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Automated Dynamic Analytical Model Improvement for Damped Structures

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16 Abstract A method is described for improving a linear nonproportionally damped analytical model of a structure. The procedure developed finds the smallest changes in the analytical model such that the improved model matches the measured modal parameters. Features of the method are: ability to properly treat complex-valued modal parameters of a damped system; applicability to realistically large structural models; and computationally efficient without involving eigensolutions and inversion of a large matrix. Although the identified analytical model is a distinct improvement over the original, it does not exactly satisfy the dynamic equation. Further research is necessary to eliminate this deficiency.			
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SUMMARY

A procedure is described for improving a linear nonproportionally damped analytical model of a structure. An approximate analytical model is assumed to be available. Given also is an incomplete set of measured frequencies, damping ratios and complex mode shapes of the structure, as may be obtained from a vibration test. A method is developed which finds the smallest changes in the analytical model such that the improved model matches the measured modal parameters. This report includes a discussion of the problem, the derivation of the algorithms and a description of the computer implementation. An example of an application of the procedure to improve an analytical model is presented.

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INTRODUCTION

With the increasing complexity and accuracy requirements of modern aerospace structures, system identification may be perceived as an attractive alternative to intuitive validation of analytical models. In recent years, within the framework of linear undamped systems, many procedures have been proposed to improve an analytical model by using measured modal parameters. Typical publications which are related to the method developed in this report are represented by Refs. 1-7.

Analytical mass and stiffness matrices are often available from some source, such as a finite element analysis. Certainly, the resulting model is only an approximate representation to the structure and contains errors of undetermined magnitude. On the other hand, vibration tests result in incomplete sets of natural frequencies and mode shapes of the structure. These data also contain errors whose magnitudes are unknown. Assuming it is desired to have an analytical model which will be consistent with the analysis and the test, Berman and Wei⁶ applied the Lagrange multiplier method of Baruch and Bar Itzhack³ to minimize a weighted matrix norm and obtained an improved model which exactly predicted the measured modal data and also minimized the changes in the analytical model.

The acceptability of the original analytical model as a baseline for further analysis depends upon the size of the minimum changes. The requirement for a small modification implies that the analytical model is a good representation of the structure. Evaluation of these changes is based on the user's judgement and statistical information regarding the changes. In contrast with more conventional procedures which require trial-and-error adjustment of the original model to match the test data, this method yields an improved model and information of possible deficiencies in the test data or analytical model. Applications of this procedure to improve the analytical models of large structural systems have been reported^{6,7}.

The procedure mentioned above, among others, is based on the assumption that the system possesses only negligible damping or that the damping is proportional and that the measured modal parameters are real. However, there are cases in which damping effects must be taken into account. For instance, ground vibration tests of a helicopter fuselage have shown that the modal damping ratios of several lowest modes are over 12 percent and the mode shapes are essentially complex⁸. Therefore, it was necessary to refine the previous method so that the more realistic, nonproportionally damped system and the corresponding modal parameters may be equally treated. In the analyses reported hereafter, the system damping can be of any type so long as it can be represented by a real symmetric matrix coefficient of the velocity vector in the dynamic equation.

Primary concerns in the present study are as follows:

1. The method must be applicable to large structural systems. Let the number of degrees of freedom be n and the number of measured modes be m . Typically, n is of the order of hundreds or thousands and m is much less than n . Methods involving extensive iteration or repeated eigensolutions may be prohibitively expensive and the convergence problem would always involve some uncertainty. Methods requiring the solution of sets of equations of order n^2 would also be impractical.

2. Besides the analytical mass (M_A) and stiffness (K_A) matrices, the present procedure requires an analytical damping matrix (C_A). Each of these analytical matrices is to be minimally changed to agree with the test data. It is acknowledged that no general rules currently exist for formulating a damping matrix as are those for mass and stiffness matrices. One way to obtain C_A in the first approximation is to represent it as a proportional matrix which is a linear combination of M_A and K_A . This step, of course, involves considerable engineering judgement and the accuracy may not be high. It should be noted that the procedure will modify this matrix to make it appropriately nonproportional if the modes are complex.

3. Unlike its undamped counterpart, a damped system has complex-valued modal matrices Ω and ϕ . In order to guarantee that the improved matrices are real symmetric, the constraints which must be imposed on the system equations render the problem considerably more complicated than would otherwise be the case.

A method was developed which is applicable to realistically large models, is computationally efficient, and satisfies all the theoretical requirements, except that further improvement of the identified stiffness matrix is required. Further research is necessary to eliminate this deficiency. The modified analytical model is, however, a distinct improvement over the original.

SYMBOLS

A	Matrix defined in Eq. (4)
B	Matrix defined in Eq. (5)
C	System damping matrix
E	Coefficient matrix, Eq. (14)
F	GH, defined in Eq. (27c)
f	Matrix defined in Eq. (15)
f_A	Matrix defined in Eq. (22c)
G	Matrix defined in Eq. (27a)
g	Matrix defined in Eq. (22a)
H	Matrix defined in Eq. (27b)
h	Matrix defined in Eq. (22b)
I	Identity matrix
K	System stiffness matrix
k	$W_K K W_K$
l	Number of test points
M	System mass matrix
m	Number of measured modes
n	Number of degrees of freedom
P	Matrix defined in Eq. (27d)
Q	Matrix defined in Eq. (27e)
R	Real part of ϕ
S	Imaginary part of ϕ
U	Real part of $\phi\Omega$
V	Imaginary part of $\phi\Omega$
W	Weighting matrix
X	Displacement vector, Eq. (1)
Y	Vector of system state variables, Eq. (3)
Z	Matrix defined in Eq. (27f)
α	Matrix consisting of Lagrange multipliers, Eq. (18)
β	Matrix consisting of Lagrange multipliers, Eq. (18)
γ	Matrix consisting of Lagrange multipliers, Eq. (18)
Δ	Matrix defined in Eq. (14b)

ϵ	Weighted matrix norm
Θ	$\gamma - \gamma^T$
Λ	Matrix consisting of Lagrange multipliers, Eq. (14b)
Φ	Rectangular complex mode shape matrix
ϕ	$W_K^{-1} \Phi$
ψ	Lagrangian function
Ω	Diagonal complex matrix consisting of damping ratios and natural frequencies

