = 0.0274Ω; RR2 = 0.0213/0.085 = 0.251Ω

R1 = R2 = 10Ω; R11 = R22 = 3.9Ω

C1 = C2 = 0.33μF

The transient waveforms of the collector-emitter voltage and bifilar winding current (i_1 in Fig. F-6) are plotted in Fig. F-7, and the corresponding simulation source program is listed in Fig. F-8. Without R11 and R22, the transient voltage is about 1160 volts.

Next, the equivalent leakage inductances LLS1 and LLS2 are changed from zero to 10μH and the peak transient voltage is plotted in Fig. F-9. The operating condition of 80% speed, 60kW output is also simulated, and the transient voltage variation is shown in Fig. F-9 using the following parameters:

Stator current = 245A

Magnetizing current = 20A

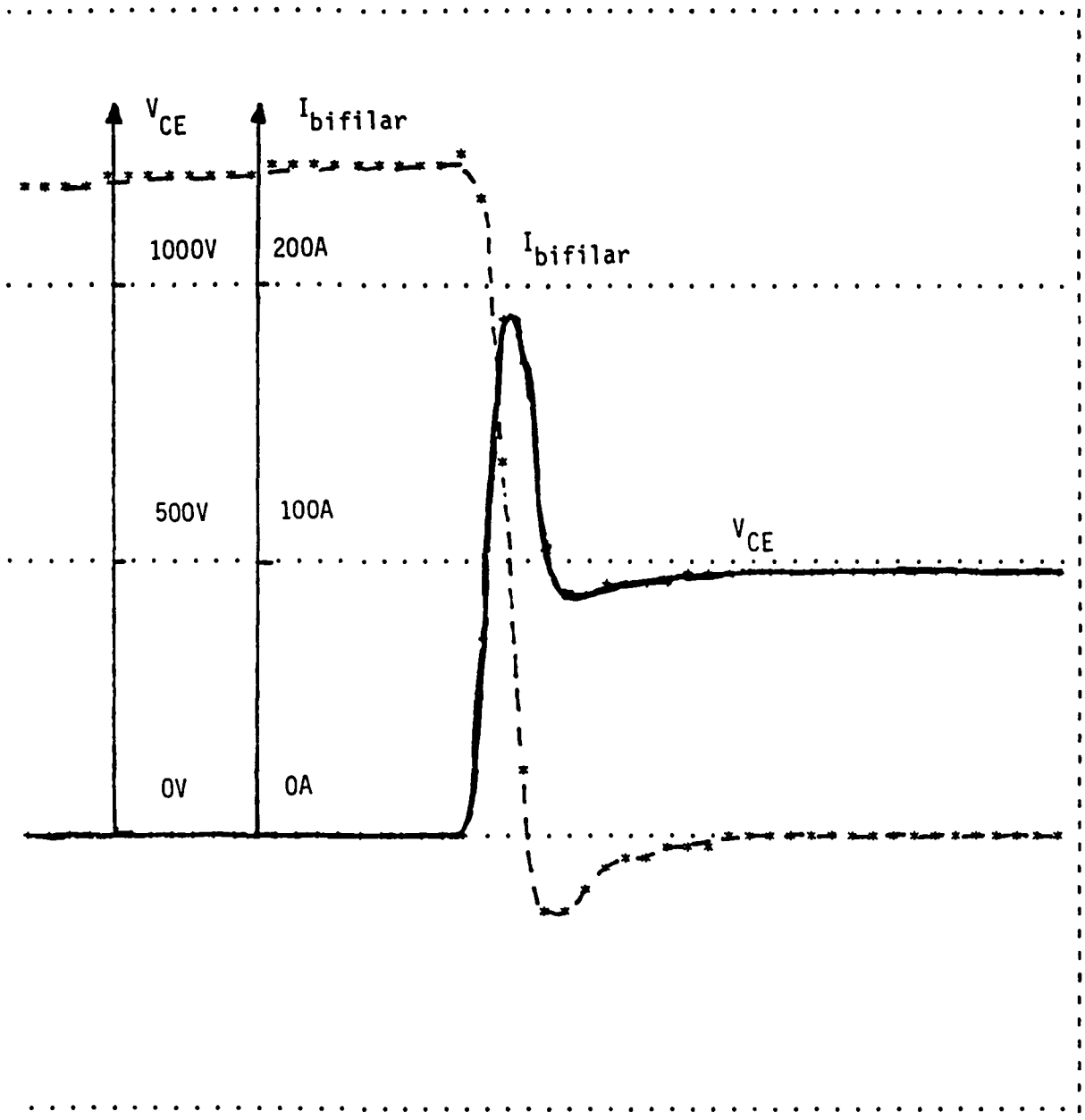
Rotor current = 225A

LM = 26.8mH; LLS1 + LMB = 294μH

RR2 = 0.0213/.02 = 1.07Ω(2% slip)

Other parameters same as for 16kW, 10% speed condition

Fig. F-9 indicates that the spike voltage is somewhat less than 1000V if the equivalent leakage inductance is 4μH.



V_{CE}	9.4260D-02	9.4330D-02
	9.4390D-02	9.4460D-02
	9.4520D-02	9.4590D-02
	9.4660D-02	9.4720D-02
	9.4790D-02	9.4850D-02
	9.4920D-02	9.4980D-02
	9.5050D-02	9.5110D-02
	9.5240D-02	9.5310D-02
	9.5370D-02	9.5440D-02
	9.5500D-02	9.5570D-02
	1.1950D-01	3.6160D+02
	9.4250D+02	8.5780D+02
	5.2650D+02	4.4460D+02
	4.4640D+02	4.5550D+02
	4.6620D+02	4.6600D+02
	4.6870D+02	4.7060D+02
	4.7150D+02	4.7200D+02
	4.7300D+02	4.7370D+02
	4.7420D+02	4.7440D+02
	4.7450D+02	4.7460D+02
	4.7470D+02	4.7470D+02
	4.7470D+02	4.7470D+02
	4.7470D+02	4.7470D+02
	4.7470D+02	4.7470D+02
	4.7470D+02	4.7470D+02
	4.7470D+02	4.7470D+02
time	0.0000D-05	0.1000D-05
	0.2000D-05	0.3000D-05
	0.4000D-05	0.5000D-05
	0.6000D-05	0.7000D-05
	0.8000D-05	0.9000D-05
	0.9000D-05	0.9100D-05
	0.9200D-05	0.9300D-05
	0.9400D-05	0.9500D-05
	0.9600D-05	0.9700D-05
	0.9800D-05	0.9900D-05
	1.0000D-04	1.0100D-04
	1.0200D-04	1.0300D-04
	1.0400D-04	1.0500D-04
	1.0600D-04	1.0700D-04
	1.0800D-04	1.0900D-04
	1.1000D-04	1.1100D-04
	1.1200D-04	1.1300D-04
	1.1400D-04	1.1500D-04
	1.1600D-04	1.1700D-04
	1.1800D-04	1.1900D-04
	1.2000D-04	1.2100D-04
	1.2200D-04	1.2300D-04
	1.2400D-04	1.2500D-04
	1.2600D-04	1.2700D-04
	1.2800D-04	1.2900D-04
	1.3000D-04	

Fig. F-7 Voltage spike simulation : 16kw, 10%speed, I_p=245A,
C1=C2=0.33 μF, LLS1=LLS2=4 μH, R11=R22=3.9Ω , R1=R2=10Ω in Fig.F-6

```
***** 1/28/85********** SPICE 2E.2 (26SEP78) *********15445436*****

      JPL PROJECT VOLTAGE SPIKE 12/18/84                                SPI00060

     INPUT LISTING                    TEMPERATURE =   27.000 DEG C

*********************************************************************


*** BIFILAR WINDING     3 TRANSFORMER MODEL                            SPI00070
***    MAIN CIPCUIT                                                    SPI00080
*      CURRENT SENSE FOR COLLECTOR CURRENT VCOLLE
       VDC1 1 0 240VOLT
       Q1 9 2 3 QNPN
       VCOLLE 1 9 1UV
****** ADJUST R11 *******
       R11 1 8 3.900OHM
       D1 8 4 DMOD
       R1 1 4 10.00HM
*  CHANGE C FOR SNUBBER //////
       C1 4 3 0.33UF
       D2 3 1 DMOD
       RS1 3 5 0.0274
*  CHANGE LLS1**************************
       LLS1 5 7 4.00UH IC=198A
*  CURRENT SENSE FOR BIFILAR WINDING VBIFI                            SPI00170
       VBIFI 7 6 1UV
*                                                                     SPI00190
       VDC2 0 11 240V
       Q2 13 12 11 QNPN
****** ADJUST R22 ******
       R22 13 16 3.900OHM
       D3 16 14 DMOD
       R2 13 14 10.00HM
*  CHANGE C FOR SNUBBER ///////
       C2 14 11 0.33UF IC=450V
       D4 11 13 DMOD
       RS2 13 15 0.0274
*  CHANGE LLS2**************************
       LLS2 15 6 4.00UF
*  CHANGE LMB**********LMB+LLS1(LLS2)=225.6UH AT 20 HZ, 294UH AT 160HZ  SPI00280
*  ********************LM=6.51MH, LLR=241UH AT 20 HZ
*  ********************LM=26.8MH, LLR=323UH AT 160 HZ
       LMB 6 21 221.6UH IC=198A
       LM 21 0 6.51MH IC=85A
       LLR 21 22 241UH IC=113A
       RR2 22 0 0.251
*** GATE DRIVE                                                         SPI00330
       RG1 2 7 1OHM
       RG2 12 17 1OHM
       VG1 7 3 PWL  (0  10V 100US 10V 105US -10 420US -10 425US 10V)
       VG2 17 11 PWL(0 -10 120US -10 125US 10V 400US 10V 405US -10)
*** DEVISE MODEL                                                       SPI00380
       .MODEL QNPN NPN(VA=2000V,VB=2000V)
       .MODEL DMOD D(BV=2000V)
***    OUTPUT                                                          SPI00410
      .PRINT TRAN V(9,3),I(VCOLLE),V(4,3),I(VBIFI),V(6)
      .PLOT TRAN V(9,3),I(VBIFI)
      .TRAN 1.0US 130US UIC
*     .OPTIONS LIMPTS=201 LIST NODE
      .END
```

Fig. F-8 Source listing of SPICE simulation program

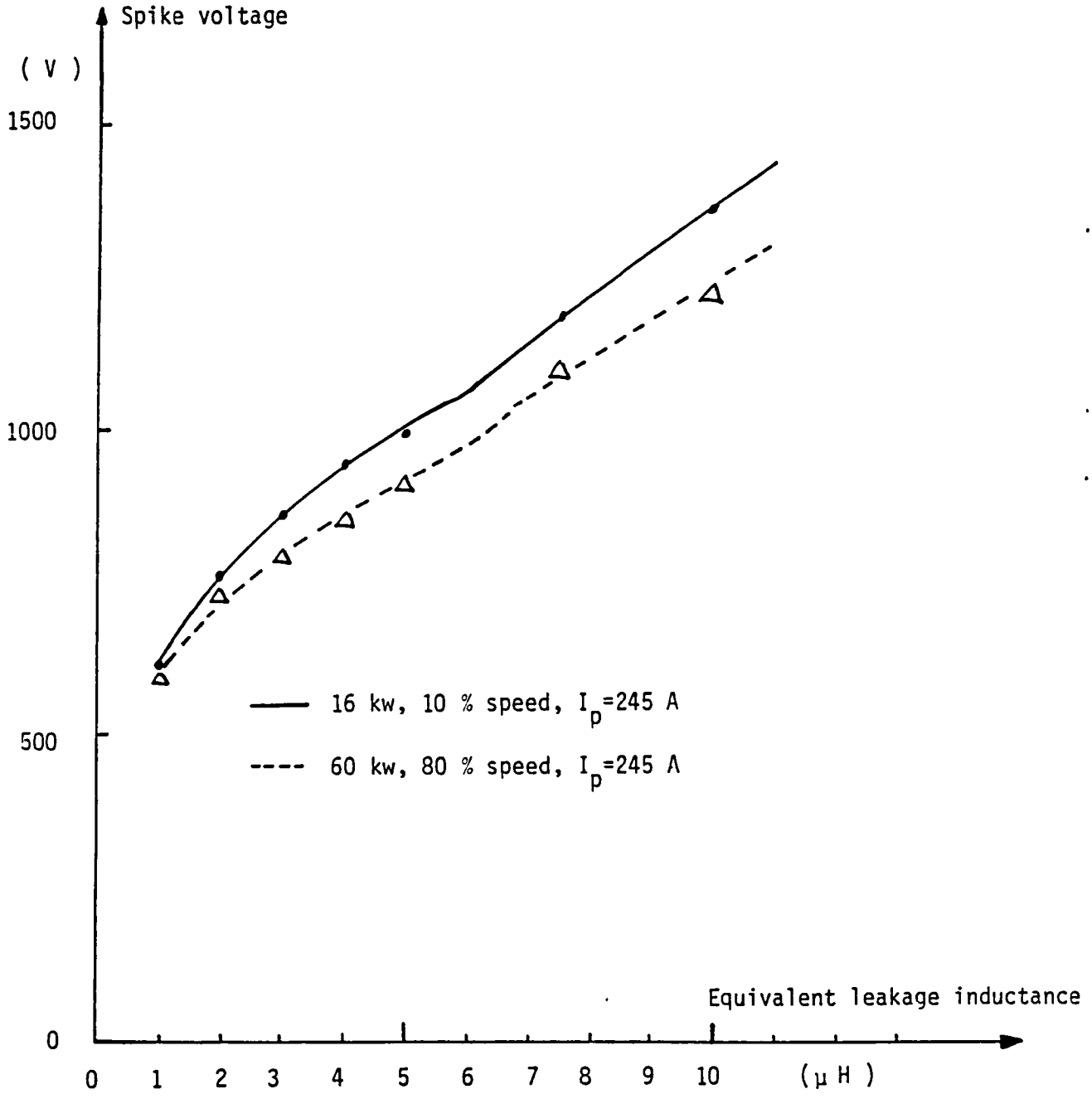


Fig. F-9 Spike voltage vs equivalent leakage inductance
(simulation results)

F.4 LOSS CALCULATION*

Switching Loss

There are four possible switching operations for which different loss calculations are required. Fig. F-10 shows the switching devices and the snubber circuit components only for the A phase. I_{A1} and I_{A2} stand for the stator currents for bifilar windings A1 and A2 respectively, and V_{C1} and V_{C2} are the voltages across capacitors C_1 and C_2 respectively. The following four cases are considered to calculate the switching loss only for switch S1.

(1) Turn-On of S1 When $I_{A2} > 0$: Initially, while switching device S1 is open, diode D4 conducts and capacitor voltage V_{C1} is charged up to $2V_{DC}$. The stored energy in this capacitor is

$$W_C = \frac{1}{2} C (2V_{DC})^2 = 2CV_{DC}^2 \qquad (17)$$

Some of this energy will be dissipated in S1 during its turn-on and some will be dissipated in the snubber R1. Assuming an exponential drop of collector-emitter voltage v_{S1} as S1 turns on,

$$v_{S1} = 2V_{DC}\, \varepsilon^{-t/\tau}$$

---

* "Analysis and Design of Optimized Snubber Circuits for dv/dt Protection in Power Thyristor Applications," S. J. Wu, GE 660.24, 2/71.

"Optimum Snubbers for Power Semiconductors," William McMurray, 1971 IAS Conference Record, pp. 885-893.

"Snubbers for Pulse-Width Modulated Bridge Converters with Power Transistors or GTOs," Tore M. Undeland, IPEC Tokyo '83 Conference Record, pp. 313-323.

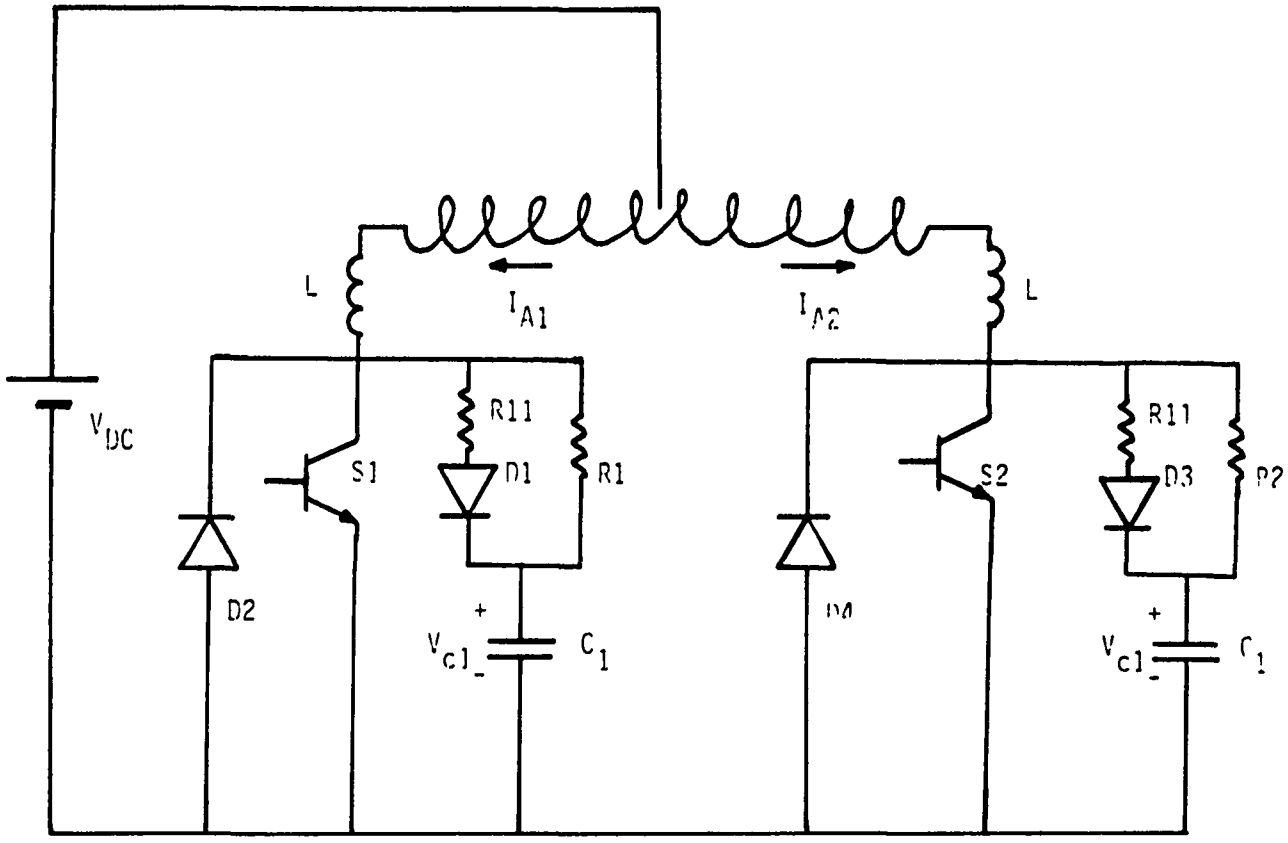


Fig. F-10 Switching devices and snubber circuits only for A phase

where τ is the exponential time constant. Then the discharging current i_S is

$$i_S = \frac{v_C - v_{S1}}{R} = \frac{2V_{DC}}{R} (\varepsilon^{-t/\tau_S} - \varepsilon^{-t/\tau}) \frac{\tau_S}{\tau_S - \tau} \tag{19}$$

where $\tau_S = RC$.

The energy dissipated in S1 is

$$W_{S1} \simeq \int_0^\infty v_{S1}\, i_{S1}\, dt = \frac{2V_{DC}^2}{R} \frac{\tau\, \tau_S}{\tau_S + \tau} \tag{20}$$

Since all of the initial stored energy in the snubber capacitor must be dissipated, the total turn-on switching energy loss is given by

(2) Turn-Off of S1 when $I_{A1} > 0$: While switch S1 is on, I_{A1} flows through S1.

The energy initially stored in the equivalent leakage L is

$$W_L = \frac{1}{2} L\, i_L^2(0) \tag{21}$$

where $i_L(0)$ is the current I_{A1} at the instant S1 turns off.
The final energy stored in C after the switch is open is

$$W_C = \frac{1}{2} C\, (2V_{DC})^2 = 2CV_{DC}^2 \tag{22}$$

An amount of energy equal to that in (22) is dissipated in the equivalent charging resistance for capacitor C1. Some of the initial stored energy in L is returned to the dc source, but the worst case turn-off switching energy loss is

$$W_L - W_C = \frac{1}{2} L\, i_L^2(0) + 2CV_{DC}^2 \tag{23}$$

(3) Turn-On of S1 When $I_{A2} > 0$: While switch S2 is on, I_{A2} flows through S2 and capacitor C1 is charged up to $2V_{DC}$. Even though S2 is turned off and the base current is supplied to switch S1, S1 is still open. If $|I_{A1}|$ is less than 48A, first capacitor C1 discharges its stored energy, and after the capacitor voltage V_{C1} minus the voltage drop across R1 goes to zero, diode D2 conducts. If $|I_{A1}|$ is larger than 48A, then diode D2 conducts very quickly after switch S2 is turned off.

The worst case energy loss is estimated to be equal to the stored energy in the capacitor. Thus, the loss energy is

$$W_C = \frac{1}{2} C (2V_{DC})^2 = 2CV_{DC}^2 \tag{24}$$

(4) Turn-Off of S1 when $I_{A1} < 0$: While S2 is open, diode D2 is on, and capacitor voltage V_{C1} is zero. When S2 turns on, I_{A1} extinguishes and V_{C1} is charged up to $2V_{DC}$.

The loss calculation is very similar to case (2), and the total loss energy is

$$W_L + W_C = \frac{1}{2} L i_L^2(0) + 2CV_{DC}^2 \tag{25}$$

On-State Loss

The total on-state loss is the integral of the instantaneous power dissipated in the four switch branches, which is

$$P_{ON} = \frac{1}{T}\left[\int_0^{t_{ON,1}} V_{S1}\, i_{S1}\, dt + \int_{t_{ON,1}}^{t_{ON,1}+t_{OFF,1}} V_{D1}\, i_{D1}\, dt + \int_{t_{ON,1}+t_{OFF,1}}^{t_{ON,1}+t_{ON,2}+t_{OFF,1}} V_{S1}\, i_{S1}\, dt + \cdots\cdots\right] \times 4$$

$$= [d(V_{S1}\, I_{S1}) + (1-d)\, V_{D1}\, I_{D1}] \times 4 \tag{26}$$

where d $\triangleq$ duty cycle of switching

$V_{S1} \triangleq$ on-state voltage drop across S1 (assumed constant)

$I_{S1} \triangleq$ average current through S1

$V_{D1} \triangleq$ diode drop (assumed constant)

$I_{D1} \triangleq$ average current through diode

and (26) assumes that the average currents over the cycle through the transistor and the diode are equal.

Summary

The total switching energy loss w_{SW} for one switching is given by adding the turn-on energy loss, (17), and the turn-off energy loss, (23) or (25).

$$w_{SW} = \frac{1}{2} L\, i_L^2(0) + 4CV_{DC}^2 \tag{27}$$

and the total switching loss over a period T is

$$P_{SW} = \frac{1}{T} \sum_{n=0}^{N} \left[\frac{1}{2} L\, i_L^2(t_n) + 4CV_{DC}^2\right] \times 4$$

$$\approx \left[\frac{1}{2} L\, I_{L,RMS}^2\, f_S + 4CV_{DC}^2\, f_S\right] \times 4 \tag{28}$$

The total on-state loss is given by (26). Thus, the total inverter loss is the sum of (26) and (28)

$$P_T = \left[\frac{1}{2} L\, I_{L,RMS}^2\, f_S + 4CV_{DC}^2\, f_S + dV_{S1} I_{S1} - (1-d)\, V_{D1} I_{D1}\right] \times 4$$

Calculated Losses

13.1 kW output at 80% speed

I_{rms} = 53.2A; I_P = 103A; f_S = 1.6 kHz

d = 0.72/2 = 0.36; V_{S1} = 1.8V; V_{D1}=0.8V

I_{ave} = 48.8/2 = 24.4A/device

On-State Loss from (26)

$$P_{on} = [0.36 \times 1.2 \times 24.4 + 0.64 \times 0.8 \times 24.4] \times 4 = 113.2$$

Switching loss from (28)

$$P_{sw} = [0.5 \times 4 \times 10^{-6} \times 53.2^2 + 4 \times 0.33 \times 10^{-6} \times 240^2] \times 1.6 \times 10^3 \times 4 = 522.8$$

Total: 636.0 W

Efficiency: 95.4%

16.4 kW output at 10% speed

I_{rms} = 200A; I_P = 245A; f_S = 3.4kHz

d = 0.6/2 = 0.3; V_{S1} = 1.8V; V_{D1} = 0.8V

I_{ave} = 197.4/2 = 98.7A/device

On-State Loss from (26)

$$P_{on} = [0.3 \times 1.8 \times 98.7 + 0.7 \times 0.8 \times 98.7] \times 4 = 434.3$$

Switching Loss from (28)

$$P_{sw} = [0.5 \times 4 + 10^{-6} \times 200^2 + 4 \times 0.33 + 10^{-6} \times 240^2] \times 3.4 \times 10^3 \times 4 = 2,122.0$$

Total: 2,556.3 W

Efficiency: 86.5%

60.5 kW Output at 80% Speed

I_{rms} = 175A ; I_P = 245A ; f_S = 160Hz

d = 0.83/2 = 0.42; V_{S1} = 18.V; V_{D1} = 0.8V

I_{ave} = 173/2 = 86.5A/device

On-State Loss from (26)

$$P_{on} = [0.42 \times 1.8 \times 86.5 + 0.58 \times 0.8 \times 86.5] \times 4 = 422.1$$

Switching Loss from (28)

$$P_{sw} = [0.5 \times 4 \times 10^{-6} \times 175^2 + 4 \times 0.33 \times 10^{-6} \times 240^2] \times 160 \times 4 = 87.9$$

Total: 510.0 W

Efficiency: 99.2%

F.5 CONTROLLER

The microprocessor based controller system must generate the required driving signals for the power transistors to implement current controlled PWM. The hardware necessary for this controller is shown in Fig. F-11 which consists of the main CPU, ROM, RAM, I/O ports and other electronic components. The controller sets the output frequency of the inverter drive, in reference to the driving command, from 0-200Hz. In addition, the controller sets the amplitude and the envelope of the controlled current at any given load and speed to provide maximum efficiency with acceptable torque pulsations. The closed loop control diagram is shown in Fig. F-12.

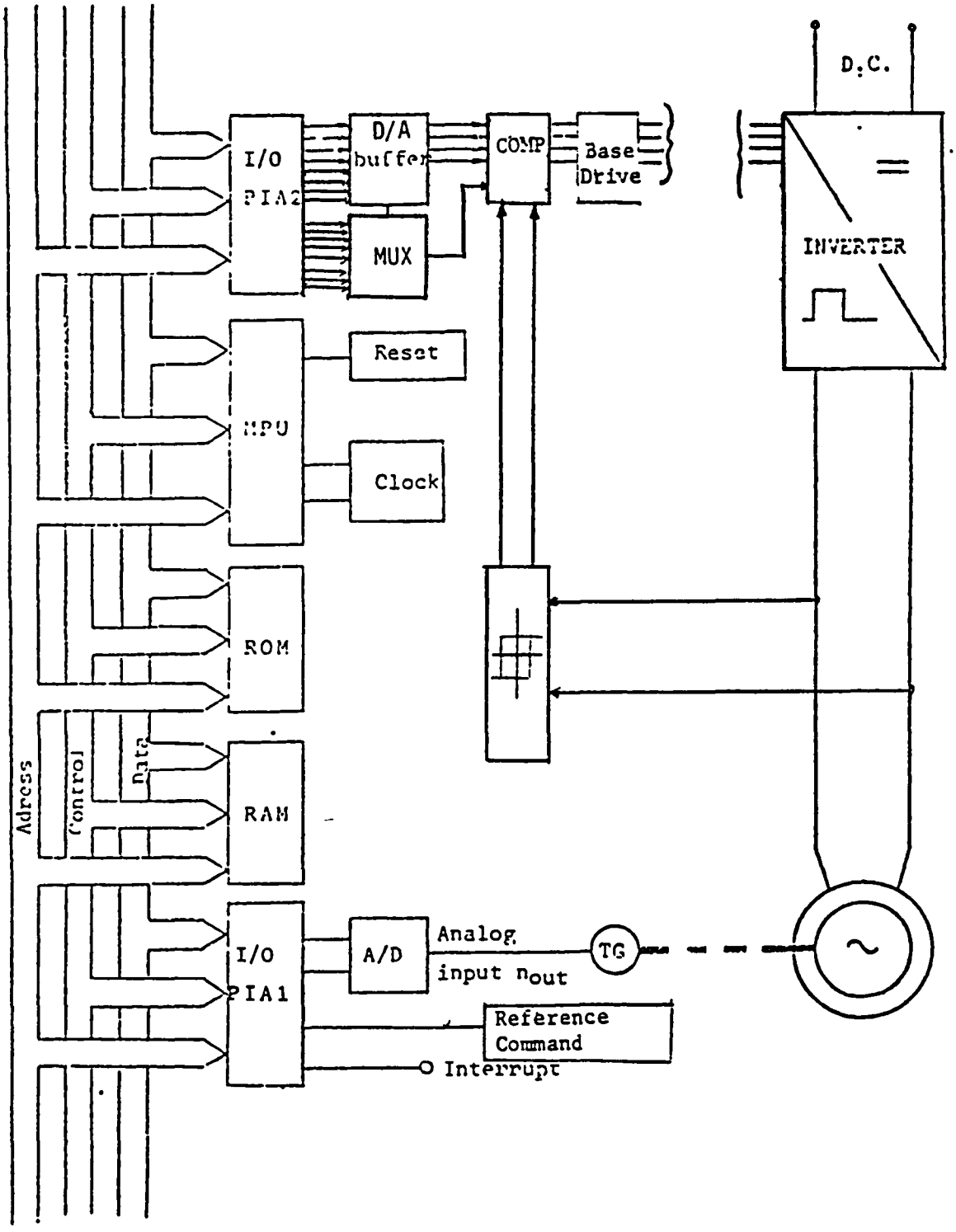


Fig. F-11 System hardware

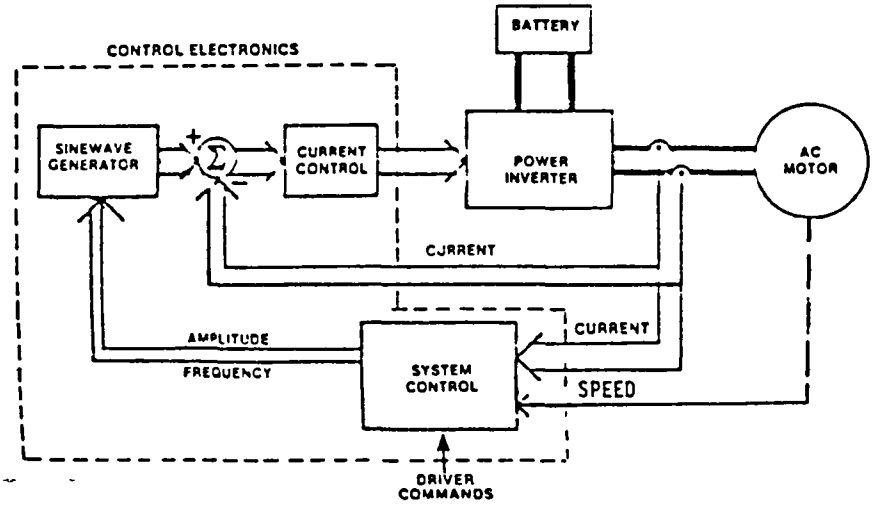


Fig. F-12 Closed loop control

APPENDIX G

SIMULATION RESULTS

G.1 DERIVATION OF EQUATIONS FOR TWO-PHASE MACHINE

The circuit diagram of the two-phase inverter/bifilar induction motor is shown in Fig. 2, section 3.0 of this report. Fig. G-1 shows the expected phase voltage and current waveforms without PWM current control. The phase relationships between the stator and rotor windings are shown in Fig. G-2.

