INFLUENCE OF THE SOURCE DISTRIBUTION ON THE AGE DISTRIBUTION OF GALACTIC COSMIC RAYS

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ABSTRACT

The age distribution of galactic cosmic rays in the diffusion approximation is calculated. The influence of the scale height of the spatial source distribution on the mean age of particles arriving at the solar system is discussed. The broader the source distribution with respect to the galactic plane, the longer the mean age. This result provides a natural explanation for the shorter mean age of secondary cosmic rays compared to primary cosmic rays necessary for the understanding of the observed secondary/primary ratio.

1. Introduction. The concept of the age distribution of cosmic ray particles at the solar system allows to reduce the solution of the full cosmic ray transport equation to the simpler leaky-box equation (Schlickeiser and Lerche 1985, OG 7.2-8). Here we consider the influence of the spatial source distribution on the resulting age distribution and mean arrival time at the solar system.

2. Age Distribution in the Diffusion Approximation. At large momenta spatial diffusion in partially stochastic magnetic fields dominates particle's escape by convection in a galactic wind. Because of the highly flattened shape of the Galaxy we may neglect spatial gradients in the galactic plane compared to gradients perpendicular (z-coordinate) to the galactic plane. For simplicity and ease of exposition, we also assume a constant diffusion coefficient throughout the Galaxy, $K(r) = K_o$. The cosmic ray age distribution can then be derived from the solution of the diffusion equation (see Schlickeiser and Lerche 1985, OG 7.2-8, equations (7) - (9)).

$$K_o \frac{\partial^2 N(z,y)}{\partial z^2} = \frac{\partial N(z,y)}{dy} \qquad (1),$$

$$N(z,y=0) = q(z) \qquad (2).$$

As boundary condition we use the free escape boundary condition

$$N_{z=+L} = N_{z=-L} = 0 \qquad (3)$$

which in cases, where the majority of particle sources do not concentrate on the disk boundary, is appropriate. We also demand symmetry of the solution relative to the galactic plane

$$N(z,y) = N(-z,y) \qquad (4).$$

The solution of equations (1) - (4) is

$$N(z,y) = \frac{1}{L} \sum_{m=1}^{\infty} c_m \sin\left[(2m-1)\pi \frac{z+L}{2L}\right] \exp(-\lambda_m^2 y) \qquad (5)$$

belonging to the eigenvalues (m = 1, 2, 3, ...)

$$\lambda_m^2 = \frac{\pi^2 K_o}{4L^2} (2m-1)^2 \qquad (6).$$

As can be seen from (6) the set of eigenvalues λ_m^2 form a monotonously increasing sequence $0 < \lambda_1^2 < \lambda_2^2 < \lambda_3^2 < \ldots$, so that for times $y \gg \lambda_1^{-2} = 4L^2/(\pi^2 K_o)$ the tail of the age distribution is pure exponentially, $N(z,y) \propto \exp(-\lambda_1^2 y)$. After these times the cosmic ray particles have been scattered so often that they "forget" their original source distribution.

Throughout this work we consider a step-like source distribution

$$q(z) = \begin{cases} 1 & \text{for } |z| < B < L \\ 0 & \text{otherwise} \end{cases} \qquad (7)$$

where B is the scale height of the sources. The expansion coefficients c_m in (5) are then determined via the initial condition (2) as

$$c_m = \frac{1}{L} \int_{-L}^{+L} dz\, q(z) \sin\left[(2m-1)\pi \frac{z+L}{2L}\right] = \frac{4(-1)^{m-1}}{\pi(2m-1)} \sin \frac{\pi(2m-1)B}{2L} \qquad (8).$$

Using (8) in (5) and normalizing N(z,y) yields the age distribution

$$P(z,y) \equiv \frac{N(z,y)}{\int_0^{\infty} dy\, N(z,y)}$$

$$= \sum_{m=0}^{\infty} \frac{(-1)^m}{(2m+1)} e^{-\frac{\pi^2 K_o (2m+1)^2 y}{4L^2}} \left[\!\left[\cos \frac{\pi(2m+1)(L+z-B)}{2L} - \cos \frac{\pi(2m+1)(L+z+B)}{2L} \right]\!\right] \cdot \begin{cases} \dfrac{4 K_o}{\pi(2BL-B^2-z^2)} & \text{for } |z| \leqq B < L \\ \dfrac{2 K_o}{\pi B(L-|z|)} & \text{for } B \leqq |z| \leqq L \end{cases} \qquad (9).$$