Superscripts

$(\dot{})$	Differentiation with respect to time
$(\bar{})$	Complex-conjugate of a matrix
T	Matrix transpose
*	Conjugate transpose of a matrix
+	Moore-Penrose inverse of a matrix

Subscripts

A	Analytical matrix
C	System damping
I	Imaginary part
K	System stiffness
M	System mass
R	Real part

THEORETICAL DEVELOPMENT

Theoretical Background

A linear structural system of n degrees of freedom is described by the matrix equation

$$M\ddot{X} + C\dot{X} + KX = 0 \quad (1)$$

where M , C and K are real symmetric constant coefficient matrices. With the state-space representation, Eq. (1) can be rewritten as

$$A\dot{Y} + BY = 0 \quad (2)$$

where

$$Y = \begin{Bmatrix} \dot{X} \\ X \\ X \end{Bmatrix} \quad (3)$$

$$A = \begin{bmatrix} 0 & M \\ M & C \end{bmatrix} \quad (4)$$

$$B = \begin{bmatrix} -M & 0 \\ 0 & K \end{bmatrix} \quad (5)$$

By the use of Eqs. (2) to (5), the following orthogonality relationships are obtained:

$$\begin{bmatrix} \Phi & \Omega \\ \Phi & \Phi \end{bmatrix}^T A \begin{bmatrix} \Phi & \Omega \\ \Phi & \Phi \end{bmatrix} = I \quad (6)$$

$$\begin{bmatrix} \Phi & \Omega \\ \Phi & \Phi \end{bmatrix}^T B \begin{bmatrix} \Phi & \Omega \\ \Phi & \Phi \end{bmatrix} = -\Omega \quad (7)$$

or equivalently,

$$\Phi^T M \Phi \Omega + \Omega \Phi^T M \Phi + \Phi^T C \Phi = I \quad (6a)$$

and

$$\Omega \Phi^T M \Phi \Omega - \Phi^T K \Phi = \Omega \quad (7a)$$

where Ω is a complex-valued diagonal matrix containing modal damping ratios (real part) and natural frequencies (imaginary part) and Φ is the corresponding mode shape matrix. In this analysis Φ is a rectangular matrix and Ω is diagonal, of order m . With these modal parameters, the dynamic equation (1) can be cast in the form of

$$M \Phi \Omega^2 + C \Phi \Omega + K \Phi = 0 \quad (8a)$$

Given Ω and Φ , it can be shown that if M , C and K satisfy Eqs. (8a) and (6) then they also satisfy Eq. (7). In other words, Eqs. (8a) and (6) imply Eq. (7), but Eqs. (8a) and (7) do not imply Eq. (6) unless Ω is nonsingular. Therefore, Eqs. (8a) and (6) have been selected to improve the analytical model since it is desired to include rigid body modes if the system has free boundary conditions.

Intuitively, it might seem appropriate to improve M , C and K by directly identifying A and B of Eqs. (4) and (5). Unfortunately, this cannot be done. The reason is that the optimization techniques would distribute variations to all variables involved and, for example, the upper-left portion of the modified matrix will no longer be precisely zero, which is certainly unacceptable. Thus, it is necessary to take the more difficult analytical approach of specifically identifying the physical matrices.

Full Mode Computation

Since in practice, the degrees of freedom measured in a vibration test are only a small portion of those specified in the analytical model, an interpolation procedure is employed to compute the modal displacements at the unmeasured degrees of freedom. This preprocessing procedure, referred to as "full-mode computation," has been discussed in Ref. 6. A slight modification is made here to include the system damping.

Let ϕ_i be the i th column of the mode shape matrix Φ . The portion of ϕ_i whose elements are measured is denoted by ϕ_{1i} and the rest by ϕ_{2i} . With this coordinate transformation, Eq. (8a) can be written as:

$$\left(\Omega_i^2 \begin{bmatrix} M_1 & M_2 \\ M_2^T & M_4 \end{bmatrix} + \Omega_i \begin{bmatrix} C_1 & C_2 \\ C_2^T & C_4 \end{bmatrix} + \begin{bmatrix} K_1 & K_2 \\ K_2^T & K_4 \end{bmatrix} \right) \begin{Bmatrix} \phi_{1i} \\ \phi_{2i} \end{Bmatrix} = 0 \quad (8b)$$

where, for the i th mode, Ω_i is the measured eigenvalue and M , C and K are partitioned as shown. At the start of the identification procedure, M , C and K are the analytical matrices M_A , C_A and K_A . It is apparent from Eq. (8b) that

$$\phi_{2i} = - (\Omega_i^2 M_4 + \Omega_i C_4 + K_4)^{-1} (\Omega_i^2 M_2^T + \Omega_i C_2^T + K_2^T) \phi_{1i} \quad (9)$$

Because analytical matrices are involved in the equation, the accuracy of ϕ_{2i} solved from Eq. (9) is expected to compare to that of M_A , C_A , or K_A , even if ϕ_{1i} is exact. Note that even if ϕ_{2i} is inexact, the improved model should exactly predict ϕ_{1i} , which is the only data actually measured. The first step in the procedure is to compute the full modes from Eq. (9). The following analysis uses the full, but incomplete, modal matrices $\Phi(n \times m)$ and $\Omega(m \times m)$.