The derivation of the simulation model for the two-phase induction motor proceeds as follows:

Let $L_{lSA} = L_{lSB} = L_{lS}$, $\quad L_{lrA} = L_{lrB} = L_{lr}$

$L_S = L_{lS} + L_m$, $\quad L_r = L_{lr} + L_m$

L_m is the mutual inductance of A-phase or B-phase

i_{AS}, i_{BS} have the frequency of supply (ω)

i_{Ar}, i_{Br} have the frequency of the slip ($\omega-\omega_r$)

$$
\begin{aligned}
V_{AS} &= R_S i_{AS} + L_{lS} \frac{di_{AS}}{dt} + \frac{d}{dt} \lambda_{AS} \\
V_{BS} &= R_S i_{BS} + L_{lS} \frac{di_{BS}}{dt} + \frac{d}{dt} \lambda_{BS} \\
V_{Ar} &= R_r i_{Ar} + L_{lr} \frac{di_{Ar}}{dt} + \frac{d}{dt} \lambda_{Ar} \\
V_{Br} &= R_r i_{Br} + L_{lr} \frac{di_{Br}}{dt} + \frac{d}{dt} \lambda_{Br}
\end{aligned}
\qquad (1)
$$

$$
\begin{aligned}
\lambda_{AS} &= L_m i_{AS} + L_m i_{Ar} \cos\theta - L_m i_{Br} \sin\theta \\
\lambda_{BS} &= L_m i_{BS} + L_m i_{Ar} \sin\theta + L_m i_{Br} \cos\theta \\
\lambda_{Ar} &= L_m i_{Ar} + L_m i_{AS} \cos\theta + L_m i_{BS} \sin\theta \\
\lambda_{Br} &= L_m i_{Br} - L_m i_{AS} \sin\theta + L_m i_{BS} \cos\theta
\end{aligned}
\qquad (2)
$$

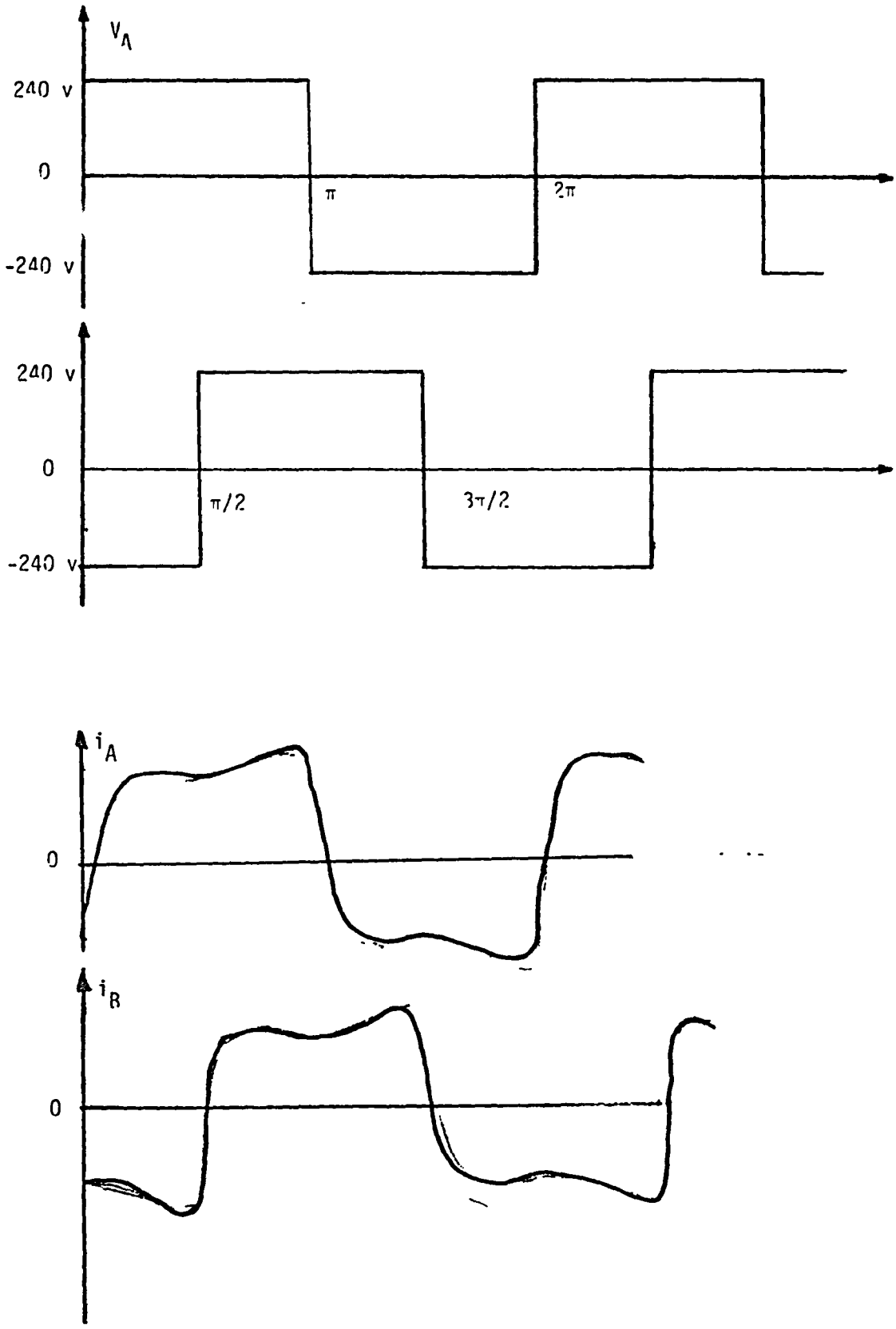


Fig. G-1 Expected Waveforms without peak current controller

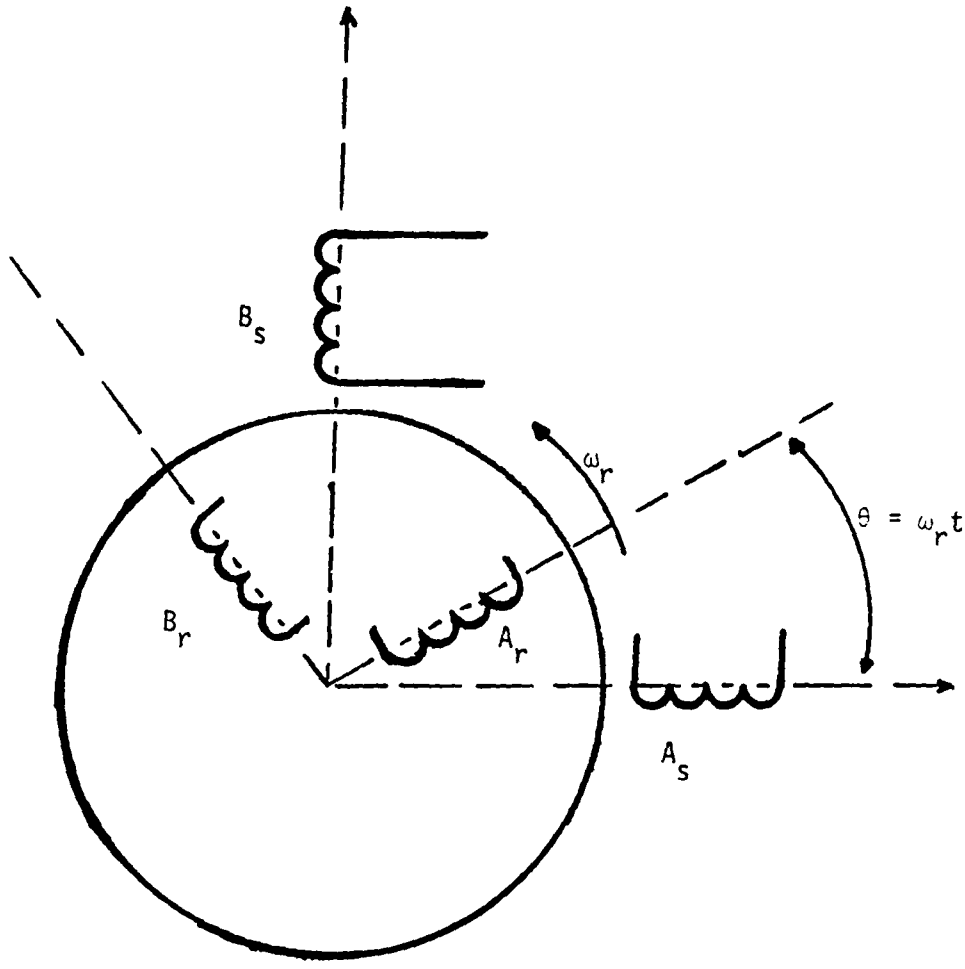


Fig. G-2 Phase relationship of stator and rotor

Equations (1) and (2) can be expressed in matrix form as follows:

$$\left|\begin{array}{c}\vec{V}_S \\ \hline \vec{V}_r\end{array}\right| = \left|\begin{array}{c|c}\bar{R}_S & 0 \\ \hline 0 & \bar{R}_r\end{array}\right| \left|\begin{array}{c}\vec{i}_S \\ \hline \vec{i}_r\end{array}\right| + \left|\begin{array}{c|c}\bar{L}_{ls} & 0 \\ \hline 0 & \bar{L}_{lr}\end{array}\right| \frac{d}{dt} \left|\begin{array}{c}\vec{i}_S \\ \hline \vec{i}_r\end{array}\right| + \frac{d}{dt} \left|\begin{array}{cc}L_1 & L_2 \\ \hline L_3 & L_4\end{array}\right| \left|\begin{array}{c}\vec{i}_S \\ \hline \vec{i}_r\end{array}\right| \qquad (3)$$

where

$$\bar{R}_S \triangleq \begin{vmatrix} R_S & 0 \\ 0 & R_S \end{vmatrix} \qquad \bar{R}_r \triangleq \begin{vmatrix} R_r & 0 \\ 0 & R_r \end{vmatrix}$$

$$\bar{L}_{ls} \triangleq \begin{vmatrix} L_{ls} & 0 \\ 0 & L_{ls} \end{vmatrix} \qquad \bar{L}_{lr} \triangleq \begin{vmatrix} L_{lr} & 0 \\ 0 & L_{lr} \end{vmatrix}$$

$$\vec{V}_S \triangleq \begin{vmatrix} V_{AS} \\ V_{BS} \end{vmatrix} \qquad \vec{V}_r \triangleq \begin{vmatrix} V_{Ar} \\ V_{Br} \end{vmatrix} \qquad \vec{i}_S \triangleq \begin{vmatrix} i_{AS} \\ i_{BS} \end{vmatrix} \qquad \vec{i}_r \triangleq \begin{vmatrix} i_{Ar} \\ i_{Br} \end{vmatrix}$$

$$L_1 \triangleq \begin{vmatrix} L_m & 0 \\ 0 & L_m \end{vmatrix} \qquad L_2 \triangleq \begin{vmatrix} L_m\cos\theta & -L_m\sin\theta \\ L_m\sin\theta & L_m\cos\theta \end{vmatrix}$$

$$L_3 \triangleq \begin{vmatrix} L_m\cos\theta & L_m\sin\theta \\ -L_m\sin\theta & L_m\cos\theta \end{vmatrix} \qquad L_4 \triangleq \begin{vmatrix} L_m & 0 \\ 0 & L_m \end{vmatrix}$$

$$\left|\begin{array}{c}\vec{V}_S \\ \hline \vec{V}_r\end{array}\right| = \left|\begin{array}{c|c}\bar{R}_S & 0 \\ \hline 0 & \bar{R}_r\end{array}\right| \left|\begin{array}{c}\vec{i}_S \\ \hline \vec{i}_r\end{array}\right| + \left|\begin{array}{c|c}\bar{L}_{ls} & 0 \\ \hline 0 & \bar{L}_{lr}\end{array}\right| \left|\begin{array}{c}\dot{\vec{i}}_S \\ \hline \dot{\vec{i}}_r\end{array}\right| + \frac{d}{dt} \left|\begin{array}{c}L_1\vec{i}_S + L_2\vec{i}_r \\ \hline L_3\vec{i}_S + L_4\vec{i}_r\end{array}\right|$$

$$= \begin{vmatrix} \bar{R}_S & \vec{i}_S \\ \bar{R}_r & \vec{i}_r \end{vmatrix} + \begin{vmatrix} \bar{L}_{1S} & \dot{\vec{i}}_S \\ \bar{L}_{1r} & \vec{i}_r \end{vmatrix} + \begin{vmatrix} \dot{L}_1\vec{i}_S + L_1\dot{\vec{i}}_S + \dot{L}_2\vec{i}_r + L_2\dot{\vec{i}}_r \\ \dot{L}_3\vec{i}_S + L_3\dot{\vec{i}}_S + \dot{L}_4\vec{i}_r + L_4\dot{\vec{i}}_r \end{vmatrix}$$

$$= \begin{vmatrix} \bar{R}_S + \dot{L}_1 & \dot{L}_2 \\ \dot{L}_3 & \bar{R}_r + \dot{L}_4 \end{vmatrix} \begin{vmatrix} \vec{i}_S \\ \vec{i}_r \end{vmatrix} + \begin{vmatrix} \bar{L}_{1s} + L_1 & L_2 \\ L_3 & \bar{L}_{1r} + L_4 \end{vmatrix} \begin{vmatrix} \dot{\vec{i}}_S \\ \vec{i}_r \end{vmatrix} \qquad (4)$$

If i_{Ar} and i_{Br} are transformed to d-q coordinates which are fixed on the stator, that is, d-q axes rotating in the reverse direction relative to the rotor, then i_{Ar} and i_{Br} with angular velocity $(\omega-\omega_r)$ become i'_{Ar} and i'_{Br} with angular velocity (ω).

$$\vec{i}'_r \triangleq \begin{vmatrix} i'_{Ar} \\ i'_{Br} \end{vmatrix} \qquad \vec{i}_r \triangleq \bar{C}\,\vec{i}'_r \qquad (5)$$

$$\vec{i}'_r \triangleq \bar{C}^{-1}\vec{i}_r \qquad \dot{\vec{i}}_r = \dot{\bar{C}}\,\vec{i}'_r + \bar{C}\,\dot{\vec{i}}_r$$

$$\bar{C} = \begin{vmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$$

$$\bar{C}^{-1} = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} \qquad \dot{\bar{C}} = \begin{vmatrix} -\omega_r\sin\theta & -\omega_r\cos\theta \\ \omega_r\cos\theta & -\omega_r\sin\theta \end{vmatrix}$$

$$\theta \triangleq \omega_{rt}$$

From (4) and (5),

$$\begin{vmatrix} \vec{V}_S \\ \bar{C}^{-1}\vec{V}_r \end{vmatrix} = \begin{vmatrix} \bar{R}_S + \dot{L}_1 & \dot{L}_2 \\ \bar{C}^{-1}\dot{L}_3 & \bar{C}^{-1}(\bar{R}_r + \dot{L}_4) \end{vmatrix} \begin{vmatrix} \vec{i}_S \\ \bar{C}\,\vec{i}_r \end{vmatrix} +$$

$$\begin{vmatrix} \bar{L}_{1s} + L_1 & L_2 \\ \bar{C}^{-1} L_3 & \bar{C}^{-1}(\bar{L}_{1r}+L_4) \end{vmatrix} \begin{vmatrix} \dot{i}_s \\ \dot{\bar{C}}\, i'_r + \bar{C}\, \dot{i}'_r \end{vmatrix}$$

$$= \begin{vmatrix} \textcircled{1} & \textcircled{2} \\ \bar{R}_s + \dot{L}_1 & \dot{L}_2\bar{C} + L_2\dot{\bar{C}} \\ \textcircled{3} & \textcircled{4} \\ \bar{C}^{-1}\dot{L}_3 & \bar{C}^{-1}(\bar{R}_r+\dot{L}_4)\bar{C} \\ & + \bar{C}^{-1}(\bar{L}_{1r}+L_4)\dot{\bar{C}} \end{vmatrix} \begin{vmatrix} i_s \\ i_r \end{vmatrix}$$

$$\begin{vmatrix} \textcircled{5} & \textcircled{6} \\ \bar{L}_{1s} + L_1 & L_2\bar{C} \\ \textcircled{7} & \textcircled{8} \\ \bar{C}^{-1}L_3 & \bar{C}^{-1}(\bar{L}_{1r}+L_4)\bar{C} \end{vmatrix} \begin{vmatrix} \dot{i}_s \\ \dot{i}'_r \end{vmatrix} \qquad (6)$$

① $\bar{R}_S + \dot{L}_1 = \begin{vmatrix} R_S & 0 \\ 0 & R_S \end{vmatrix}$ Therefore, $\dot{L}_1 = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$

② $\dot{L}_2\bar{C} + L_2\dot{\bar{C}} = \begin{vmatrix} -\omega_r L_m \sin\theta & -\omega_r L_m \cos\theta \\ \omega_r L_m \cos\theta & -\omega_r L_m \sin\theta \end{vmatrix} \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$

$$+ \begin{vmatrix} L_m\cos\theta & -L_m\sin\theta \\ L_m\sin\theta & L_m\cos\theta \end{vmatrix} \begin{vmatrix} -\omega_r\sin\theta & -\omega_r\cos\theta \\ \omega_r\cos\theta & -\omega_r\sin\theta \end{vmatrix} = |0|$$

③ $\bar{C}^{-1}\dot{L}_3 = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} \begin{vmatrix} -\omega_r L_m \sin\theta & \omega_r L_m \cos\theta \\ -\omega_r L_m \cos\theta & -\omega_r L_m \sin\theta \end{vmatrix} = \begin{vmatrix} 0 & \omega_r L_m \\ -\omega_r L_m & 0 \end{vmatrix}$

④ $\bar{C}^{-1}(\bar{R}_r + \dot{L}_4)\,\bar{C} + \bar{C}^{-1}(\bar{L}_{lr} + L_4)\,\dot{\bar{C}}$

$$= \begin{vmatrix} R_r & 0 \\ 0 & R_r \end{vmatrix} + \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} \begin{vmatrix} L_r & 0 \\ 0 & L_r \end{vmatrix} \begin{vmatrix} -\omega_r\sin\theta & \omega_r\cos\theta \\ -\omega_r\cos\theta & -\omega_r\sin\theta \end{vmatrix}$$

$$= \begin{vmatrix} R_r & 0 \\ 0 & R_r \end{vmatrix} + \begin{vmatrix} 0 & \omega_r L_r \\ -\omega_r L_r & 0 \end{vmatrix} = \begin{vmatrix} R_r & \omega_r L_r \\ -\omega_r L_r & R_r \end{vmatrix}$$

⑤ $\bar{L}_{ls} + L_1 = \begin{vmatrix} L_{ls} & 0 \\ 0 & L_{ls} \end{vmatrix} + \begin{vmatrix} L_m & 0 \\ 0 & L_m \end{vmatrix} = \begin{vmatrix} L_S & 0 \\ 0 & L_S \end{vmatrix}$

⑥ $L_2\bar{C} = \begin{vmatrix} L_m\cos\theta & -L_m\sin\theta \\ L_m\sin\theta & L_m\cos\theta \end{vmatrix} \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} = \begin{vmatrix} L_m & 0 \\ 0 & L_m \end{vmatrix}$

⑦ $\bar{C}^{-1}L_3 = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} \begin{vmatrix} L_m\cos\theta & L_m\sin\theta \\ -L_m\sin\theta & L_m\cos\theta \end{vmatrix} = \begin{vmatrix} L_m & 0 \\ 0 & L_m \end{vmatrix}$

⑧ $\bar{C}^{-1}(\bar{L}_{lr} + L_4)\bar{C} = \begin{vmatrix} L_r & 0 \\ 0 & L_r \end{vmatrix}$

So equation ⑥ becomes

$$\begin{vmatrix} \vec{V}_S \\ \bar{C}^{-1}\vec{V}_r \end{vmatrix} = \left|\begin{array}{cc|cc} R_S & 0 & & 0 \\ 0 & R_S & & \\ \hline & & R_r & \omega_r L_r \\ 0 & \omega_r L_m & -\omega_r L_r & R_r \end{array}\right| \begin{vmatrix} \vec{i}_S \\ \vec{i}_r \end{vmatrix} + \quad +$$

$$\left|\begin{array}{cc|cc} L_S & 0 & L_m & 0 \\ 0 & L_S & 0 & L_m \\ \hline L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{array}\right| \left|\begin{array}{c} \dot{\vec{i}}_S \\ \hline \dot{\vec{i}}'_r \end{array}\right| \qquad (7)$$

$$\dot{\vec{i}}_S = \left|\begin{array}{c} \dot{i}_{AS} \\ \dot{i}_{BS} \end{array}\right| \qquad \dot{\vec{i}}'_r = \left|\begin{array}{c} \dot{i}'_{Ar} \\ \dot{i}'_{Br} \end{array}\right| \qquad \dot{i}_{AS} \triangleq \frac{di_{AS}}{dt} = pi_{AS} \cdot \cdot \cdot \cdot ,$$

If the machine has a squirrel-cage rotor, then

$$\bar{C}^{-1} \vec{V}_r = \left|\begin{array}{c} 0 \\ 0 \end{array}\right|$$

Therefore

$$\left|\begin{array}{c} V_{AS} \\ V_{BS} \\ 0 \\ 0 \end{array}\right| = \left|\begin{array}{cccc} R_S+L_Sp & 0 & L_mp & 0 \\ 0 & R_S+L_Sp & 0 & L_mp \\ L_mp & \omega_rL_m & R_r+L_rp & \omega_rL_r \\ -\omega_rL_m & L_mp & -\omega_rL_r & R_r+L_rp \end{array}\right| \left|\begin{array}{c} i_{AS} \\ i_{BS} \\ i'_{Ar} \\ i'_{Br} \end{array}\right| \qquad (8)$$

The commonly used torque equation is

$$T = pL_m (i_{BS} i'_{Ar} - i_{AS} i'_{Br}) \qquad (9)$$

Equations (8) and (9) are the mathematical model for simulation of the two-phase machine. Equation (8) can be changed to the form

$$\left|\begin{array}{c} V_{AS} \\ V_{BS} \\ 0 \\ 0 \end{array}\right| = \left|\begin{array}{cccc} R_S & 0 & 0 & 0 \\ 0 & R_S & 0 & 0 \\ 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & R_r \end{array}\right| \left|\begin{array}{c} i_{AS} \\ i_{BS} \\ i'_{Ar} \\ i'_{Br} \end{array}\right| + \left|\begin{array}{cccc} L_{ls}+L_m & 0 & L_m & 0 \\ 0 & L_{ls}+L_m & 0 & L_m \\ L_m & 0 & L_{lr}+L_m & 0 \\ 0 & L_m & 0 & L_{lr}+L_m \end{array}\right| \left|\begin{array}{c} \dot{i}_{AS} \\ \dot{i}_{BS} \\ \dot{i}'_{Ar} \\ \dot{i}'_{Br} \end{array}\right|$$

$$+\begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \omega_r L_m & 0 & \omega_r(L_{1r}+L_m) \\ -\omega_r L_m & 0 & -\omega_r(L_{1r}+L_m) & 0 \end{vmatrix} \begin{vmatrix} i_{AS} \\ i_{BS} \\ i'_{Ar} \\ i'_{Br} \end{vmatrix} \tag{10}$$

where

$$\begin{vmatrix} L_S & 0 & L_m & 0 \\ 0 & L_S & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{vmatrix}^{-1} \triangleq \frac{1}{L_S L_r - L_m^2} \begin{vmatrix} L_r & 0 & -L_m & 0 \\ 0 & L_r & 0 & -L_m \\ -L_m & 0 & L_S & 0 \\ 0 & -L_m & 0 & L_S \end{vmatrix}$$

$$L_S \triangleq L_{1S} + L_m$$

$$L_r \triangleq L_{1r} + L_m$$

Then, (10) may be written

$$\begin{vmatrix} i_{AS} \\ i_{BS} \\ i'_{Ar} \\ i'_{Br} \end{vmatrix} = \frac{1}{L_S L_r - L_m^2} \begin{vmatrix} L_r & 0 & -L_m & 0 \\ 0 & L_r & 0 & -L_m \\ -L_m & 0 & L_S & 0 \\ 0 & -L_m & 0 & L_S \end{vmatrix} \begin{vmatrix} V_{AS} \\ V_{BS} \\ V'_{Ar}(0) \\ V'_{Br}(0) \end{vmatrix}$$

$$- \frac{1}{L_S L_r - L_m^2} \begin{vmatrix} L_r R_s & -L_m^2 \omega_r & -L_m R_r & -L_m L_r \omega_r \\ \omega_r L_m^2 & L_r R_S & L_m L_r \omega_r & -L_m R_r \\ -L_m R_S & L_S L_m \omega_r & L_S R_r & L_S L_r \omega_r \\ -L_S L_m \omega_r & -L_m R_S & -L_S L_r \omega_r & L_S R_r \end{vmatrix} \begin{vmatrix} i_{AS} \\ i_{BS} \\ i'_{Ar} \\ i'_{Br} \end{vmatrix} \tag{11}$$

or

$$\dot{i}_{AS} = \frac{1}{L_S L_r - L_m^2} \left(L_r V_{AS} - L_r R_S i_{AS} + L_m^2 \omega_r i_{BS} + L_m R_r i'_{Ar} + L_m L_r \omega_r i'_{Br}\right)$$

$$\dot{i}_{BS} = \frac{1}{L_S L_r - L_m^2} \left(L_r V_{BS} - \omega_r L_m^2 i_{AS} - L_r R_S i_{BS} - L_m L_r \omega_r i'_{Ar} + L_m R_r i'_{Br}\right)$$

$$\dot{i}'_{Ar} = \frac{1}{L_S L_r - L_m^2} \left(-L_m V_{AS} + L_m R_S i_{AS} - L_S L_m \omega_r i_{BS} - L_S R_r i'_{Ar} - L_S L_r \omega_r i'_{Br}\right)$$

$$\dot{i}'_{Br} = \frac{1}{L_S L_r - L_m^2} \left(-L_m V_{BS} + L_S L_m \omega_r i_{AS} + L_m R_S i_{BS} + L_S L_r \omega_r i'_{Ar} - L_S R_r i'_{Br}\right)$$

Let

$A_{22} \triangleq L_S/A_{50}$

$A_{26} = 2\pi L_r$

$A_{31} = 2\pi L_m$

$A_{36} = L_r/A_{50}$

$A_{38} = L_m/A_{50}$

$A_{39} = P \times A_{31}/2\pi$

$A_{40} = V_{AS} - R_S i_{AS}$

$A_{41} = (A_{31}\, i_{BS} + A_{26}\, i'_{Br}) \times \omega_r$

$A_{42} = (A_{31}\, i_{AS} + A_{26}\, i'_{Ar}) \times \omega_r$

$A_{47} = A_{42} - R_r i'_{Br}$

$A_{50} = L_S L_r - L_m^2$

$A_{53} = V_{BS} - R_S i_{BS}$

$A_{61} = -A_{41} - R_r i'_{Ar}$

Therefore

$$\begin{aligned}
\dot{i}_{AS} &= A_{40} \times A_{36} - A_{61} \times A_{38} \triangleq x\ (1)\\
\dot{i}_{BS} &= A_{53} \times A_{36} - A_{47} \times A_{38} \triangleq x\ (4)\\
\dot{i}_{Ar} &= A_{61} \times A_{22} - A_{40} \times A_{38} \triangleq x\ (2)\\
\dot{i}_{Br} &= A_{47} \times A_{22} - A_{53} \times A_{38} \triangleq x\ (3)
\end{aligned} \qquad (12)$$

$$T = PL_m (i_{BS}\, i'_{Ar} - i_{AS}\, i'_{Br}) = A_{39} (i_{BS}\, i'_{Ar} - i_{AS}\, i'_{Br}) = A_{83}$$

The simulation diagram for (12) is shown in Fig. G-3.

The following sections of this appendix include simulation programs and results for different operating points, using the mathematical model of (12).

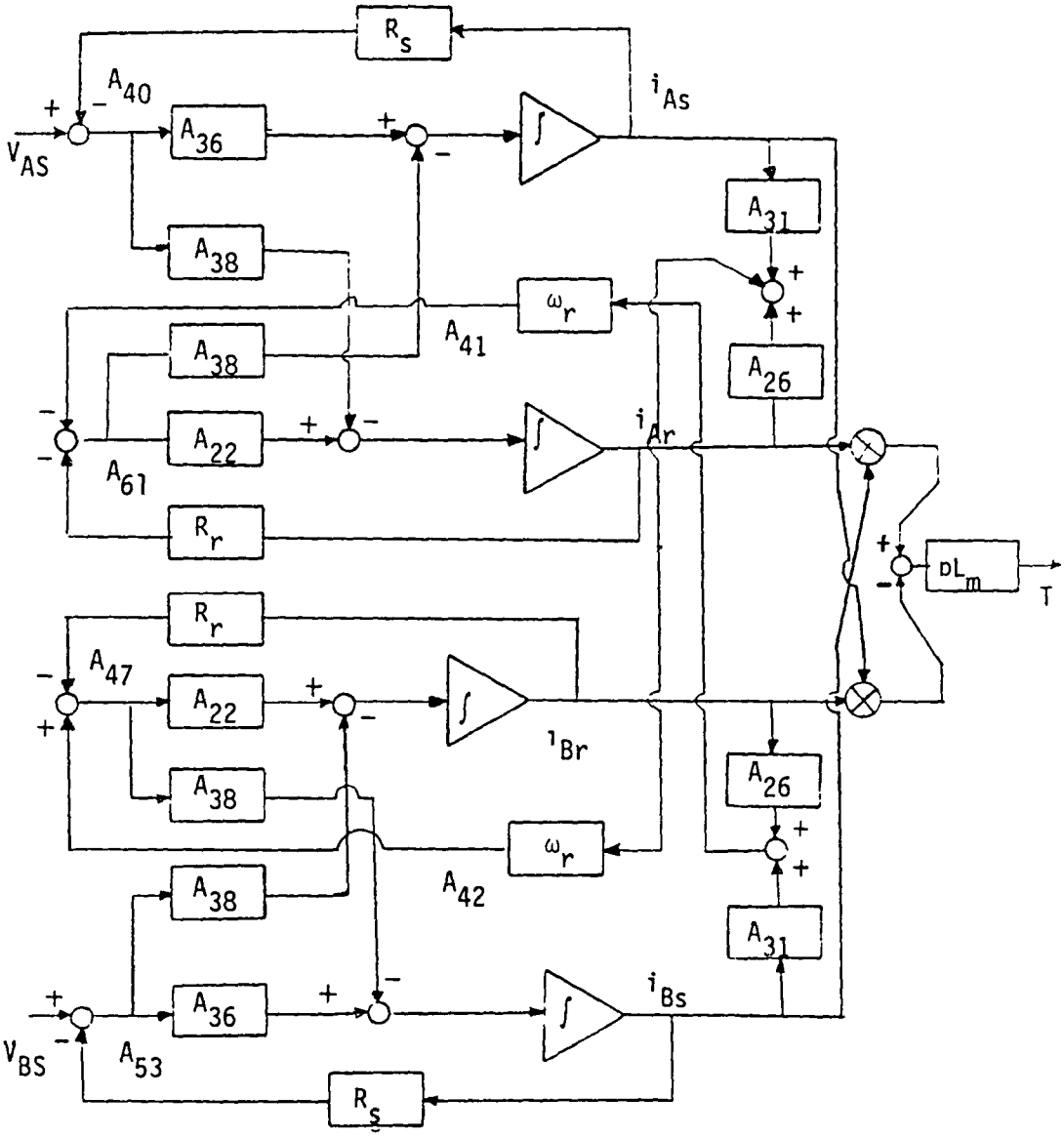


Fig. G-3 Simulation diagram of two-phase induction motor

G.2 TWO-PHASE SIMULATION RESULTS FOR 10% SPEED, 16 KW

Source Program

Plots

- Stator Voltage
- Stator Current
- Output Power
- Current Spectrum
- Power Spectrum

Partial Listing of Plot Data

```
//LS       JOB (XXXXLH),'GOODARZI',
// PASSWORD=XXXX
/*JOBPARM       T=5,L=6
//*ROUTE PRINT UMVMA.R12
//    EXEC    WATFIV
//GO.SYSIN  DD  *
$JOB           ,NOLIST,T=(5,0)
C*********************************************************************
C                                                                     *
C                 10% SPEED,16KW  JPL PROJECT                         *
C                                                                     *
C*********************************************************************
      DIMENSION AR1(800),AR2(800),AR3(800),Y(20),X(20),GA(20),PE(20),
     1          XX(4,20),H(9),ZZ1(20),ZZ2(20),ZZ3(20),IFL(7),AR4(800),
     2          AR5(800),AR6(800),AR7(800),AR8(800)
      LOGICAL*1 LR1(800),LR2(800),LR3(800),LR4(800),LR5(800),LR6(800),
     1          LR7(800),LR8(800)
      REAL LOS,LOR,LM,IAS,IBS,ICS,IAR,IBR,ICR,IDS,IQS,IDR,IQR,LMDQ,INS,
     1     POUT,IAREF,IBREF,L1,L2,M,IMAX,IMAGD,IMAGQ,IBU,
     2     MAG1(512),MAG2(512)
      COMPLEX  F1(512),F2(512)
C     WRITE(6,50)
C  50 FORMAT(/,3X,'TIME',8X,'VDS',8X,'IDS',8X,'VQS',8X,'IQS',8X,
C    4        'IDR',9X,'POUT')
      IMAX=205.
      SLIP=.085
      E=240.
      DI=40
      NRUN=1
      ISTOP=0
      KEEP=1
      IFL(1)=-1
      NINT=7
      IPNT1=0
      IPNT2=0
      IPNT3=0
      IPNT4=0
      IPNT5=0
      IPNT6=0
      IPNT7=0
C  *********************************************************************
C     PARAMETERS FOR TWOPHASE INDUCTION MOTOR
C  *********************************************************************
      LOS=.2256E-03
      LOR=.2410E-03
      LM=6.5100E-03
      RS=.0274
      RR=.0213
      PAI=3.141593
      P=1.
      FREQ=20.
      OMEGA=2*PAI*FREQ
      OMEGR=OMEGA*(1.-SLIP)
      L1=LOS+LM
      L2=LOR+LM
```

```
      M=LM
      FACT=1./(L1*L2-M*M)
      FACT1=L2*FACT
      FACT2=L2*RS*FACT
      FACT22=L1*RR*FACT
      FACT3=M*M*OMEGR*FACT
      FACT4=M*L2*OMEGR*FACT
      FACT44=M*L1*OMEGR*FACT
      FACT5=M*RR*FACT
      FACT55=M*RS*FACT
      FACT6=M*FACT
      FACT7=L1*L2*OMEGR*FACT
    1 IFL(2)=-1
    2 CALL RKFOUR(NINT,Y,X,GA,PE,XX,H,ZZ1,ZZ2,ZZ3,TIME,PNTS,KEEP,IFL)
      IF(IFL(4))3,4,5
    3 INIT=1
      TIME1=-1
      NC=1
      NCC=1
      TIME=0.
      FINTIM=2.0500
      PTMAX=800.
      Y(1)=0.
      Y(2)=0.
      Y(3)=0.
      Y(4)=0.
      Y(5)=0.0
      Y(6)=0.0
      Y(7)=0.0
      AREAIN=0.0
      ARAOUT=0.0
      H(1)=0.2000E-04
C     H(7)=-1
      VDS=0.0
      VQS=0.0
      IN=0
      SUM=0.
      SUM1=0.
      NUMBER=0.
      SW=0.0
      SWITCH=0.0
      PSW=0.0
      TOTSW=0.0
      IAE=-1
      IBE=-1
      DO 33 I=1,512
      F1(I)=(0.0,0.0)
      F2(I)=(0.0,0.0)
   33 CONTINUE
C
    4 IF(TIME-TIME1)7,8,7
    7 TIME1=TIME
C
      DEG=FREQ*TIME
      IDEG=DEG
```

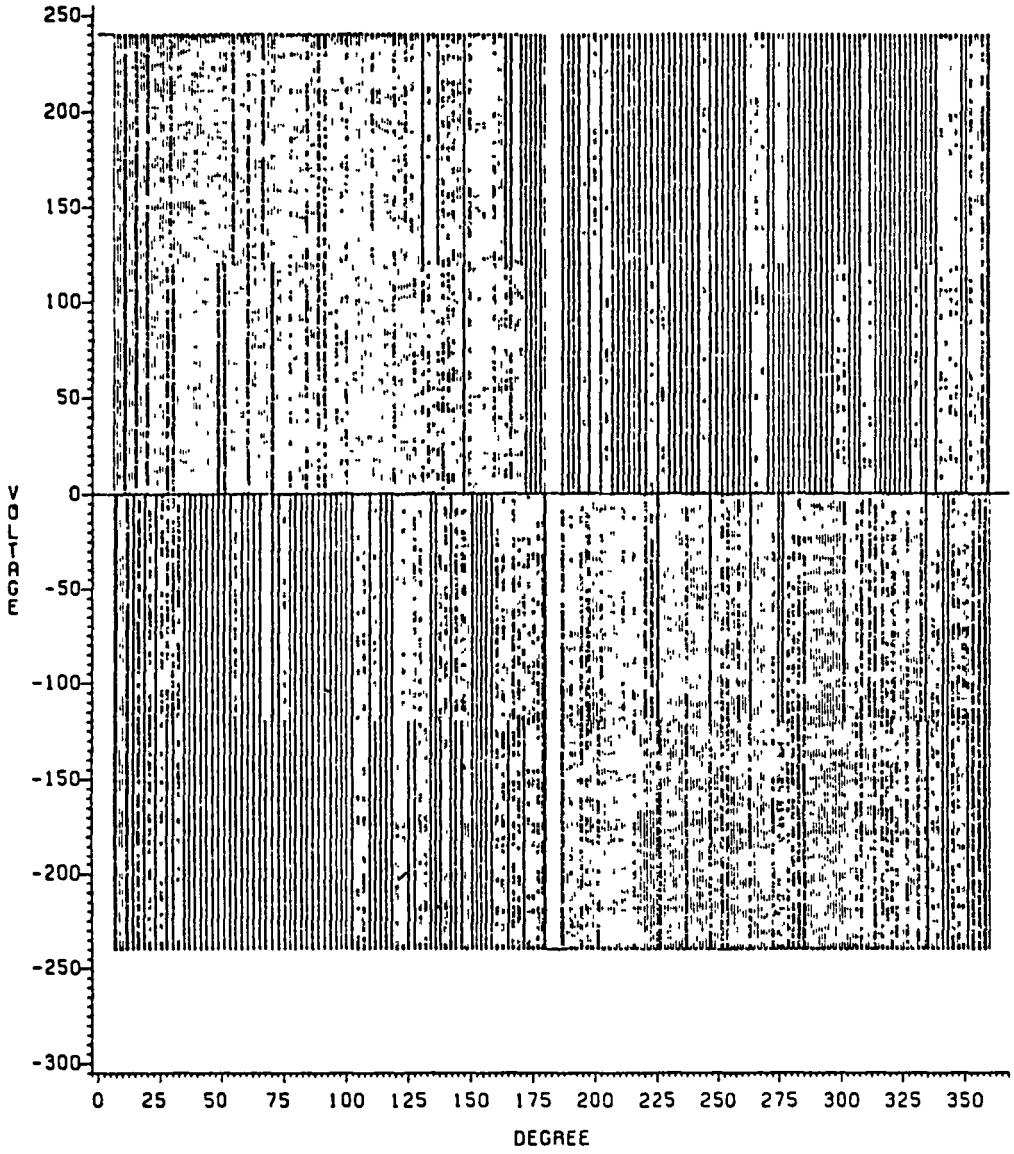
```
      DEG=(DEG-IDEG)*360.
C
      IAREF=IMAX*SIN(OMEGA*TIME)
      IBREF=IMAX*SIN(OMEGA*TIME-PAI/2.)
C
      IF(IAREF.GT.0.0) IAREF=IMAX
      IF(IAREF.LT.0.00) IAREF=-IMAX
      IF(IBREF.GT.0.0) IBREF=IMAX
      IF(IBREF.LT.0.00) IBREF=-IMAX
C
      DIA=Y(1)-IAREF
      DIB=Y(2)-IBREF
C
      VDS=E
      VQS=E
      IF(IAREF.LT.0)   VDS=-E
      IF(IBREF.LT.0)   VQS=-E
C
      IF(TIME.LT.0.35000) GOTO 8
      IF(DIA.GT.DI.AND.IAREF.GT.0.0)     GOTO 24
      IF(DIA.GT.DI)     IAE=1
      GOTO 27
 24   IAE=1
      IF(TIME.LT.2.0000) GOTO 27
      SW=SW+1
      PSW=ABS((240.*Y(1)*5.5E-06)/(6*0.050000))+PSW
 27   IF(DIA.LT.(-DI))  IAE=0
      IF(VDS.GT.0.0.AND.IAE.EQ.1)  GOTO 18
      IF(VDS.LT.0.0.AND.IAE.EQ.0)  GOTO 18
      GOTO 22
 18   VDS=-VDS
      IF(TIME.LT.2.0000) GOTO 22
      SWITCH=SWITCH+1.
22    IF(DIB.GT.DI)     IBE=1
      IF(DIB.LT.(-DI))  IBE=0
      IF(VQS.GT.0.0.AND.IBE.EQ.1)  GOTO 19
      IF(VQS.LT.0.0.AND.IBE.EQ.0)  GOTO 19
      GOTO 23
19    VQS=-VQS
23    IF(TIME.GT.1.90000.AND.TIME.LT.1.9001) AREAIN=Y(6)
      IF(TIME.GT.1.90000.AND.TIME.LT.1.9001) ARAOUT=Y(5)
      IF(TIME.GT.1.90000.AND.TIME.LT.1.9001) ARMS=Y(7)
C
      IF(TIME.LT.2.0000) GOTO 8
      TOTSW=TOTSW+1.
C
    8 CONTINUE
C
      X(1)=FACT1*VDS-FACT2*Y(1)+FACT3*Y(2)+FACT4*Y(4)+FACT5*Y(3)
      X(2)=FACT1*VQS-FACT2*Y(2)-FACT3*Y(1)-FACT4*Y(3)+FACT5*Y(4)
      X(3)=-FACT6*VDS+FACT55*Y(1)-FACT44*Y(2)-FACT7*Y(4)-FACT22*Y(3)
      X(4)=-FACT6*VQS+FACT55*Y(2)+FACT44*Y(1)+FACT7*Y(3)-FACT22*Y(4)
      X(5)=LM*(Y(2)*Y(3)-Y(1)*Y(4))*OMEGR/.15
      X(6)=(Y(1)*VDS+Y(2)*VQS)/.15
      X(7)=Y(1)*Y(1)/.15
```

