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DIFFUSION-CONVECTION FUNCTION OF COSMIC RAYS

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ABSTRACT

This paper presents the fundamental properties and some numerical results of the solution of the diffusion equation of an impulsive cosmic-ray point source in an uniform, unbounded and spherically symmetrical moving medium.

I. INTRODUCTION The diffusion-convection(D-C) function is an elementary composite function of the solution of the D-C equation for the particles injected impulsively from a diffusive point source into a uniform, unbounded and spherically symmetrical medium. It is the analytic solution derived by the dimensional method for the propagation equation of solar cosmic rays in the heliosphere, i.e. the interplanetary space.[1] Because of the introduction of convection effect of solar wind, a nonhomogeneous term appears in the propagation equation, it is difficult to express its solution in terms of the ordinary special functions. The research made so far has led to a solution containing only the first order approximation of the convection effect. Undoubtedly it is insufficient to study only the propagation of the particle with not too high energy. The solution we get with the dimensional method, up to date, is the most general solution of propagation equation in a uniform, unbounded and spherically symmetrical medium. It includes the higher orders of approximation for the convection effect, and has been used in discussing various kinds of propagation effects of solar cosmic ray, making propagation corrections and evaluating the equivalent diffusion coefficients.[1] The solution may also have its value for reference in the discussion of the diffusion in the ordinary moving medium.

It is necessary to point out that this solution is valid only in the case of a constant K. For the low energy particles, the energy of particles and also their diffusion coefficients change as a result of the adiabatic expension loss and other processes. In this case, such limitations can be retrieved by the equivalent diffusion coefficient reflecting the average nature of propagation space.

The diffusion equation in a uniform, unbounded and spherically symmetrical medium is

$$r^2\frac{\partial^2 U}{\partial r^2}+2r\frac{\partial U}{\partial r}-\frac{r^2}{K}\frac{\partial U}{\partial t}=\frac{Vr^2}{K}\frac{\partial U}{\partial r}+\frac{2CVr}{K}U \qquad (1.1)$$

where U is the number density of diffusing particle, C is a constant relating to the energy spectrum index. In the space far from the source, the solution satisfying condition

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$U(r,t=o)=\frac{N}{4\pi r^2}\delta(r-r_0)$ and Eq. (1.1) is[1]

$$U(r,t)=\frac{N}{(4\pi r^2)^{3/2}}Z^{3/2}\exp\left[-\frac{(r-Vt)^2}{4Kt}\right]\phi(X,Z), \qquad (1.2)$$

where ϕ is 'diffusion-convection function' which the paper is going to discuss in detail. It can be expressed as a function of two dimensionless parameters as follows: Diffusion parameter $Z=r^2/(4Kt)$, Convection parameter for time $X=(2C-1)V(t/K)^{1/2}$. Disregarding ϕ, solution (1.2) is like a kind of the diffusion caused by a source moving outward with the convective velocity V. The equation of ϕ is

$$r^2\frac{\partial^2\phi}{\partial r^2}+r\frac{\partial\phi}{\partial r}\left(2-\frac{r^2}{Kt}\right)-\frac{r^2}{K}\frac{\partial\phi}{\partial t}=(2C-1)\frac{Vr}{K}\phi \qquad (1.3)$$

In case of no convection, $V=0$ and $\phi=1$, solution(1.2) is a function of a single parameter Z. When convective effect is not too strong, ϕ can be expanded into a power series of power series of parameter X: $\phi(X,Z)=\sum_{l=0}^{\infty}X^l F(l,Z)$, (1.4)

F(l,Z) is called the l-th order influence function of convection, and its equation is

$$F''(l,Z)+\left[-1+\frac{3}{2}Z^{-1}\right]F'(l,Z)-\frac{1}{2}Z^{-1}F(l,Z)=\frac{1}{2}Z^{-3/2}F(l-1,Z) \qquad (1.5)$$

Seeing that the solution must be finite at the origin, $Z\sim 0$, expression of F(l,Z) can be taken as

$$F(l,Z)=\sum_{m=0}^{l}Z^{m/2}\sum_{k=0}^{\infty}A(n,m,k)Z^k,\quad n=l-m. \qquad (1.6)$$