Improvement of Mass and Damping Matrices

The basic equation to be used for improving analytical mass and damping matrices is the orthogonality relationship, Eq. (6a). Since $m < n$, given Φ and Ω , there are infinite sets of $M(n \times n)$ and $C(n \times n)$ satisfying Eq. (6a). A unique set of mass and damping matrices will result if, in addition to satisfying the orthogonality relationship, a norm

$$\epsilon = ||W_M (M - M_A) W_M|| + ||W_C (C - C_A) W_C|| \quad (10)$$

is also minimized, where $W_M(n \times n)$ and $W_C(n \times n)$ are arbitrary real symmetric, nonsingular weighting matrices. Let $\Phi = R + iS$ and $\Phi\Omega = U + iV$; then the real and imaginary parts of (6a) are separated into

$$R^T M U + U^T M R - S^T M V - V^T M S + R^T C R - S^T C S = I \quad (11a)$$

$$R^T M V + V^T M R + S^T M U + U^T M S + R^T C S + S^T C R = 0 \quad (11b)$$

Define the Lagrangian function

$$\begin{aligned} \Psi = \epsilon + \sum_{i=1}^m \sum_{j=1}^m (\Lambda_R)_{ij} (R^T M U + U^T M R - S^T M V - V^T M S + R^T C R - S^T C S - I)_{ij} \\ + \sum_{i=1}^m \sum_{j=1}^m (\Lambda_I)_{ij} (R^T M V + V^T M R + S^T M U + U^T M S + R^T C S + S^T C R)_{ij} \end{aligned} \quad (12)$$

where Λ_R and Λ_I are Lagrangian multipliers. Differentiating Ψ with respect to M and C , respectively, and equating the results to 0 yield

$$\begin{aligned} M = M_A - \frac{1}{2} W_M^{-2} (R \Lambda_R U^T + U \Lambda_R R^T - S \Lambda_R V^T - V \Lambda_R S^T + R \Lambda_I V^T + V \Lambda_I R^T \\ + S \Lambda_I U^T + U \Lambda_I S^T) W_M^{-2} \end{aligned} \quad (13a)$$

$$C = C_A - \frac{1}{2} W_C^{-2} (R \Lambda_R R^T - S \Lambda_R S^T + R \Lambda_I S^T + S \Lambda_I R^T) W_C^{-2} \quad (13b)$$

Obviously, requiring Λ_R and Λ_I to be symmetric in Eqs. (13a, b) is equivalent to imposing the constraints of M and C to be symmetric in Eqs. (11a, 11b). The Lagrange multipliers $(\Lambda_R)_{ij}$ and $(\Lambda_I)_{ij}$ may be solved numerically from the following set of equations obtained by substituting Eqs. (13a, b) into Eqs. (11a, b),

$$\left[\begin{array}{c|c} (E_{11})_{ik\ell j} & (E_{12})_{ik\ell j} \\ \hline (E_{21})_{ik\ell j} & (E_{22})_{ik\ell j} \end{array} \right] \left\{ \begin{array}{c} (\Lambda_R)_{k\ell} \\ (\Lambda_I)_{k\ell} \end{array} \right\} = \left\{ \begin{array}{c} (\Delta_R)_{ij} \\ (\Delta_I)_{ij} \end{array} \right\} \quad (14a)$$

which, for simplification, may be written as

$$E \Lambda = \Delta \quad (14b)$$

where

$$\Delta_R = 2 (I - R^T M_A U - U^T M_A R + S^T M_A V + V^T M_A S - R^T C_A R + S^T C_A S) \quad (14c)$$

$$\Delta_I = 2 (R^T M_A V + V^T M_A R + S^T M_A U + U^T M_A S + R^T C_A S + S^T C_A R) \quad (14d)$$

The submatrices E_{11} , E_{12} , E_{21} and E_{22} are functions of R , S , U , V , W_M^{-2} and W_C^{-2} and their explicit function forms are given in the Appendix. In Eq. (14), a repeated index in a product denotes summation over the range of the index.

Since Λ_R and Λ_I are symmetric, there are $m(m+1)$ unknowns along with $m(m+1)$ equations in Eq. (14). In the implementation of the technique, the coefficient matrix E which has $m(m+1) \times m(m+1)$ elements is formulated by a FORTRAN program. The improved mass and damping matrices, M and C , result when Λ_R and Λ_I are solved from Eq. (14) and are numerically substituted into Eqs. (13a, b).

In this section, a numerical method has been used to solve $m(m+1)$ equations. In practice the number of measured modes is small enough to make this solution economically practical.

Improvement of Stiffness Matrix

Given the full modal matrices Φ and Ω , the improved M and C , and the analytical stiffness matrix K_A , an improved stiffness matrix K must satisfy the dynamic equation

$$\Phi^T k = - (\Omega^2 \Phi^T M + \Omega \Phi^T C) W_K \quad (15)$$

or

$$\Phi^T k = f \quad (15a)$$

and the constraints

$$k = k^* \quad (16a)$$

$$k = k^T \quad (16b)$$

in which an asterisk denotes the conjugate transpose of a matrix, $\phi = W_K^{-1} \Phi$, $k = W_K K W_K$ and W_K ($n \times n$) is an arbitrary, real symmetric, nonsingular weighting matrix. The constraints, Eqs. (16a, b), guarantee the improved stiffness

matrix to be real symmetric. A unique set of K will result if k satisfies Eqs. (15), (16a,b) and also minimizes the norm

$$\epsilon = ||W_K (K - K_A) W_K|| = ||k - k_A|| \quad (17)$$

Define

$$\begin{aligned} \Psi = \epsilon + \sum_{i=1}^m \sum_{j=1}^n & \left[\alpha_{ij} (\phi^T k - f)_{ij} + \alpha_{ij}^* (\phi^T k - f)_{ij}^* \right] \\ & + \sum_{i=1}^n \sum_{j=1}^n \left[\beta_{ij} (k - k^*)_{ij} + \beta_{ij}^* (k^* - k)_{ij} + \gamma_{ij} (k - k^T)_{ij} \right. \\ & \left. + \gamma_{ij}^* (k - k^T)_{ij}^* \right] \end{aligned} \quad (18)$$

Note that it is no longer practical to solve the equations numerically for the Lagrange multipliers β and γ because their order is n . A closed-form representation for the improved stiffness matrix is necessary. Differentiating Ψ with respect to k and equating the result to zero yield

$$(k - k_A)^* + \phi\alpha + \beta - \beta^* + \gamma - \gamma^T = 0 \quad (19)$$