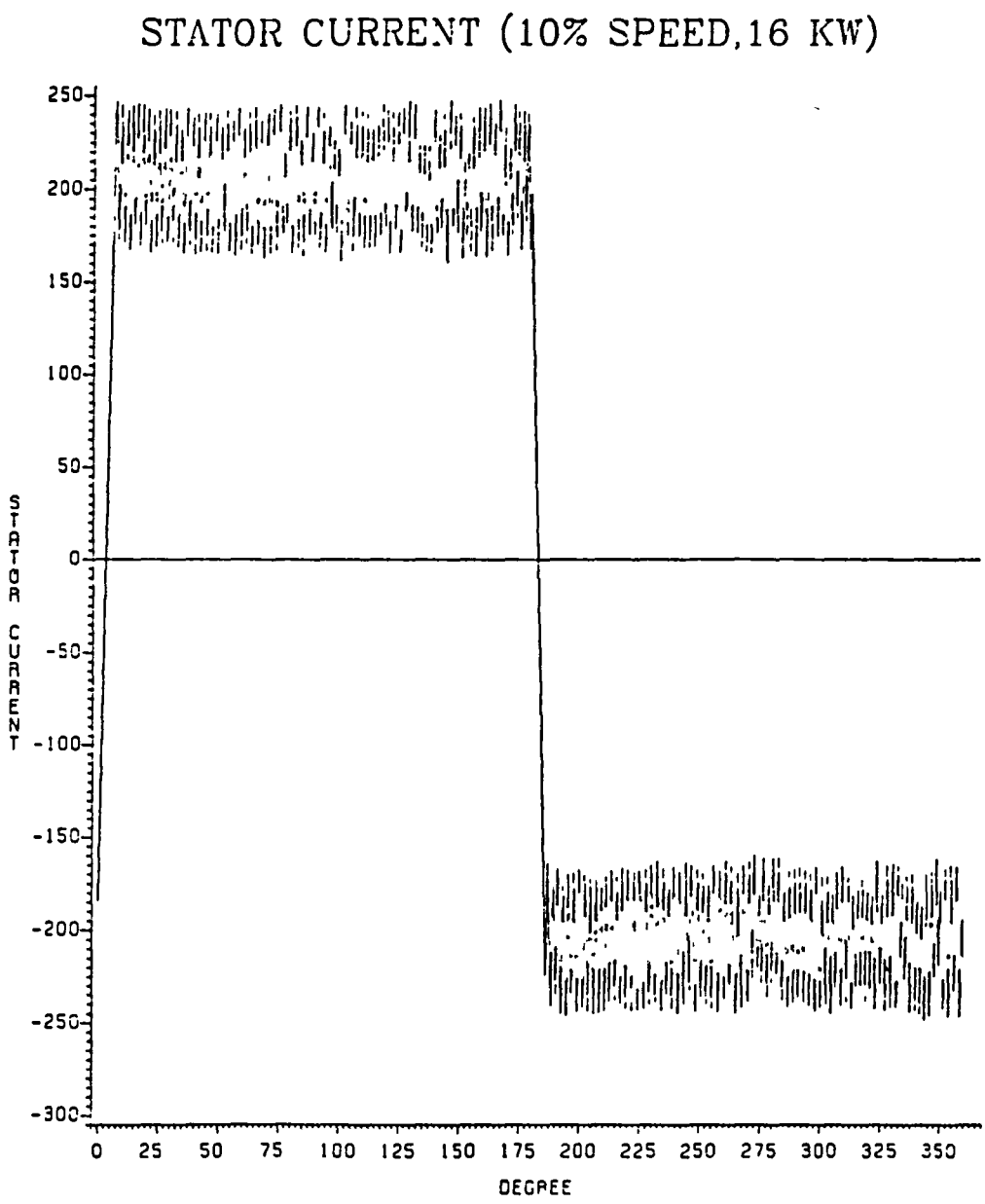
```
C
      GOTO 2
    5 IPR=0
      NC=NC-1
      IF(NC)9,9,10
    9 NC=NCC
      IPR=1
   10 IF(TIME.GE.FINTIM) ISTOP=1
      IF((IPR+ISTOP).EQ.0) GOTO 11
C
      IF(TIME.LT.2.0000)GOTO 11
      IDS=Y(1)
      IQS=Y(2)
      IDR=Y(3)
      IQR=Y(4)
C
      IMAGD=IDS+IDR
      IMAGQ=IQS+IQR
C
      TQ=LM*(IQS*IDR-IDS*IQR)
      POUT=TQ*OMEGR
C     PIN=IDS*VDS+IQS*VQS
C     SUM1=SUM1+PIN
C     SUM=SUM+POUT
C     NUMBER=NUMBER+1.
      IF(TIME.LT.2.00) GOTO 38
      IN=IN+1
C     F1(IN)=CMPLX(Y(1),0.0)
C     F2(IN)=CMPLX(POUT,0.0)
  38  CONTINUE
      WRITE(6,250)DEG,IDS,VDS,POUT
      IF(TIME.LT.2.00035) POUT=0.0
      CALL FPLOT(800,IPNT1,AR1,LR1,ISTOP,1,1,TIME,IDS)
      CALL FPLOT(800,IPNT2,AR2,LR2,ISTOP,1,1,TIME,VDS)
C     CALL FPLOT(800,IPNT4,AR4,LR4,ISTOP,1,2,TIME,IQS)
C     CALL FPLOT(800,IPNT4,AR4,LR4,ISTOP,2,2,TIME,VQS)
      CALL FPLOT(800,IPNT3,AR3,LR3,ISTOP,1,2,TIME,IMAGD)
C     CALL FPLOT(800,IPNT3,AR3,LR3,ISTOP,2,2,TIME,IDR)
      CALL FPLOT(800,IPNT3,AR3,LR3,ISTOP,1,1,TIME,POUT)
C
   11 INIT=0
      IF(ISTOP)1,1,6
    6 CONTINUE
  250 FORMAT(' ',1X,F6.1,3(F9.1,2X))
C     AVER=SUM/NUMBER
C     AVER1=SUM1/NUMBER
C     EFF=AVER/AVER1
      PIN=Y(6)-AREAIN
      POUT=Y(5)-ARAOUT
      RMS=(Y(7)-ARMS)**0.5
C     ****************************************************************
C     NOTE .7 IS THE VOLTAGE DROP ACROSS THE DIODE
C     1.85 IS THE CE VOLTAGE OF THE TRANSISTOR
C     SWITCH IS THE # OF INTERVAL OF THE SWITCHING (DIODE CONDUCTION)
C     TOTSW IS TOTOAL # OF INTEGRATION INTERVALS
```

```
C     PDIODE IS THE LOSS OF DIODE DURING CONDUCTION
C     PTRANS IS THE ON TIME LOSS OF THE TRANSISTOR
C     PSW IS THE SWITCHING LOSS OF EACH TRANSISTOR (E*I*TSW/T)
C     TOTSL IS THE TOTAL LOSS OF THE SWITCH
C      ************************************************
      PDIODE=(SWITCH/TOTSW)*0.7*IMAX/PAI
      PTRANS=(1-SWITCH/TOTSW)*1.85*IMAX/PAI
      TOTSL=PDIODE+PTRANS+PSW
      TOT4SL=4*TOTSL
      DTRIO=SWITCH/TOTSW
      EFFIO=POUT/(TOT4SL+PIN)
C
      WRITE(6,702)POUT,EFFIO,SLIP,IMAX,DTRIO,RMS
      WRITE(6,703)SW,PSW,PTRANS,PDIODE,TOT4SL,IN
  702 FORMAT(///,' POWER OUT=',F8.1,2X,'EFFIO=',F5.3,2X,'SLIP=',
     1F7.5,2X,'IMAX=',F6.1,2X,'DTRIO=',F7.1,2X,'I(RMS)=',F6.2,///)
  703 FORMAT(/,'#SW/CYC=',F5.1,6X,'PSW=',F6.1,3X,'PTRANS=',F5.1,
     1 2X,'PDIODE=',F4.1,2X,'TOT4SL=',F6.1,5X,'IN=',I5,///////)
      WRITE(6,705)
 705  FORMAT('1',3X,'Z',9X,'I HARMONIC ',9X,'P HARMONIC',//)
C706  FORMAT(2X,F5.1,5X,F12.8,5X,F15.8)
      ISTOP=0.0
CC    CALL FFT(F1,9)
CC    CALL FFT(F2,9)
CC    DO 401 I=1,150
CC    MAG1(I)=CABS(F1(I))/512.
CC    MAG2(I)=CABS(F2(I))/512.
CC    Z=FLOAT(I-1)
CC    IF (I.EQ.1) MAG1(2)=270.
CC    AMP1=MAG1(I)/MAG1(2)
CC    AMP2=MAG2(I)/MAG2(I)
C     WRITE(6,706) Z,AMP1,AMP2
CC    IF(I.EQ.150) ISTOP=1
C
C     CALL FPLOT(800,IPNT5,AR5,LR5,ISTOP,1,1,Z,AMP1)
C     CALL FPLOT(800,IPNT6,AR6,LR6,ISTOP,1,1,Z,AMP2)
CC401 CONTINUE
      STOP
      END
       SUBROUTINE FFT(X,M)
       COMPLEX X(512),U,W,T
       N=2**M
       NV2=N/2
       NM1=N-1
       J=1
       DO 7 I=1,NM1
       IF(I.GT.J) GOTO 5
       T=X(J)
       X(J)=X(I)
       X(I)=T
   5   K=NV2
   6   IF(K.GE.J)GO TO 7
       J=J-K
       K=K/2
       GO TO 6
```

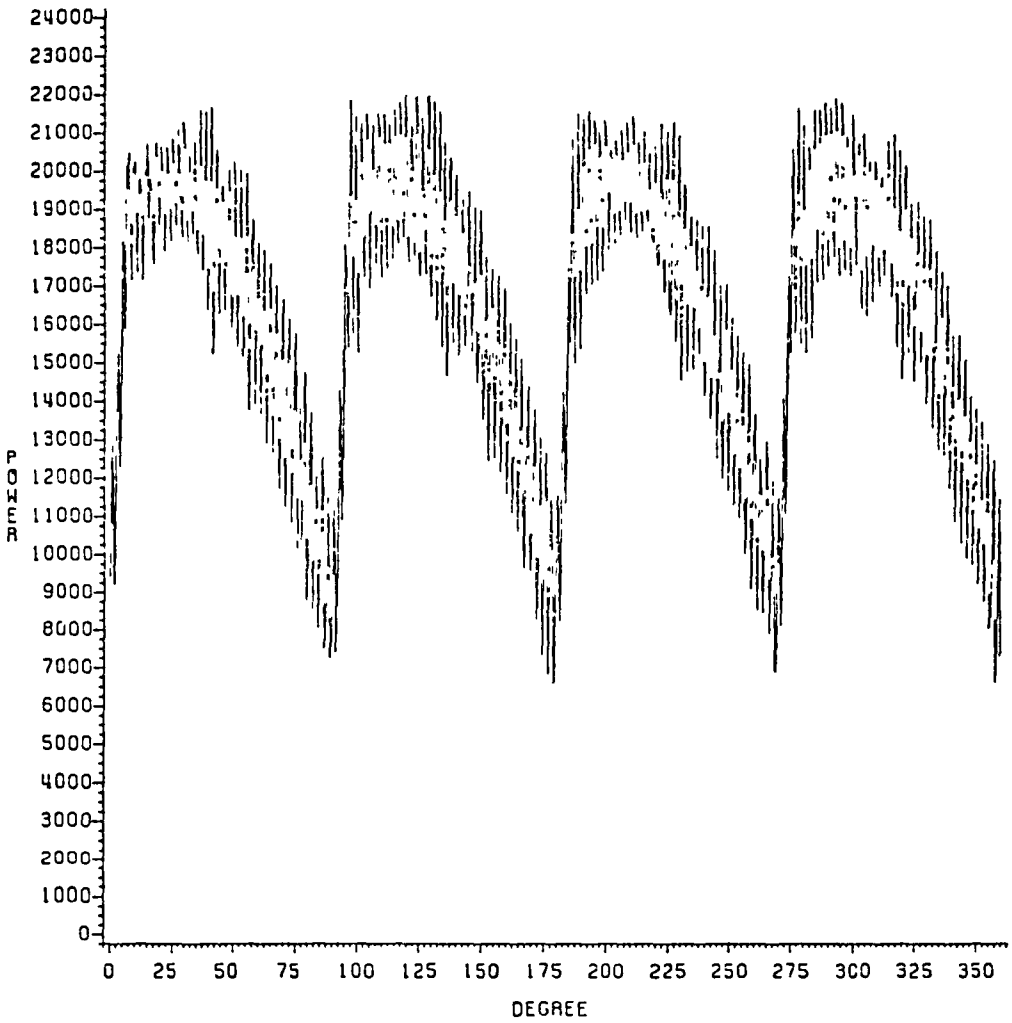
```
   7    J=J+K
        PI=3.14159265358979
        DO 20 L=1,M
        LE=2**L
        LE1=LE/2
        U=(1.0,0.0)
        Z=COS(PI/FLOAT(LE1))
        Y=-SIN(PI/FLOAT(LE1))
        W=CMPLX(Z,Y)
        DO 20 J=1,LE1
        DO 10 I=J,N,LE
        IP=I+LE1
        T=X(IP)*U
        X(IP)=X(I)-T
 10     X(I)=X(I)+T
 20     U=U*W
        RETURN
        END
$ENTRY
1         I   CURRENT OF PHASE A STATOR
1         V   VOLTAGE OF PHASE A
1         P   POWER VS TIME
1         I   CURRENT FREQUENCY SPECTRUM
1         P   POWER   FREQUENCY SPECTRUM
$STOP
/*
//
```

STATOR VOLTAGE (10% SPEED,16KW)

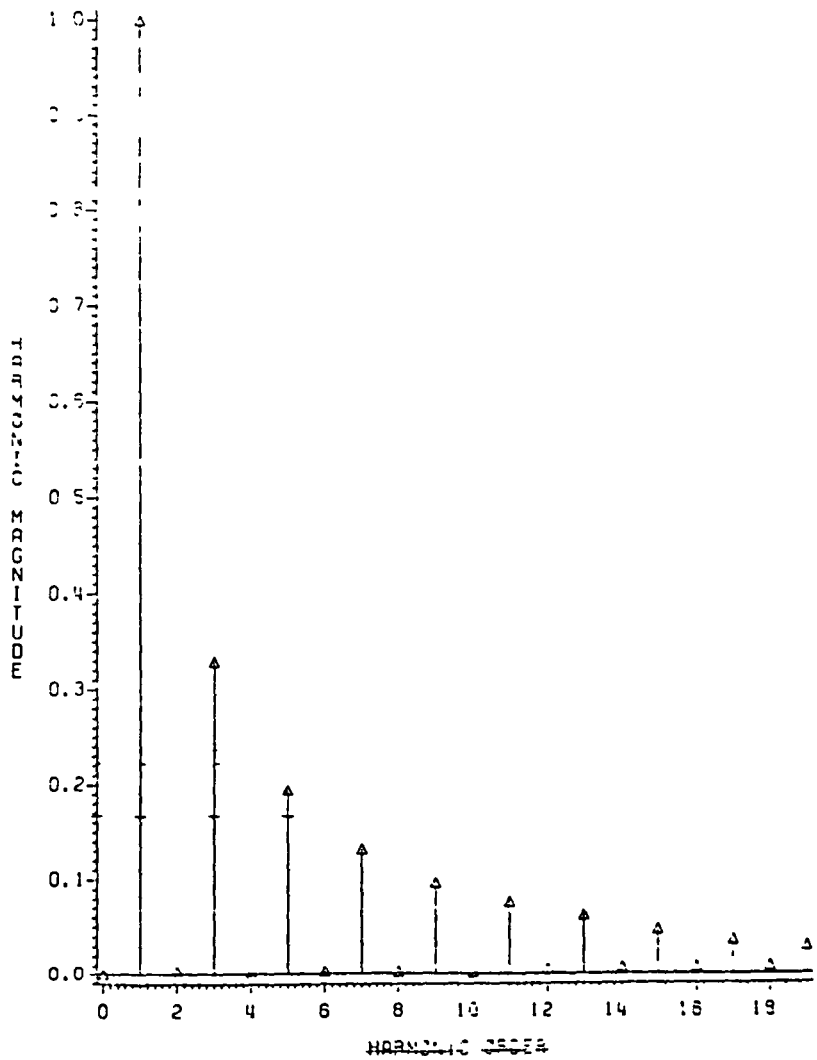



STATOR CURRENT (10% SPEED,16 KW)
STATOR CURRENT
250
200
150
100
50
0
-50
-100
-150
-200
-250
-300
0
25
50
75
100
125
150
175
200
225
250
275
300
325
350
DEGREE

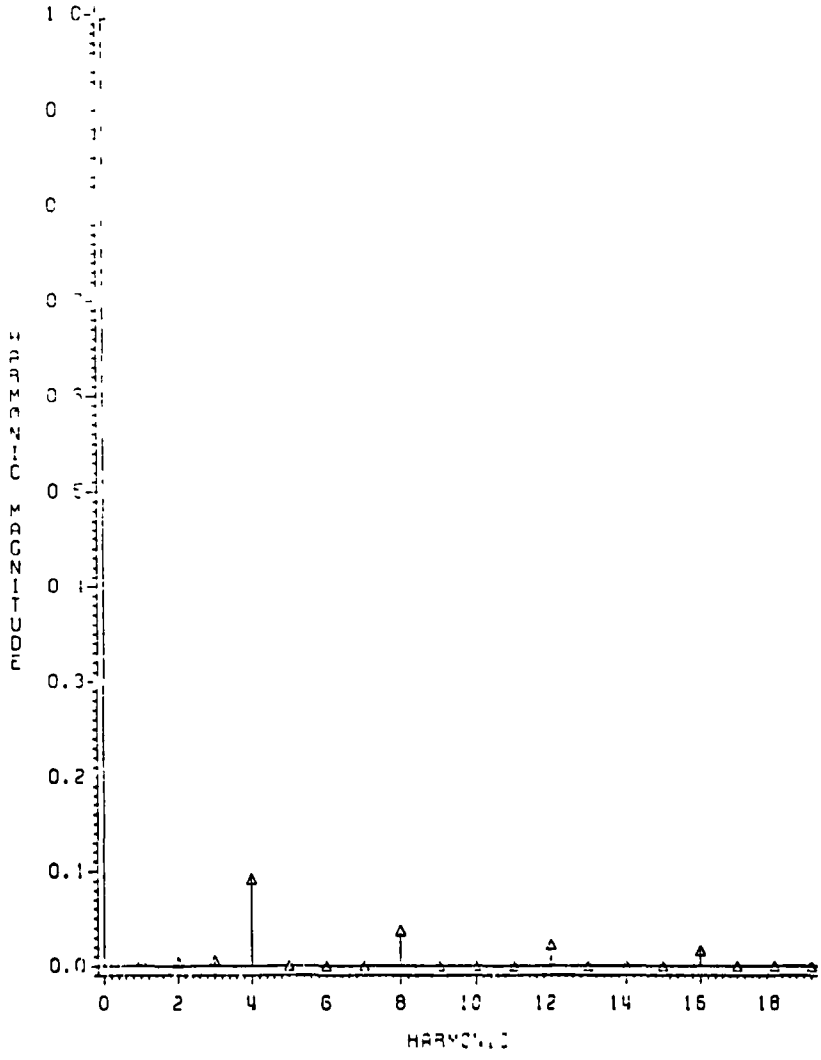
OUTPUT POWER (10% SPEED,16KW)



CURRENT SPECTRUM (10% SPEED.16 KW)



POWER SPECTRUM (10% SPEED,16KW)



```
//PLOTLS   JOB (XXXXLH),'GOODARZI',
// PASSWORD=XXXX
/*JOBPARM R=900,B=99000
// EXEC PLOTSAS
//SYSIN DD *
*********************************************************************
*                                                                   *
*             10% SPEED 16KW JPL PROJECT                            *
*             PLOT   DATA                                           *
* DEGREE   CURRENT   VOLTAGE    POWER                               *
*********************************************************************
          GOPTIONS  DEVICE=VARIAN;
          DATA ONE;
          INPUT X Y1-Y3;
CARDS;
    0.1   -184.3      240.0     9429.2
    0.4   -164.8      240.0    10725.8
    0.6   -145.3      240.0    12025.3
    0.9   -126.0      240.0    12876.4
    1.2   -106.7      240.0    11486.5
    1.5    -87.6      240.0    10119.5
    1.8    -68.5      240.0     9224.4
    2.0    -49.6      240.0    10584.4
    2.3    -30.8      240.0    11946.4
    2.6    -12.1      240.0    13310.3
    3.0     14.2      240.0    15241.9
    3.3     32.7      240.0    14390.6
    3.6     51.0      240.0    13116.9
    3.9     69.3      240.0    12308.7
    4.1     87.5      240.0    13723.0
    4.4    105.5      240.0    15138.2
    4.7    123.5      240.0    16553.9
    5.1    148.7      240.0    18556.9
    5.4    166.4      240.0    17790.5
    5.7    184.0      240.0    16608.7
    5.9    201.6      240.0    15884.9
    6.2    219.0      240.0    17345.3
    6.5    236.3      240.0    18805.1
    6.8    246.6     -240.0    20059.5
    7.1    221.9     -240.0    20278.0
    7.4    197.4     -240.0    20476.7
    7.6    172.9     -240.0    20227.3
    7.9    183.4      240.0    18895.5
    8.2    200.7      240.0    17797.6
    8.5    217.9      240.0    17147.4
    8.8    235.0      240.0    18629.2
    9.0    245.1     -240.0    19888.2
    9.3    220.3     -240.0    20028.4
    9.6    195.5     -240.0    20148.9
    9.9    170.9     -240.0    20249.9
   10.2    181.1      240.0    19392.7
   10.5    198.2      240.0    18367.0
   10.7    215.3      240.0    17363.8
   11.0    232.2      240.0    18449.1
   11.3    242.1     -240.0    19709.7
```

11.6	217.1	-240.0	19770.4
11.9	192.1	-240.0	19811.7
12.1	167.3	-240.0	19833.7
12.4	177.4	240.0	19028.3
12.7	194.3	240.0	18080.2
13.0	211.2	240.0	17154.6
13.3	227.9	240.0	18265.2
13.6	244.6	240.0	19777.7
13.8	226.3	-240.0	20012 9
14.1	201.2	-240.0	19975.6
14.4	176.1	-240.0	19919 0
14.7	185.9	240.0	20750.0
15.0	202.7	240.0	19869.6
15.2	219.3	240.0	19011.5
15 5	235.8	240.0	18567.9
15.8	245.3	-240.0	19823.9
16.1	219.9	-240 0	19724.2
16.4	194.6	-240 0	19605 5
16.6	169.4	-240.0	19467.9
16.9	179.0	240.0	18766 9
17.2	195.6	240.0	17975 6
17.5	212.1	240.0	17588.6
17.8	228 5	240.0	19119.3
18.1	244.8	240.0	20645.3
18.3	226.1	-240.0	20742.3
18.6	200.6	-240.0	20536.9
18.9	175.2	-240.0	20312.8
19.2	184.7	240.0	19658.1
19.5	201.0	240.0	18939.4
19.7	217.3	240 0	18242.4
20.0	233.5	240.0	19408.4
20.3	242.6	-240.0	20641.3
20.6	216.9	-240.0	20371.6
20.9	191.2	-240.0	20083.4
21.2	165.6	-240.0	19776.9
21.4	175.0	240.0	19173.0
21.7	191.2	240.0	18531.4
22.0	207.3	240.0	17911.2
22.3	223.4	240.0	19091.0
22.6	239.4	240.0	20622.7
22.8	220.4	-240.0	20582.2
23.1	194.6	-240.0	20211.5
23.4	168.8	-240.0	19822.9
23.7	178.0	240.0	19269 9
24.0	194.0	240.0	18705.7
24.3	210.0	240.0	18162.5
24.5	225.9	240.0	19353.9
24.8	241.7	240.0	20883.5
25.1	222.5	-240.0	20772.9
25.4	196.5	-240.0	20319.0
25.7	170.6	-240.0	19847.5
25.9	179.6	240.0	19345.0
26.2	195.5	240.0	18857.4
26.5	211.3	240.0	18390.6
26.8	227.0	240.0	19590.5

27.1	242.6	240.0	21114.3
27.3	223.3	-240.0	20932.6
27.6	197.2	-240.0	20395.0
27.9	171.1	-240.0	19517.4
28.2	179.9	240.0	18747.8
28.5	195.7	240.0	18342.1
28.8	211.3	240.0	18274.2
29.0	226.9	240.0	19796 9
29.3	242.3	240.0	21311.4
29.6	222.9	-240.0	21057.7
29.9	196.6	-240.0	20436.0
30.2	170.4	-240.0	19488.9
30.4	179.0	240.0	18782.5
30.7	194.6	240.0	18451.9
31.0	210.1	240.0	18141.3
31.3	225.6	240.0	19354.8
31.6	240.9	240.0	20862.5
31.8	221.3	-240.0	20541.9
32.1	194.9	-240.0	19841.8
32.4	168.5	-240.0	19125.3
32.7	177.1	240.0	18775.6
33.0	192.5	240.0	18519.2
33.3	207.9	240.0	18282.4
33.5	223.2	240.0	19495.2
33.8	238.4	240.0	20986.8
34.1	218.7	-240.0	20592.9
34.4	192.1	-240.0	19808.4
34.7	165.6	-240.0	18729.2
34.9	174.1	240.0	18154.0
35.2	189.4	240.0	17976.7
35.5	204.6	240.0	17818.6
35.8	219.8	240.0	19033.6
36.1	234.9	240.0	20511.6
36.4	243.0	-240.0	21592.9
36.6	216.1	-240.0	20728.4
36.9	189.4	-240.0	19585.0
37.2	169.8	240.0	17517.2
37.5	185.0	240.0	17418.1
37.8	200.1	240.0	17595.1
38.0	215.2	240.0	19061.1
38.3	230.2	240.0	20516.4
38.6	238.2	-240.0	21564.1
38.9	211.2	-240.0	20615.2
39.2	184.4	-240.0	18415.1
39.5	164.7	240.0	16380.2
39.7	179.8	240.0	16363.8
40.0	194.9	240.0	17572.0
40.3	209.8	240.0	19012.7
40.6	224.7	240.0	20442.1
40.9	239.6	240.0	21860.2
41.1	219.5	-240.0	20066.5
41.4	192.5	-240.0	17645.8
41.7	165.6	-240.0	15240.1
42.0	173.7	240.0	16036.4
42.3	188.7	240.0	17460.4

42.6	203.6	240.0	18872.6
42.8	218.4	240.0	20273.1
43.1	233.1	240.0	20567.1
43.4	240.8	-240.0	20240.8
43.7	213.7	-240.0	18064.5
44.0	186.6	-240.0	16961.5
44.2	166.6	240.0	16264.5
44.5	181.5	240.0	17652.0
44 8	196.3	240.0	19027.3
45.1	211.1	240.0	19368.4
45.4	225.8	240.0	19519.8
45.6	240.4	240 0	19688.1
45.9	220.1	-240.0	18738.5
46.2	192.9	-240.0	17551.2
46.5	165 8	-240.0	16350.9
46 8	173.6	240 0	17277.2
47.1	188.4	240.0	17670.4
47.3	203.1	240.0	17888 2
47.6	217.7	240.0	18122 4
47.9	232.3	240 0	19289.0
48.2	239.8	-240.0	20194.6
48.5	212.4	-240.0	18921 6
48.7	185.2	-240.0	17636.0
49.0	165.0	240 0	15900 9
49.3	179.7	240 0	16184 6
49 6	194.3	240.0	16484.1
49 9	208.9	240.0	17640.4
50.2	223.4	240.0	18953.6
50 4	237 8	240 0	20253.1
50.7	217.4	-240.0	19337.1
51.0	190.0	-240 0	17171.5
51.3	169.7	240.0	15304.2
51.6	184.4	240.0	15666.2
51.8	198.9	240.0	16196.5
52.1	213.4	240.0	17490.5
52.4	227.8	240.0	18770.3
52.7	242.1	240.0	20035.9
53.0	221.6	-240.0	19054.5
53.2	194.1	-240.0	17473.1
53.5	166.8	-240.0	15187.1
53.8	174.4	240.0	15162.9
54.1	188.9	240.0	15602.2
54.4	203.4	240.0	16721.9
54.7	217.7	240.0	17962.5
54.9	232.1	240.0	19188.5
55 2	239.3	-240.0	19946.9
55.5	211.8	-240.0	17808.1
55.8	184.3	-240.0	15561.1
56.1	163.9	240.0	13787.5
56.3	178.4	240.0	14420.9
56.6	192.8	240.0	15639.9
56.9	207.2	240.0	16843.9
57.2	221.5	240.0	18032.8
57.5	235.8	240.0	18663.3
57.7	242.9	-240.0	18738.1

58.0	215.3	-240.0	16643.1
58.3	187.8	-240.0	15071.4
58.6	167.3	240.0	13950.0
58.9	181.7	240 0	15114.6
59.2	196.1	240.0	16263.9
59.4	210.4	240.0	16933.3
59.7	224.6	240.0	17520.7
60.0	238.8	240.0	18121.2
60.3	218.1	-240.0	16509.8
60.6	190.5	-240.0	14869.4
60.8	170.0	240.0	13684 0
61.1	184.3	240.0	14804.2
61.4	198.7	240.0	15908.6
61.7	212.9	240.0	16623.4
62.0	227.1	240.0	17273.6
62.3	241.2	240.0	17936.3
62.5	220.4	-240.0	16351.2
62.8	192.8	-240.0	14643.3
63.1	165.3	-240.0	12927.2
63.4	172.7	240.0	13536.2
63.7	187.0	240.0	14596.3
63.9	201.3	240.0	15357.6
64.2	215.5	240.0	16071.8
64.5	229.6	240.0	16797.7
64.8	243.7	240.0	17584.9
65.1	222.9	-240.0	16272.8
65.4	195.2	-240.0	14483.9
65.6	167.6	-240.0	12687.6
65.9	175.0	240.0	13190 5
66.2	189.3	240.0	13960.5
66 5	203.5	240.0	14741.6
66.8	217.7	240.0	15533 6
67.0	231.8	240.0	16493.3
67.3	238.9	-240.0	16998.4
67.6	211.1	-240.0	15136.6
67.9	183.5	-240.0	13243.6
68.2	162.9	240.0	11709.8
68.5	177.2	240.0	12549.6
68.7	191.5	240.0	13399.8
69.0	205.7	240.0	14336.7
69.3	219.9	240.0	15274.1
69.6	233.9	240.0	16194.4
69.9	241.0	-240.0	16616.5
70.1	213.2	-240.0	14638.9
70.4	185.5	-240.0	12682.5
70.7	165.0	240.0	11220.4
71.0	179.3	240.0	12133.1
71.3	193.5	240.0	13030.0
71.5	207.7	240.0	13909.7
71.8	221.9	240.0	14779.9
72.1	235.9	240.0	15687.0
72.4	243.0	-240.0	16130.0
72.7	215.2	-240.0	14223.4
73.0	187.5	-240.0	12321.3
73.2	166.9	240.0	10813.7

73.5	181.2	240.0	11659.6
73.8	195.5	240.0	12488.1
74.1	209.7	240.0	13325.2
74.4	223.8	240.0	14290.8
74.6	237.9	240.0	15265.0
74.9	244.9	-240.0	15774.6
75.2	217.2	-240.0	13759.3
75.5	189.5	-240.0	11706 8
75.8	168.9	240.0	10123.0
76.1	183.2	240.0	10932.5
76.3	197.5	240.0	11950.6
76.6	211.7	240.0	12976.6
76.9	225.8	240.0	14010.3
77.2	205.1	-240.0	12438 5
77.5	177.5	-240.0	10341.8
77.7	184.8	240.0	10596.5
78 0	199.1	240.0	11596.1
78.3	213.3	240.0	12659.1
78.6	227.5	240.0	13729.3
78.9	241.6	240.0	14741.3
79.1	220.8	-240.0	13070 5
79.4	193.1	-240.0	10926.0
79.7	165.6	-240.0	8778.8
80.0	173.0	240.0	9049.0
80.3	187.4	240.0	10157.1
80.6	201.7	240.0	11271.8
80.8	215.9	240.0	12392.8
81.1	230.1	240.0	13104.1
81.4	244.2	240.0	13716.0
81.7	223.5	-240.0	11973.1
82.0	195.8	-240.0	10210.1
82.2	168.3	-240.0	8557.5
82.5	175.8	240.0	9258.8
82.8	190.2	240.0	10336.2
83.1	204.5	240.0	10920.4
83.4	218.8	240.0	11486.3
83.6	233.0	240.0	12033.9
83.9	212.3	-240.0	10770.6
84.2	184.8	-240.0	9168.2
84.5	164.4	240.0	8052.6
84.8	178.8	240.0	9157.0
85.1	193.2	240.0	9689.1
85.3	207.6	240.0	10202.8
85.6	221.9	240.0	10698.1
85.9	236.2	240.0	11782.5
86.2	243.4	-240.0	12529.6
86.5	215.8	-240.0	10985.4
86.7	188.3	-240.0	9335.4
87.0	167.9	240.0	7512.7
87.3	182.4	240.0	7977.9
87.6	196.9	240.0	8424.7
87.9	211.3	240.0	8990.4
88.2	225.7	240.0	10237.1
88.4	240.0	240.0	11487.9
88.7	219.4	-240.0	10463.3

