THE MODEL-INDEPENDENCE OF COSMIC RAY SOURCE DETERMINATIONS

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ABSTRACT

The direct inversion method of Margolis (1983) is used to explore the dependence of $Z \leq 28$ source abundance determinations on the choice of the pathlength distribution. The source abundances do not depend strongly on the form of the truncation used, although some truncation at the lower energies (compared to a leaky box) is necessary. The decrease of mean grammage with increasing energy is required by the observations. The effects of errors and the use of other secondary to primary ratios is discussed.

1. Introduction. In the past, discussions of the source abundances of cosmic rays have always been in the context of a particular propagation history (pathlength distribution) and a sequence of calculations designed to correct an initial estimate of the source abundances to match observations. Margolis (1983) showed that the propagation equations of the GCR through the interstellar medium permit a direct inversion. This inversion method is used here to determine the source abundances of the Galactic Cosmic Rays in a model-independent manner. The errors of observation set the criteria for consistency between models. The implications of this method for determining the pathlength distribution (PLD) and the abundances are discussed.

2. The Analytic Procedure. Since the details of the analysis are presented in detail elsewhere, only a brief outline is provided for the reader's convenience. The steady-state diffusion equation governing the propagation of the cosmic rays can be separated into 2 equations, one governing the spatial distribution of the particles, and the other the nuclear physics of fragmentation and decay. This split is frequently referred to as the "weighted slab method." Neglecting energy loss reduces the composition equations to a set of linear equations with energy appearing as a parameter. By expressing the solution in terms of matrix exponentials, Margolis (1983, 1985) showed that the observed abundances and the source abundances could be related for an arbitrary PLD. A graphical analysis of these solutions allows not only the determination of the "best" source abundances from a set of observations, but also the errors associated with the determinations based on those measurements.

3. The Overall Source Abundances. Since cosmic ray observations sample different energies, a separate decomposition can be produced for each energy. Previous determinations have not yielded a single source composition for all energies (Dwyer *et al.*, 1981) nor a single choice of propagation parameters covering all energies (Garcia-Muñoz *et al.*, 1981, 1984; Cesarsky, Koch, and Perron, 1981; Ormes and Protheroe, 1981). Such variations are also seen here. For discussion purposes, consider the truncated exponential distribution of Tsao, Shapiro, and Silberberg (1973), whose shape is that of a linear rise to some grammage Λ_t matched to an exponential tail of scale Λ_e. Simultaneous use of the ratios B/C and (Sc–Mn)/Fe allows the determination of a single decomposition and a single set of source abundances for each energy. For observations, use the 9 energies above 2.54 GeV/nucleon from the HEAO C2 experiment (Engelmann *et al.*, 1983). The resulting plots of Λ_e and Λ_t in Figure 1 show a relatively uniform variation of the escape length and some fluctuations in the truncation length. These results suggest that the truncation is significant but does not vary with energy. Figure 1 also shows an alternate set of tracks, determined by looking at the ratio pair B/C and V/Fe. The escape lengths match reasonably well, but this second set suggests a truncation which decreases with energy. How, then, should these results be compared, and what confidence can be placed on these results?

An advantage of this formulation is the direct calculation of estimated errors from the errors of observation, if reasonable values for these are known. For the data used here, the formal errors at the source are generally smaller than the scatter of the computed values. The true errors are probably somewhat larger. The Silicon measurements, whose average was used to normalize the observations (Engelmann *et al.*, 1983) show a scatter 4 times the size of the formal error of the average. A comparison of the two sets of source abundances described above shows that, at worst, the difference