The reason for defining Lagrangian function in the form of Eq. (18) is now clear since if Ψ is differentiated with respect to k^* rather than k , the same equation, Eq. (19), is obtained. Taking the conjugate transpose of Eq. (19) and adding it to Eq. (19) result in

$$2(k - k_A) + \phi\alpha + (\phi\alpha)^* + \theta + \theta^* = 0 \quad (20)$$

where $\theta = \gamma - \gamma^T$ and the relationship $k - k_A = (k - k_A)^*$ has been used in the equation. Premultiplying Eq. (20) by ϕ^T , applying Eq. (15), and noting that $\phi^T \phi$ is nonsingular, then

$$\alpha = -(\phi^T \phi)^{-1} \left[2(f - f_A) + \phi^T (\theta + \theta^*) \right] - (\phi^T \phi)^{-1} \phi^T \alpha^* \phi^* \quad (21)$$

The Lagrange multipliers α and α^* can be eliminated by substituting Eq. (21) into Eq. (20) which, after some algebraic manipulations, results in

$$k = k_A + g + h \quad (22)$$

where

$$g = \phi (\phi^T \phi)^{-1} (f - f_A) + \left[\phi (\phi^T \phi)^{-1} (f - f_A) \right]^* - \frac{1}{2} \phi (\phi^T \phi)^{-1} \left\{ \phi^T (f - f_A)^* + \left[\phi^T (f - f_A)^* \right]^* \right\} \phi (\phi^T \phi)^{-1} \quad (22a)$$

$$h = -\frac{1}{2} \left[I - \phi (\phi^T \phi)^{-1} \phi^T \right] (\theta + \theta^*) \left[I - \phi (\phi^T \phi)^{-1} \phi^T \right]^* \quad (22b)$$

$$f_A = \phi^T k_A \quad (22c)$$

Applying the symmetric condition Eq. (16b) to Eq. (22) yields

$$g + h = g^T + h^T \quad (23)$$

Premultiply and postmultiply Eq. (23) by ϕ^* and ϕ , respectively, and from Eq. (22b), note that $h^T \phi = 0$. Then

$$\phi^* h \phi = \phi^* (g^T - g) \phi = Q \quad (24)$$

Solving Eq. (24) for $\theta + \theta^*$

$$W_K^{-1} (\theta + \theta^*) W_K^{-1} = -2 (\phi^* Z)^+ Q \left[(\phi^* Z)^+ \right]^* \quad (25)$$

in which $(\phi^* Z)^+$ is the Moore-Penrose inverse⁹ of $\phi^* Z$.

Therefore, the improved stiffness matrix becomes

$$K = K_A + \left[(Z^* \phi)^+ Z^* \right]^* Q (Z^* \phi)^+ Z^* - W_K^{-2} F - (W_K^{-2} F)^* + \frac{1}{2} W_K^{-2} P W_K^{-2} \quad (26)$$

where

$$G = \phi (\phi^T W_K^{-2} \phi)^{-1} \quad (27a)$$

$$H = \Omega^2 \phi^T M + \Omega \phi^T C + \phi^T K_A \quad (27b)$$

$$F = G H \quad (27c)$$

$$P = G \left[\phi^T H^* + (\phi^T H^*)^* \right] G^* \quad (27d)$$

$$Q = \phi^* \left\{ W_K^{-2} (F - \bar{F}) + \left[W_K^{-2} (F - \bar{F}) \right]^* + \frac{1}{2} W_K^{-2} (P^T - P) W_K^{-2} \right\} \phi \quad (27e)$$

$$Z = I - W_K^{-2} G \phi^T \quad (27f)$$

The matrix inversions required for the evaluation of the improved damping matrix are W_K^{-2} and $(\phi^T W_K^{-2} \phi)^{-1}$. Neither will cause difficulty since the first one is a chosen weighting matrix and the other is of order m .

Two important features are related to the selection of weighting matrices. First, the present identification is based on the minimum-norm approach, thus, the chosen weighting matrices reflect, in a sense, the confidence level in each element of the original analytical model. Second, the improved mass and damping matrices are used to identify the stiffness matrix. For large structural systems, it is quite impractical to specify confidence levels for all elements of system matrices. A convenient and physically reasonable choice is to use $M_A^{-1/2}$ and $C_A^{-1/2}$ as the weighting matrices for mass and damping identification and then $M^{-1/2}$ for stiffness identification. Other options, provided in the "Computer Implementation" (see page 19), are identity matrix and user supplied diagonal matrices.

Note that the analysis up to Eq. (23) is exact. Numerical solution for θ from Eq. (23) would be prohibitive because the unknown matrix is of order n . Eq. (24) is obtained through pre- and post-multiplying h by ϕ^* and ϕ . The rank of $\phi^* h \phi$ in Eq. (24) is at most m (the rank of ϕ) which is in general less than that of h . It is believed that the analysis is only approximate after Eq. (24) because of the reduction in rank due to the pre and post multiplications. The inexact orthogonality check in the example are explained by this consideration. However, it appears that a similar procedure is exact in case of undamped system identification¹⁰. Further research in this area is required.

COMPUTER IMPLEMENTATION

General Description

Fig. 1 summarizes the computer program designed for improving mass, damping and stiffness matrices described in this report. A separate program, AMIR, is used to reorder analytical matrices on sequential files to place test degrees of freedom in the upper left of each matrix (see Eq. (8b)) before invoking the present program.

The program performs the following functions:

(1) Input and validate data. The data input include n , m , λ , test modal parameters (natural frequencies, damping ratios and mode shapes) and weighting matrices (optional).

(2) Full mode computation. If $m < n$, Eq. (9) is solved to compute the full modes. A user option is available to ignore the frequency dependent terms (Guyan reduction) or using LDL^T (see Ref. 6) decomposition to solve Eq. (9) exactly.

(3) Normalize the mode shapes by using the orthogonality relationship, Eq. (6a).

(4) Formulate E and Δ in Eq. (14) and solve for Lagrange multipliers. The CPU time spent in performing this job depends on the number of measured modes, m , and may be the most time consuming step if m is not small.

(5) Compute the improved mass and damping matrices by using Eq. (13) and check the orthogonality relationship, Eq. (6).

(6) Compute the improved stiffness matrix by using Eqs. (26) and (27) and check the orthogonality relationship, Eq. (7).