G.3 TWO-PHASE SIMULATION RESULTS FOR 80% SPEED, 60 KW

Source Program

Plots

- Stator Voltage and Current
- Output Power
- Current Spectrum
- Power Spectrum

Listing of Plot Data

```
//OPTIM    JOB (XXXXLH),'GOODARZI',
// PASSWORD=????
/*JOBPARM      T=4,L=6
//*ROUTE PRINT UMVMA.EEPRI
//    EXEC    WATFIV
//GO.SYSIN  DD  *
$JOB
C*********************************************************************
C                                                                    *
C            80% SPEED 60KW   JPL PROJECT                            *
C                                                                    *
C*********************************************************************
      DIMENSION AR1(800),AR2(800),AR3(800),Y(20),X(20),GA(20),PE(20),
     1          XX(4,20),H(9),ZZ1(20),ZZ2(20),ZZ3(20),IFL(7),AR4(800),
     2          AR5(800),AR6(800),AR7(800),AR8(800)
      LOGICAL*1 LR1(800),LR2(800),LR3(800),LR4(800),LR5(800),LR6(800),
     1          LR7(800),LR8(800)
      COMPLEX   F1(512),F2(512)
      REAL LOS,LOR,LM,IAS,IBS,ICS,IAR,IBR,ICR,IDS,IQS,IDR,IQR,LMDQ,INS,
     1     POUT,IAREF,IBREF,L1,L2,M,IMAX,IMAGD,IMAGQ,IBU,
     2          MAG1(256),MAG2(256)
      IMAX=245
      DI=15.0
      SLIP=0.02
C     WRITE(6,50)
C  50 FORMAT(/,3X,'TIME',8X,'VDS',8X,'IDS',8X,'VQS',8X,'IQS',8X,
C    4        'IDR',9X,'POUT')
      NRUN=1
      ISTOP=0
      KEEP=1
      IFL(1)=-1
      NINT=7
      IPNT1=0
      IPNT2=0
      IPNT3=0
      IPNT4=0
      IPNT5=0
      IPNT6=0
      IPNT7=0
      E=240.
C  *******************************************************************
C     PARAMETERS FOR TWOPHASE INDUCTION MOTOR
C  *******************************************************************
      LOS=.2940E-03
      LOR=.3230E-03
      LM=26.800E-03
      RS=.0274
      RR=.0213
      PAI=3.141593
      P=1.
      FREQ=160.
      OMEGA=2.0*PAI*FREQ
      OMEGR=OMEGA*(1.-SLIP)
      L1=LOS+LM
      L2=LOR+LM
```

```
      M=LM
      FACT=1./(L1*L2-M*M)
      FACT1=L2*FACT
      FACT2=L2*RS*FACT
      FACT22=L1*RR*FACT
      FACT3=M*M*OMEGR*FACT
      FACT4=M*L2*OMEGR*FACT
      FACT44=M*L1*OMEGR*FACT
      FACT5=M*RR*FACT
      FACT55=M*RS*FACT
      FACT6=M*FACT
      FACT7=L1*L2*OMEGR*FACT
    1 IFL(2)=-1
    2 CALL RKFOUR(NINT,Y,X,GA,PE,XX,H,ZZ1,ZZ2,ZZ3,TIME,PNTS,KEEP,IFL)
      IF(IFL(4))3,4,5
    3 INIT=1
      TIME1=-1
      NC=1
      NCC=1
      TIME=0.
      FINTIM=0.2500
      PTMAX=800.
      Y(1)=0.
      Y(2)=0.
      Y(3)=0.
      Y(4)=0.
      Y(5)=0.0
      Y(6)=0.0
C     Y(7)=0.0
      AREAIN=0.0
      ARAOUT=0.0
      H(1)=.050E-04
C     H(7)=-1
      VDS=0.0
      VQS=0.0
      SUM=0.
      IN=0
      SUM1=0.
      NUMBER=0.
      SW=0.0
      DI1=DI
      SWITCH=0.0
      PSW=0.0
      TOTSW=0.0
      IAE=-1
      IBE=-1
      DO 61 I=1,256
      F1(I)=(0.0,0.0)
      F2(I)=(0.0,0.0)
 61   CONTINUE
C
    4 IF(TIME-TIME1)7,8,7
    7 TIME1=TIME
C
      DEG=FREQ*TIME
```

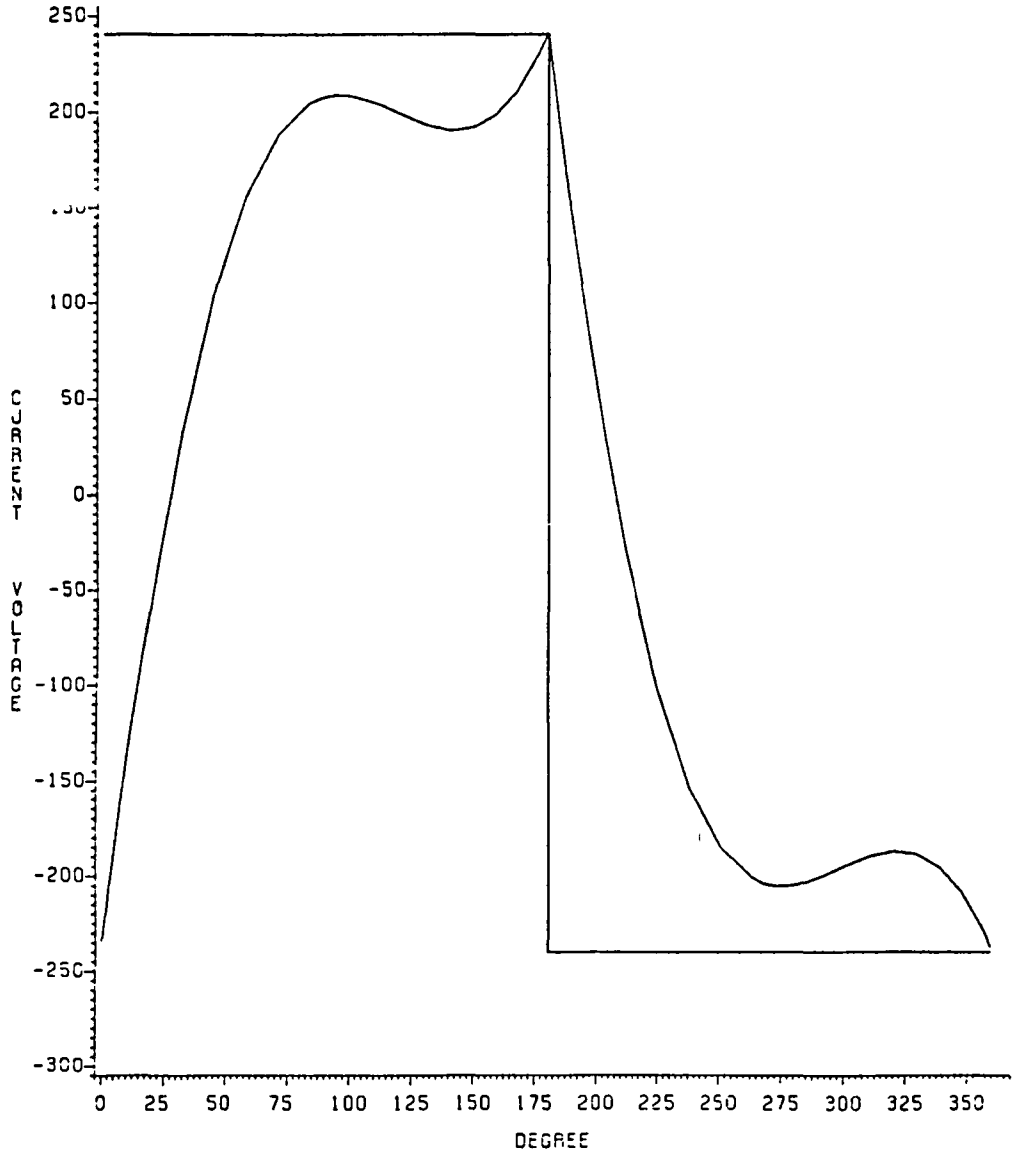
```
      IDEG=DEG
      DEG=(DEG-IDEG)*360.
C
      IAREF=IMAX*SIN(OMEGA*TIME)
      IBREF=IMAX*SIN(OMEGA*TIME-PAI/2.)
C
      IF(IAREF.GT.0.0) IAREF=IMAX
      IF(IAREF.LT.0.00) IAREF=-IMAX
      IF(IBREF.GT.0.0) IBREF=IMAX
      IF(IBREF.LT.0.00) IBREF=-IMAX
C
      DIA=Y(1)-IAREF
      DIB=Y(2)-IBREF
C
      VDS=E
      VQS=E
      IF(IAREF.LT.0)   VDS=-E
      IF(IBREF.LT.0)   VQS=-E
C
      IF(TIME.LT.0.02000) GOTO 8
      IF(TIME.GT.0.00000) GOTO 23
      ISW=0
      IF(DIA.LT.DI1)ISW=1
      IF(DIA.GT.DI) DI1=DIA
      IF(DIA.GT.DI.AND.IAREF.GT.0.0.AND.ISW.EQ.0)     GOTO 24
      IF(DIA.GT.DI)     IAE=1
      GOTO 27
 24   IAE=1
      IF(TIME.LT.0.24375) GOTO 27
      SW=SW+1
      PSW=ABS((240.*Y(1)*5.5E-06)/(6*6.25E-03))+PSW
 27   IF(DIA.LT.(-DI))  IAE=0
      IF(VDS.GT.0.0.AND.IAE.EQ.1)  GOTO 18
      IF(VDS.LT.0.0.AND.IAE.EQ.0)  GOTO 18
      GOTO 22
 18   VDS=-VDS
      IF(TIME.LT.0.24375) GOTO 22
      SWITCH=SWITCH+1.
22    IF(DIB.GT.DI)     IBE=1
      IF(DIB.LT.(-DI))  IBE=0
      IF(VQS.GT.0.0.AND.IBE.EQ.1)  GOTO 19
      IF(VQS.LT.0.0.AND.IBE.EQ.0)  GOTO 19
      GOTO 23
19    VQS=-VQS
23    IF(TIME.GT.0.23125.AND.TIME.LT.0.2312612)AREAIN=Y(6)
      IF(TIME.GT.0.23125.AND.TIME.LT.0.2312612)ARAOUT=Y(5)
      IF(TIME.GT.0.23125.AND.TIME.LT.0.2312612)ARMS=Y(7)
C
      IF(TIME.LT.0.24375) GOTO 8
      TOTSW=TOTSW+1.
C
 8    CONTINUE
C
      X(1)=FACT1*VDS-FACT2*Y(1)+FACT3*Y(2)+FACT4*Y(4)+FACT5*Y(3)
      X(2)=FACT1*VQS-FACT2*Y(2)-FACT3*Y(1)-FACT4*Y(3)+FACT5*Y(4)
```

```
      X(3)=-FACT6*VDS+FACT55*Y(1)-FACT44*Y(2)-FACT7*Y(4)-FACT22*Y(3)
      X(4)=-FACT6*VQS+FACT55*Y(2)+FACT44*Y(1)+FACT7*Y(3)-FACT22*Y(4)
      X(5)=LM*(Y(2)*Y(3)-Y(1)*Y(4))*OMEGR/.01875
      X(6)=(Y(1)*VDS+Y(2)*VQS)/.01875
      X(7)=Y(1)*Y(1)/.01875
C
      GOTO 2
    5 IPR=0
      NC=NC-1
      IF(NC)9,9,10
    9 NC=NCC
      IPR=1
   10 IF(TIME.GE.FINTIM) ISTOP=1
      IF((IPR+ISTOP).EQ.0) GOTO 11
C
      IF(TIME.LT.0.23125)  GOTO 11
      IDS=Y(1)
      IQS=Y(2)
      IDR=Y(3)
      IQR=Y(4)
C
      IMAGD=IDS+IDR
      IMAGQ=IQS+IQR
C
      TQ=LM*(IQS*IDR-IDS*IQR)
      POUT=TQ*OMEGR
C     PIN=IDS*VDS+IQS*VQS
C     SUM1=SUM1+PIN
C     SUM=SUM+POUT
C     NUMBER=NUMBER+1.
      IF(TIME.LT.0.24375) GOTO 31
       IN=IN+1
CC     F1(IN)=CMPLX(Y(1),0.0)
CC     F2(IN)=CMPLX(POUT,0.0)
 31    CONTINUE
      WRITE(6,250)DEG,IDS,VDS,POUT
C     IF(TIME.LT.0.19250) POUT=0.0
      CALL FPLOT(800,IPNT1,AR1,LR1,ISTOP,1,1,TIME,IDS)
      CALL FPLOT(800,IPNT2,AR2,LR2,ISTOP,1,1,TIME,VDS)
C     CALL FPLOT(800,IPNT4,AR4,LR4,ISTOP,1,2,TIME,IQS)
C     CALL FPLOT(800,IPNT4,AR4,LR4,ISTOP,2,2,TIME,VQS)
C     CALL FPLOT(800,IPNT3,AR3,LR3,ISTOP,1,2,TIME,IMAGD)
C     CALL FPLOT(800,IPNT3,AR3,LR3,ISTOP,2,2,TIME,IDR)
      CALL FPLOT(800,IPNT5,AR5,LR5,ISTOP,1,1,TIME,POUT)
C
   11 INIT=0
      IF(ISTOP)1,1,6
    6 CONTINUE
  250 FORMAT(' ',F6.1,3(F9.2,2X))
C     AVER=SUM/NUMBER
C     AVER1=SUM1/NUMBER
C     EFF=AVER/AVER1
      PIN=Y(6)-AREAIN
      POUT=Y(5)-ARAOUT
      RMS=(Y(7)-ARMS)**0.5
```

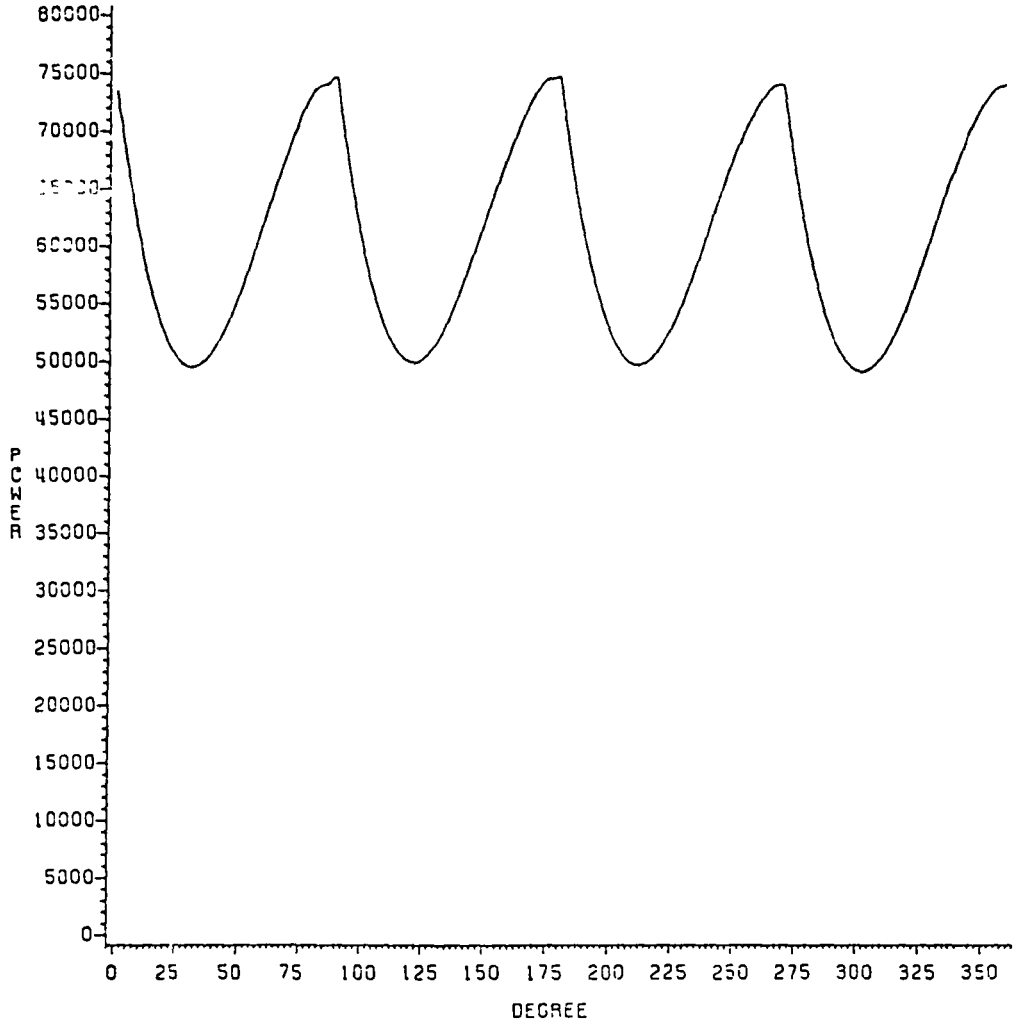
```
C     ******************************************************************
C     NOTE .7 IS THE VOLTAGE DROP ACROSS THE DIODE
C     1.85 IS THE CE VOLTAGE OF THE TRANSISTOR
C     SWITCH IS THE # OF INTERVAL OF THE SWITCHING (DIODE CONDUCTION)
C     TOTSW IS TOTOAL # OF INTEGRATION INTERVALS
C     PDIODE IS THE LOSS OF DIODE DURING CONDUCTION
C     PTRANS IS THE ON TIME LOSS OF THE TRANSISTOR
C     PSW IS THE SWITCHING LOSS OF EACH TRANSISTOR (E*I*TSW/T)
C     TOTSL IS THE TOTAL LOSS OF THE SWITCH
C      ************************************************
      PDIODE=(SWITCH/TOTSW)*0.7*IMAX/PAI
      PTRANS=(1-SWITCH/TOTSW)*1.85*IMAX/PAI
      TOTSL=PDIODE+PTRANS+PSW
      TOT4SL=4*TOTSL
      DTRIO=SWITCH/TOTSW
C     EFFIO=POUT/(TOT4SL+PIN)
      EFFIO=POUT/PIN
      SW=SW+SW
C
      WRITE(6,702)POUT,EFFIO,SLIP,IMAX,DTRIO,RMS
      WRITE(6,703)SW,PSW,PTRANS,PDIODE,TOT4SL,IN
  702 FORMAT(///,' POWER OUT=',F8.1,2X,'EFFIO=',F5.3,2X,'SLIP=',
     1F7.5,2X,'IMAX=',F6.1,2X,'DTRIO=',F7.1,2X,'I(RMS)=',F7.1,//)
  703 FORMAT(/,'#SW/CYC=',F5.1,8X,'PSW=',F6.1,3X,'PTRANS=',F5.1,
     1 2X,'PDIODE=',F4.1,2X,'TOT4SL=',F6.1,5X,'IN=',I5,///////)
      WRITE(6,705)
  705  FORMAT(1X,'Z',11X,'I HARMONIC ',15X,'P HARMONIC',//)
C706  FORMAT(1X,F5.1,5X,F12.8,5X,F15.8)
      ISTOP=0.0
CC    CALL FFT(F1,8)
CC    CALL FFT(F2,8)
CC    DO 401 I=1,128
CC    MAG1(I)=CABS(F1(I))/256.
CC    MAG2(I)=CABS(F2(I))/256.
CC    Z=FLOAT(I-1)
CC    IF (I.EQ.1) MAG1(2)=275.
CC    AMP1=MAG1(I)/MAG1(2)
CC    AMP2=MAG2(I)/MAG2(1)
C     WRITE(6,706) Z,AMP1,AMP2
CC    IF(I.EQ.128) ISTOP=1
C      IF(AMP2.GT.300) AMP2=0.0
C
C     CALL FPLOT(800,IPNT4,AR4,LR4,ISTOP,1,1,Z,AMP1)
C     CALL FPLOT(800,IPNT3,AR3,LR3,ISTOP,1,1,Z,AMP2)
  401  CONTINUE
      STOP
      END
       SUBROUTINE FFT(X,M)
       COMPLEX X(256),U,W,T
       N=2**M
       NV2=N/2
       NM1=N-1
       J=1
       DO 7 I=1,NM1
       IF(I.GT.J) GOTO 5
```

```
      T=X(J)
      X(J)=X(I)
      X(I)=T
   5  K=NV2
   6  IF(K.GE.J)GO TO 7
      J=J-K
      K=K/2
      GO TO 6
   7  J=J+K
      PI=3.14159265358979
      DO 20 L=1,M
      LE=2**L
      LE1=LE/2
      U=(1.0,0.0)
      Z=COS(PI/FLOAT(LE1))
      Y=-SIN(PI/FLOAT(LE1))
      W=CMPLX(Z,Y)
      DO 20 J=1,LE1
      DO 10 I=J,N,LE
      IP=I+LE1
      T=X(IP)*U
      X(IP)=X(I)-T
  10  X(I)=X(I)+T
  20  U=U*W
      RETURN
      END
$ENTRY
1       A   CURRENT OF PHASE A OF STATOR
1       V   VOLTAGE OF PHASE A OF STATOR
1       P   POWER VS TIME
1       I   CURRENT HARMONIC MAGNITUDE VS HARMONIC ORDER
1       P   POWER HARMONIC MAGNITUDE VS HARMONIC ORDER
$STOP
/*
//
```

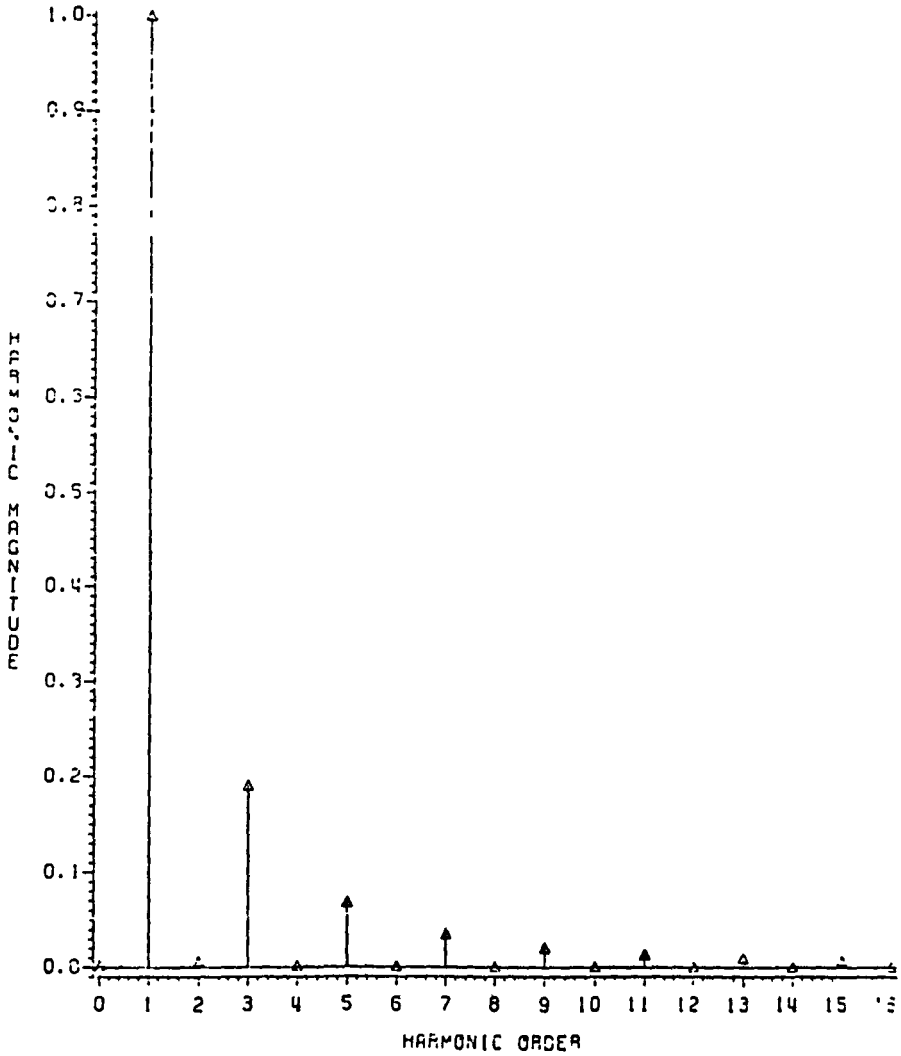
VOLTAGE & CURRENT (80% S,60KW,MOTOR OPERATION)



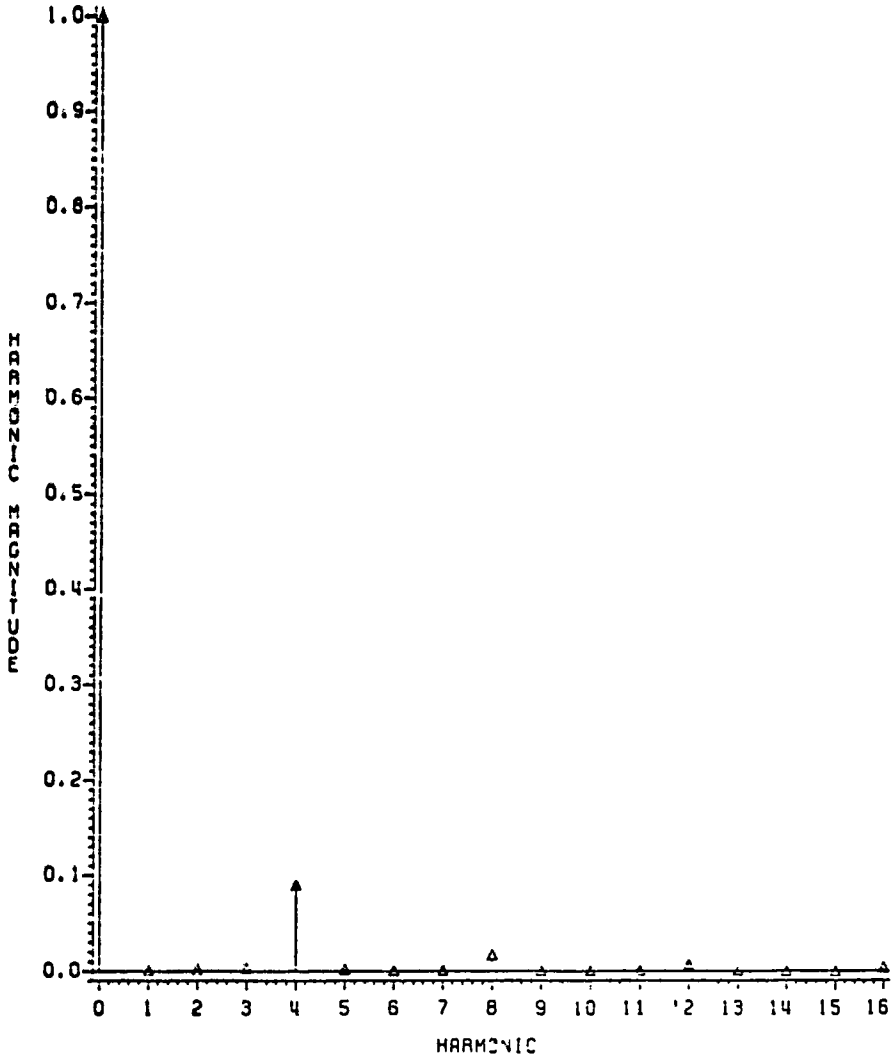
OUTPUT POWER (80% S,60KW, MOTOR OPERATION)



CURRENT SPECTRUM (80% SPEED,60 KW)



POWER SPECTRUM (80% SPEED,60KW)



```
FILE: MOTOR    PLOT     A   UNIVERSITY OF MISSOURI COMPUTER NETWORK                    PAGE 001

//MOTORP   JOB (XXXXLH),'GOODARZI',
// PASSWORD=XXXX
/*JOBPARM R=900,B=99000
// EXEC PLOTSAS
//SYSIN DD *
          GOPTIONS  DEVICE=VARIAN;
          DATA ONE;
          INPUT X Y1-Y3;
*************************************************************************
*                                                                       *
*              80% SPEED,60KW  JPL PROJECT                              *
*              PLOT DATA                                                *
*         |         |          |                                        *
* DEGREE |CURRENT |VOLTAGE   |POWER                                     *
*********|********|**********|****************************************
CARDS;
   0 4  -233.96      240.00   73530 25
   0.9  -227.92      240.00   72584.00
   1.5  -221.93      240 00   71657.56
   2.1  -215.98      240.00   70751.13
   2.9  -207.63      240.00   69503.44
   4.1  -195.96      240 00   67807 31
   5.7  -179.75      240.00   65544.56
   8.0  -157.41      240.00   62615.67
  11.2  -126.97      240.00   59009 33
  15.8   -86.28      240.00   54955.57
  22.4   -33.54      240.00   51216.40
  31.6    31.40      240.00   49532.14
  44.6   104.19      240.00   52894.97
  57.6   155.75      240.00   60361.47
  70.6   188.23      240.00   68587.13
  83.7   204.60      240.00   74058.00
  88.3   207.21      240.00   74693.56
  89.9   207.80      240.00   74708.81
  90.3   207.92      240.00   74138.25
  90.7   208.03      240.00   73465.38
  91.1   208.13      240.00   72802.44
  91.5   208.22      240.00   72149.25
  92.1   208.33      240.00   71242.75
  92.9   208.45      240.00   69995.00
  94.1   208.57      240.00   68298.13
  95.7   208.61      240.00   66034.44
  98.0   208.44      240.00   63103.30
 101.2   207.81      240.00   59492.22
 105.8   206.24      240.00   55429.19
 112.4   203.07      240.00   51671.75
 121.6   197.75      240.00   49950.17
 130.8   192.98      240.00   51714.76
 140.0   190.43      240.00   55985.92
 149.2   191.68      240.00   61636.06
 158.4   198.18      240.00   67448.50
 167.6   211.24      240.00   72177.50
 176.8   231.96      240.00   74603.25
 179.1   238.45      240.00   74714.19
 179.9   240.87      240.00   74698.63
```

```
 180.5   236.11    -240.00   73889.38
 181.1   230.08    -240.00   72941.25
 181.7   224.10    -240.00   72012.88
 182.2   218.16    -240.00   71104.63
 183.1   209.82    -240.00   69854.19
 184.2   198.17    -240.00   68153.75
 185.8   181.99    -240.00   65884.75
 188.1   159.68    -240.00   62946.13
 191.4   129.29    -240.00   59323.89
 196.0    88.69    -240.00   55243.95
 202.5    36.05    -240.00   51460.83
 211.7   -28.73    -240.00   49697.88
 224.7  -101.31    -240.00   52920.96
 237.8  -152.69    -240.00   60219.04
 250.8  -185.00    -240.00   68248.75
 263.8  -201.25    -240.00   73508.50
 268.4  -203.82    -240.00   74067.63
 269.6  -204.24    -240.00   74071.44
 270.0  -204.37    -240.00   74058.94
 270.4  -204.49    -240.00   73484.50
 270.8  -204.60    -240.00   72807.69
 271.2  -204.70    -240.00   72140.88
 271.6  -204.79    -240.00   71484.19
 272.2  -204.90    -240.00   70572.56
 273.0  -205.02    -240.00   69317.56
 274.1  -205.13    -240.00   67611.13
 275.8  -205.17    -240.00   65333.88
 278.1  -205.00    -240.00   62384.63
 281.3  -204.37    -240.00   58749.62
 285.9  -202.80    -240.00   54655.75
 292.4  -199.63    -240.00   50861.50
 301.6  -194.33    -240.00   49102.36
 310.9  -189.59    -240.00   50845.82
 320.1  -187.09    -240.00   55110.38
 329.3  -188.39    -240.00   60769.29
 338.5  -194.96    -240.00   66604.94
 347.7  -208.09    -240.00   71369.31
 356.9  -228.90    -240.00   73842.38
 359.2  -235.41    -240.00   73966.63
 359.8  -237.12    -240.00   73962.13
;
          PROC GPLOT;
          PLOT Y1*X Y2*X /OVERLAY ;
          SYMBOL1 I=JOIN L=1;
          SYMBOL2 I=JOIN L=13;
          LABEL X=DEGREE
                Y1=CURRENT   VOLTAGE;
         TITLE VOLTAGE & CURRENT (80% S,60KW,MOTOR OPERATION);
          PROC GPLOT;
          PLOT Y3*X /VAXIS=0 TO 80000 BY 5000;
          SYMBOL1 I=SPLINE;
          LABEL X=DEGREE
                Y3= POWER ;
          TITLE OUTPUT POWER (80% S,60KW, MOTOR OPERATION);
/*
```

G.4 TWO-PHASE SIMULATION RESULTS FOR 80% SPEED, 12 KW

Source Program

Plots

- Stator Voltage
- Stator Current
- Output Power
- Current Spectrum
- Power Spectrum

Partial Listing of Plot Data

```
//OPTIM    JOB (XXXXLH),'JPL PROJ',
// PASSWORD=XXXX
/*JOBPARM      T=2,L=4
//*ROUTE PRINT UMVMA.EEPR3
//    EXEC    WATFIV
//GO.SYSIN  DD  *
$JOB
      DIMENSION AR1(800),AR2(800),AR3(800),Y(20),X(20),GA(20),PE(20),
     1          XX(4,20),H(9),ZZ1(20),ZZ2(20),ZZ3(20),IFL(7),AR4(800),
     2          AR5(800),AR6(800),AR7(800),AR8(800)
      LOGICAL*1 LR1(800),LR2(800),LR3(800),LR4(800),LR5(800),LR6(800),
     1          LR7(800),LR8(800)
      COMPLEX   F1(512),F2(512)
      REAL LOS,LOP,LM,IAS,IBS,ICS,IAR,IBR,ICR,IDS,IQS,IDR,IQR,LMDQ,INS,
     1     POUT,IAREF,IBREF,L1,L2,M,IMAX,IMAGD,IMAGQ,IBU,
     2          MAG1(256),MAG2(256)
      IMAX=65.0
      DI=38.0
      SLIP=0.00725
C*********************************************************************
C                                                                     *
C            80% SPEED,12KW         JPL PROJECT                     *
C                                                                     *
C*********************************************************************
C     WRITE(6,50)
C  50 FORMAT(/,3X,'TIME',8X,'VDS',8X,'IDS',8X,'VQS',8X,'IQS',8X,
C    4        'IDR',9X,'POUT')
      NRUN=1
      ISTOP=0
      KEEP=1
      IFL(1)=-1
      NINT=7
      IPNT1=0
      IPNT2=0
      IPNT3=0
      IPNT4=0
      IPNT5=0
      IPNT6=0
      IPNT7=0
      E=240.
C  ******************************************************************
C     PARAMETERS FOR TWOPHASE INDUCTION MOTOR
C  ******************************************************************
      LOS=.2940E-03
      LOR=.3230E-03
      LM=26.800E-03
      RS=.0274
      RR=.0213
      PAI=3.141593
      P=1.
      FREQ=160.
      OMEGA=2.0*PAI*FREQ
      OMEGR=OMEGA*(1.-SLIP)
      L1=LOS+LM
      L2=LOR+LM
```