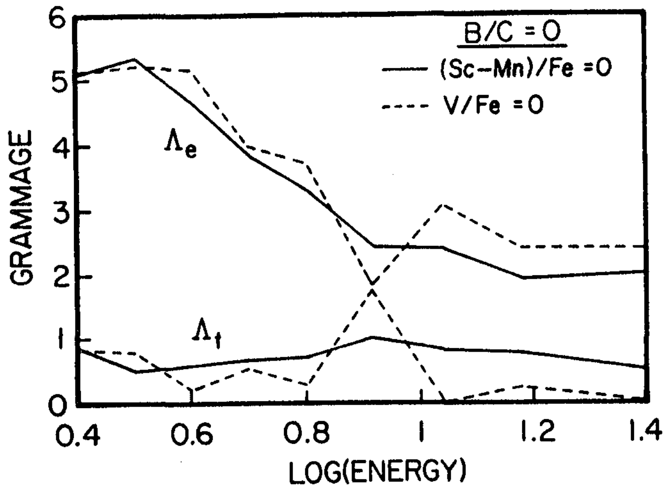

Figure 1. The parameters Λ_e and Λ_t are plotted as functions of energy per nucleon for two different model selection criteria. Note that the decrease of Λ_e is relatively uniform as a function of energy, but that the variation of the truncation Λ_t is quite noisy. Both sets of parameters are determined using the null B/C source ratio. The solid lines are the parameters determined by the null contour of the sub-Fe/Fe source ratio, and the dashed lines were based on the null V/Fe source ratio.

between the separate determinations is only twice the formal error. Given that, it seems reasonable to produce the GCRS abundances shown in Table 1. These were formed from the simple averages of the two sets of 9 decompositions, and the formal errors shown are derived under the assumption of independence. The table also shows the errors derived from the scatter of the individual source abundances, which shall be denoted as the informal errors. These two estimates serve as confidence indicators.

Comparison of the abundances with both the formal and informal errors shows quite clearly the distinction between well- and poorly-determined elements. The formal errors, when compared with the abundances, indicate the significance to which the source abundances are determined within the propagation model. The informal errors, when compared with the formal errors, indicate the variation of the abundances across the separate determinations. The informal error is the better indication of the significance of a source abundance, but as noted above, the intrinsic scatter of the original normalization is about 4 times the formal, statistical error. C ($Z = 6$) and Fe ($Z = 26$), for example, have abundances which stand out quite clearly from the errors. The abundances of F ($Z = 9$) and K ($Z = 19$), though negative, are comparable to the errors. Significant values cannot be determined for those elements whose values are clearly consistent with zero abundance. The difficulty comes in assessing a level of significance relative to the errors. The scatter of the normalization suggests that an abundance is significant only when its value is more than 4 times its associated error. The abundances which are significant under this criterion are marked with an asterisk (*).

4. Discussion. The contrast between the variations in the parameters selected and the abundances calculated can be understood from the complementary variations displayed in Figure 1. The mirroring of changes in the general trends of the escape and truncations lengths imply a well-defined average grammage and a non-pathological PLD underlying the abundances. The observations can be matched approximately by several models. Despite some uncertainty in the PLD, the source abundances vary slowly. This uncertainty suggests two questions. How much error does the choice of a particular PLD introduce? To what extent can the shape of the PLD be investigated using these techniques?

The choice of the form of the PLD need not be a significant source of error. The "no short pathlengths" model employed by Garcia-Muñoz *et al.*, (1984) yields an alternate sequence, with a truncation parameter about half that of the corresponding parameter for the linear rise model; this seems almost intuitive when the linear rise model is viewed as including a triangular distribution with half the area of the rectangle *excluded* by the no short paths model. The more extreme form of the truncation does not significantly change the source abundances estimated here. The abundance values of Table 1 also compare quite well with others derived from the same measurements using other distributions and other methods (Koch-Miramond *et al.*, 1983). (Note again that the errors presented

The Cosmic Ray Source Abundances			
Atomic Number	Abundance Si ≡ 10^3	Formal Error	Informal Error
4	10.2	15.2	51.7
5	1.7	26.2	12.0
6 *	4270.0	43.0	212.0
7 *	374.0	24.5	24.6
8 *	5050.0	44.6	143.0
9	-5.6	3.6	7.1
10 *	660.0	9.7	26.9
11 *	60.8	4.9	9.1
12 *	1050.0	11.7	32.7
13 *	117.0	5.3	8.0
14 *	1000.0	11.1	0.0
15	7.8	2.5	2.1
16 *	143.0	5.5	8.3
17	2.8	2.7	2.7
18 *	15.5	3.6	2.5
19	-1.1	3.5	5.4
20 *	63.7	4.9	5.9
21	-3.2	2.5	3.7
22	-11.8	4.3	8.5
23	0.9	3.1	2.4
24	11.1	4.7	7.8
25 *	20.5	3.8	3.9
26 *	958.0	12.7	34.5
27	3.1	1.0	0.6
28 *	55.1	3.3	4.5