The mean age at position z is determined from (9) as

$$\langle T\rangle(z) \equiv \int_0^{\infty} dy\, y\, P(z,y) =$$

$$= \begin{cases} \dfrac{1}{12K_o} \left[\!\left[5B(2L-B)-z^2 + \dfrac{4B(2L^3+4LB^2-5BL^2-B^3)}{B(2L-B)-z^2} \right]\!\right] & \text{for } |z| \leqq B \\[2ex] \dfrac{2L^2+2L|z|-z^2-B^2}{6K_o} & \text{for } B \leqq |z| \leqq L \end{cases} \qquad (10).$$

Equations (9) and (10) generalize results of Owens (1976) who has only considered the cases $B = 0$ and $B = L$. For $B = L$ equation (10a) coincides with Owens' equation (16a); in the case $B = 0$ Owens' expression (16c) differs ours (10b).

3. Mean Age of Cosmic Rays Arriving at the Solar System. The solar system is located very near to the galactic plane $z_s \cong 0$. For small values of $|z| << \sqrt{B(2L-B)}$ (10) reduces to

$$\langle T\rangle(z \cong 0) \;\cong\; \frac{L^2}{3K_o}\left(1 + \frac{1}{2}\,\frac{B}{L} - \frac{1}{4}\left(\frac{B}{L}\right)^2\right) \qquad (11)$$

which is shown in Figure 1. It can be seen that the mean age of cosmic rays arriving at the solar system depends significantly on the scale height of cosmic sources: the broader the source distribution with respect to the galactic plane, the longer the mean age.

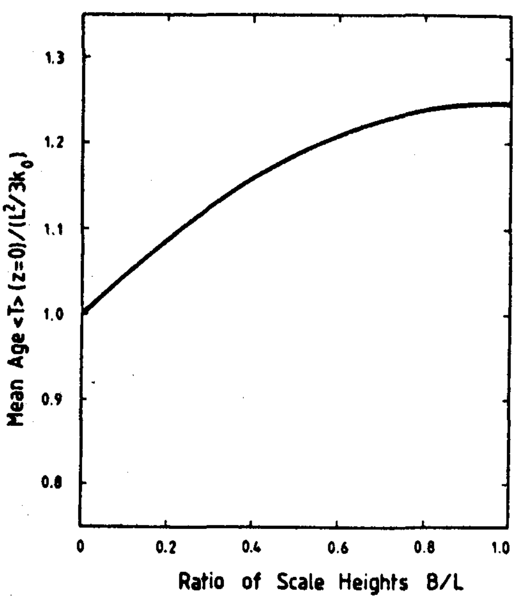

Fig. 1: Mean age of cosmic rays arriving at the solar system $\langle T\rangle(z=0)$ in units of $L^2/(3K_o)$ as a function of B/L. K_o is the value of the spatial diffusion coefficient, L the half-thickness of the cosmic ray confinement volume, and B the scale height of the cosmic ray source distribution. The larger the value of B, the larger the mean age.

This important result offers a natural explanation for the shorter mean age of secondaries compared to primaries necessary for the understanding of the observed secondary/primary ratio (see Lerche and Schlickeiser 1985, OG 8.3-2). Primary cosmic rays are continuously accelerated from the thermal background plasma which has a scale height of $B_F = 5\ T_6$ Kpc, if associated with the hot coronal gas (McKee and Ostriker 1977) where T_6 is the gas temperature in units of 10^6 K. Secondaries are mainly produced in dense molecular clouds where primary cosmic rays fragment in interactions with gas atoms, so their source scale height is given by the scale height of galactic molecular clouds, $B_S \cong 60$ pc (Burton 1976). For the ratio of lifetimes we then find

$$\Omega = \frac{\langle T\rangle_S\,(z=0)}{\langle T\rangle_F\,(z=0)} = \frac{4L^2 + 2B_S L - B_S{}^2}{4L^2 + 2B_F L - B_F{}^2} \tag{12}$$

which equals $\Omega = 0.87$ for $T_6 = 0.4$ and $L = 5$ kpc, and $\Omega = 0.97$ for $T_6 = 0.04$ and $L = 2$ kpc. As can be seen from (12) the smallest value of Ω is $\Omega_{min} = 0.8$ for $B_S \cong 0$, $B_F = L$.

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