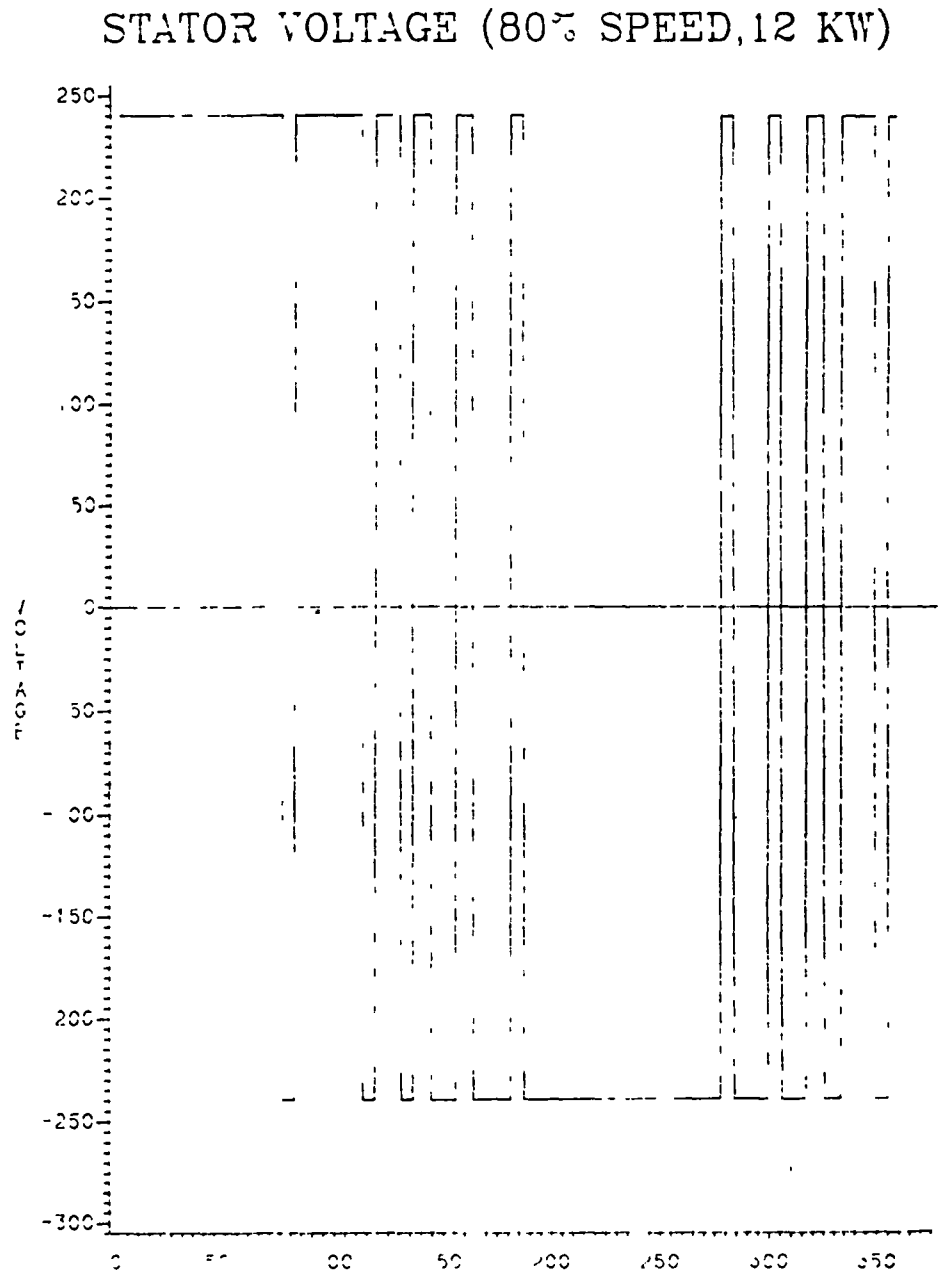
```
      M=LM
      FACT=1./(L1*L2-M*M)
      FACT1=L2*FACT
      FACT2=L2*RS*FACT
      FACT22=L1*RR*FACT
      FACT3=M*M*OMEGR*FACT
      FACT4=M*L2*OMEGR*FACT
      FACT44=M*L1*OMEGR*FACT
      FACT5=M*RR*FACT
      FACT55=M*RS*FACT
      FACT6=M*FACT
      FACT7=L1*L2*OMEGR*FACT
    1 IFL(2)=-1
    2 CALL RKFOUR(NINT,Y,X,GA,PE,XX,H,ZZ1,ZZ2,ZZ3,TIME,PNTS,KEEP,IFL)
      IF(IFL(4))3,4,5
    3 INIT=1
      TIME1=-1
      NC=1
      NCC=1
      TIME=0.
      FINTIM=0.2500
      PTMAX=800.
      Y(1)=0.
      Y(2)=0.
      Y(3)=0.
      Y(4)=0.
      Y(5)=0.0
      Y(6)=0.0
C     Y(7)=0.0
      AREAIN=0.0
      ARAOUT=0.0
      H(1)=.050E-04
C     H(7)=-1
      VDS=0.0
      VQS=0.0
      SUM=0.
      SUMI=0.
      IN=0
      SUM1=0.
      NUMBER=0.
      SW=0.0
      DI1=DI
      SWITCH=0.0
      PSW=0.0
      TOTSW=0.0
      IAE=-1
      IBE=-1
      DO 61 I=1,256
      F1(I)=(0.0,0.0)
      F2(I)=(0.0,0.0)
 61   CONTINUE
C
    4 IF(TIME-TIME1)7,8,7
    7 TIME1=TIME
C
```

```
      DEG=FREQ*TIME
      IDEG=DEG
      DEG=(DEG-IDEG)*360.
C
      IAREF=IMAX*SIN(OMEGA*TIME)
      IBREF=IMAX*SIN(OMEGA*TIME-PAI/2.)
C
C     IF(IAREF.GT.0.0) IAREF=IMAX
C     IF(IAREF.LT.0.00) IAREF=-IMAX
C     IF(IBREF.GT.0.0) IBREF=IMAX
C     IF(IBREF.LT.0.00) IBREF=-IMAX
C
      DIA=Y(1)-IAREF
      DIB=Y(2)-IBREF
C
      VDS=E
      VQS=E
      IF(IAREF.LT.0)   VDS=-E
      IF(IBREF.LT.0)   VQS=-E
C
      IF(TIME.LT.0.02000) GOTO 8
C     IF(TIME.GT.0.00000) GOTO 23
      ISW=0
      IF(DIA.LT.DI1)ISW=1
      IF(DIA.GT.DI) DI1=DIA
      IF(DIA.GT.DI.AND.IAREF.GT.0.0.AND.ISW.EQ.0)     GOTO 24
      IF(DIA.GT.DI)     IAE=1
      GOTO 27
 24   IAE=1
      IF(TIME.LT.0.24375) GOTO 27
      SW=SW+1
      PSW=ABS((240.*Y(1)*5.5E-06)/(6*6.25E-03))+PSW
 27   IF(DIA.LT.(-DI))  IAE=0
      IF(VDS.GT.0.0.AND.IAE.EQ 1)  GOTO 18
      IF(VDS.LT.0.0.AND.IAE.EQ.0)  GOTO 18
      GOTO 22
 18   VDS=-VDS
      IF(TIME.LT.0.24375) GOTO 22
      SWITCH=SWITCH+1.
22    IF(DIB GT.DI)     IBE=1
      IF(DIB.LT.(-DI))  IBE=0
      IF(VQS.GT.0.0.AND.IBE.EQ.1)  GOTO 19
      IF(VQS.LT.0.0.AND.IBE.EQ.0)  GOTO 19
      GOTO 23
19    VQS=-VQS
23    IF(TIME.GT.0.23125.AND.TIME.LT.0.2316612)AREAIN=Y(6)
      IF(TIME.GT.0.23125.AND.TIME.LT.0.2316612)ARAOUT=Y(5)
      IF(TIME.GT.0.23125.AND.TIME.LT.0.2316612)ARMS=Y(7)
C
      IF(TIME.LT.0.24375) GOTO 8
      TOTSW=TOTSW+1.
C
 8    CONTINUE
C
      X(1)=FACT1*VDS-FACT2*Y(1)+FACT3*Y(2)+FACT4*Y(4)+FACT5*Y(3)
```

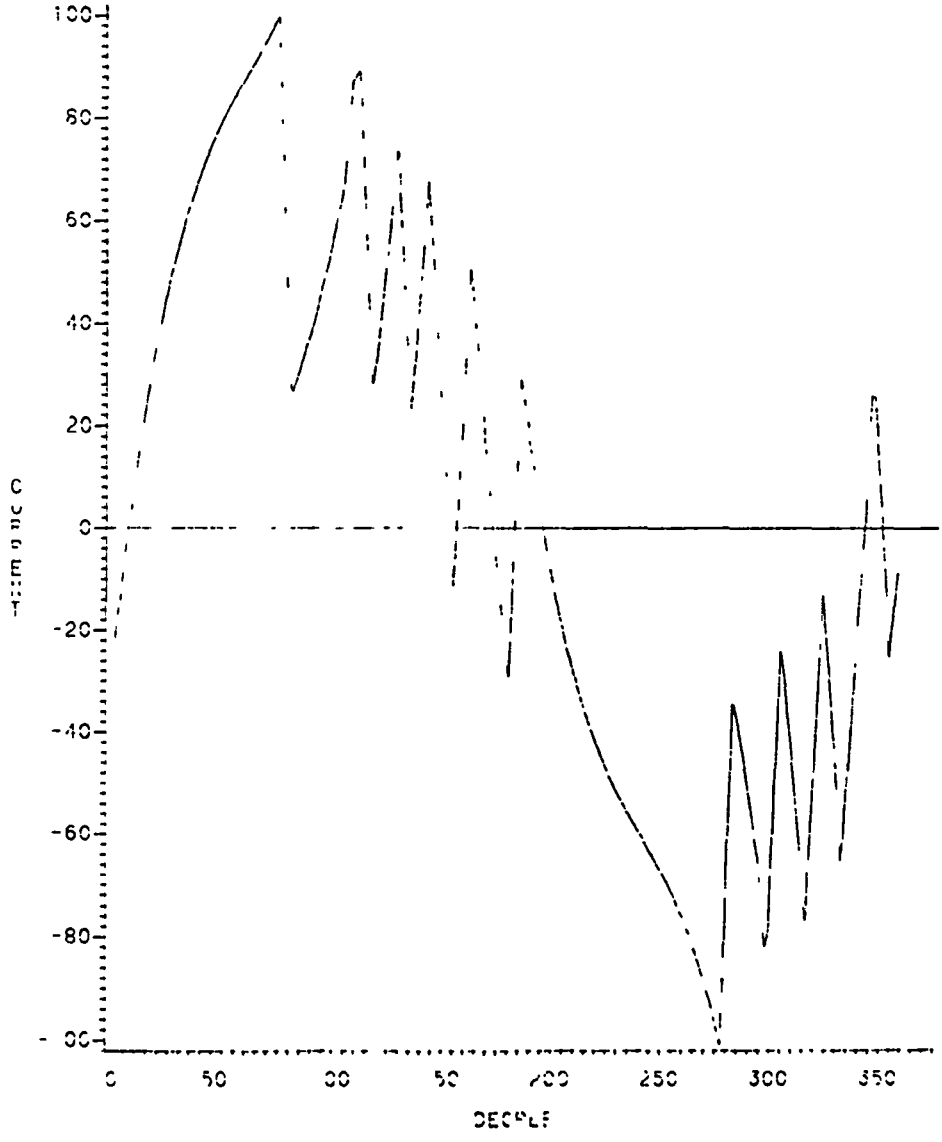
```
      X(2)=FACT1*VQS-FACT2*Y(2)-FACT3*Y(1)-FACT4*Y(3)+FACT5*Y(4)
      X(3)=-FACT6*VDS+FACT55*Y(1)-FACT44*Y(2)-FACT7*Y(4)-FACT22*Y(3)
      X(4)=-FACT6*VQS+FACT55*Y(2)+FACT44*Y(1)+FACT7*Y(3)-FACT22*Y(4)
      X(5)=LM*(Y(2)*Y(3)-Y(1)*Y(4))*OMEGR/.01875
      X(6)=(Y(1)*VDS+Y(2)*VQS)/.01875
      X(7)=Y(1)*Y(1)/.01875
C
      GOTO 2
    5 IPR=0
      NC=NC-1
      IF(NC)9,9,10
    9 NC=NCC
      IPR=1
   10 IF(TIME.GE.FINTIM) ISTOP=1
      IF((IPR+ISTOP).EQ.0) GOTO 11
C
      IF(TIME.LT.0.24375)  GOTO 11
      IDS=Y(1)
      IQS=Y(2)
      IDR=Y(3)
      IQR=Y(4)
C
      IMAGD=IDS+IDR
      IMAGQ=IQS+IQR
C
      TQ=LM*(IQS*IDR-IDS*IQR)
      POUT=TQ*OMEGR
C     PIN=IDS*VDS+IQS*VQS
C     SUM1=SUM1+PIN
C     SUM=SUM+POUT
C     NUMBER=NUMBER+1.
      IF(TIME.LT.0.24375) GOTO 31
      IF(DEG.GT.180.) GOTO 41
       SUMI=SUMI+Y(1)
 41    IN=IN+1
       F1(IN)=CMPLX(Y(1),0.0)
       F2(IN)=CMPLX(POUT,0.0)
 31    CONTINUE
      WRITE(6,250)DEG,IDS,VDS,POUT
C     IF(TIME.LT.0.24400) POUT=0.0
      CALL FPLOT(800,IPNT1,AR1,LR1,ISTOP,1,1,TIME,IDS)
C     CALL FPLOT(800,IPNT2,AR2,LR2,ISTOP,1,1,TIME,VDS)
C     CALL FPLOT(800,IPNT4,AR4,LR4,ISTOP,1,2,TIME,IQS)
C     CALL FPLOT(800,IPNT4,AR4,LR4,ISTOP,2,2,TIME,VQS)
C     CALL FPLOT(800,IPNT3,AR3,LR3,ISTOP,1,2,TIME,IMAGD)
C     CALL FPLOT(800,IPNT3,AR3,LR3,ISTOP,2,2,TIME,IDR)
      CALL FPLOT(800,IPNT5,AR5,LR5,ISTOP,1,1,TIME,POUT)
C
   11 INIT=0
      IF(ISTOP)1,1,6
    6 CONTINUE
  250 FORMAT(' ',F6.1,3(F9.2,2X))
C     AVER=SUM/NUMBER
      AVERI=2*SUMI/IN
C     EFF=AVER/AVERI
```

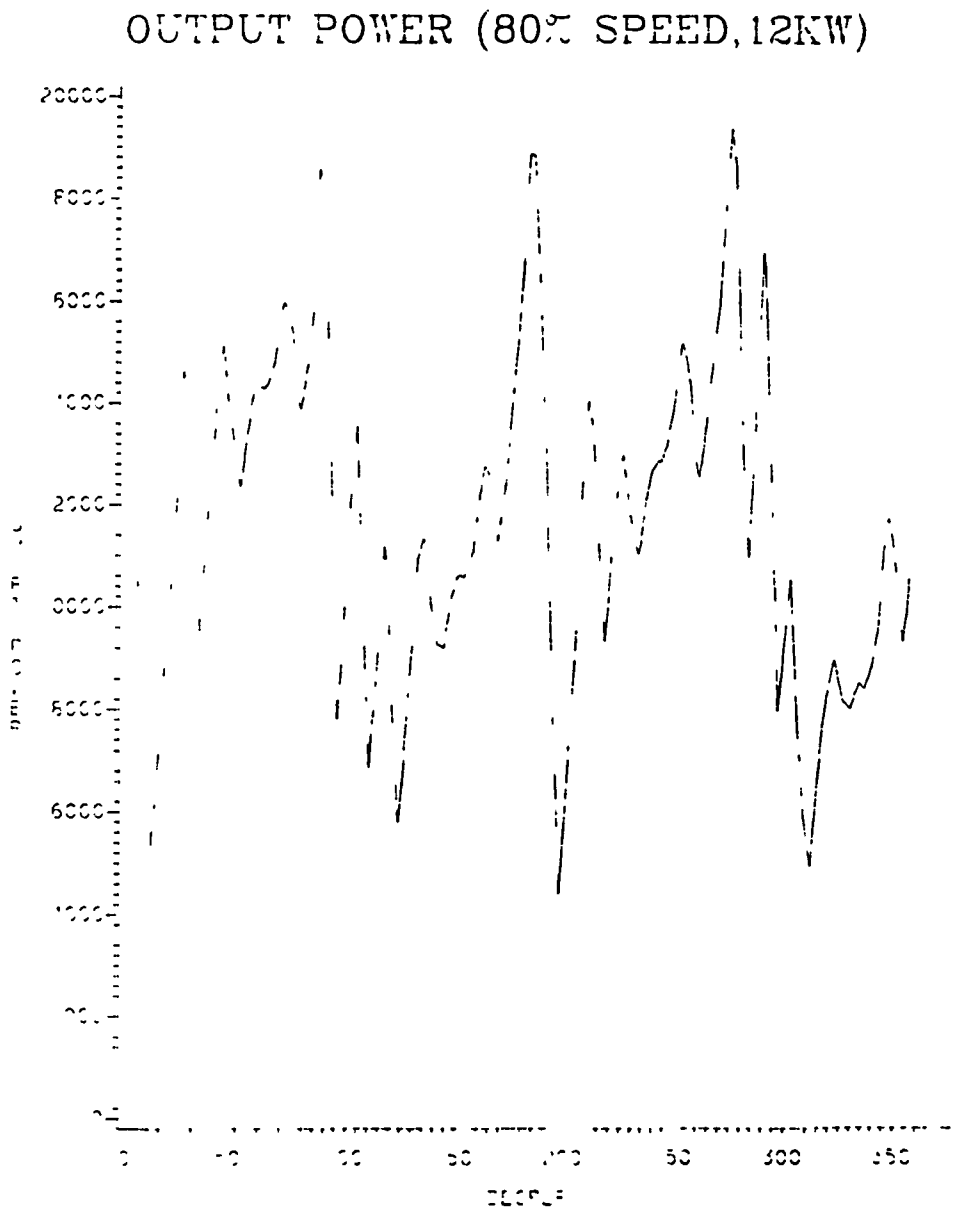
```
      PIN=Y(6)-AREAIN
      POUT=Y(5)-ARAOUT
      RMS=(Y(7)-ARMS)**0.5
C     ******************************************************************
C     NOTE .7 IS THE VOLTAGE DROP ACROSS THE DIODE
C     1.85 IS THE CE VOLTAGE OF THE TRANSISTOR
C     SWITCH IS THE # OF INTERVAL OF THE SWITCHING (DIODE CONDUCTION)
C     TOTSW IS TOTOAL # OF INTEGRATION INTERVALS
C     PDIODE IS THE LOSS OF DIODE DURING CONDUCTION
C     PTRANS IS THE ON TIME LOSS OF THE TRANSISTOR
C     PSW IS THE SWITCHING LOSS OF EACH TRANSISTOR (E*I*TSW/T)
C     TOTSL IS THE TOTAL LOSS OF THE SWITCH
C      ************************************************
      PDIODE=(SWITCH/TOTSW)*0.7*IMAX/PAI
      PTRANS=(1-SWITCH/TOTSW)*1.85*IMAX/PAI
      TOTSL=PDIODE+PTRANS+PSW
      TOT4SL=4*TOTSL
      DTRIO=1-(SWITCH/TOTSW)
C     EFFIO=POUT/(TOT4SL+PIN)
      EFFIO=POUT/PIN
      SW=SW+SW
C
      WRITE(6,702)POUT,EFFIO,SLIP,IMAX,DI,RMS,AVERI,DTRIO
      WRITE(6,703)SW,PSW,PTRANS,PDIODE,TOT4SL,IN
  702 FORMAT(///,' POWER OUT=',F6.0,4X,'EFFIO=',F5.3,2X,'SLIP=',
     1F7.5,2X,'IMAX=',F4.0,2X,'DI=',F3.0,4X,
     2'I(RMS)=',F5.1,3X,'I(AVE)=',F5.1,3X,'DTRIO=',F5.3,/////)
  703 FORMAT(/,'#SW/CYC=',F5.1,8X,'PSW=',F6.1,3X,'PTRANS=',F5.1,
     1 2X,'PDIODE=',F4.1,2X,'TOT4SL=',F6.1,5X,'IN=',I5,///////)
CC    WRITE(6,705)
C705  FORMAT(1X,'Z',11X,'I HARMONIC ',15X,'P HARMONIC',//)
C706  FORMAT(1X,F5.1,5X,F12.8,5X,F15.8)
CC    ISTOP=0.0
CC    CALL FFT(F1,8)
CC    CALL FFT(F2,8)
CC    DO 401 I=1,128
CC    MAG1(I)=CABS(F1(I))/256.
CC    MAG2(I)=CABS(F2(I))/256.
CC    Z=FLOAT(I-1)
CC    IF (I.EQ.1) MAG1(2)=200.
CC    AMP1=MAG1(I)/MAG1(2)
CC    AMP2=MAG2(I)/MAG2(1)
CC    WRITE(6,706) Z,AMP1,AMP2
CC    IF(I.EQ.128) ISTOP=1
C     IF(AMP2.GT.300) AMP2=0.0
C
CC    CALL FPLOT(800,IPNT4,AR4,LR4,ISTOP,1,1,Z,AMP1)
CC    CALL FPLOT(800,IPNT3,AR3,LR3,ISTOP,1,1,Z,AMP2)
C401  CONTINUE
      STOP
      END
       SUBROUTINE FFT(X,M)
       COMPLEX X(256),U,W,T
       N=2**M
       NV2=N/2
```

```
      NM1=N-1
      J=1
      DO 7 I=1,NM1
      IF(I.GT.J) GOTO 5
      T=X(J)
      X(J)=X(I)
      X(I)=T
   5  K=NV2
   6  IF(K.GE.J)GO TO 7
      J=J-K
      K=K/2
      GO TO 6
   7  J=J+K
      PI=3.14159265358979
      DO 20 L=1,M
      LE=2**L
      LE1=LE/2
      U=(1.0,0.0)
      Z=COS(PI/FLOAT(LE1))
      Y=-SIN(PI/FLOAT(LE1))
      W=CMPLX(Z,Y)
      DO 20 J=1,LE1
      DO 10 I=J,N,LE
      IP=I+LE1
      T=X(IP)*U
      X(IP)=X(I)-T
 10   X(I)=X(I)+T
 20   U=U*W
      RETURN
      END
$ENTRY
1       A   CURRENT OF PHASE A OF STATOR
1       P   POWER VS TIME
1       I   CURRENT HARMONIC MAGNITUDE VS HARMONIC ORDER
1       P   POWER HARMONIC MAGNITUDE VS HARMONIC ORDER
1       V   VOLTAGE OF PHASE A OF STATOR
$STOP
/*
//
```

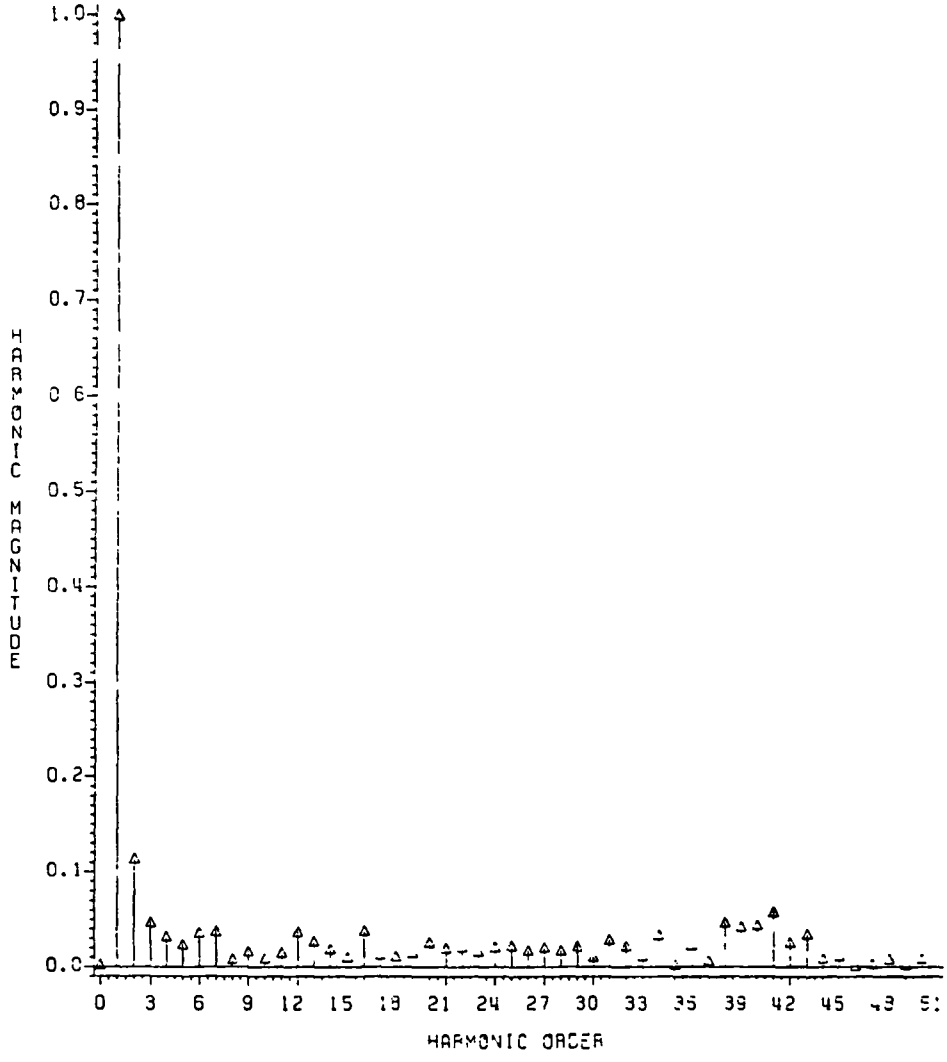

STATOR VOLTAGE (80% SPEED,12 KW)
250
200
50
00
50
0
50
- 00
-150
200
-250
-300
VOLTAGE

STATOR CURRENT (80% SPEED,12 KW)

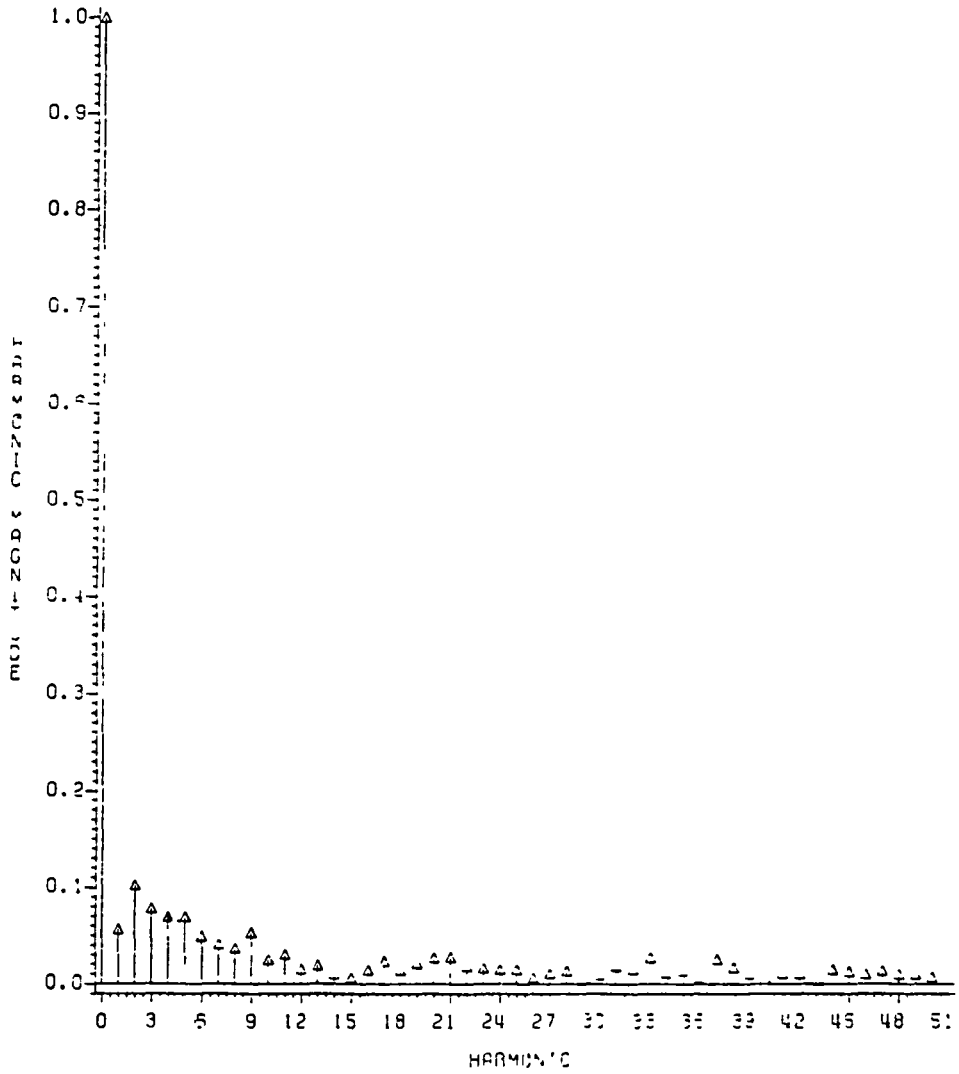



OUTPUT POWER (80% SPEED, 12KW)

CURRENT SPECTRUM (80% SPEED,12 KW)



POWER SPECTRUM (80% SPEED,12KW)



```
//PLOTHS   JOB (XXXXLH),'JPL PROJ',
// PASSWORD=
/*JOBPARM R=900,B=99000
// EXEC PLOTSAS
//SYSIN DD *
          GOPTIONS  DEVICE=CAL1077;
          DATA ONE;
          INPUT X Y1-Y3;
CARDS;
****************************************
```

DATA FOR 80% SPEED 60 KW OPERATION

DEGREE	CURRENT	VOLTAGE	POWER
1.1	-22.81	240.00	5920.99
1.6	-20.59	240.00	6231.94
2.2	-18.39	240.00	6548.00
2.8	-16.23	240.00	6868.89
3.6	-13.22	240.00	7330.49
4.7	-9.07	240.00	7997.52
6.4	-3.40	240.00	8965.61
8.7	4.21	240.00	10373.63
11.9	14.19	240.00	12412.63
16.5	26.83	240.00	15306.36
23.0	41.97	240.00	19177.87
23.4	42.81	240.00	18808.73
23.8	43.65	240.00	18330.34
24.2	44.48	240.00	17862.74
24.6	45.29	240.00	17405.93
25.2	46.42	240.00	16778.39
26.0	47.99	240.00	15927.89
27.1	50.13	240.00	14799.00
28.8	53.03	240.00	13350.31
29.2	53.73	240.00	13531.00
29.6	54.42	240.00	13812.66
30.0	55.10	240.00	14090.93
30.4	55.77	240.00	14365.63
30.9	56.71	240.00	14747.93
31.8	58.00	240.00	15275.69
32.9	59.76	240.00	15995.06
34.5	62.14	240.00	16954.89
36.8	65.29	240.00	18187.43
40.1	69.33	240.00	19653.47
40.5	69.81	240.00	19466.14
40.9	70.28	240.00	19217.89
41.3	70.74	240.00	18979.47
41.7	71.20	240.00	18750.66
42.3	71.83	240.00	18443.44
43.1	72.71	240.00	18042.04
44.2	73.91	240.00	17539.42
45.8	75.54	240.00	16957.45
48.1	77.71	240.00	16387.70

48.5	78.08	240.00	16533.59
48.9	78.45	240.00	16717.02
49.3	78.81	240.00	16893.27
49.8	79.17	240.00	17062.34
50.3	79.67	240.00	17288.99
51.1	80.36	240.00	17584.35
52.3	81.33	240.00	17950.66
53.9	82.65	240.00	18363.02
56.2	84.45	240.00	18727.66
56.6	84.76	240.00	18682.68
57.0	85.07	240.00	18627.90
57.4	85.38	240.00	18581.06
57.8	85.69	240.00	18542.06
58.4	86.12	240.00	18500.16
59.2	86.73	240.00	18467.24
60.3	87.58	240.00	18472.39
62.0	88.78	240.00	18580.52
64.3	90.48	240.00	18928.00
64.7	90.79	240.00	18958.41
65.1	91.09	240.00	18969 84
65.5	91.39	240.00	18971.66
65.9	91.70	240.00	18963.81
66.5	92.13	240.00	18936.08
67.3	92.75	240.00	18863.47
68.4	93.63	240.00	18693.46
70.0	94.91	240.00	18316.52
70.4	95.23	240.00	18347.82
70.8	95.56	240.00	18413.83
71.2	95.89	240.00	18485.64
71.7	96.22	240.00	18563.12
72.2	96.69	240.00	18682.33
73.0	97.38	240.00	18869.82
74.2	98.36	240.00	19171.70
75.8	99.80	240.00	19668.84
76.4	93.83	-240.00	18545.96
76.9	86.57	-240.00	17178.11
77.5	79.32	-240.00	15829.75
78.1	72.09	-240.00	14501.10
78.9	61.89	-240.00	12656.22
80.1	47.51	-240.00	10116.46
81.7	27.30	-240.00	6666.26
82.2	26.69	240.00	6761.41
82.8	27.39	240.00	7121.00
83.4	28.10	240.00	7488.83
84.0	28.83	240.00	7864.62
84.8	29.87	240.00	8409.14
85.2	30.41	240.00	8298.47
85.6	30.95	240.00	8100.25
86.0	31.49	240.00	7890.95
86.4	32.04	240.00	7670.58
87.0	32.84	240.00	7339.98
87.8	33.98	240.00	6834.56
88.9	35.65	240.00	6043.88
90.5	38.12	240.00	4773.80
91.0	38.76	240.00	4903.55

91.4	39.40	240.00	5129.91
91.8	40.05	240.00	5358.91
92.2	40.71	240.00	5590.45
92.7	41.56	240.00	5922.09
93.6	43.03	240.00	6398.95
94.7	45.03	240.00	7087.78
96.3	47.97	240.00	8087.09
98.6	52.39	240.00	9539.93
101.9	59.18	240.00	11643.59
106.4	69.95	240.00	14630.51
112.9	87.72	240.00	18636.06
113.5	82.94	-240.00	18127.67
114.1	76.89	-240.00	17470.94
114.7	70.88	-240.00	16835.94
115.2	64.89	-240.00	16222.70
116.0	56.49	-240.00	15392.52
117.2	44.71	-240.00	14292.68
118.8	28.28	-240.00	12885.57
119.4	29.04	240.00	13154.08
120.0	31.12	240.00	13562.98
120.5	33.23	240.00	13964.82
121.1	35.37	240.00	14359.29
121.9	38.44	240.00	14904.15
123.1	42.88	240.00	15647.38
124.7	49.35	240.00	16640.20
127.0	58.90	240.00	17917.97
130.2	73.20	240.00	19445.14
130.8	69.33	-240.00	19197.31
131.4	64.19	-240.00	18870.05
131.9	59.09	-240.00	18562.40
132.5	54.02	-240.00	18273.94
133.3	46.92	-240.00	17898.85
134.5	37.00	-240.00	17433.36
136.1	23.22	-240.00	16903.72
136.7	24.93	240.00	17099.50
137.2	27.96	240.00	17353.89
137.8	31.02	240.00	17594.44
138.4	34.11	240.00	17321.02
139.2	38.54	240.00	18117.16
140.3	44.90	240.00	18486.34
142.0	54.12	240.00	18906.48
144.3	67.58	240.00	19289.04
144.8	64.52	-240.00	19189.06
145.4	60.20	-240.00	19071.40
146.0	55.92	-240.00	18969.96
146.5	51.68	-240.00	18884.70
147.4	45.74	-240.00	18791.24
148.5	37.46	-240.00	18712.46
150.1	26.02	-240.00	18705.63
152.4	10.35	-240.00	18893.12
155.7	-10.80	-240.00	19553.66
156.2	-7.91	240.00	19597.46
156.8	-3.69	240.00	19600.96
157.4	0.55	240.00	19584.65
158.0	4.83	240.00	19548.38