II. MAIN METHODS A(n,m,k) is a multiple series relating to m, and k, the lowest recurrence values for m and k are all zero. For the derivation of the general recurrence formula of A(n,m,k), it is instructive to evaluate first from the lower-order coefficient A(n,m-1,k) the next one A(n,m,k) byrecurring k. Then, the general formula of A(n,m,k) can be yielded as:

$$A(n,m,k)=A(n)\,a(n,m)\left[\begin{matrix}\frac{n}{2}+m\\ \frac{m+1}{2},\ \frac{m+3}{2}\end{matrix}\right]_k S(n,m,k) \qquad (2.1)$$

$$a(n,m)=\frac{(\frac{n}{2})_m}{(m+1)!},\ n\neq 0;\quad a(0,m)=\frac{2(m-1)!}{(m+1)},\ m\neq 0;\ a(0,0)=1 \qquad (2.2)$$

$$S(n,m,k)=\sum_{k_1=0}^{k}\frac{S(n,m-1,k_1)}{(\frac{n}{2}+m-1+k_1)(\frac{m}{2}+k_1)} \qquad (2,3)$$

$$S(n,m_0,k)=1,\ k=0,1,2,\cdots,\ m_0=0,\ n\neq 0,\ m_0=1,\ n=0, \qquad (2.4)$$

where coefficient A(n) is determined by the initial condition.

The multiple series s(n,m,k) can be transformed into a series composed of the higher-order Γ function (function ψ and G).

From the initial condition at $t=o$ and the asymptotic value (2Υ) of function as $Z\to\infty$, the recurrence formula of coefficient A(l) can be yielded :

$$\sigma_l=\sum_{m=0}^{l-1}\frac{A(n)}{\Gamma(\frac{n}{2})}\left(\frac{1}{2}\right)^m S(n,m)+A(0)\left(\frac{1}{2}\right)^{l-1}S(0,l)=0,\quad n=l-m,\ A(0)=1 \qquad (2.5)$$

The higher order coefficient A is difficult to express in a simple form. However, it is quite convenient to calculate A(1) with the recurrence formulae (2.3) and (2.5) by computer. It can be shown that A(1) is an alternative sequence, and its absolute value decreases when 1 increases.

Formula (1.5) is a nonhomogeneous equation. Its nonhomogeneous terms are composed of the same functions of lower order. The formula of function F(1,Z) can be derived from formulae (1.6) and (2.1):

$$F(l,Z) = \sum_{m=0}^{l} Z^{m/2}\, a(n,m)\, A(n)\, f(n,m,Z) \qquad (2.6)$$

The asymptotic value of f(n,m,Z) when t = o or $Z \to \infty$ is

$$f(n,m,Z)_{Z\to\infty} \sim \Gamma(\tfrac{3}{2})\, Z^{\frac{n-3}{2}} \exp(Z) \frac{(m+1)!}{\Gamma(n/2+m)} (\tfrac{1}{2})^m S(n,m) \qquad (2.7)$$

Therefrom the recurrence formula (2.5) of coefficient A(1) can be formulated.

III. DIFFUSION-CONVECTION FUNCTION $\phi(X,Z)$

For convenience of discussion, function ϕ can be rewritten as

$$\phi = \sum_{n+m\geq 0}^{\infty} A(n)\, a(n,m)\, X^n Y^m f(n,m,Z) \qquad (3.1)$$

where Y is another dimensionless parameter, namely, the convection parameter for space: $Y = (2C-1)\frac{Vr}{2K} = XZ^{\frac{1}{2}}$ (3.2)

Fig. 1. illustrates the value of n for calculation of ϕ as a function of Y and T, when the relative magnitude of the last term os less than 10 . It can be seen that the term number of convergence increases with the value of Y and T. Function ϕ is probably divergent as X > 3.oo. So, we often restrict our discussion to propagation problem within the range X ≤ 3.oo.

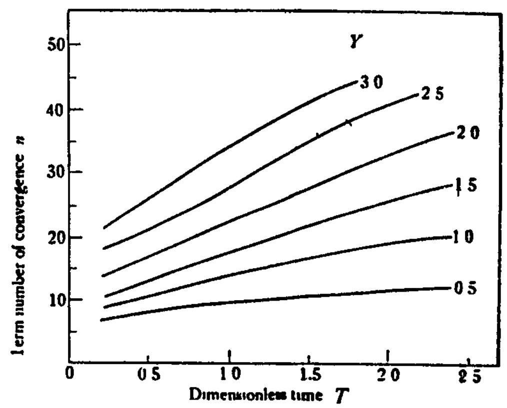


Fig. 1.