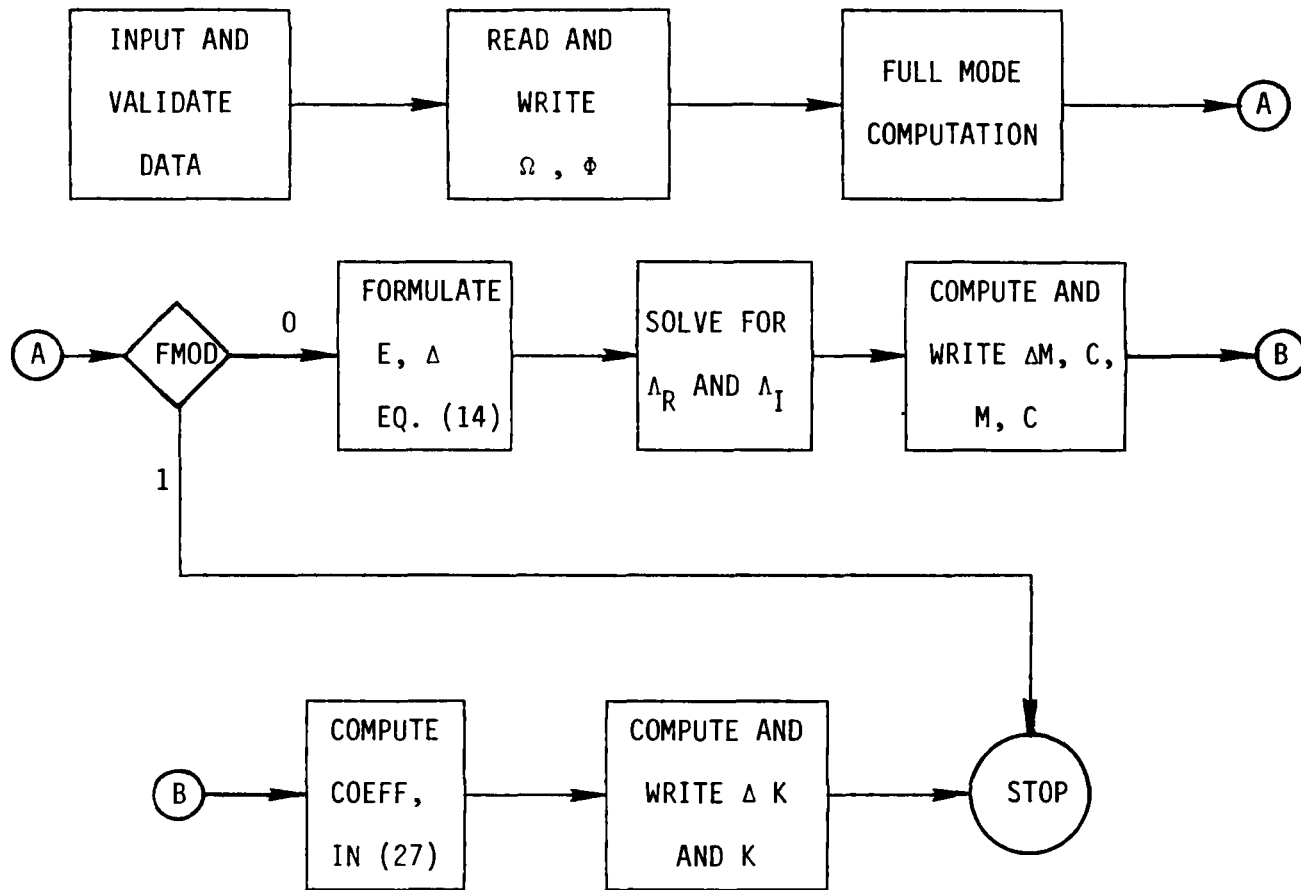


Figure 1. Program AMIMP.

(7) The improved mass, damping and stiffness matrices are listed and placed on sequential files and statistical data is computed and listed. This data consists of: RMS of original matrix, RMS of changes, the ratio of the preceding, the mean absolute ratio of the diagonal changes to the original diagonals, the RMS of the changes divided by the corresponding diagonal elements, i.e., the square root of the mean of

$$\left(\sum_{i,j} (M - M_A)_{ij}^2 / (M_{A_{ii}} \times M_{A_{jj}}) \right)^{1/2}.$$

In addition, the 50 largest changes are printed.

Fourteen sequential files are used in the program, one for the input data, 3 for analytical matrices (M_A , C_A , K_A), 3 for improved matrices (M , C , K) and the rest are working files. At the completion of the implementation, the analytical matrices will be preserved and three files will contain the improved matrices.

Numerical Schemes

(1) The LDL^T decomposition used to compute full modes is the same as that described in Ref. 6. This algorithm performs lower-diagonal-lower transpose decomposition of a symmetric matrix and is designed for large matrices which do not fit in core. For details, see Ref. 6.

(2) E in Eq. (14b) is a full matrix containing $m(m+1) \times m(m+1)$ elements. Let $E(p,q)$ be the (p,q) th element of matrix E and $(E_{rs})_{ik\ell j}$ be the (i, k, ℓ, j) th element of E_{rs} ($r=1,2; s=1,2$) as shown in the Appendix. Then the coefficient matrix E is formulated as follows

$$E(p,q) = (E_{rs})_{ik\ell j}$$

where

$$\begin{aligned} p &= (i(i-1)/2) + j && \text{for } r=1 \\ &= (i(i-1)/2) + j + m(m+1)/2 && \text{for } r=2 \end{aligned}$$

$$\begin{aligned}
q &= (k(k-1)/2) + \ell && \text{for } s=1 \\
&= (k(k-1)/2) + \ell + m(m+1)/2 && \text{for } s=2
\end{aligned}$$

$$i, j, k, \ell = 1 \text{ to } m$$

The vector Δ in Eq. (14b) contains $m(m+1)$ elements. If $\Delta(p)$ denotes the p th element of Δ and $(\Delta_R)_{ij}$, $(\Delta_I)_{ij}$ the (i,j) th element of Δ_R , Δ_I respectively, then

$$\Delta(p) = (\Delta_R)_{ij} \text{ or } (\Delta_I)_{ij}$$

where

$$\begin{aligned}
p &= (i(i-1)/2) + j && \text{for } \Delta(p) = (\Delta_R)_{ij} \\
&= (i(i-1)/2) + j + m(m+1)/2 && \text{for } \Delta(p) = (\Delta_I)_{ij}
\end{aligned}$$

An LU decomposition algorithm is then applied to solve Λ from Eq. (14b).

