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COSMIC RAY MODULATION IN A RANDOM ANISOTROPIC MAGNETIC FIELD

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Inhomogeneities of the interplanetary magnetic field can be divided into small-scale and large-scale ones/I,2/, as may be required by the character of the problem of cosmic ray (CR) propagation. CR propagation in stochastic magnetic fields is of diffusion character. The main contribution into the scattering of CR particles is made by their interaction with inhomogeneities of the magnetic field H(r,t) which have characteristic dimensions l of the order of Larmor radius $R=cp/eH$ of particle (p is the absolute value of particle momentum, e is particle charge, c is velocity of light). Scattering of particles on such inhomogeneities leads to their diffusion mostly along a magnetic field with characteristic dimensions of variation in space exceeding the mean free path Λ of particles before their scattering. In view of this fact the inhomogeneities of the interplanetary field can be divided into small-scale (l) and large-scale ($l<\Lambda$) ones, and the random magnetic field can be represented as a sum of the small-scale $\mathcal{H}_1$ (r,t) and the large-scale $\mathcal{H}$ (r,t) fields. The boundary value l_0 of the inhomogeneities can be determined by several lengths Λ of the mean free path of particles and l_0 can be taken as the principal scale of the small-scale turbulence. The choice of the scale L_0 of the large-scale turbulence depends on a concrete formulation of the problem.

It is necessary to divide a random field into small-scale and large-scale parts because one cannot describe the whole of the magnetic field turbulence spectrum on a single basis in terms of the local isotropy. As the scale of fluctuations increases, the statistic distribution of these fluctuations becomes, apparently, nonisotropic in space.

The diffusion coefficient along the large-scale field H for $R\ll l_0$ can be written in the form /2/

$$\varkappa_{\parallel}=\frac{v\Lambda_{\parallel}}{3}, \quad \Lambda_{\parallel}=\frac{\nu(\nu+2)\Gamma\left(\frac{\nu-1}{2}\right)}{3\sqrt{\pi}(\nu+1)\Gamma(\nu/2)}(cp/e)^{2-\nu} l_0^{\nu-1}\frac{\tilde{H}^{\nu}}{\langle\mathcal{H}_1^2\rangle}; \quad \nu>1 \qquad \text{(I)}$$

where $\tilde{H} = |\tilde{\vec{H}}| = |\vec{H}_0 + \vec{\mathcal{H}}|$, H_0 is a regular magnetic field, ν is the index of the frequency spectrum of the small-scale turbulence, $\Gamma(x)$ is gamma-function, V is particle velocity, $\Lambda_{\parallel}$ is the mean free path of a particle along the field H before scattering.

The transport equation in the case of the field $\tilde{\vec{H}}$ has the form /2,3/

$$\frac{\partial N}{\partial t} = \nabla_\alpha \mathcal{H}_{\alpha\lambda} \nabla_\lambda N - (\vec{u}\vec{\nabla})N + (\vec{\nabla}\vec{u})\frac{p}{3}\frac{\partial N}{\partial p} \tag{2}$$

where $N(r,p,t)$ is CR particle concentration, u is solar wind velocity, $\nabla_\alpha \equiv \partial/\partial r_\alpha$, $\mathcal{H}_{\alpha\lambda}$ is tensor diffusion coefficient which for a strong field $H^2 \gg \langle \mathcal{H}_1^2 \rangle$ $R \ll \Lambda_{\parallel}$, can be written as

$$\mathcal{H}_{\alpha\lambda} = \mathcal{H}_{\parallel} h_\alpha h_\lambda \tag{3}$$

where $\vec{h} = \tilde{\vec{H}}/\tilde{H}$. If the field $\tilde{\vec{H}}$ is represented by a sum of the random $\vec{\mathcal{H}}$ and the regular $\vec{H}_0$ fields, the expression (3) for $\langle \mathcal{H}_1^2 \rangle \ll H_0$ can be rewritten as

$$\mathcal{H}_{\alpha\lambda} = \mathcal{H}_{\parallel}^0 (n_\alpha n_\lambda + n_\alpha \mathcal{H}_\lambda H_0^{-1} + n_\lambda \mathcal{H}_\alpha H_0^{-1} + (\nu-2) n_\alpha n_\lambda n_\mu H_0^{-1} \mathcal{H}_\mu) \tag{4}$$

where $\vec{n} = \vec{H}_0/H_0$, $\mathcal{H}_0^{\parallel} = \mathcal{H}_{\parallel} \tilde{H}^{-\nu} / H_0^{\nu}$.