Table 1. The Cosmic Ray Source Abundances determined using the truncated exponential distribution. The asterisks mark those elements with clearly significant abundances. The Formal Errors are those calculated from the errors of the 18 separate energy/model combinations. The Informal Errors represent the scatter of those separate points.

here do *not* include the cross section errors, but are only the measurement errors as modified by the effects of propagation.) There is a well-defined source composition which can be related to the observations by a PLD whose average grammage decreases with increasing energy. Can anything more specific be determined about the form of the PLD?

This method might allow some discrimination between distributions. The internal scatter, the informal error, of the determined abundances differs among distributions. The consistency of a run of abundances with energy is an important measure of the quality of a particular decomposition. The informal errors, examined over all charges, could offer a quantitative reason to select one distribution shape over another. General features of the PLD might be distinguished on a comparative basis; while not truly model-independent, such a procedure does not introduce bias beyond that involved in formulating the propagation problem. The observations used here do not provide sufficient discrimination to distinguish between different forms of truncation, but only to confirm the necessity for some at the lower energies.

Determinations of a variable truncation in a strictly phenomenological way do not explain the propagation of the cosmic ray nuclei. With some decrease in the associated errors, the methods described here should be able to provide a test for quantitative theories such as that of Margolis (1981) and Margolis and Bussard (1983) for the origin of the energy-dependent PLD. They point out that scattering from Alfvén waves generated by the flux of cosmic rays leads naturally to a truncation which decreases with energy and an average grammage which would decrease and then level off with increasing energy. Taking the results of Garcia-Muñoz *et al.* (1984) as evidence for a variable truncation, one must look to the overall variation of grammage with energy as the next key test. The prediction of Margolis and Bussard that the average grammage should level off as energy increases is suggested, but not proved, by the curves of Figure 1. In the future, more sensitive decompositions at higher energy will resolve this question.

Much of the sensitivity of any decomposition depends on the range of elements through the variation of the total inelastic cross section. Although the various types of distributions lead to different values of precision, the increases in accuracy are comparable for models in the range allowed by the observations. The potential accuracy for the linear rise, truncated exponential models used here is increased by only about 20% if an S/P ratio based on the Te-Ba peak elements is substituted for the sub-Fe/Fe ratio. A ratio based on Pt-Pb group elements might permit a 25% increase compared to the sub-Fe/Fe ratios, all other factors being equal.

Unfortunately, all other factors are not equal. Currently available observations at the higher charges are so much less precise that the overall selectivity is much poorer. The general increase of partial cross sections with atomic number helps a little, but the errors associated with poorly determined cross sections and multiple primary contributions to secondary elements outweigh all such gains. This also limits the utility of the obvious generalization of the two-parameter search to more complex distributions. At present, the search for the GCRS would be aided most by a more reliable normalization of the observed abundances across the dynamic ranges of charge and energy.

5. Summary. The source abundances of the Galactic Cosmic Rays can be calculated by a reasonably model-independent procedure. The decomposition depends on the relations between the production of secondary elements and the relative abundances of the primaries. The analytic nature of the procedure allows the straightforward assignment of errors to the source abundances calculated from the errors associated with the observations. Although a model for the PLD must be used in the procedure, there is no effective restriction on the form of the model, and the dispersion of the source abundances calculated at several energies can be used as a measure of quality to select between distributions. The technique has been demonstrated by calculating the source abundances of the nuclides $Z \leq 28$. The source abundances are consistent with those determined by others using different methods. The average grammage traversed by the cosmic rays decreases with increasing energy. The PLD required by the observations is deficient in short pathlengths at lower energies. There are indications that the truncation decreases with energy, but the errors associated with the observations are a little too large to make a definitive assessment at this time.

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