(3) The following algorithm (Ref. 11) is used to compute the Moore-Penrose inverse (A^+) of an arbitrary rectangular matrix A ($r \times s$), $r \geq s$.

1. Compute $C=A^*A$. Note that the size of $C(s \times s)$ is no larger than that of $A(r \times s)$.
2. Use LU decomposition to find nonsingular matrices P and Q so that

$$PC^2Q = I_0 = \left[\begin{array}{c|c} I & 0 \\ \hline 0 & 0 \end{array} \right]$$

where I is the identity matrix of the rank of C^2 .

3. Compute $S = QI_0P$.
4. Compute $C^+ = (CS)(CS)C$.

5. Compute $A^+ = C^+A^*$.

Since $(A^T)^+ = (A^+)^T$, it is A^T , rather than A , which must be used to compute A^+ for A ($r \times s$) in steps 1 to 5, if $r < s$.

User Options

There are some user options in the program:

(1) Input/Output

ITYP = 0 normal input

1 special NASTRAN input

IRITE = 0 no matrix output

1 diagonal element output, with ratio changes

2 full matrices (analytical and changes)

(2) Analytical damping matrix

ICOP = 1 user supplied matrix

2 proportional damping matrix (can be null matrix)

If ICOP = 2, then $C_A = COEM \times M_A + COEK \times K_A$, where COEM and COEK are user supplied coefficients.

(3) Full mode computation

FDEP = 0 frequency independent (Guyan reduction)

= 1 frequency dependent

FMOD = 0 full program execution

= 1 stop when the mode computation finishes

(4) Weighting matrices

IWM = 0 $W_M = I$

= 1 $W_M^{-2} = M_A$

= 2 $W_M^{-2} =$ user supplied diagonal matrix

IWC = 0 $W_C = I$

= 1 $W_C^{-2} = C_A$

= 2 $W_C^{-2} =$ user supplied diagonal matrix

IWK = 0 $W_K = I$

= 1 $W_K^{-2} = M$

= 2 $W_K^{-2} =$ user supplied diagonal matrix

A NUMERICAL EXAMPLE

A 10 DOF lumped mass model and the corresponding analytical mass, stiffness and damping matrices are shown in Fig. 2. Eigenvalues and eigenvectors of the system were obtained by using Kaman's computer program, DYSCO. Fig. 3 lists computed eigensolutions of the first two modes ($m=2$) with five test points ($l=5$). The "measured" modes Ω and Φ were simulated by perturbing up to 10% the computed Ω and the real part of computed Φ and multiplying the imaginary part of computed Φ by 100. With Ω , Φ , M_A , C_A and K_A , the procedure described in this report was then applied to improve the analytical model. The results are shown in Figs. 4 and 5. In Fig. 4, the diagonal elements of M_A and M and the percentage change of these diagonal elements are listed on the first three columns. Columns 4 and 5 are maximum absolute changes of the elements beyond the diagonal element in each row and the percentage change with respect to the corresponding diagonal element in M_A . In this example, both the maximum percentage change of column 3 (28%) and column 5 (33%) appear in the 3rd row of the mass matrix. For the damping matrix, the maximum percentage change of the diagonal elements is 8.5%. Three columns (9 to 11) which list the sub-diagonal element in each row of C_A and C and the percentage change with respect to corresponding C_A element are also included. Columns 12 and 13 give the maximum absolute change of element beyond the diagonal and sub-diagonal ones in each row. Similarly, percentage changes of the diagonal, sub-diagonal elements and the maximum change in each row are given in Fig. 5 for stiffness matrix.

Given in Fig. 6 is a partial validation of improved model by checking the orthogonality relationships, Eqs. (6a) and (7a). As shown in the table, the improved mass and damping matrices excellently satisfy Eq. (6a) with numerical errors of order 10^{-8} . The improved mass and stiffness matrices, however, do not satisfy Eq. (7a) as well. The resultant matrix is very close to the measured Ω except that the second damping ratio here reads -0.01414 rather than 0.00916. The off-diagonal terms are in error by about 10^{-3} . In this regard, improvement of stiffness matrix is only partially successful.

An eigenanalysis has been performed on the improved model. For comparison, Fig. 7 lists the first two modes of the improved model, as well as the measured modes duplicated from Fig. 3. It is noted that the imaginary part of Ω and real part of Φ of the improved model are almost identical to their measured correspondence. However, the real part of Ω and imaginary part of Φ are in error. Again, this is attributable to an imperfect improvement of the stiffness matrix.

$$n = 10, m = 2, l = 5$$

COMPUTED MODES

MEASURED MODES

$$\Omega = \begin{bmatrix} -0.002486 + 0.4986i & \\ & -0.009555 + 0.9774i \end{bmatrix} \quad \Omega = \begin{bmatrix} -0.002586 + 0.5286i & \\ & -0.009155 + 0.9374i \end{bmatrix}$$

$$\Phi = \begin{bmatrix} 0.1119-1.46E-6i & -0.2159-1.6E-7i \\ 0.3301-3.99E-6i & -0.6064-4.2E-7i \\ 0.7008-6.54E-6i & -0.9239-4.7E-7i \\ 0.8513-6.34E-6i & -0.9062-3.9E-7i \\ 0.9830-2.35E-6i & -0.1779+9.0E-8i \end{bmatrix} \quad \Phi = \begin{bmatrix} 0.1189-1.46E-4i & -0.2259-1.6E-5i \\ 0.3401-3.99E-4i & -0.6364-4.2E-5i \\ 0.6808-6.54E-4i & -0.9539-4.7E-5i \\ 0.8913-6.34E-4i & -0.8762-3.9E-5i \\ 0.9530-2.35E-4i & -0.1879+9.0E-6i \end{bmatrix}$$

Figure 3. Computed and measured (perturbed) modes.