The coefficient $\mathcal{H}_{\alpha\lambda}$ in (2) depends on the random field $\mathcal{H}(r,t)$ and, hence, the function $N[\mathcal{H}(r,t)$; $(r,p,t)]$ itself is a functional of the random field, and if eq. (2) is averaged over a large-scale random field, we can use the standard averaging procedure /4/. As a result, we are led to the following equation for the mean CR concentration $\langle N \rangle$

$$\frac{\partial \langle N \rangle}{\partial t} = \nabla_\alpha \bar{\mathcal{H}}_{\alpha\lambda} \nabla_\lambda \langle N \rangle - (\vec{u}\vec{\nabla})\langle N \rangle + \frac{p}{3}\vec{\nabla}\vec{u}\frac{\partial \langle N \rangle}{\partial p}, \tag{5}$$

where

$$\bar{\mathcal{H}}_{\alpha\lambda} = \mathcal{H}_{\parallel} n_\alpha n_\lambda + \bar{\mathcal{H}}_{\alpha\lambda}^{\perp} + \mathcal{H}_{\alpha\lambda}^{\parallel,\perp},$$
$$\bar{\mathcal{H}}_{\parallel} = \mathcal{H}_{\parallel}^0 \left\{ 1 + H_0^{-2} (\nu-1)^2 n_\beta n_\gamma \int d\vec{k}\, \mathcal{B}_{\beta\gamma}(\vec{k}) \right\}, \tag{6}$$

$$\bar{\mathcal{H}}_{\alpha\lambda}^{\perp} = \mathcal{H}_{\parallel}^0 H_0^{-2} \Delta_{\alpha\beta} \Delta_{\lambda\gamma} \int d\vec{k}\, \mathcal{B}_{\beta\gamma}(\vec{k}); \; \Delta_{\alpha\beta} = \delta_{\alpha\beta} - n_\alpha n_\beta, \tag{7}$$

$$\bar{\varkappa}^{\parallel\perp}_{\alpha\lambda}=\varkappa^{0}_{\parallel}(\Delta_{\alpha\beta}n_{\gamma}n_{\lambda}+\Delta_{\lambda\gamma}n_{\beta}n_{\alpha})\int d\vec{k}\,\mathcal{B}_{\beta\gamma}(\vec{k}) \qquad (8)$$

Here $\mathcal{B}_{\alpha\lambda}(\vec{k})$ is a Fourier-transform of the correlation tensor of a random magnetic field. When deriving (5), we used the hypothesis of frozen turbulence.

For an isotropic probability distribution of fluctuations of the field $\mathcal{H}$ (r,t) we have

$$\bar{\varkappa}_{\parallel}=\varkappa^{0}_{\parallel}\left(1+\frac{(\nu-1)^{2}}{3}\frac{\langle\mathcal{H}^{2}\rangle}{H_0^{2}}\right), \qquad (9)$$

$$\bar{\varkappa}^{\perp}_{\alpha\lambda}=\frac{1}{3}\varkappa^{0}_{\parallel}\frac{\langle\mathcal{H}^{2}\rangle}{H_0^{2}}\Delta_{\alpha\lambda};\quad \varkappa^{\parallel,\perp}_{\alpha\lambda}=0, \qquad (10)$$

$$\langle\mathcal{H}^{2}\rangle=8\pi\int_{L_0}^{l_0^{-1}}dk\,k^{2}\mathcal{B}(k). \qquad (11)$$

Here L_0 is the maximum dimension of large-scale inhomogeneities.

The coefficient $\varkappa^{\perp}_{\alpha\lambda}$ describes particle transport across a regular magnetic field under the action of a large-scale random field. Its expression for the isotropic case agrees with the analogous expression obtained in the papers by Toptygin /2/ in the averaging of the kinetic equation for $\varkappa^{0}_{\parallel}=\varkappa_{\parallel}$. In our case, however $\varkappa_{\parallel}$ depends on the random field $\mathcal{H}$, and to compare it with the value of $\varkappa^{0}_{\parallel}$ one should average $\varkappa_{\parallel}$ over an ensemble of a large-scale random field. As a result we have $\langle\varkappa_{\parallel}\rangle=\varkappa^{0}_{\parallel}(1+\nu\langle\mathcal{H}^{2}\rangle/H_0^{2})$. Such a renormalization of the diffusion coefficient will not affect noticeably the values of $\varkappa^{\perp}_{\alpha\lambda}$ and $\varkappa^{\parallel,\perp}_{\alpha\lambda}$ in which $\varkappa^{0}_{\parallel}$ is replaced by $\langle\varkappa_{\parallel}\rangle$ without an essential change in their values, but the coefficient of longitudinal diffusion changes the form. So, for the expression (9) we have

$$\bar{\varkappa}_{\parallel}=\langle\varkappa_{\parallel}\rangle\left\{1-\frac{1}{3}\frac{\langle\mathcal{H}^{2}\rangle}{H_0^{2}}(3\nu-(\nu-1)^{2})\right\}, \qquad (12)$$

from which one can conclude that an isotropic large-scale turbulence changes particle diffusion along the regular field if the spectrum of small-scale turbulence does not fall very rapidly, namely, if the index of the spectrum ν is not larger than $\nu \approx 4.8$. Usually ν = I - 2/2/.