G.5 TWO-PHASE SIMULATION RESULTS FOR POWER REGENERATION AT 80% SPEED

Plots

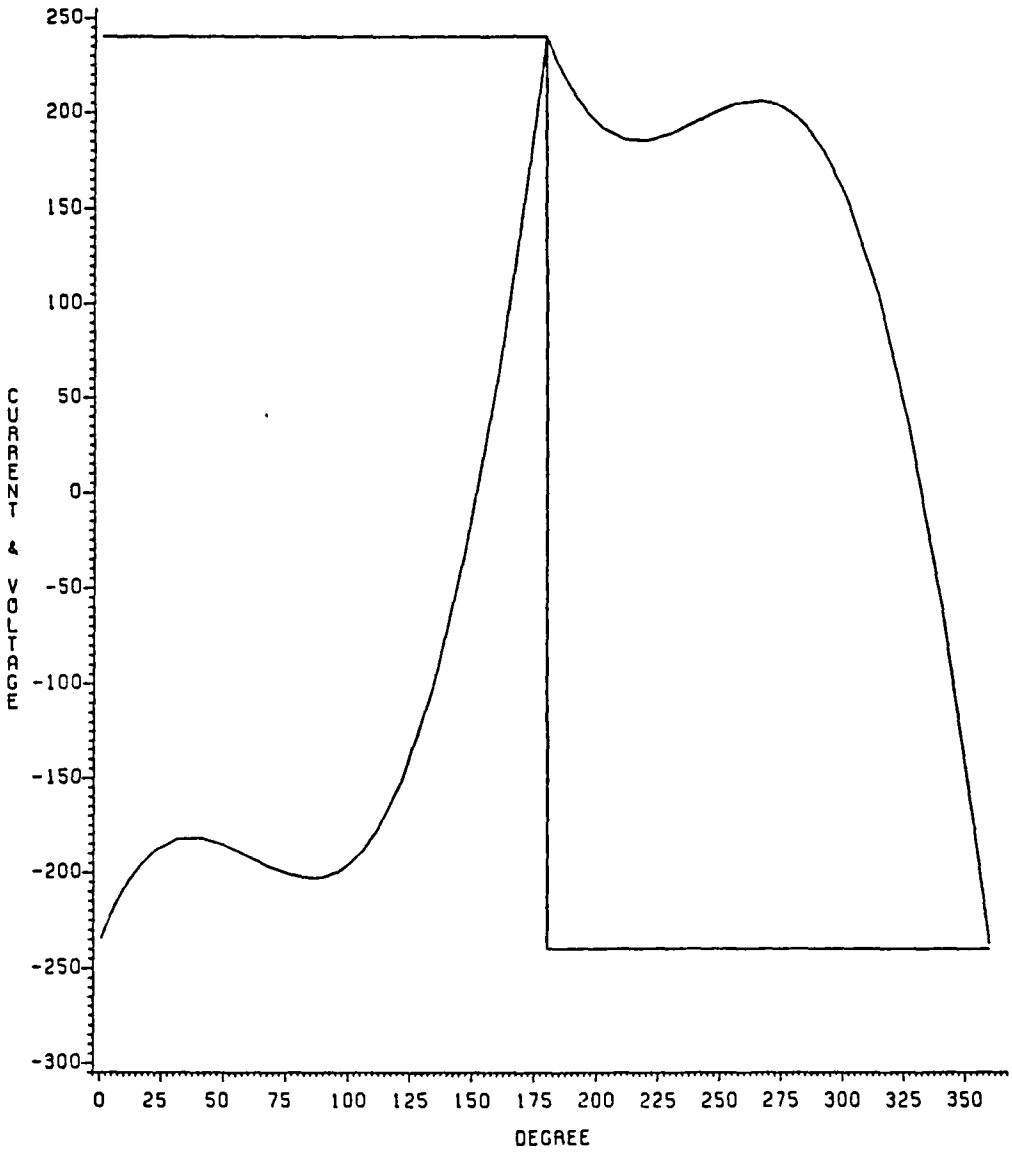
Stator Voltage and Current

Output Power

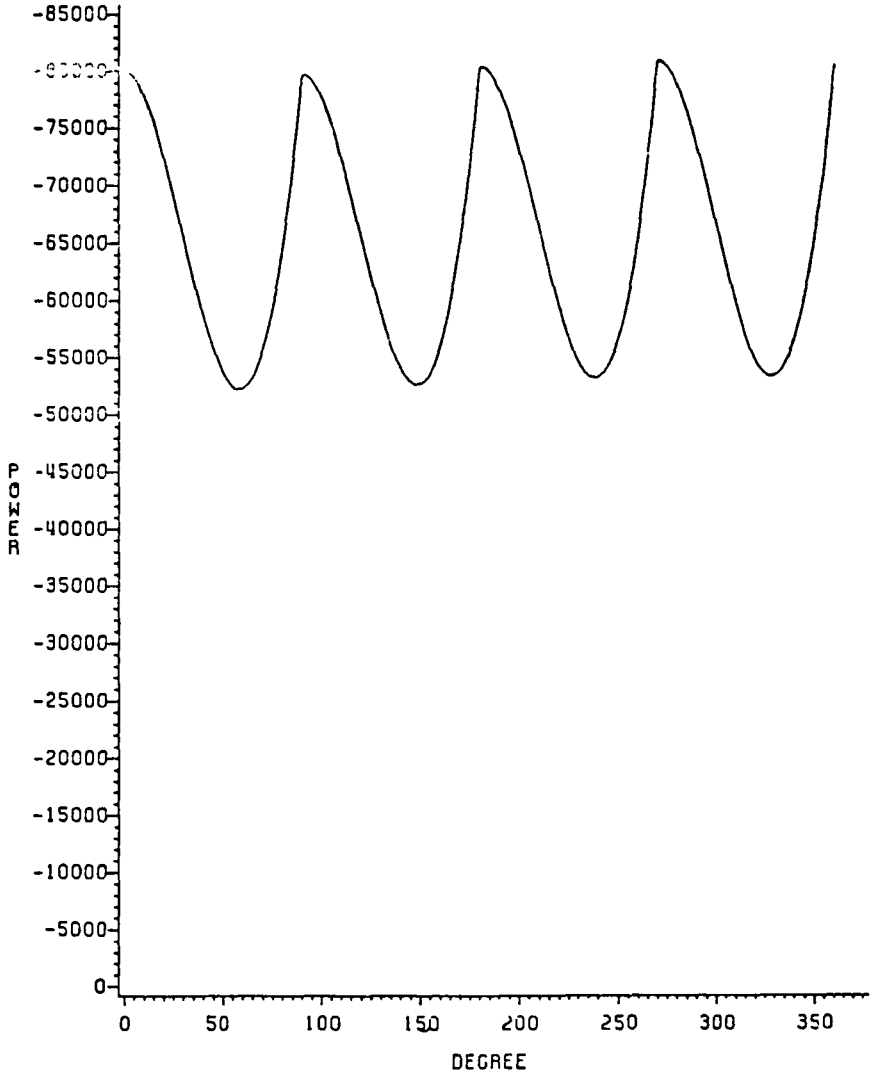
Listing of Plot Data

Comment: Source Program is the same as for G.3 except the slip frequency is negative.

VOLTAGE & CURRENT (80% SPEED,REGENERATIVE)



POWER (80% SPEED ,REGENERATION)



FILE: REG PLOT A UNIVERSITY OF MISSOURI COMPUTER NETWORK PAGE 001

```
//REGP     JOB (XXXXLH),'GOODARZI',
// PASSWORD=XXXX
/*JOBPARM R=900,B=99000
// EXEC PLOTSAS
//SYSIN DD *
          GOPTIONS  DEVICE=VARIAN;
          DATA ONE;
          INPUT X Y1-Y3;
*********************************************************************
*
*                REGENERATION MODE @ 80% SPEED  JPL PROJECT
*                PLOT DATA
*        |         |          |
* DEGREE |VOLTAGE  | CURRENT  |POWER
*********|********|**********|****************************************
CARDS;
   0.5  -234.49     240.00  -79927.88
   1.1  -232.59     240.00  -79913.88
   1.6  -230.73     240.00  -79883.56
   2.2  -228.91     240.00  -79837.13
   3.0  -226.41     240.00  -79744.81
   4.2  -223.00     240.00  -79562.31
   5.8  -218.45     240.00  -79205.38
   8.1  -212.52     240.00  -78515.63
  11.3  -205.15     240.00  -77208.75
  16.0  -196.63     240.00  -74807.44
  22.5  -188.09     240.00  -70622.75
  31.7  -182.09     240.00  -64028.68
  40.9  -181.74     240 00  -57917.55
  50.1  -185.33     240.00  -53617.25
  59.3  -191.03     240.00  -52301.88
  68.5  -197.03     240.00  -54937.41
  77.7  -201.52     240.00  -62237.88
  86.9  -202.77     240.00  -74629.75
  89.2  -202.38     240.00  -78543.88
  89.8  -202.24     240.00  -79573.25
  90.2  -202.12     240.00  -79703.56
  90.6  -201.99     240.00  -79704.81
  91.0  -201.85     240.00  -79698.00
  91.4  -201.71     240.00  -79682.94
  92.0  -201.48     240.00  -79647.94
  92.8  -201.11     240.00  -79571.56
  94.0  -200.52     240.00  -79411.38
  95.6  -199.52     240.00  -79085.44
  97.9  -197.77     240.00  -78438.06
 101.2  -194.58     240.00  -77188.06
 105.8  -188.51     240.00  -74862.19
 112.3  -176.55     240.00  -70770.81
 121.5  -152.17     240.00  -64283.78
 134.5  -101.16     240.00  -56195.74
 147.5   -29.33     240.00  -52653.23
 160.6    63.87     240.00  -56617.87
 173.6   177.76     240.00  -70080.00
 178.2   222.68     240.00  -77304.44
 179.8   239.11     240.00  -80170.69
```

```
 180.4   238.47    -240.00  -80355.13
 181.0   236.56    -240.00  -80352.50
 181.5   234.69    -240.00  -80333.38
 182.1   232 87    -240.00  -80298.25
 182.9   230.35    -240.00  -80221.63
 184.1   226.93    -240.00  -80061.06
 185.7   222.35    -240.00  -79734.38
 188.0   216.40    -240.00  -79086.13
 191.3   208.99    -240.00  -77834.75
 195.9   200.42    -240.00  -75505.56
 202.4   191.81    -240.00  -71410.00
 211.6   185.73    -240.00  -64915.04
 220.8   185.32    -240.00  -58869.37
 230.0   188.85    -240.00  -54598.85
 239.2   194.52    -240.00  -53279.17
 248.4   200.51    -240.00  -55878.30
 257.6   205.00    -240.00  -63112.98
 266.9   206.26    -240.00  -75411.81
 269.2   205.88    -240.00  -79299.13
 270.0   205.67    -240.00  -80751.25
 270.4   205.55    -240.00  -80879.50
 270.8   205.42    -240.00  -80878.31
 271.2   205.27    -240.00  -80868.75
 271.6   205.12    -240.00  -80851.06
 272.2   204.88    -240.00  -80812.31
 273.0   204.51    -240.00  -80730.44
 274.1   203.90    -240.00  -80562.56
 275.8   202.88    -240.00  -80225.38
 278.1   201.10    -240.00  -79561.69
 281.3   197.86    -240.00  -78288.13
 285.9   191.72    -240.00  -75926.63
 292.4   179.65    -240.00  -71782.19
 301.6   155.10    -240.00  -65216.10
 314.7   103.83    -240.00  -57010.73
 327.7    31.73    -240.00  -53344.49
 340.7   -61.74    -240.00  -57181.73
 353.7  -175.89    -240.00  -70511.56
 358.3  -220.89    -240.00  -77688.31
 360.0  -237.35    -240.00  -80537.50
;
          PROC GPLOT;
          PLOT Y1*X Y2*X /OVERLAY;
          SYMBOL1 I=JOIN L=1;
          SYMBOL2 I=JOIN L=13;
          LABEL X=DEGREE
                Y1=CURRENT & VOLTAGE  ;
         TITLE VOLTAGE & CURRENT (80% SPEED,REGENERATIVE);

          PROC GPLOT;
          PLOT Y3*X /VAXIS=0 TO -85000 BY -5000;
          SYMBOL1 I=SPLINE;
          LABEL X=DEGREE
                Y3= POWER ;
          TITLE REGENERATIVE POWER (80% SPEED );
```

G.6 THREE-PHASE SIMULATION RESULTS

Source Program

Plots

Stator Voltage and Current

Output Power

Listing of Plot Data

```
FILE: JPL330   WATFIV   A   UNIVERSITY OF MISSOURI COMPUTER NETWORK                    PAGE 001

//B6330 JOB (XXXXEE),'ABBAS  GOODARZI',CLASS=A,MSGLEVEL=(0,0),          JPL00010
// PASSWORD=XXXX                                                        JPL00020
/*JOBPARM T=2 ,L=2                                                     JPL00030
/*ROUTE PRINT UMVMA.EEPR3                                              JPL00040
//     EXEC     WATFIV                                                  JPL00050
//GO.SYSIN DD *                                                        JPL00060
$JOB                                                                   JPL00070
      DIMENSION AR1(800),AR2(800),AR3(800),Y(20),X(20),GA(20),PE(20),  JPL00080
     1XX(4,20),H(9),ZZ1(20),ZZ2(20),ZZ3(20),IFL(7),AR4(800)            JPL00090
      LOGICAL*1 LR1(800),LR2(800),LR3(800),LR4(800)                    JPL00100
      REAL L(6,6),LR(6,6),D(6,6),LI(6,6), WKAREA(54),LOS,LOR,LM,THETA,C,JPL00110
     1IAS,IBS,ICS,IAR,IBR,ICR,IER,RS,RR,DD,EE,FF,TQ,AA,BB,CC,E,OMEGR,   JPL00120
     2DL(6,6),IAREF,IBREF,ICREF,IMAX                                   JPL00130
C/ PASSWORD=XXXX,MSGCLASS=S                                            JPL00140
C*JOBPARM T=3 ,L=3                                                     JPL00150
C*ROUTE PRINT UMMVSA.RO                                                JPL00160
C  50 FORMAT(/,2X,'TIME',8X,'VA',8X,'IAS',8X,'IBS',8X,'ICS',8X,        JPL00170
C     1     'IAR',8X,'IBR',8X,'ICR',5X,'POWER',7X,'ANGLE')             JPL00180
      IMAX=100.                                                        JPL00190
      SLIP=0.0125                                                      JPL00200
      DI=10.                                                           JPL00210
      E=240.                                                           JPL00220
      NRUN=1                                                           JPL00230
      ISTOP=0                                                          JPL00240
      KEEP=1                                                           JPL00250
      IFL(1)=-1                                                        JPL00260
      NINT=9                                                           JPL00270
      IPNT1=0                                                          JPL00280
      IPNT2=0                                                          JPL00290
      IPNT3=0                                                          JPL00300
      IPNT4=0                                                          JPL00310
C***********************************************************           JPL00320
C                 MOTOR PARAMETERS                                     JPL00330
C      FOR BRIDGE INVERTER (JPL)  DATA    SIMPLIFIED MODEL             JPL00340
C***********************************************************           JPL00350
      LOS=75.360E-06                                                   JPL00360
      LOR=150.E-06                                                     JPL00370
      LM=4200.E-06                                                     JPL00380
      RS=0.007345                                                      JPL00390
      RR=0.004813                                                      JPL00400
      PAI=3.14159                                                      JPL00410
      PI2=2.*PAI/3.                                                    JPL00420
      THETA=0.                                                         JPL00430
      FREQ=330.                                                        JPL00440
      OMEGA=2*PAI*FREQ                                                 JPL00450
      OMEGR=OMEGA*(1.-SLIP)                                            JPL00460
      WRITE(6,64)                                                      JPL00470
      WRITE(6,66)LOS,LOR                                               JPL00480
      WRITE(6,69)RS,RR                                                 JPL00490
      WRITE(6,68)LM                                                    JPL00500
      WRITE(6,71)SLIP                                                  JPL00510
      WRITE(6,72)FREQ                                                  JPL00520
      WRITE(6,64)                                                      JPL00530
C     WRITE(6,50)                                                      JPL00540
64    FORMAT(' ',///,20X,'******MOTOR PARAMETERS******',///)           JPL00550
```

```
66    FORMAT(' ',20X,'LOS=',F9.7,5X,'LOR=',F9.7)                          JPL00560
68    FORMAT(' ',20X,'LM=',F9.7)                                          JPL00570
69    FORMAT(' ',20X,'RS=',F8.6,5X,'RR=',F8.6)                            JPL00580
71    FORMAT(' ',20X,'SLIP=',F6.4)                                        JPL00590
72    FORMAT(' ',20X,'FREQUENCY=',F6.1)                                   JPL00600
      WS=LM+LOS                                                           JPL00610
      WR=LM+LOR                                                           JPL00620
      T=SQRT(3.)/2.*OMEGR*LM                                              JPL00630
      Z=-0.5*LM                                                           JPL00640
      U=SQRT(3.)/3.*OMEGR*(3./2.*LM+LOR)                                  JPL00650
      DO 100 I=1,6                                                        JPL00660
      DO 100 J=1,6                                                        JPL00670
      L(I,J)=Z                                                            JPL00680
      LR(I,J)=0.                                                          JPL00690
      D(I,J)=0.                                                           JPL00700
  100 CONTINUE                                                            JPL00710
      DO 200 I=1,3                                                        JPL00720
      L(I,I)=WS                                                           JPL00730
      LR(I,I)=RS                                                          JPL00740
  200 CONTINUE                                                            JPL00750
      DO 201 I=4,6                                                        JPL00760
      L(I,I)=WR                                                           JPL00770
      LR(I,I)=RR                                                          JPL00780
  201 CONTINUE                                                            JPL00790
      L(1,4)=LM                                                           JPL00800
      L(2,5)=LM                                                           JPL00810
      L(3,6)=LM                                                           JPL00820
      L(4,1)=LM                                                           JPL00830
      L(5,2)=LM                                                           JPL00840
      L(6,3)=LM                                                           JPL00850
      LR(4,2)=T                                                           JPL00860
      LR(4,3)=-T                                                          JPL00870
      LR(4,5)=U                                                           JPL00880
      LR(4,6)=-U                                                          JPL00890
      LR(5,1)=-T                                                          JPL00900
      LR(5,3)=T                                                           JPL00910
      LR(5,4)=-U                                                          JPL00920
      LR(5,6)=U                                                           JPL00930
      LR(6,1)=T                                                           JPL00940
      LR(6,2)=-T                                                          JPL00950
      LR(6,4)=U                                                           JPL00960
      LR(6,5)=-U                                                          JPL00970
      DO 400 I=1,6                                                        JPL00980
      DO 400 J=1,6                                                        JPL00990
  400 LI(I,J)=L(I,J)                                                      JPL01000
      CALL INVERS (LI,6)                                                  JPL01010
      CALL MATMUL (LI,LR,6,6,6,D)                                         JPL01020
    1 IFL(2)=-1                                                           JPL01030
    2 CALL RKFOUR(NINT,Y,X,GA,PE,XX,H,ZZ1,ZZ2,ZZ3,TIME,PNTS,KEEP,IFL)     JPL01040
      IF(IFL(4))3,4,5                                                     JPL01050
    3 INIT=1                                                              JPL01060
      TIME1=-1                                                            JPL01070
      NC=1                                                                JPL01080
      NCC=3                                                               JPL01090
      TIME=0.                                                             JPL01100
```

```
      FINTIM=0.060505                                                    JPL01110
      Y(1)=0.                                                            JPL01120
      Y(2)=0.                                                            JPL01130
      Y(3)=0.                                                            JPL01140
      Y(4)=0.                                                            JPL01150
      Y(5)=0.                                                            JPL01160
      Y(6)=0.                                                            JPL01170
      Y(7)=0.0                                                           JPL01180
      Y(8)=0 0                                                           JPL01190
      Y(9)=0.0                                                           JPL01200
      H(1)=0.1E-4                                                        JPL01210
      E2=E/2.                                                            JPL01220
      IAE=-1                                                             JPL01230
      IBE=-1                                                             JPL01240
      ICE=-1                                                             JPL01250
      UPIN=0.0                                                           JPL01260
      UPOUT=0.0                                                          JPL01270
      UPLOSS=0.0                                                         JPL01280
      POWER=0.0                                                          JPL01290
C     SUM=0.0                                                            JPL01300
C     PIN=0.0                                                            JPL01310
C     RMS1=0.0                                                           JPL01320
C     RMS2=0.0                                                           JPL01330
C     COUNT=0.0                                                          JPL01340
       SW=0.0                                                            JPL01350
       PSW=0.0                                                           JPL01360
       PERIOD=1./FREQ                                                    JPL01370
       TF=FINTIM-PERIOD                                                  JPL01380
    4 IF(TIME-TIME1)7,8,7                                                JPL01390
    7 TIME1=TIME                                                         JPL01400
C*********************************************************************** JPL01410
C*********************************************************************** JPL01420
C                                                                        JPL01430
C                                                                        JPL01440
C      REFFERENCE SIGNAL FOR GENERATING INPUT VOLTAGE                    JPL01450
C      AND THE CURENT CONTROLER REFRENCE SIGNAL                          JPL01460
C                                                                        JPL01470
C*********************************************************************** JPL01480
C*********************************************************************** JPL01490
      DEG=FREQ*TIME                                                      JPL01500
      IDEG=DEG                                                           JPL01510
      DEG=(DEG-IDEG)*360.                                                JPL01520
C                                                                        JPL01530
      VA=160.*SIN(OMEGA*TIME)                                            JPL01580
      VB=160.*SIN(OMEGA*TIME-PI2)                                        JPL01590
      VC=160.*SIN(OMEGA*TIME+PI2)                                        JPL01600
      IF(TIME.LT.2.8890909) GOTO 88                                      JPL01650
      VA0=E2                                                             JPL01540
      VB0=E2                                                             JPL01550
      VC0=E2                                                             JPL01560
C                                                                        JPL01570
      IAREF=IMAX*SIN(OMEGA*TIME)                                         JPL01580
      IBREF=IMAX*SIN(OMEGA*TIME-PI2)                                     JPL01590
      ICREF=IMAX*SIN(OMEGA*TIME+PI2)                                     JPL01600
C                                                                        JPL01610
```

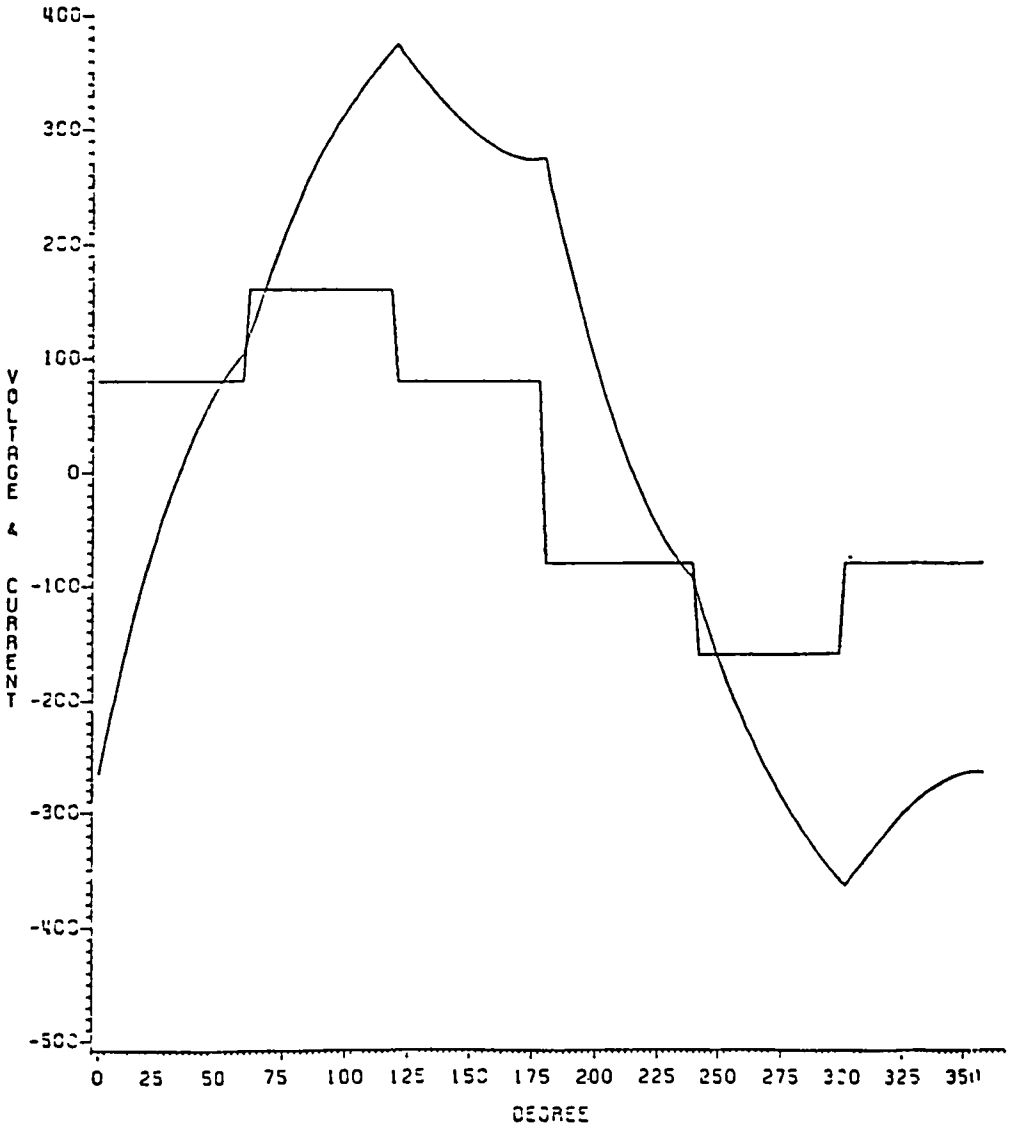
```
      IF(IAREF.LT.0.0)  VA0=-E2                                          JPL01620
      IF(IBREF.LT.0.0)  VB0=-E2                                          JPL01630
      IF(ICREF.LT.0.0)  VC0=-E2                                          JPL01640
      IF(TIME.LT.2.8890909) GOTO 87                                      JPL01650
C                                                                        JPL01660
12    DIA=Y(1)-IAREF                                                     JPL01670
      DIB=Y(2)-IBREF                                                     JPL01680
      DIC=Y(3)-ICREF                                                     JPL01690
C                                                                        JPL01700
      IF(DIA.GT.DI)     IAE=1                                            JPL01710
      IF(TIME.LT.TF.AND.DIA.GT.DI) GOTO 13                               JPL01720
      SW=SW+1.                                                           JPL01730
      PSW=ABS((240.*Y(1)*5.5E-06)/(6*PERIOD))+PSW                        JPL01740
 13   IF(DIA.LT.(-DI))  IAE=0                                            JPL01750
      IF(VA0.GT.0.0.AND.IAE.EQ.1)  VA0=-VA0                              JPL01760
      IF(VA0.LT.0.0.AND.IAE.EQ.0)  VA0=-VA0                              JPL01770
C                                                                        JPL01780
      IF(DIB.GT.DI)    IBE=1                                             JPL01790
      IF(DIB.LT.(-DI)) IBE=0                                             JPL01800
      IF(VB0.GT.0.0.AND.IBE.EQ.1)  VB0=-VB0                              JPL01810
      IF(VB0.LT.0.0.AND IBE.EQ.0)  VB0=-VB0                              JPL01820
C                                                                        JPL01830
      IF(DIC.GT.DI)    ICE=1                                             JPL01840
      IF(DIC.LT.(-DI)) ICE=0                                             JPL01850
      IF(VC0.GT.0.0.AND.ICE.EQ.1)  VC0=-VC0                              JPL01860
      IF(VC0.LT.0.0.AND.ICE.EQ.0)  VC0=-VC0                              JPL01870
C                                                                        JPL01880
87    VN=(VA0+VB0+VC0)/3.                                                JPL01890
      VA=VA0-VN                                                          JPL01900
      VB=VB0-VN                                                          JPL01910
      VC=VC0-VN                                                          JPL01920
88     CONTINUE                                                          JPL01930
      IF(TIME.GT.0.03318.AND.TIME.LT.0.03323)   UPIN=Y(7)                JPL01940
      IF(TIME.GT.0.03318.AND.TIME.LT.0 03323)   UPOUT=Y(8)               JPL01950
      IF(TIME.GT.0.03318.AND.TIME.LT.0.03323)   UPLOSS=Y(9)              JPL01960
C                                                                        JPL01970
      THETA=OMEGR*TIME                                                   JPL01980
      DD=SIN(THETA)                                                      JPL01990
      EE=SIN(THETA+2.*PAI/3.)                                            JPL02000
      FF=SIN(THETA-2.*PAI/3.)                                            JPL02010
C                                                                        JPL02020
      IAR=2./3.*((COS(THETA)+0.5)*Y(4)+(COS(THETA-2.*PAI/3.)+0.5)*Y(5)   JPL02030
     . 1+(COS(THETA+2.*PAI/3.)+0.5)*Y(6))                                JPL02040
C                                                                        JPL02050
    8 CONTINUE                                                           JPL02060
C                                                                        JPL02070
      X(1)=-D(1,1)*Y(1)-D(1,2)*Y(2)-D(1,3)*Y(3)-D(1,4)*Y(4)-D(1,5)*Y(5)  JPL02080
     1      -D(1,6)*Y(6)+LI(1,1)*VA+LI(1,2)*VB+LI(1,3)*VC                JPL02090
      X(2)=-D(2,1)*Y(1)-D(2,2)*Y(2)-D(2,3)*Y(3)-D(2,4)*Y(4)-D(2,5)*Y(5)  JPL02100
     1      -D(2,6)*Y(6)+LI(2,1)*VA+LI(2,2)*VB+LI(2,3)*VC                JPL02110
      X(3)=-D(3,1)*Y(1)-D(3,2)*Y(2)-D(3,3)*Y(3)-D(3,4)*Y(4)-D(3,5)*Y(5)  JPL02120
     1      -D(3,6)*Y(6)+LI(3,1)*VA+LI(3,2)*VB+LI(3,3)*VC                JPL02130
      X(4)=-D(4,1)*Y(1)-D(4,2)*Y(2)-D(4,3)*Y(3)-D(4,4)*Y(4)-D(4,5)*Y(5)  JPL02140
     1      -D(4,6)*Y(6)+LI(4,1)*VA+LI(4,2)*VB+LI(4,3)*VC                JPL02150
      X(5)=-D(5,1)*Y(1)-D(5,2)*Y(2)-D(5,3)*Y(3)-D(5,4)*Y(4)-D(5,5)*Y(5)  JPL02160
```