MASS					DAMPING							
M_A	DIA. M	%	MAX. Δ	%	DIA. C_A	DIA. C	%	SUB. C_A	SUB. C	%	MAX. Δ	%
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
2	2.022	1.1	0.129	6.5	.6	.601	1.4				.005	1
4	4.175	4.4	0.367	9.2	.3	.307	2.3	-.2	-.198	1	.013	5
2	2.560	28	0.654	33	.3	.320	6.8	-.1	-.088	12	-.017	-6
4	4.763	19	0.654	16	.3	.325	8.5	-.2	-.177	12	-.021	-7
2	2.502	25	0.626	31	.3	.312	4.0	-.1	-.082	18	-.017	-6
4	3.985	-.1	-.409	-10	.3	.298	-1	-.2	-.198	1	-.009	-3
2	1.767	-12	-.345	-17	.3	.301	0.7	-.1	-.108	-8	-.015	-5
4	3.913	-2	0.521	-13	.3	.310	3.4	-.2	-.195	2.5	-.021	-7
2	2.006	0.3	-.308	-15	.3	.304	1.3	-.1	-.094	6	-.012	-4
4	4.003	0.1	-.123	-3	.6	.601	1.0	-.2	-.198	1	-.005	1

Figure 4. Improved mass and damping matrices.

STIFFNESS							
DIA. K_A (1)	DIA. K (2)	% (3)	SUB. K_A (4)	SUB. K (5)	% (6)	MAX. Δ (7)	% (8)
30	30.05	.2				-.24	-1
15	15.43	3	-10	-9.85	2	-.63	-4
15	16.18	8	-5	-4.33	13	-.52	-3
15	15.34	2	-10	-9.23	8	-.42	3
15	15.32	2	-5	-5.55	-11	-.63	-4
15	15.19	1	-10	-9.31	7	-.49	-3
15	14.82	-1	-5	-5.33	-7	.60	4
15	15.03	.2	-10	-10.1	-1	.53	4
15	15.05	.3	-5	-4.95	1	-.28	-2
30	30.01	.03	-10	-9.98	.2	-.11	.4

Figure 5. Improved stiffness matrix.

Element	$\phi^T M \phi \Omega + \Omega \phi^T M \phi + \phi^T C \phi$		$\Omega \phi^T M \phi \Omega - \phi^T K \phi$	
	Real	Imaginary	Real	Imaginary
1,1	1.0 (1)*	-1.3×10^{-8} (0)	$-2.538 (-2.537) \times 10^{-3}$	$5.286 (5.286) \times 10^{-1}$
1,2	7.2×10^{-9} (0)	5.8×10^{-10} (0)	3.263×10^{-3} (0)	8.24×10^{-6} (0)
2,1	1.7×10^{-8} (0)	-1.1×10^{-9} (0)	3.263×10^{-3} (0)	8.24×10^{-6} (0)
2,2	1.0 (1)	-3.6×10^{-9} (0)	$-1.414 (0.9155) \times 10^{-2}$	$9.374 (9.374) \times 10^{-1}$

*Exact values are given in parentheses.

Figure 6. Orthogonality check for the improved model.

MEASURED MODES

FIRST TWO MODES OF IMPROVED MODEL

$$\Omega = \begin{bmatrix} -0.002586 + 0.5286i & \\ & -0.009155 + 0.9374i \end{bmatrix}$$

$$\Omega = \begin{bmatrix} -0.002537 + 0.5286i & \\ & -0.01415 + 0.9374i \end{bmatrix}$$

$$\Phi = \begin{bmatrix} 0.1189-1.46E-4i & -0.2259-1.6E-5i \\ 0.3401-3.99E-4i & -0.6364-4.2E-5i \\ 0.6808-6.54E-4i & -0.9539-4.7E-5i \\ 0.8913-6.34E-4i & -0.8762-3.9E-5i \\ 0.9530-2.35E-4i & -0.1879+9.0E-6i \end{bmatrix}$$

$$\Phi = \begin{bmatrix} 0.1189-1.29E-3i & -0.2258-.791E-2i \\ 0.3400-3.49E-3i & -0.6363-2.28E-2i \\ 0.6807-5.58E-3i & -0.9539-3.81E-2i \\ 0.8913-6.59E-3i & -0.8764-4.57E-2i \\ 0.9530-3.81E-3i & -0.1887-2.30E-2i \end{bmatrix}$$

Figure 7. Measured modes vs modes of improved model.

CONCLUDING REMARKS

A method is developed to find the smallest changes in the analytical model of a nonproportionally damped structure so that the improved model is compatible with the test modal parameters. The only assumptions about the system damping are that it is viscous and can be represented by a real symmetric matrix. The reported procedure aims at applications to large structural systems and has used methods which are numerically feasible.

The following conclusions can be drawn from previous discussion:

1) Given an incomplete set of modal parameters and an analytical model of a structure, application of this procedure will lead to an improved model. Changes between two models serve as a useful guideline for the validity of the original model being a good representation of the structure. The engineering judgement required using this approach is to establish acceptable levels of change. This procedure is, therefore, efficient and objective.

(2) A numerical experiment using arbitrary perturbations in modal parameters found no numerical sensitivities.

(3) The improved stiffness matrix obtained from the present procedure, although better than the original one, is considered not completely satisfactory since M and C satisfy the orthogonality relationship, Eq. (6a), but M and K do not exactly fit the orthogonality relationship, Eq. (7a). Further research is required to overcome this deficiency.

(4) The improved matrices are filled using the present procedure and may imply non-existent coupling. This effect may be insignificant if the resulting coupling terms are small. Additional constraints are required and further research is necessary to keep this problem of a solvable size.