Abscissa is the dimensionless time T, ordinate is the number of terms summing up the function ϕ accurate to 10^{-7}.

IV. VARIATION OF DENSITY U

Variation of D-C function with the convection parameter for space Y and the dimensionless time T(= $4Kt/r^2$) have already been shown in Fig. 1. of [2] . Plotted here is the variation of diffusing particles density U with T under the different conditions of solar wind convections (Fig. 2.). A set of curves on the left side are the results calculated by Formula (1.2), taking $2C - 1 = 4$, or the differential-momentum-spectrum index of particles to be 5.5. Such case corresponds to that of the isotropic propagation. But in fact, propagation of cosmic rays particles in interplanetary space is anisotropic, the diffusion coefficient along the magnetic line $K_{\parallel}$ is different from the transverse one $K_{\perp}$. In this case, propaga- tion equation becomes

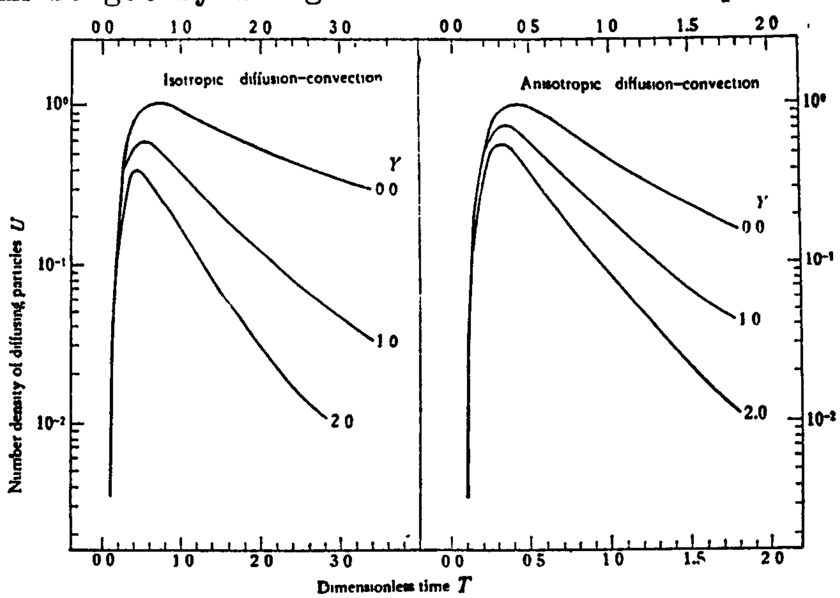
$$U = \frac{N}{(4\pi r^2)^{3/2}} Z^{5/2} \frac{K_{\parallel}}{K_{\perp}} \left(\frac{\theta}{\sin\theta}\right)^{\frac{1}{2}} \exp\left[-Z\left(1+\frac{K_{\parallel}}{K_{\perp}}\theta^2\right)\right] \cdot \exp\left[H-\frac{H^2}{4Z}\right] \Phi(X,Z) \quad (4.1)$$

where is the magnetic azimuth of observation point from the source, and H is the modulation parameter, $H = \frac{Vr}{2K_{\parallel}}$ (4.2)

Substitute $k_{\parallel}$ for K when Z is calculated.

Solution (4.1) of the anisotropic propagation equation is drawn by a set of curves of the right side of Fig2 when $\theta = 0°$. It can be shown that the intensity of diffusing particles decreases and the maximum time moves up as the convection effect intensifies. Besides, the time scale of the isotropic D-C propagation is longer than that of the anisotropic one. Hence, it is possible to form an over estimation of equivalent diffusion coefficient if observational data is fitted merely by the former. However, better results can be got by using the model of anisotropic propagation.

Fig.2



(a) Isotropic D-C (b) anisotropic D-C

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[2] Zhang, Gongliang, SCIENTIA SINICA, 22(1979), 934.