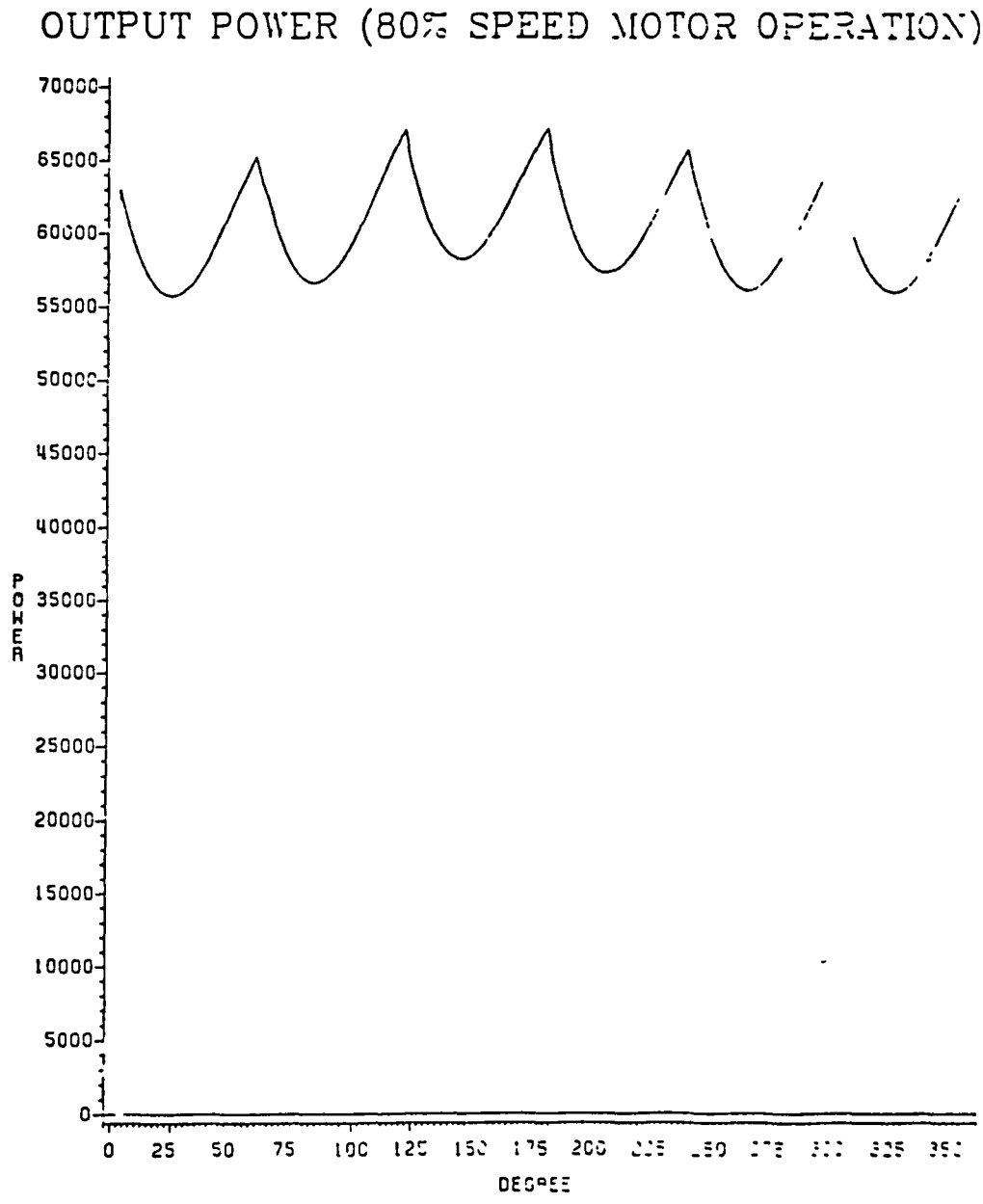
```
     1     -D(5,6)*Y(6)+LI(5,1)*VA+LI(5,2)*VB+LI(5,3)*VC                  JPL02170
      X(6)=-D(6,1)*Y(1)-D(6,2)*Y(2)-D(6,3)*Y(3)-D(6,4)*Y(4)-D(6,5)*Y(5) JPL02180
     1     -D(6,6)*Y(6)+LI(6,1)*VA+LI(6,2)*VB+LI(6,3)*VC                  JPL02190
      X(7)=(VA*Y(1)+VB*Y(2)+VC*Y(3))/.02632                              JPL02200
      X(8)=POWER/.02632                                                  JPL02210
      X(9)=3.*(Y(1)*Y(1)*RS+IAR*IAR*RR)/.02632                           JPL02220
C                                                                        JPL02230
      GOTO 2                                                             JPL02240
    5 IPR=0                                                              JPL02250
      NC=NC-1                                                            JPL02260
      IF(NC)9,9,10                                                       JPL02270
    9 NC=NCC                                                             JPL02280
      IPR=1                                                              JPL02290
   10 IF(TIME.GE.FINTIM)ISTOP=1                                          JPL02300
      IF((IPR+ISTOP).EQ.0) GOTO 11                                       JPL02310
C                                                                        JPL02320
C                                                                        JPL02330
      IAR=2./3.*((COS(THETA)+0.5)*Y(4)+(COS(THETA-2.*PAI/3.)+0.5)*Y(5)   JPL02340
     1+(COS(THETA+2.*PAI/3.)+0.5)*Y(6))                                  JPL02350
      IBR=2./3.*((COS(THETA+2.*PAI/3.)+0.5)*Y(4)+(COS(THETA)+0.5)*Y(5)   JPL02360
     1+(COS(THETA-2.*PAI/3.)+0.5)*Y(6))                                  JPL02370
      ICR=2./3.*((COS(THETA-2.*PAI/3.)+0.5)*Y(4)+(COS(THETA+2.*PAI/3.)   JPL02380
     1+0.5)*Y(5)+(COS(THETA)+0.5)*Y(6))                                  JPL02390
      TQ=(DD*Y(1)*IAR+FF*Y(2)*IAR+EE*Y(3)*IAR+EE*Y(1)*IBR                JPL02400
     1   +DD*Y(2)*IBR+FF*Y(3)*IBR+FF*Y(1)*ICR+EE*Y(2)*ICR                JPL02410
     1   +DD*Y(3)*ICR)*(-LM)                                             JPL02420
      POWER=TQ*OMEGR                                                     JPL02430
      IF(TIME.LT.0.05756) GOTO 11                                        JPL02440
      IAS=Y(1)                                                           JPL02450
      IBS=Y(2)                                                           JPL02460
      ICS=Y(3)                                                           JPL02470
      IAR=Y(4)                                                           JPL02480
      IBR=Y(5)                                                           JPL02490
      ICR=Y(6)                                                           JPL02500
      WRITE(6,250)TIME,VA,Y(1),Y(2),Y(3),Y(4),Y(5),Y(6),POWER,DEG        JPL02510
C     PIN=(VA*IAS+VB*IBS+VC*ICS)+PIN                                     JPL02520
C     SUM=SUM+POWER                                                      JPL02530
C     RMS1=RMS1+(IAS*IAS)                                                JPL02540
C     RMS2=RMS2+(IAR*IAR)                                                JPL02550
C     COUNT=COUNT+1.                                                     JPL02560
      IF( TIME.LT.0.057597) POWER=0.0                                    JPL02570
      CALL FPLOT(800,IPNT1,AR1,LR1,ISTOP,1,2,DEG,IAS)                    JPL02580
      CALL FPLOT(800,IPNT1,AR1,LR1,ISTOP,2,2,DEG,VA)                     JPL02590
C     CALL FPLOT(800,IPNT2,AR2,LR2,ISTOP,1,2,DEG,IBS)                    JPL02600
C     CALL FPLOT(800,IPNT2,AR2,LR2,ISTOP,2,2,DEG,VB)                     JPL02610
C     CALL FPLOT(800,IPNT3,AR3,LR3,ISTOP,1,2,DEG,ICS)                    JPL02620
C     CALL FPLOT(800,IPNT3,AR3,LR3,ISTOP,2,2,DEG,VC)                     JPL02630
      CALL FPLOT(800,IPNT4,AR4,LR4,ISTOP,1,1,DEG,POWER)                  JPL02640
   11 INIT=0                                                             JPL02650
      IF(ISTOP)1,1,6                                                     JPL02660
    6 CONTINUE                                                           JPL02670
  250 FORMAT(F8.6,8(F8.0,3X),F5.1)                                       JPL02680
C  12 FORMAT(' ',A20,/,6(F12.6,3X))                                      JPL02690
      POWERI=Y(7)-UPIN                                                   JPL02700
      POWERO=Y(8)-UPOUT                                                  JPL02710
```

```
      PLOSS=Y(9)-UPLOSS                                                 JPL02720
      EFFIO=POWERO/POWERI                                               JPL02730
C     AVER=SUM/COUNT                                                    JPL02740
C     PIN=PIN/COUNT                                                     JPL02750
C     RMS1=(RMS1/COUNT)**.5                                             JPL02760
C     RMS2=(RMS2/COUNT)**.5                                             JPL02770
C     PLOST=(RMS1*RMS1)*RS+(RMS2*RMS2)*RR                               JPL02780
C     EFF=AVER/PIN                                                      JPL02790
      EFFL=(POWERI-PLOSS)/POWERI                                        JPL02800
      EFFPSW=(POWERI-PLOSS)/(POWERI+PSW*3)                              JPL02810
      WRITE(6,266)POWERI,POWERO,EFFIO,SLIP,IMAX,EFFL                    JPL02820
 266  FORMAT(' ',///,'*POWER IN=',F8.1,4X,'POWER OUT=',F8.1,4X,         JPL02830
     1  'EFF(IO)=',F5.3,4X,'SLIP=',F7.5,4X,'IMAX='                      JPL02840
     1,F5.1,3X,F5.3,'1')                                               JPL02850
      WRITE(6,267)SW,PSW,EFFPSW                                         JPL02860
 267  FORMAT(' ','SW=',F6.1,'PSW/LEG=',F8.1,'EFFPSW=',F8.4,////)        JPL02870
C                                                                       JPL02880
      STOP                                                              JPL02890
      END                                                               JPL02900
C                                                                       JPL02910
      SUBROUTINE INVERS(ZZBUS,N)                                        JPL02920
      DOUBLE PRECISION ZBUS(6,6)                                        JPL02930
      DIMENSION ZZBUS(6,6)                                              JPL02940
      DO 1001 I=1,N                                                     JPL02950
      DO 1001 J=1,N                                                     JPL02960
      ZBUS(I,J)=ZZBUS(I,J)                                              JPL02970
1001  CONTINUE                                                          JPL02980
      DO 1070 I=1,N                                                     JPL02990
      TRY=ZBUS(I,I)                                                     JPL03000
      IF(ABS(TRY))1000,1090,1000                                        JPL03010
1000  DO 1040 J=1,N                                                     JPL03020
      IF(I-J)1010,1040,1010                                             JPL03030
1010  DO 1030 K=1,N                                                     JPL03040
      IF(I-K) 1020,1030,1020                                            JPL03050
1020  ZBUS(J,K)=ZBUS(J,K)-ZBUS(J,I)*ZBUS(I,K)/ZBUS(I,I)                 JPL03060
1030  CONTINUE                                                          JPL03070
1040  CONTINUE                                                          JPL03080
      DO 1060 K=1,N                                                     JPL03090
      IF(K-I)1050,1060,1050                                             JPL03100
1050  ZBUS(K,I)=ZBUS(K,I)/ZBUS(I,I)                                     JPL03110
      ZBUS(I,K)=ZBUS(I,K)/ZBUS(I,I)                                     JPL03120
1060  CONTINUE                                                          JPL03130
      ZBUS(I,I)=-1.0/ZBUS(I,I)                                          JPL03140
1070  CONTINUE                                                          JPL03150
      DO 1080 I=1,N                                                     JPL03160
      DO 1080 J=1,N                                                     JPL03170
      ZBUS(I,J)=-ZBUS(I,J)                                              JPL03180
1080  CONTINUE                                                          JPL03190
      DO 1002 I=1,N                                                     JPL03200
      DO 1002 J=1,N                                                     JPL03210
      ZZBUS(I,J)=ZBUS(I,J)                                              JPL03220
1002  CONTINUE                                                          JPL03230
      RETURN                                                            JPL03240
1090  WRITE(6,1100)                                                     JPL03250
1100  FORMAT('ERROR IN MATRIX INVERSE')                                 JPL03260
```

```
      RETURN                                                        JPL03270
      END                                                           JPL03280
C                                                                   JPL03290
      SUBROUTINE MATMUL (A,B,M,L,N,C)                               JPL03300
      DIMENSION A(M,L),B(L,N),C(M,N)                                JPL03310
      DO 100 I=1,M                                                  JPL03320
      DO 100 J=1,N                                                  JPL03330
      C(I,J)=0.                                                     JPL03340
      DO 100 K=1,L                                                  JPL03350
  100 C(I,J)=C(I,J)+A(I,K)*B(K,J)                                   JPL03360
      RETURN                                                        JPL03370
      END                                                           JPL03380
$ENTRY                                                              JPL03390
1        AV  IAS & VA VS. TIME                                      JPL03400
1        P   POWER OF THE MOTOR                                     JPL03410
$STOP                                                               JPL03420
/*                                                                  JPL03430
//                                                                  JPL03440
```

VOLTAGE & CURRENT (80% SPEED,MOTOR OPERATION)




OUTPUT POWER (80% SPEED MOTOR OPERATION)
70000
65000
60000
55000
50000
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
POWER
0
25
50
75
100
125
150
175
200
DEGREE

```
FILE: THREE    PLOT     A   UNIVERSITY OF MISSOURI COMPUTER NETWORK                    PAGE 001

//THREEP   JOB (XXXXLH),'GOODARZI',
// PASSWORD=XXXX
/*JOBPARM R=900,B=99000
// EXEC PLOTSAS
//SYSIN DD *
          GOPTIONS  DEVICE=VARIAN;
          DATA ONE;
          INPUT X Y1-Y3;
*************************************************************************
*                                                                       *
*      THREE PHASE PLOT DATA  JPL PROJECT                               *
*                                                                       *
*                                                                       *
*************************************************************************
CARDS;
  2 4   -238.1        80 0     62992 7
  4 8   -215.4        80 0     61387 7
  7 2   -193 3        80.0     59997 3
  9.5   -172 0        80 0     58817.8
 11.9   -151.4        80 0     57843 1
 14.3   -131.4        80 0     57066.1
 16.6   -112.2        80.0     56478.4
 19.0    -93.8        80.0     56071 7
 21.4    -76.1        80.0     55836 0
 23.8    -59.1        80 0     55761.7
 26.1    -42.9        80.0     55837.9
 28.5    -27.5        80.0     56052.8
 30.9    -12 8        80.0     56394.6
 33.2      1.1        80 0     56850.5
 35.6     14.2        80.0     57407.2
 38.0     26.5        80.0     58051.1
 40.4     38.1        80.0     58768.1
 42.7     48.9        80.0     59543.6
 45.1     59.0        80.0     60362.7
 47.5     68.3        80.0     61210.5
 49.8     76.8        80.0     62070.9
 52.2     84.6        80.0     62929.0
 54.6     91.6        80.0     63768.9
 56.9     97.9        80 0     64574.7
 59.3    103.5        80 0     65330.1
 61.7    120.9       160.0     63515.1
 64.1    135.4       160.0     62315.5
 66.4    151.3       160.0     60915.1
 68.8    166.6       160.0     59724.7
 71.2    181.2       160.0     58741.3
 73.5    195.1       160.0     57956.0
 75.9    208.5       160.0     57359.9
 78.3    221.2       160.0     56947.8
 80.7    233.4       160.0     56707.7
 83.0    244.9       160.0     56629 4
 85.4    256.0       160.0     56705.8
 87.8    266.5       160.0     56921.1
 90.1    276.6       160.0     57265.6
 92.5    286.2       160.0     57726.8
 94.9    295.3       160.0     58291.0
```

97.3	304.1	160.0	58944.8
99.6	312.4	160.0	59674.0
102.0	320.5	160.0	60464.4
104.4	328.1	160.0	61300.7
106.7	335.5	160.0	62168.1
109.1	342.7	160.0	63051.2
111.5	349.6	160.0	63934.3
113.8	356.2	160.0	64802.2
116.2	362.7	160.0	65638.1
118.6	369.1	160.0	66426.8
121.0	375.3	80.0	67152.6
123.3	367.0	80.0	65000.2
125.7	360.7	80.0	63487.0
128.1	354.4	80.0	62183.1
130.4	348.1	80.0	61083.0
132.8	341.9	80.0	60180.0
135.2	335.7	80.0	59467.0
137.5	329.7	80.0	58935.5
139.9	323.9	80.0	58577.0
142.3	318.2	80.0	58381.4
144.7	312.7	80.0	58338.2
147.0	307.5	80 0	58437.0
149.4	302.6	80.0	58665.8
151.8	297.9	80.0	59012.7
154.1	293.6	80.0	59464.2
156.5	289.6	80 0	60007.5
158.9	286.1	80.0	60627.9
161 3	282.9	80.0	61312.5
163.6	280.2	80.0	62046.2
166.0	277.9	80.0	62814 4
168.4	276.1	80 0	63602.2
170.7	274.8	80.0	64393.3
173 1	274.0	80.0	65172.8
175.5	273.8	80.0	65923.1
177.9	274.2	80.0	66630.7
180.2	275.1	-80.0	67280.4
182.6	247.6	-80.0	65003.4
185.0	225.0	-80.0	63374.9
187.3	203.0	-80.0	61957.1
189.7	181.7	-80.0	60747.6
192.1	161.1	-80.0	59738.1
194.4	141.2	-80.0	58922.0
196.8	122.0	-80.0	58291.2
199.2	103.6	-80.0	57837.4
201.6	85.9	-80.0	57551.2
203.9	69.0	-80.0	57422.9
206.3	52.9	-80.0	57441.2
208.7	37.5	-80.0	57594.9
211.0	22.9	-80.0	57871.9
213.4	9.0	-80.0	58259.6
215.8	-4.1	-80.0	58744.8
218.2	-16.4	-80.0	59314.1
220.5	-27.9	-80.0	59953.4
222.9	-38.7	-80.0	60648.3
225.3	-48.7	-80.0	61384.1

227.6	-57.9	-80.0	62145.7
230.0	-66.4	-80.0	62918.1
232.4	-74.1	-80.0	63685.0
234.8	-81.2	-80.0	64431.7
237.1	-87.4	-80.0	65142.2
239.5	-93.0	-80.0	65801.0
241.9	-110.3	-160 0	63900.4
244.2	-124.8	-160 0	62610.4
246.6	-140.7	-160 0	61121.6
249.0	-155.9	-160 0	59842 6
251.3	-170.5	-160 0	58768.7
253.7	-184.4	-160.0	57891.7
256.1	-197.7	-160.0	57205 8
258.5	-210.4	-160.0	56703 2
260.8	-222.6	-160.0	56370.8
263.2	-234.1	-160.0	56202.0
265.6	-245.1	-160 0	56186 1
267.9	-255.7	-160.0	56310.3
270.3	-265.7	-160.0	56563.8
272.7	-275.3	-160 0	56934 4
275.1	-284.4	-160 0	57409.3
277.4	-293.2	-160.0	57973.1
279.8	-301.5	-160.0	58615.2
282.2	-309.5	-160.0	59318 3
284.5	-317.2	-160.0	60068 8
286.9	-324.6	-160.0	60851.5
289.3	-331.7	-160.0	61651 4
291.6	-338.6	-160.0	62452 3
294.0	-345.3	-160.0	63240 0
296.4	-351.8	-160 0	63997 6
298.8	-358.2	-160.0	64710.0
301.1	-364.4	-80.0	65360.8
303.5	-356.1	-80.0	63149 2
305.9	-349.8	-80.0	61577.4
308.2	-343.5	-80.0	60218 0
310.6	-337.2	-80 0	59065.0
313.0	-331.0	-80 0	58112 4
315.4	-324.9	-80.0	57352.7
317.7	-318.9	-80.0	56777.5
320.1	-313.1	-80.0	56377.9
322.5	-307.4	-80 0	56145.2
324.8	-302.0	-80.0	56068.4
327.2	-296.8	-80.0	56135.7
329.6	-291.9	-80.0	56336.4
331.9	-287.3	-80.0	56657.9
334.3	-283.0	-80.0	57088.6
336.7	-279.1	-80.0	57613 4
339.1	-275.5	-80.0	58219.1
341.4	-272.4	-80.0	58892.3
343.8	-269.7	-80.0	59617.1
346.2	-267.5	-80.0	60378.2
348.5	-265.7	-80.0	61162.9
350.9	-264.5	-80.0	61956 4
353.3	-263.8	-80.0	62739.2
355.7	-263.6	-80.0	63499.7

```
358.0   -264.0       -80.0     64218.1
;
          PROC GPLOT;
          PLOT Y1*X Y2*X /OVERLAY;
          SYMBOL1 I=JOIN L=1;
          SYMBOL2 I=JOIN L=13;
          LABEL X=DEGREE
                Y1=VOLTAGE &  CURRENT ;
         TITLE VOLTAGE & CURRENT (80% SPEED,MOTOR OPERATION);
          PROC GPLOT;
          PLOT Y3*X/VREF=0;
          SYMBOL1 I=SPLINE;
          LABEL X=DEGREE
                Y3= POWER ;
          TITLE OUTPUT POWER (80% SPEED MOTOR OPERATION).
/*
//
```

APPENDIX H

DERIVATION OF INDUCTION MOTOR d-q EQUATIONS IN STATIONARY REFERENCE FRAME

DERIVATION OF INDUCTION MOTOR d-q EQUATIONS IN STATIONARY REFERENCE FRAME

EE400 Research Problem Report

(4) Credit Hours

by

Hadi Tehrani

MSEE Candidate

February 21, 1984

for

Department of Electrical and Computer Engineering

University of Missouri-Columbia

Supervisor: Dr. R. G. Hoft, Professor of Electrical Engineering

TABLE OF CONTENTS

1.0 OBJECT

For the majority of practical ac machines, including ideal induction machines, a great simplification is obtained by expressing the equations in a new reference frame and introducing certain currents and voltages which are different from, but are related to, the actual ones. The currents can have a physical meaning in that they can be considered to flow in windings acting along two axes at right angles, called the direct and quadrature axes. In this paper a two-axis d-q theory of the induction motor in a stationary reference frame is developed.

2.0 INDUCTION MOTOR EQUATIONS IN THREE PHASE VARIABLES

It is assumed that the induction motor is ideal and symmetrical, which means:

1. Uniform air gap
2. Linear magnetic circuit
3. Identical stator windings which produce a sinusoidal MMF wave in space
4. No changes in resistances due to temperature and frequency changes

The stator circuits are oriented along the a_s, b_s and c_s axes which are 120° apart. The rotor circuits are along the a_r, b_r and c_r axes which are also 120° apart. They rotate relative to the stationary stator axes with an angular velocity $\omega_r = \frac{d\theta}{dt}$, which is the motor shaft velocity, and the angle θ is the angle between the stator and rotor.

The basic KVL equations for the three phase induction motor are - for the stator

$$\begin{vmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{vmatrix} = \begin{vmatrix} R_{as}\, i_{as} + L_{las} \frac{di_{as}}{dt} + \frac{d\lambda_{as}}{dt} \\ R_{bs}\, i_{bs} + L_{lbs} \frac{di_{bs}}{dt} + \frac{d\lambda_{bs}}{dt} \\ R_{cs}\, i_{cs} + L_{lcs} \frac{di_{cs}}{dt} + \frac{d\lambda_{cs}}{dt} \end{vmatrix} \tag{2.1}$$

$$V_{bs} = R_{bs}\, i_{bs} + L_{lbs} \frac{di_{bs}}{dt} + \frac{d\lambda_{bs}}{dt} \tag{2.2}$$

$$V_{cs} = R_{cs}\, i_{cs} + L_{lcs} \frac{di_{cs}}{dt} + \frac{d\lambda_{cs}}{dt} \tag{2.3}$$

and for the rotor

$$V_{ar} = R_{ar}\, i_{ar} + L_{lar} \frac{di_{ar}}{dt} + \frac{d\lambda_{ar}}{dt} \tag{2.4}$$

$$V_{br} = R_{br}\, i_{br} + L_{lbr} \frac{di_{br}}{dt} + \frac{d\lambda_{br}}{dt} \tag{2.5}$$

$$V_{cr} = R_{cr}\, i_{cr} + L_{lcr} \frac{di_{cr}}{dt} + \frac{d\lambda_{cr}}{dt} \tag{2.6}$$

where

$V_{as} \triangleq$ instantaneous line to neutral voltage of stator Phase a

$V_{bs} \triangleq$ " " " " " " " Phase b

$V_{cs} \triangleq$ " " " " " " " Phase c

$V_{ar} \triangleq$ " " " " " " rotor Phase a

$V_{br} \triangleq$ " " " " " " " Phase b

$V_{cr} \triangleq$ " " " " " " " Phase c

$R_{as} \triangleq$ line to neutral resistance of stator Phase a

$R_{bs} \triangleq$ " " " " " " Phase b

$R_{cs} \triangleq$ " " " " " " Phase c

$R_{ar} \triangleq$ line to neutral resistance of rotor Phase a

$R_{br} \triangleq$ " " " " " " Phase b

$R_{cr} \triangleq$ " " " " " " Phase c

$i_{as} \triangleq$ instantaneous phase or line current in stator Phase a

$i_{bs} \triangleq$ " " " " " " Phase b

$i_{cs} \triangleq$ " " " " " " Phase c

$i_{ar} \triangleq$ " " " " " " rotor Phase a

$i_{br} \triangleq$ " " " " " " " Phase b

$i_{cr} \triangleq$ " " " " " " " Phase c

$\lambda_{as} \triangleq$ instaneous flux linkage of stator Phase a with air gap magnetic field

$\lambda_{bs} \triangleq$ " " " " " Phase b " " " " "

$\lambda_{cs} \triangleq$ " " " " " Phase c " " " " "

$\lambda_{ar} \triangleq$ " " " " rotor Phase a " " " " "

$\lambda_{br} \triangleq$ " " " " " Phase b " " " " "

$\lambda_{cr} \triangleq$ " " " " " Phase c " " " " "

$L_{las} \triangleq$ leakage inductance of stator Phase a

$L_{lbs} \triangleq$ " " " " Phase b

$L_{lcs} \triangleq$ " " " " Phase c

$L_{lar} \triangleq$ " " " " Phase a

$L_{lbr} \triangleq$ " " " " Phase b

$L_{lcr} \triangleq$ " " " " Phase c

Fig. 2.1 shows the three phase axes of the stator and rotor.

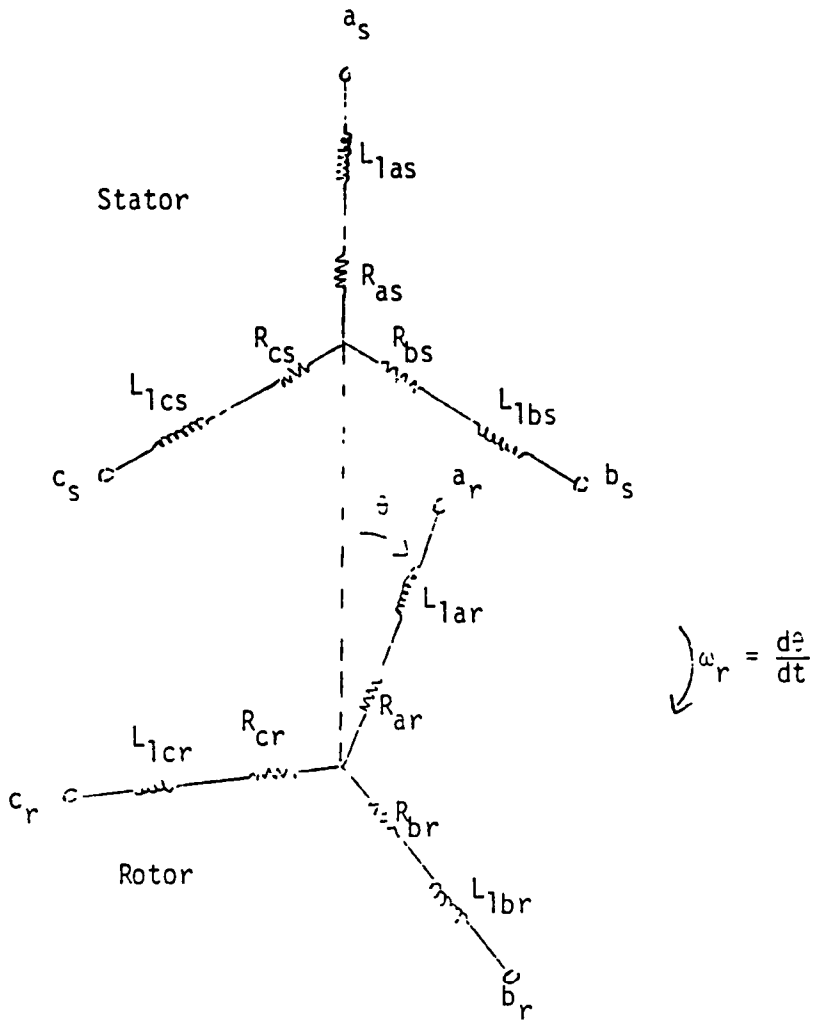


Fig. 2.1

Because of the symmetrical and balanced windings:

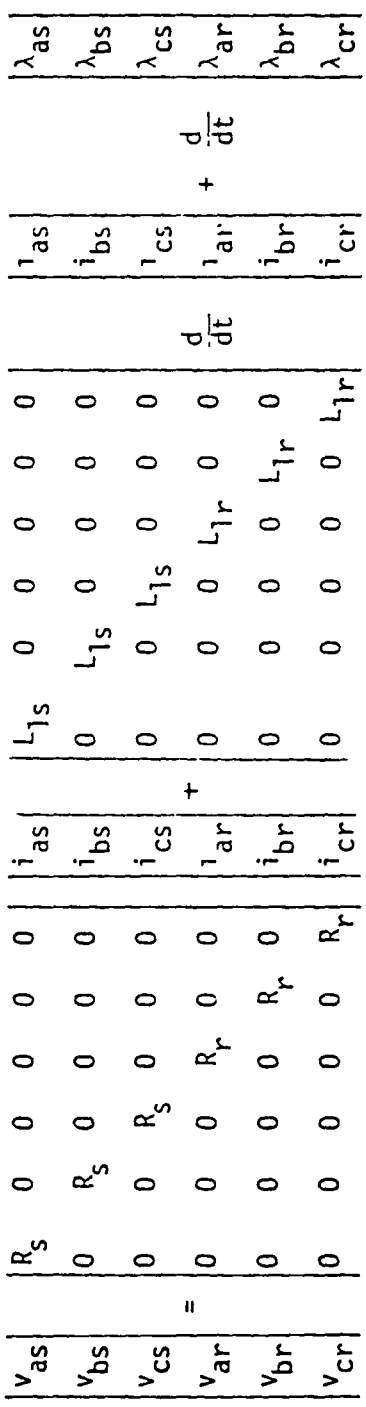
$R_{as} = R_{bs} = R_{cs} = R_s$ (stator resistance/phase)

$R_{ar} = R_{br} = R_{cr} = R_r$ (rotor resistance/phase)

$L_{las} = L_{lbs} = L_{lcs} = L_{ls}$ (stator leakage inductance/phase)

$L_{lar} = L_{lbr} = L_{lcr} = L_{lr}$ (rotor leakage inductance/phase)

The basic equations can be shown in matrix form as



$$
\begin{vmatrix} v_{as} \\ v_{bs} \\ v_{cs} \\ v_{ar} \\ v_{br} \\ v_{cr} \end{vmatrix} = \begin{vmatrix} R_s & 0 & 0 & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 & 0 & 0 \\ 0 & 0 & R_s & 0 & 0 & 0 \\ 0 & 0 & 0 & R_r & 0 & 0 \\ 0 & 0 & 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & 0 & 0 & R_r \end{vmatrix} \begin{vmatrix} i_{as} \\ i_{bs} \\ i_{cs} \\ i_{ar} \\ i_{br} \\ i_{cr} \end{vmatrix} + \begin{vmatrix} L_{ls} & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{ls} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{lr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{lr} & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{lr} \end{vmatrix} \frac{d}{dt} \begin{vmatrix} i_{as} \\ i_{bs} \\ i_{cs} \\ i_{ar} \\ i_{br} \\ i_{cr} \end{vmatrix} + \frac{d}{dt} \begin{vmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \\ \lambda_{ar} \\ \lambda_{br} \\ \lambda_{cr} \end{vmatrix} \tag{2.7}
$$

where the variables written in vector form are

$$\vec{v}_s = \begin{vmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{vmatrix} \quad (2.9a) \qquad \vec{v}_r = \begin{vmatrix} v_{ar} \\ v_{br} \\ v_{cr} \end{vmatrix} \quad (2.8a)$$

$$\vec{i}_s = \begin{vmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{vmatrix} \quad (2.9b) \qquad \vec{i}_r = \begin{vmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{vmatrix} \quad (2.8b)$$

$$\vec{\lambda}_s = \begin{vmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{vmatrix} \quad (2.9c) \qquad \vec{\lambda}_r = \begin{vmatrix} \lambda_{ar} \\ \lambda_{br} \\ \lambda_{cr} \end{vmatrix} \quad (2.8c)$$

Next, the equation for the total flux linkage for each stator and rotor phase and the air gap magnetic field is written as follows

$$\vec{\lambda} = L\vec{i} \qquad (2.10)$$

where

$$\lambda_{as} = L_{ms}\, i_{as} + L_{ms}\, i_{bs} \cos(2\pi/3) + L_{ms}\, i_{cs} \cos(-2\pi/3) + L_{mr}\, i_{ar} \cos\theta + L_{mr}\, i_{br} \cos(\theta+2\pi/3) + L_{mr}\, i_{cr} \cos(\theta-2\pi/3) \qquad (2.11)$$

$$\lambda_{bs} = L_{ms}\, i_{as} \cos(-2\pi/3) + L_{ms}\, i_{bs} + L_{ms}\, i_{cs} \cos(2\pi/3) + L_{mr}\, i_{ar} \cos(\theta-2\pi/3) + L_{mr}\, i_{br} \cos\theta + L_{mr}\, i_{cr} \cos(\theta+2\pi/3) \qquad (2.12)$$

$$\lambda_{cs} = L_{ms}\, i_{as} \cos(2\pi/3) + L_{ms}\, i_{bs} \cos(-2\pi/3) + L_{ms}\, i_{cs}$$
$$L_{mr}\, i_{ar} \cos(\theta+2\pi/3) + L_{mr}\, i_{br} \cos(\theta-2\pi/3) + L_{mr}\, i_{cr} \cos\theta \tag{2.13}$$

$$\lambda_{ar} = L_{ms}\, i_{as} \cos\theta + L_{ms}\, i_{bs} \cos(\theta-2\pi/3) + L_{ms}\, i_{cs} \cos(\theta+2\pi/3)$$
$$L_{mr}\, i_{ar} + L_{mr}\, i_{br} \cos(-2\pi/3) + L_{mr}\, i_{cr} \cos(-2\pi/3) \tag{2.14}$$

$$\lambda_{br} = L_{ms}\, i_{as} (\theta+2\pi/3) + L_{ms}\, i_{bs} \cos\theta + L_{ms}\, i_{cs} \cos(\theta-2\pi/3)$$
$$L_{mr}\, i_{ar} \cos(2\pi/3) + L_{mr}\, i_{br} + L_{mr}\, i_{cr} \cos(-2\pi/3) \tag{2.15}$$

$$\lambda_{cr} = L_{ms}\, i_{as} \cos(\theta-2\pi/3) + L_{ms}\, i_{bs} \cos(\theta+2\pi/3) + L_{ms}\, i_{cs} \cos\theta$$
$$L_{mr}\, i_{ar} \cos(-2\pi/3) + L_{mr}\, i_{br} \cos(2\pi/3) + L_{mr}\, i_{ar} \tag{2.16}$$

and

$L_{ms} \triangleq$ magnetizing inductance/stator phase of air gap magnetic field

$L_{mr} \triangleq$ magnetizing inductance/rotor phase of air gap magnetic field

Equations (2.11)-(2.16) can be written in the matrix equation form of (2.17).

For simplicity, (2.17) can be divided into four 3X3 as follows

$$|L_1| = \begin{vmatrix} L_{ms} & -1/2L_{ms} & -1/2L_{ms} \\ -1/2L_{ms} & L_{ms} & -1/2L_{ms} \\ -1/2L_{ms} & -1/2L_{ms} & L_{ms} \end{vmatrix} \tag{2.18a}$$

$$|L_2| = \begin{vmatrix} L_{mr}\cos\theta & L_{mr}\cos(\theta+2\pi/3) & L_{ms}\cos(\theta+2\pi/3) \\ L_{mr}\cos(\theta-2\pi/3) & L_{mr}\cos\theta & L_{mr}\cos(\theta+2\pi/3) \\ L_{mr}\cos(\theta+2\pi/3) & L_{mr}\cos(\theta-2\pi/3) & L_{mr}\cos\theta \end{vmatrix} \tag{2.18b}$$

$$|L_3| = \begin{vmatrix} L_{ms}\cos\theta & L_{ms}\cos(\theta-2\pi/3) & L_{ms}\cos(\theta+2\pi/3) \\ L_{ms}\cos(\theta+2\pi/3) & L_{ms}\cos\theta & L_{ms}\cos(\theta-2\pi/3) \\ L_{ms}\cos(\theta-2\pi/3) & L_{ms}\cos(\theta+2\pi/3) & L_{ms}\cos\theta \end{vmatrix} \tag{2.18c}$$

$$
\begin{vmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \\ \lambda_{ar} \\ \lambda_{br} \\ \lambda_{cr} \end{vmatrix} = \begin{vmatrix} L_{ms} & -1/2L_{ms} & -1/2L_{ms} & L_{mr}\cos\theta & L_{mr}\cos(\theta+2\pi/3) & L_{mr}\cos(\theta-2\pi/3) \\ -1/2L_{ms} & L_{ms} & -1/2L_{ms} & L_{mr}\cos(\theta-2\pi/3) & L_{mr}\cos\theta & L_{mr}\cos(\theta+2\pi/3) \\ -1/2L_{ms} & -1/2L_{ms} & L_{ms} & L_{mr}\cos(\theta+2\pi/3) & L_{mr}\cos(\theta-2\pi/3) & L_{mr}\cos\theta \\ L_{ms}\cos\theta & L_{ms}\cos(\theta-2\pi/3) & L_{ms}\cos(\theta+2\pi/3) & L_{mr} & -1/2L_{mr} & -1/2L_{mr} \\ L_{ms}\cos(\theta+2\pi/3) & L_{ms}\cos\theta & L_{ms}\cos(\theta-2\pi/3) & -1/2L_{mr} & L_{mr} & -1/2L_{mr} \\ L_{ms}\cos(\theta-2\pi/3) & L_{ms}\cos(\theta+2\pi/3) & L_{ms}\cos\theta & -1/2L_{mr} & -1/2L_{mr} & L_{mr} \end{vmatrix} \begin{vmatrix} i_{as} \\ i_{bs} \\ i_{cs} \\ i_{ar} \\ i_{br} \\ i_{cr} \end{vmatrix}
$$

(2.17)

$$|L_4| = \begin{vmatrix} L_{mr} & -1/2L_{mr} & -1/2L_{mr} \\ -1/2L_{mr} & L_{mr} & -1/2L_{mr} \\ -1/2L_{mr} & -1/2L_{mr} & L_{mr} \end{vmatrix} \tag{2.18d}$$

Then (2.17) can be written

$$\begin{vmatrix} \vec{\lambda}_s \\ \vec{\lambda}_r \end{vmatrix} = \begin{vmatrix} L_1 & L_2 \\ L_3 & L_4 \end{vmatrix} \begin{vmatrix} \vec{i}_s \\ \vec{i}_r \end{vmatrix} \tag{2.19}$$

Since this induction motor has no neutral connection

$$i_{as} + i_{bs} + i_{cs} = 0 \tag{2.20a}$$

$$i_{ar} + i_{br} + i_{cr} = 0 \tag{2.20b}$$

Therefore

$$i_{as} = -i_{bs} - i_{cs} \tag{2.21a}$$

$$i_{ar} = -i_{br} - i_{cr} \tag{2.21b}$$

If these equations are substituted into (2.17), matrixes L_1 and L_4 become

$$|L_1| = \begin{vmatrix} 3/2L_{ms} & 0 & 0 \\ 0 & 3/2L_{ms} & 0 \\ 0 & 0 & 3/2L_{ms} \end{vmatrix} \tag{2.22}$$

$$|L_4| = \begin{vmatrix} 3/2L_{mr} & 0 & 0 \\ 0 & 3/2L_{mr} & 0 \\ 0 & 0 & 3/2L_{mr} \end{vmatrix} \tag{2.23}$$

It is more convenient to write the resistance matrix and the leakage inductance matrix of (2.7) as follows

$$|R| = \begin{vmatrix} R_s & 0 \\ 0 & R_r \end{vmatrix} \qquad (2.24) \qquad\qquad |L_l| = \begin{vmatrix} L_{ls} & 0 \\ 0 & L_{lr} \end{vmatrix} \qquad (2.25)$$

where

$$|R_s| = \begin{vmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{vmatrix} \qquad (2.24a) \qquad\qquad |L_{ls}| = \begin{vmatrix} L_{ls} & 0 & 0 \\ 0 & L_{ls} & 0 \\ 0 & 0 & L_{ls} \end{vmatrix} \qquad (2.25a)$$

$$|R_r| = \begin{vmatrix} R_r & 0 & 0 \\ 0 & R_r & 0 \\ 0 & 0 & R_r \end{vmatrix} \qquad (2.24b) \qquad\qquad |L_{lr}| = \begin{vmatrix} L_{lr} & 0 & 0 \\ 0 & L_{lr} & 0 \\ 0 & 0 & L_{lr} \end{vmatrix} \qquad (2.25b)$$

By substituting (2.19), (2.24) and (2.25) into equation (2.7), the induction motor equations can be written

$$[\vec{V}_s] = [R_s]\,[\vec{i}_s] + [L_l]\,d/dt\,[\vec{i}_s] + d/dt\,[L_2]\,[\vec{i}_r] + [L_{ls}]\,d/dt\,[\vec{i}_s] \qquad (2.26)$$

$$[\vec{V}_r] = [R_r]\,[\vec{i}_r] + d/dt\,[L_3]\,[\vec{i}_s] + [L_4]\,d/dt\,[\vec{i}_r] + [L_{lr}]\,d/dt\,[\vec{i}_r] \qquad (2.27)$$

Equations (2.26) and (2.27) can be written in matrix form

$$\begin{vmatrix} \vec{V}_s \\ \vec{V}_r \end{vmatrix} = \begin{vmatrix} [R_s] & \vec{i}_s \\ [R_r] & \vec{i}_r \end{vmatrix} + \begin{vmatrix} [L_{ls}] & \\ & [L_{lr}] \end{vmatrix} P \begin{vmatrix} \vec{i}_s \\ \vec{i}_r \end{vmatrix} + P \begin{vmatrix} [L_1] & [L_2] \\ [L_3] & [L_4] \end{vmatrix} \begin{vmatrix} \vec{i}_s \\ \vec{i}_r \end{vmatrix} \qquad (2.28)$$

where

$P = d/dt$

3.0 TRANSFORMATION TO STATIONARY d-q REFERENCE FRAME

In this report, the following symbols are used:

d-q => stationary reference frame (fixed with respect to the stator)

D-Q => rotating reference frame (rotating at rotor speed - fixed with respect to the rotor)

3.1 Definition of d-q Stationary Reference Frame

For the d-q stationary reference frame, the d-axis is in line with the a-axis of the stator as shown in Fig. 3.1. Then,

$$v_d = v_{as} + v_{bs} \cos (120°) + v_c \cos (-120°)$$

$$v_q = v_b \sin (120°) + v_c \sin (-120°)$$

or

$$\begin{vmatrix} v_d \\ v_q \end{vmatrix} = \begin{vmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{vmatrix} \begin{vmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{vmatrix} \tag{3.1}$$

3.2 Definition of D-Q Rotating Reference Frame

The D-Q reference frame rotates relative to the d-q stationary reference frame, where θ is the angle between the d-axis and the D-axis and

$$\theta = \int \omega_r \, dt; \qquad \omega_r = \frac{d\theta}{dt}$$

where ω_r is the rotor speed in radians/second. This is shown in Fig. 3.2.

$$\begin{vmatrix} v_d \\ v_q \end{vmatrix} = \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} \begin{vmatrix} v_D \\ v_Q \end{vmatrix} \tag{3.2a}$$

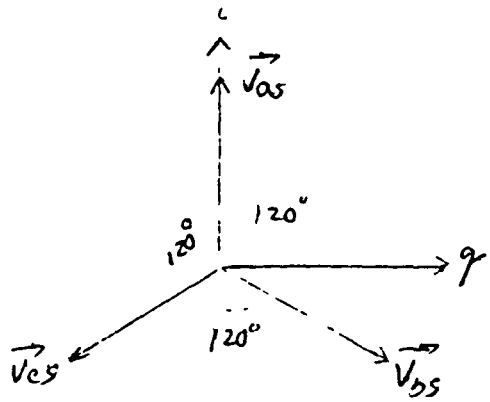

$\vec{V}_{as}$
120°
120°
120°
q
$\vec{V}_{cs}$
$\vec{V}_{bs}$

Fig. 3.1

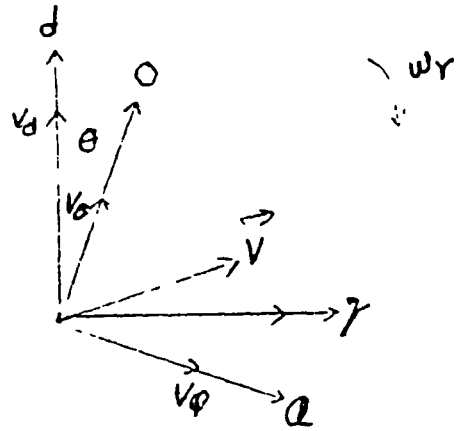

d
O
V_d
θ
V_q
$\vec{V}$
γ
V_φ
a
ω_r

Fig. 3.2

$$\begin{vmatrix} v_D \\ v_Q \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} \begin{vmatrix} v_d \\ v_q \end{vmatrix} \tag{3.2b}$$

The $\vec{v}$ is rotating clockwise at angular velocity ω_r.