APPENDIX

Explicit Function Forms of Coefficients in Eq. (14)

$$(E_{11})_{iklj} = (e_{11})_{iklj} + (1-\delta_{kl}) (e_{11})_{ilkj}$$

$$(E_{12})_{iklj} = (e_{12})_{iklj} + (1-\delta_{kl}) (e_{12})_{ilkj}$$

$$(E_{21})_{iklj} = (e_{21})_{iklj} + (1-\delta_{kl}) (e_{21})_{ilkj}$$

$$(E_{22})_{iklj} = (e_{22})_{iklj} + (1-\delta_{kl}) (e_{22})_{ilkj}$$

$$(e_{11})_{ilkj} =$$

$$\begin{aligned} & - (M_{RR})_{ik} (M_{UU})_{lj} - (M_{RU})_{ik} (M_{RU})_{lj} + (M_{RS})_{ik} (M_{VU})_{lj} + (M_{RV})_{ik} (M_{SU})_{lj} \\ & - (M_{UR})_{ik} (M_{UR})_{lj} - (M_{UU})_{ik} (M_{RR})_{lj} + (M_{US})_{ik} (M_{VR})_{lj} + (M_{UV})_{ik} (M_{SR})_{lj} \\ & + (M_{SR})_{ik} (M_{UV})_{lj} + (M_{SU})_{ik} (M_{RV})_{lj} - (M_{SS})_{ik} (M_{VV})_{lj} - (M_{SV})_{ik} (M_{SV})_{lj} \\ & + (M_{VR})_{ik} (M_{US})_{lj} + (M_{VU})_{ik} (M_{RS})_{lj} - (M_{VS})_{ik} (M_{VS})_{lj} - (M_{VV})_{ik} (M_{SS})_{lj} \\ & - (C_{RR})_{ik} (C_{RR})_{lj} + (C_{RS})_{ik} (C_{SR})_{lj} + (C_{SR})_{ik} (C_{RS})_{lj} - (C_{SS})_{ik} (C_{SS})_{lj} \end{aligned}$$

$$(e_{12})_{ilkj} =$$

$$\begin{aligned} & - (M_{RR})_{ik} (M_{VU})_{lj} - (M_{RV})_{ik} (M_{RU})_{lj} - (M_{RS})_{ik} (M_{UU})_{lj} - (M_{RU})_{ik} (M_{SU})_{lj} \\ & - (M_{UR})_{ik} (M_{VR})_{lj} - (M_{UV})_{ik} (M_{RR})_{lj} - (M_{US})_{ik} (M_{UR})_{lj} - (M_{UU})_{ik} (M_{SR})_{lj} \\ & + (M_{SR})_{ik} (M_{VV})_{lj} + (M_{SV})_{ik} (M_{RV})_{lj} + (M_{SS})_{ik} (M_{UV})_{lj} + (M_{SU})_{ik} (M_{SV})_{lj} \\ & + (M_{VR})_{ik} (M_{VS})_{lj} + (M_{VV})_{ik} (M_{RS})_{lj} + (M_{VS})_{ik} (M_{US})_{lj} + (M_{VU})_{ik} (M_{SS})_{lj} \\ & - (C_{RR})_{ik} (C_{SR})_{lj} - (C_{RS})_{ik} (C_{RR})_{lj} + (C_{SR})_{ik} (C_{SS})_{lj} + (C_{SS})_{ik} (C_{RS})_{lj} \end{aligned}$$

$$(e_{21})_{iklj} =$$

$$\begin{aligned} & (M_{RR})_{ik} (M_{UV})_{lj} + (M_{RU})_{ik} (M_{RV})_{lj} - (M_{RS})_{ik} (M_{VV})_{lj} - (M_{RV})_{ik} (M_{SV})_{lj} \\ & + (M_{VR})_{ik} (M_{UR})_{lj} + (M_{VU})_{ik} (M_{RR})_{lj} - (M_{VS})_{ik} (M_{VR})_{lj} - (M_{VV})_{ik} (M_{SR})_{lj} \\ & + (M_{SR})_{ik} (M_{UU})_{lj} + (M_{SU})_{ik} (M_{RU})_{lj} - (M_{SS})_{ik} (M_{VU})_{lj} - (M_{SV})_{ik} (M_{SU})_{lj} \\ & + (M_{UR})_{ik} (M_{US})_{lj} + (M_{UU})_{ik} (M_{RS})_{lj} - (M_{US})_{ik} (M_{VS})_{lj} - (M_{UV})_{ik} (M_{SS})_{lj} \\ & + (C_{RR})_{ik} (C_{RS})_{lj} - (C_{RS})_{ik} (C_{SS})_{lj} + (C_{SR})_{ik} (C_{RR})_{lj} - (C_{SS})_{ik} (C_{SR})_{lj} \end{aligned}$$

$$(e_{22})_{iklj} =$$

$$\begin{aligned} & (M_{RR})_{ik} (M_{VV})_{lj} + (M_{RV})_{ik} (M_{RV})_{lj} + (M_{RS})_{ik} (M_{UV})_{lj} + (M_{RU})_{ik} (M_{SV})_{lj} \\ & + (M_{VR})_{ik} (M_{VR})_{lj} + (M_{VV})_{ik} (M_{RR})_{lj} + (M_{VS})_{ik} (M_{UR})_{lj} + (M_{VU})_{ik} (M_{SR})_{lj} \\ & + (M_{SR})_{ik} (M_{VU})_{lj} + (M_{SV})_{ik} (M_{RU})_{lj} + (M_{SS})_{ik} (M_{UU})_{lj} + (M_{SU})_{ik} (M_{SU})_{lj} \\ & + (M_{UR})_{ik} (M_{VS})_{lj} + (M_{UV})_{ik} (M_{RS})_{lj} + (M_{US})_{ik} (M_{US})_{lj} + (M_{UU})_{ik} (M_{SS})_{lj} \\ & + (C_{RR})_{ik} (C_{SS})_{lj} + (C_{RS})_{ik} (C_{RS})_{lj} + (C_{SR})_{ik} (C_{SR})_{lj} + (C_{SS})_{ik} (C_{RR})_{lj} \end{aligned}$$

where

$$\delta_{k\ell} = \text{Dirac delta function}$$

and for P, Q denoting R, S, U, or V,

$$M_{PQ} = P^T W_M^{-2} Q$$

$$C_{PQ} = P^T W_C^{-2} Q$$

Note that $M_{PQ} = M_{QP}^T$ and $C_{PQ} = C_{QP}^T$.

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