Large-scale turbulence of a magnetic field in interplanetary space is apparently anisotropic, i.e. spatial probability distribution of a random magnetic field has a distinguished direction e. As working models of anisotropic turbulence we take its two limiting cases: two-dimensional and one-dimensional fluctuation distribution. In the case of two-dimensional turbulence, the correlation tensor $\mathcal{B}_{\alpha\lambda}$ will be written in the form

$$\mathcal{B}^{\perp}_{\alpha\lambda}(\bar{k}) = \mathcal{B}^{\perp}(k_{\perp}^{2})\left(\delta_{\alpha\lambda} - \frac{k_{\alpha}^{\perp}k_{\lambda}^{\perp}}{k_{\perp}^{2}}\right)\delta(\bar{k}_{\parallel}); \qquad (13)$$

and for the one-dimensional model we have

$$\mathcal{B}^{\parallel}_{\alpha\lambda}(\bar{k}) = \mathcal{B}^{\parallel}(k_{\parallel}^{2})\,\delta(k_{\perp}^{2})\,(\delta_{\alpha\lambda} - e_{\alpha}e_{\lambda}); \; k_{\parallel} = \bar{k}\bar{e},\; k_{\perp} = |\bar{k} - \bar{k}_{\parallel}| \qquad (14)$$

where $\mathcal{B}^{\perp}$ and $\mathcal{B}^{\parallel}$ are corresponding spectral functions.

For two-dimensional turbulence with a distinguished direction $\bar{e} \parallel \bar{n}$ the values of the diffusion coefficient are

$$\bar{\mathscr{X}}_{\parallel} = \langle \mathscr{X}_{\parallel} \rangle \left\{1 - \nu \frac{\langle \mathscr{H}^{2} \rangle}{H_0^{2}}\right\}; \; \bar{\mathscr{X}}^{\perp}_{\alpha\lambda} = \langle \mathscr{X}_{\parallel} \rangle_{\ell_0} \frac{\langle \mathscr{H}_{\parallel}^{2} \rangle}{H_0^{2}} \Delta_{\alpha\lambda} \qquad (15)$$

$$\mathscr{X}^{\parallel,\perp}_{\alpha\lambda} = 0; \quad \langle \mathscr{H}_{\parallel}^{2} \rangle = 2\pi \int_{L_0^{-1}} dk_{\perp} k_{\perp} \mathcal{B}^{\perp}(k_{\perp}^{2})$$

L_0 is the principal scale of inhomogeneities in the direction transverse to H_0.

In the model of one-dimensional turbulence with a distinguished direction along the solar wind velocity, $\bar{e} \parallel \bar{u}$, we have

$$\bar{\mathscr{X}}^{\parallel} = \langle \mathscr{X}_{\parallel} \rangle \left\{1 - \left(\nu(\nu-1)^{2}\frac{u_{\perp}^{2}}{u^{2}}\right)\frac{\langle \mathscr{H}_{\perp}^{2} \rangle}{H_0^{2}}\right\}; \; \bar{\mathscr{X}}^{\perp}_{\alpha\lambda} = \langle \mathscr{X}_{\parallel} \rangle_{\ell_0} \frac{\langle \mathscr{H}_{\perp}^{2} \rangle}{H_0^{2}} \left\{\frac{1}{3}\Delta_{\alpha\lambda} - \frac{u_{\alpha}^{\perp}u_{\lambda}^{\perp}}{u^{2}}\right\},$$

$$\bar{\mathscr{X}}^{\parallel,\perp}_{\alpha\lambda} = \langle \mathscr{X}_{\parallel} \rangle \frac{\langle \mathscr{H}_{\perp}^{2} \rangle}{H_0^{2}} (\nu - 1) \frac{u_{\parallel}}{u^{2}} (u_{\alpha}^{\perp} n_{\lambda} + n_{\alpha} u_{\lambda}^{\perp}); \; \langle \mathscr{H}_{\perp}^{2} \rangle = 2\pi \int_{(L_0^{u})^{-1}} dk_{\parallel} \mathcal{B}^{\parallel}(k_{\parallel}^{2});$$

$$k_{\parallel} = (\bar{k}\bar{u})\,u,$$

where $u_{\parallel}$ and $\bar{u}_{\perp}$ are a parallel and a perpendicular components of the solar wind velocity vector relative to the regular magnetic field direction, L_0^{u} is the principal scale of inhomogeneities along u. In this case the cross diffusion coefficient $\mathscr{X}^{\parallel,\perp}_{\alpha\lambda}$ is nonzero, whereas in all the rest of the considered cases it is equal to zero. The latter model of large-scale turbulence is apparently a most realistic reflection of the situation in interplanetary space with fluctuations going radially from the Sun, with systems of discontinuities and waves following one another in the far regions of solar wind.

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