3.3 Induction Motor d-q-0 Variables

For simplicity in the definitions of the d-q and D-Q reference frames, only two axes were considered. In general, it is desirable to consider three coordinate axes for two reasons. Matrix transformation methods can be used to transform variables from one reference frame to another, but only if the transformation matrices are square and nonsingular matrices. Thus, for three phase situations with three phase variables, it is necessary to consider transformations to coordinate axes where three variables exist. For balanced and symmetrical cases, it is often sufficient to consider only d-q variables. However, the three d-q-0 variables are necessary in the most general situations.

In the remainder of this report we are concerned with the d-q-0 reference frame. Again, this is a reference frame fixed with respect to the stator. The 0-variables will not only make it possible to use matrix transformation techniques, but they also will have important physical significance in many cases.

For this report, the d-q-0 transformation from stator three phase voltages or currents is defined as follows

$$\begin{vmatrix} v_{ds} \\ v_{qs} \\ v_{0s} \end{vmatrix} \triangleq \frac{2}{3} \begin{vmatrix} 1 & \cos 2\pi/3 & \cos(-2\pi/3) \\ 0 & \sin 2\pi/3 & \sin(-2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{vmatrix} \begin{vmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{vmatrix} \tag{3.3a}$$

$$\begin{vmatrix} i_{ds} \\ i_{qs} \\ i_{0s} \end{vmatrix} \triangleq \frac{2}{3} \begin{vmatrix} 1 & \cos 2\pi/3 & \cos(-2\pi/3) \\ 0 & \sin 2\pi/3 & \sin(-2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{vmatrix} \begin{vmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{vmatrix} \tag{3.3b}$$

Equations (3.3a) and (3.3b) transform the three phase stator line-to-neutral voltages or line currents to d-q-0 coordinate voltages or currents. It should be noted that

$$i_{0s} = 1/3\ (i_{as} + i_{bs} + i_{cs}) \tag{3.4a}$$

and thus i_{0s} is zero for all three wire machines where no neutral is present. This is a consequence of Kirchoff's current law. However,

$$v_{0s} = 1/3\ (v_{as} + v_{bs} + v_{cs}) \tag{3.4b}$$

is zero when balanced and symmetrical three phase line-to-neutral voltages are involved, but it is not zero in general.

Equations (3.3a) or (3.3b) apply to the transformation of stator three phase to d-q-0 quantities for all circuit variables including flux linkages as well as the voltages and currents.

The d-q-0 transformation for rotor quantities is defined as follows

$$\begin{vmatrix} v_{dr} \\ v_{qr} \\ v_{0r} \end{vmatrix} \triangleq \frac{2}{3} \begin{vmatrix} \cos\theta & \cos(\theta + 2\pi/3) & \cos(\theta - 2\pi/3) \\ \sin\theta & \sin(\theta + 2\pi/3) & \sin(\theta - 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{vmatrix} \begin{vmatrix} v_{ar} \\ v_{br} \\ v_{cr} \end{vmatrix} \tag{3.5}$$

Equation (3.5) transforms the rotor quantities, which are given in a three phase coordinate frame rotating with angular velocity ω_r, to the d-q-0 reference frame fixed with respect to the stator. The same transformation as in (3.5) is used to transform currents and flux linkages. Fig. 3.3 illus-

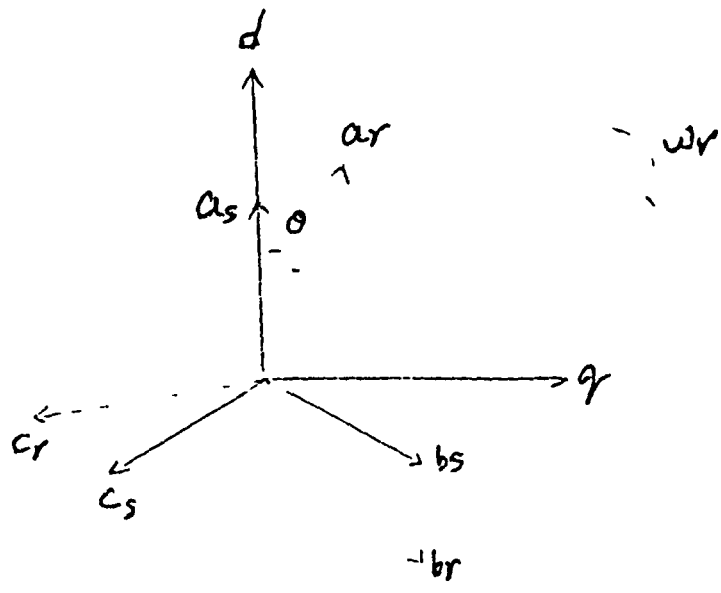


Fig. 3.3

trates this relationship. The 0-axis is orthogonal to the d and q-axes, and it may be considered directed into the paper.

A number of additional transformation relations are given as follows

for stator variables

$$|\vec{v}_s| = |A_s|^{-1} |v^s_{dq0}| \tag{3.6a}$$

$$|\vec{i}_s| = |A_s|^{-1} |i^s_{dq0}| \tag{3.6b}$$

$$|\vec{\lambda}_s| = |A_s|^{-1} |\lambda^s_{dq0}| \tag{3.6c}$$

and for stator variables

$$|\vec{v}_r| = |A_r|^{-1} |v^r_{dq0}| \tag{3.7a}$$

$$|\vec{i}_r| = |A_r|^{-1} |i^r_{dq0}| \tag{3.7b}$$

$$|\vec{\lambda}_r| = |A_r|^{-1} |\lambda^r_{dq0}| \tag{3.7c}$$

where

$$|A_s| = \frac{2}{3} \begin{vmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{vmatrix} \tag{3.8}$$

$$|A_s|^{-1} = \begin{vmatrix} 1 & 0 & 1 \\ -1/2 & \sqrt{3}/2 & 1 \\ -1/2 & -\sqrt{3}/2 & 1 \end{vmatrix} \tag{3.9}$$

$$|A_r| = \frac{2}{3}\begin{vmatrix} \cos\theta & \cos(\theta + 2\pi/3) & \cos(\theta - 2\pi/3) \\ \sin\theta & \sin(\theta + 2\pi/3) & \sin(\theta - 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{vmatrix} \tag{3.10}$$

$$|A_r|^{-1} = \begin{vmatrix} \cos\theta & \sin\theta & 1 \\ \cos(\theta + 2\pi/3) & \sin(\theta + 2\pi/3) & 1 \\ \cos(\theta + -2\pi/3) & \sin(\theta - 2\pi/3) & 1 \end{vmatrix} \tag{3.11}$$

From (2.7), the motor voltage equations are

$$|\vec{v}_s| = R_s\,|\vec{i}_s| + P\,L_{ls}\,|\vec{i}_s| + P|\vec{\lambda}_s| \tag{3.12}$$

$$|\vec{v}_r| = R_r\,|\vec{i}_r| + P\,L_{lr}\,|\vec{i}_r| + P|\vec{\lambda}_r| \tag{3.13}$$

Substituting (3.6) and (3.7) into (3.12) and (3.13) respectively gives

$$|A_s|^{-1}\,|v^s_{dq0}| = R_s\,|A_s|^{-1}\,|i^s_{dq0}| + L_{ls}\,P\,|A_s|^{-1}\,|i^s_{dq0}| + P\,|A_s|^{-1}\,|\lambda^s_{dq0}| \tag{3.14}$$

$$|A_r|^{-1}\,|v^r_{dq0}| = R_r\,|A_r|^{-1}\,|i^r_{dq0}| + L_{lr}\,P\,|A_r|^{-1}\,|i^r_{dq0}| + P\,|A_r|^{-1}\,|\lambda^r_{dq0}| \tag{3.15}$$

Equation (3.15) can be expanded as

$$|A_r|^{-1}\,|v^r_{dq0}| = R_r\,|A_r|^{-1}\,|i^r_{dq0}| + L_{lr}\,|PA_r|^{-1}\,|\lambda^r_{dq0}| + L_{lr}\,|A_r|^{-1}\,P\{|\lambda^r_{dq0}|\} + \{P\,|A_r|^{-1}\}\,|\lambda^r_{dq0}| + |A_r|^{-1}\,\{P\,|\lambda^r_{dq0}|\} \tag{3.16}$$

Equation (3.14) need not be expanded because the matrix $|A_s|^{-1}$ is constant.

It can be shown that [1]

$$P|A_r|^{-1} = -|A_r|^{-1}\{P|A_r|\}|A_r|^{-1} \tag{3.17}$$

Substituting (3.17) into (3.16) gives

$$\begin{aligned}|A_r|^{-1}|v^r_{dq0}| = {} & R_r|A_r|^{-1}|i^r_{dq0}| + L_{lr}\{-|A_r|^{-1}\{P|A_r|\}|A_r|^{-1}\}|i^r_{dq0}| \\ & + L_{lr}A_r^{-1}|P|i^r_{dq0}|| + \{-|A_r|^{-1}\{P|A_r|\}|A_r|^{-1}\}|\lambda^r_{dq0}| \\ & + |A_r|^{-1}\{P|\lambda^r_{dq0}|\}\end{aligned} \tag{3.18}$$

Multiplying both sides of (3.14) and (3.18) by $|A_s|$ and $|A_r|$ respectively gives

$$|v^s_{dq0}| = R_s|i^s_{dq0}| + L_{ls}P|i^s_{dq0}| + P|\lambda^s_{dq0}| \tag{3.19}$$

$$\begin{aligned}|v^r_{dq0}| = {} & R_r|i^r_{dq0}| + L_{lr}P|i^r_{dq0}| - L_{lr}\{P|A_r|\}|A_r|^{-1}|i^r_{dq0}| \\ & + P|\lambda^r_{dq0}| - \{P|A_r|\}|A_r|^{-1}|\lambda^r_{dr0}|\end{aligned} \tag{3.20}$$

The expressions $\{P|A_r|\}|A_r|^{-1}$ in (3.20) can be evaluated using matrix algebra, which is shown in Appendix I. The result is

$$\{P|A_r|\}|A_r|^{-1} = \begin{vmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{vmatrix}\frac{d\theta}{dt} \tag{3.21}$$

where

$$\frac{d\theta}{dt} \triangleq \omega_r$$

and

$$|H| \triangleq \begin{vmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \tag{3.22}$$

Substituting (3.21) into (3.20) gives

$$|v^r_{dq0}| = R_r\,|i^r_{dq0}| + L_{lr}\,P\,|i^r_{dq0}| + L_{lr}\,\omega_r\,|H|\,|i^r_{dq0}|$$

$$+ P\,|\lambda^r_{dq0}| + \omega_r\,|H|\,|\lambda^r_{dq0}| \tag{3.23}$$

Equations (3.19) and (3.23) are the motor voltage equations in the d-q-0 frame.

Now, the flux linkage equations are expressed in terms of d-q-0 current as follows. Matrix (2.19) can be written

$$|\vec{\lambda}_s| = |L_1|\,|\vec{i}_s| + |L_2|\,|\vec{i}_r| \tag{3.24}$$

$$|\vec{\lambda}_r| = |L_3|\,|\vec{i}_s| + |L_4|\,|\vec{i}_r| \tag{3.25}$$

Substituting (3.6c) and (3.6b) into (3.24) and (3.7c) and (3.7b) into (3.25) gives

$$|A_s^{-1}|\,|\lambda^s_{dq0}| = |L_1|\,|A_s|^{-1}\,|i^s_{dq0}| + |L_2|\,|A_r|^{-1}\,|i^r_{dq0}| \tag{3.26}$$

$$|A_r^{-1}|\,|\lambda^r_{dq0}| = |L_3|\,|A_s|^{-1}\,|i^s_{dq0}| + |L_4|\,|A_r|^{-1}\,|i_{dq0}| \tag{3.27}$$

Multiplying both sides of (3.26) by A_s and (3.27) by A_r gives

$$|\lambda^s_{dq0}| = |A_s|\,|L_1|\,|A_s|^{-1}\,|i^s_{dq0}| + |A_s|\,|L_2|\,|A_r|^{-1}\,|i^r_{dq0}| \tag{3.28}$$

$$|\lambda^r_{dq0}| = |A_r|\,|L_3|\,|A_s|^{-1}\,|i^s_{dq0}| + |A_r|\,|L_4|\,|A_r|^{-1}\,|i^r_{dqo}| \tag{3.29}$$

where

$$|A_s|\ |L_2|\ |A_r|^{-1} = 3/2\ L_{mr} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \tag{3.30a}$$

$$|A_r|\ |L_3|\ |A_s|^{-1} = 3/2\ L_{ms} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \tag{3.30b}$$

$$|A_s|\ |L_1|\ |A_s|^{-1} = 3/2\ L_{ms} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \tag{3.31a}$$

$$|A_r|\ |L_4|\ |A_r|^{-1} = 3/2\ L_{mr} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \tag{3.31b}$$

as shown in Appendix II. Substituting (3.30) and (3.31) into (3.28) and (3.29), respectively, gives

$$|\lambda^s_{dq0}| = 3/2\ L_{ms}\ |E|\ |i^s_{dq0}| + 3/2\ L_{mr}\ |E|\ |i^r_{dq0}| \tag{3.32}$$

$$|\lambda^r_{dq0}| = 3/2\ L_{ms}\ |E|\ |i^s_{dq0}| + 3/2\ L_{mr}\ |E|\ |i^r_{dq0}| \tag{3.33}$$

where

$$|E| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \tag{3.34}$$

Equations (3.32) and (3.33) can be written in matrix form using (3.34)

$$\begin{vmatrix}\lambda_{ds}\\ \lambda_{qs}\\ \lambda_{0s}\\ \lambda_{dr}\\ \lambda_{qr}\\ \lambda_{0r}\end{vmatrix} = 3/2 \begin{vmatrix} L_{ms} & 0 & 0 & L_{mr} & 0 & 0\\ 0 & L_{ms} & 0 & 0 & L_{mr} & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ L_{ms} & 0 & 0 & L_{mr} & 0 & 0\\ 0 & L_{ms} & 0 & 0 & L_{mr} & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{vmatrix} \begin{vmatrix} i_{ds}\\ i_{qs}\\ i_{0s}\\ i_{dr}\\ i_{qr}\\ i_{0r}\end{vmatrix} \tag{3.35}$$

Equations (3.32) and (3.33) can be substituted into (3.19) and (3.23) yielding

$$[v^s_{dq0}] = R_s [i^s_{dq0}] + L_{ls} P [i^s_{dq0}] + 3/2 P L_{ms} [E] [i^s_{dq0}] + 3/2 P L_{mr} [E] [i^r_{dq0}] \tag{3.36}$$

$$[v^r_{dq0}] = R_r [i^r_{dq0}] + L_{lr} P [i^r_{dq0}] + L_{lr} \omega_r [H] [i^r_{dq0}] + 3/2 P L_{ms} [E] [i^s_{dq0}] + 3/2 P L_{mr} [E] [i^r_{dq0}] + 3/2 \omega_r [H] L_{ms} [E] [i^s_{dq0}] + 3/2 \omega_r [H] L_{mr} [E] [i^r_{dq0}] \tag{3.37}$$

where the motor equations are written in terms of d-q-0 currents. The final matrix in d-q-0 coordinates is found by writing (3.36) and (3.37) in matrix form

$$\begin{vmatrix}v_{ds}\\ v_{qs}\\ v_{0s}\\ v_{dr}\\ v_{qr}\\ v_{0r}\end{vmatrix} = \begin{vmatrix} R_s+PL_s & 0 & 0 & \frac{3}{2}PL_{mr} & 0 & 0\\ 0 & R_s+PL_s & 0 & 0 & \frac{3}{2}PL_{mr} & 0\\ 0 & 0 & R_s+PL_{ls} & 0 & 0 & \\ \frac{3}{2}PL_{ms} & \frac{3}{2}\omega_r L_{Lms} & 0 & R_r+PL_r & \omega_r L_r & 0\\ -\frac{3}{2}\omega_r L_{ms} & \frac{3}{2}PL_{ms} & 0 & -\omega_r L_r & R_r+PL_r & 0\\ 0 & 0 & 0 & 0 & 0 & R_r+PL_{lr}\end{vmatrix} \begin{vmatrix} i_{ds}\\ i_{qs}\\ i_{0s}\\ i_{dr}\\ i_{qr}\\ i_{0r}\end{vmatrix} \tag{3.38}$$

where

$$L_s = L_{ls} + 3/2\ L_{ms}$$

$$L_r = L_{lr} + 3/2\ L_{mr}$$

For the symmetrical three phase induction machine with wye-connected windings having no neutral connections, the 0-components disappear so that d-q-0 coordinates become d-q coordinates, reducing (3.38) to

$$\begin{vmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{vmatrix} = \begin{vmatrix} R_s+PL_s & 0 & \frac{3}{2}PL_{mr} & 0 \\ 0 & R_s+PL_s & 0 & \frac{3}{2}PL_{mr} \\ \frac{3}{2}PL_{ms} & \frac{3}{2}\omega_r L_{ms} & R_r+PL_s & \omega_r L_r \\ -\frac{3}{2}\omega_r L_{ms} & \frac{3}{2}PL_{ms} & -\omega_r L_r & R_r+PL_s \end{vmatrix} \begin{vmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{vmatrix} \qquad (3.39)$$

4.0 RESULTS

Matrix methods have been used to derive the d-q equations for the induction machine. The resulting matrix equation (3.39) is much simpler than the conventional three phase representation. Only four voltage and current variables are involved instead of six. Also, the representation of (3.39) is very useful in control analysis using state variable methods and optimization techniques.

APPENDIX I

Equation (3.21) is derived as follows.

From (3.10),

$$P\,|A_r| = 2/3 \begin{vmatrix} -\sin\theta & -\sin(\theta + 2\pi/3) & -\sin(\theta - 2\pi/3) \\ \cos\theta & \cos(\theta + 2\pi/3) & \cos(\theta - 2\pi/3) \\ 0 & 0 & 0 \end{vmatrix} \frac{d\theta}{dt}$$

Therefore, $[P\,|A_r|]\,|A_r|^{-1}$ can be expressed

$$[P\,|A_r|]\,|A_r|^{-1} = 2/3 \begin{vmatrix} -\sin\theta & -\sin(\theta + 2\pi/3) & -\sin(\theta - 2\pi/3) \\ \cos\theta & \cos(\theta + 2\pi/3) & \cos(\theta - 2\pi/3) \\ 0 & 0 & 0 \end{vmatrix} \frac{d\theta}{dt} \times$$

$$\begin{vmatrix} \cos\theta & \sin\theta & 1 \\ \cos(\theta + 2\pi/3) & \sin(\theta + 2\pi/3) & 1 \\ \cos(\theta - 2\pi/3) & \sin(\theta - 2\pi/3) & 1 \end{vmatrix}$$

Multiplying the first row of $P\,|A_r|$ by the first column of $|A_r|^{-1}$ yields

$$-\sin\theta\cos\theta - \sin(\theta + 2\pi/3)\cos(\theta + 2\pi/3) - \sin(\theta - 2\pi/3)\cos(\theta - 2\pi/3)$$

$$= -\left|1/2 \sin 2\theta + 1/2 \sin 2(\theta - 2\pi/3) + 1/2 \sin 2(\theta + 2\pi/3)\right|$$

$$= -1/2\left|\sin 2\theta + \sin(2\theta + 2\pi/3) + \sin(2\theta - 2\pi/3)\right|$$

$$= -1/2\left|\sin 2\theta + 2 \sin 2\theta (\cos 2\pi/3)\right|$$

$$= -1/2\left|\sin 2\theta - \sin 2\theta\right| = 0$$

Next, multiplying the first row of $P\,|A_r|$ by the second column of $|A_r|^{-1}$ gives

$$-\sin\theta^2 - \sin^2(\theta + 2\pi/3) - \sin^2(\theta - 2\pi/3)$$

$$= -\left|1/2 - 1/2 \cos 2\theta + 1/2 - 1/2 \cos 2(\theta + 2\pi/3) + 1/2 - 1/2 \cos 2(\theta - 2\pi/3)\right.$$

$$= -3/2 + 1/2\left|\cos 2\theta + \cos(2\theta + 2\pi/3) + \cos(2\theta - 2\pi/3)\right.$$

$$= -3/2 + 1/2\left|\cos 2\theta + 2(\cos 2\theta)(\cos 2\pi/3)\right|$$

$$= -3/2 + 1/2\left|\cos 2\theta - \cos 2\theta\right| = -3/2$$

Finally, the first row of P $|A_r|$ is multiplied by the third column of $|A_r|^{-1}$ which yields

$$- \sin \theta - \sin(\theta + 2\pi/3) - \sin (\theta - 2\pi/3) = 0$$

Thus, the first row of (3.21) has been derived. The second and third rows of (3.21) are derived in a similar manner.

APPENDIX II

Equation (3.30) is derived as follows, using $|A_s|$ from (3.8), $|L_2|$ from (2.18b) and A_r from (3.10).

$$|A_s|\ |L_2|\ |A_r| =$$

$$= 2/3 \begin{vmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{vmatrix} L_{mr} \begin{vmatrix} \cos\theta & \cos(\theta+2\pi/3) & \cos(\theta-2\pi/3) \\ \cos(\theta-2\pi/3) & \cos\theta & \cos(\theta+2\pi/3) \\ \cos(\theta+2\pi/3) & \cos(\theta-2\pi/3) & \cos\theta \end{vmatrix}$$

$$X \begin{vmatrix} \cos\theta & \sin\theta & 1 \\ \cos(\theta+2\pi/3) & \sin(\theta+2\pi/3) & 1 \\ \cos(\theta-2\pi/3) & \sin(\theta-2\pi/3) & 1 \end{vmatrix}$$

First, $|A_s|\ |L_2|$ is determined. Multiplying the first row of $|A_s|$ by the first, second and third columns of $|L_2|$

$$\cos\theta - 1/2\ \cos(\theta-2\pi/3) - 1/2\ \cos(\theta+2\pi/3)$$

$$\cos\theta - 1/4\ \cos\theta - \sqrt{3}/4\sin\theta + 1/4\cos\theta + \sqrt{3}/4\sin\theta$$

$$(1 + 1/4 + 1/4)\ \cos\theta = 3/2\ \cos\theta \qquad (1)$$

$$\cos(\theta+2\pi/3) - 1/2\ \cos\theta - 1/2\ \cos(\theta-2\pi/3)$$

$$-1/2\ \cos\theta - 3/2\ \sin\theta - 1/2\ \cos\theta + 1/4\ \cos\theta - \sqrt{3}/4\ \sin\theta$$

$$(-1/2 - 1/2 + 1/4)\cos\theta - 3\ (\sqrt{3}/4)\ \sin\theta = -\ 3/4\cos\theta - 3(\sqrt{3}/4)\sin\theta \qquad (2)$$

$$\cos(\theta-2\pi/3) - 1/2\ \cos(\theta+2\pi/3) - 1/2\cos\theta$$

$$-1/2\cos\theta + \sqrt{3}/2\ \sin\theta + 1/4\ \cos\theta + \sqrt{3}/4\ \sin\theta - 1/2\ \cos\theta$$

$$(-1/2 - 1/2 + 1/4)\ \cos\theta + 3(\sqrt{3}/4)\ \sin\theta = -3/4\ \cos\theta + 3(\sqrt{3}/4)\ \sin\theta \qquad (3)$$

Multiplying the second row of $|A_s|$ by the first, second and third columns of $|L_2|$ gives

$$3/2\ \cos(\theta-2\pi/3) - \sqrt{3}/2\ \cos\theta(\pi+2\pi/3)$$

$$- \sqrt{3}/4\ \cos\theta + 3/4\sin\theta + \sqrt{3}/4\ \cos\theta + 3/4\ \sin\theta$$

$$= 3/2\ \sin\theta \qquad (4)$$

$\sqrt{3}/2\cos\theta - \sqrt{3}/2\theta(-1/2\ \cos\theta + \sqrt{3}/2\ \sin\theta)$

$\sqrt{3}/2\cos\theta + \sqrt{3}/4\ \cos\theta - 3/4\ \sin\theta$

$= 3(\sqrt{3}/4)\ \cos\theta - 3/4\ \sin\theta$ (5)

$\sqrt{3}/2\ \cos\ (\theta+2\pi/3) - \sqrt{3}/2\cos\theta$

$- 3/4\ \cos\theta - 3/4\ \sin\theta - \sqrt{3}/2\cos\theta$

$= -3(\sqrt{3}/4)\ \cos\theta - 3/4\ \sin\theta$ (6)

Multiplying the third row of $|A_s|$ by the first, second and third column of $|L_2|$ gives

$1/2\ \ \cos\theta + \cos\ (\theta-2\pi/3) + \cos\ (\theta+2-/3)$

$1/2\ \ \cos\theta - 1/2\cos\theta + \sqrt{3}/2\ \sin\theta\ \ - 1/2\cos\theta\ \ - \sqrt{3}/2\ \sin\theta$

$1 - 1/2 - 1/2 = 0$ (7)

$1/2\ |\ \ \ |\ = 0$ (8)

$1/2\ |\ \ \ |\ = 0$ (9)

The resulting $A_s\ \ L_2$ is now multiplied by A_r^{-1}:

$$2/3\ L_{mr} \begin{vmatrix} 3/2\cos & -3/4\cos\ -\ 3(\ 3/4)\sin & -3/4\cos\ +\ 3(\ 3/4)\sin \\ 3/2\sin & 3(\ 3/4)\cos\ -\ 3/4\sin & -3(\ 3/4)\cos\ -\ 3/4\sin \\ 0 & 0 & 0 \end{vmatrix}$$

$$X \begin{vmatrix} \cos\theta & \sin\theta & 1 \\ \cos(\theta+2\pi/3) & \sin(\theta+2\pi/3) & 1 \\ \cos(\theta-2\pi/3) & \sin(\theta-2\pi/3) & 1 \end{vmatrix} \quad (10)$$

The first row of $|A_s|\ |L_2|$ is now multiplied by the first, second and third columns of $|A_r|^{-1}$

$$3/2\cos^2\theta + (-3/4\cos\theta - 3(\sqrt{3}/4)\sin\theta)(-1/2\cos\theta - \sqrt{3}/2\sin\theta)$$
$$+ (-3/4\cos\theta + 3(\sqrt{3}/4)\sin\theta)(-1/2\cos\theta + \sqrt{3}/2\sin\theta)$$
$$3/2\cos^2\theta + \sqrt{3}/8\cos^2\theta + 3(\sqrt{3}/8)\sin\theta\cos\theta + 3(\sqrt{3}/8)\sin\theta\cos\theta + 9/8\sin^2\theta$$
$$+ \sqrt{3}/8\cos^2\theta - 3(\sqrt{3}/8)\sin\theta\cos\theta - 3(\sqrt{3}/8)\sin\theta\cos\theta - 9/8\sin^2\theta$$
$$3/2\cos^2\theta + 3/8\cos^2\theta + 3(\sqrt{3}/8)\sin\theta\cos\theta + 3(\sqrt{3}/8)\sin\theta\cos\theta + 9/8\sin^2\theta$$
$$+ 3/8\cos^2\theta - 3(\sqrt{3}/8)\sin\theta\cos\theta + 3(\sqrt{3}/8)\sin\theta\cos\theta + 9/8\sin^2\theta$$
$$3/2 + 3/8 + 3/8 = 9/4\cos^2\theta + 9/4\sin^2\theta = 9/4 \qquad (11)$$

$$3/2\sin\theta\cos\theta + (-3/4\cos\theta - 3(\sqrt{3}/4)\sin\theta)(-1/2\sin\theta + \sqrt{3}/2\cos\theta)$$
$$+ (-3/4\cos\theta + 3(\sqrt{3}/4)\sin\theta)(-1/2\sin\theta - \sqrt{3}/2\cos\theta)$$

$$3/2\sin\theta\cos\theta + 3/8\cos\theta\sin\theta - 3(\sqrt{3}/8)\cos^2\theta + 3(\sqrt{3}/8)\sin^2\theta$$
$$-9/8\sin\theta\cos\theta + 3/8\cos\theta\sin\theta + 3(\sqrt{3}/8)\cos^2\theta - 3(\sqrt{3}/8)\sin^2\theta$$
$$-9/8\sin\theta\cos\theta = 0 \qquad (12)$$

$$= 0 \qquad (13)$$

The second row of $|A_s|\ |L_2|$ is now multiplied by the first, second and third columns of $|A_r|^{-1}$

$$3/2\sin\theta\cos\theta + ((3\sqrt{3}/4)\cos\theta - 3/4\sin\theta)(-1/2\cos\theta - 3/2\sin\theta)$$
$$+ (-3(\sqrt{3}/4)\cos\theta - 3/4\sin\theta)(-1/2\cos\theta + \sqrt{3}/2\sin\theta)$$
$$3/2\sin\theta\cos\theta + -3(\sqrt{3}/8)\cos^2\theta - 9/8\sin\theta\cos\theta + 3/8\sin\theta\cos\theta$$
$$+ 3(\sqrt{3}/8)\sin^2\theta + 3(\sqrt{3}/8)\cos^2\theta - 9/8\sin\theta\cos\theta + 3/8\sin\theta\cos\theta -$$
$$3(\sqrt{3}/8)\sin^2\theta$$
$$3/2\sin\theta\cos\theta - 3(\sqrt{3}/8)\cos^2\theta - 9/8\sin\theta\cos\theta + 3/8\sin\theta\cos\theta$$
$$+ 3(\sqrt{3}/8)\sin^2\theta + 3(\sqrt{3}/8)\cos^2\theta - 9/8\sin\theta\cos\theta + 3/8\sin\theta\cos\theta - 3(\sqrt{3}/8)\sin^2\theta$$
$$= 0 \qquad (14)$$

$$3/2\sin^2\theta + (3(\sqrt{3}/4)\cos\theta - 3/4\sin\theta)(-1/2\sin\theta + \sqrt{3}/2\cos\theta)$$
$$+ (-3(\sqrt{3}/4)\cos\theta - 3/4\sin\theta)(-1/2\sin\theta - \sqrt{3}/2\cos\theta)$$

$$3/2\sin^2\theta + (3(\sqrt{3}/8)\sin\theta\cos\theta + 9/8\cos^2\theta + 3/8\sin^2\theta -$$

$$3(\sqrt{3}/8)\sin\theta\cos\theta + 3(\sqrt{3}/8)\sin\theta\cos\theta + 9/8\cos^2\theta + 3/8\sin^2\theta + 3(\sqrt{3}/8)\sin\theta\cos\theta$$

$$3/2\sin^2\theta - 3(\sqrt{3}/8)\sin\theta\cos\theta - 9/8\cos^2\theta + 3/8\sin^2\theta - 3(\sqrt{3}/8)\sin\theta\cos\theta$$

$$+ 3(\sqrt{3}/8)\sin\theta\cos\theta + 9/8\cos^2\theta + 3/8\sin^2\theta + 3(\sqrt{3}/8)\sin\theta\cos\theta$$

$$(9/4\sin^2\theta + 9/4\cos^2\theta) = 9/4$$

The third row of $|A_s|\ |L_2|$, multiplied by the first, second and third columns of $|A_r|^{-1}$, gives zero in each case.

Therefore,

$$|A_s|\ |L_2|\ |A_r|^{-1} = \begin{vmatrix} 3/2L_{mr} & 0 & 0 \\ 0 & 3/2L_{mr} & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

Equation (3.31) can be derived in the same manner.

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