ANGULAR RESOLUTION OF AIR SHOWER ARRAY-TELESCOPES

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ABSTRACT

A fundamental limit on the angular resolution of air shower array-telescopes is set by the finite number of shower particles coupled with the finite thickness of the particle swarm. Consequently the angular resolution which can be achieved in practice depends in a determinate manner on the size and number of detectors in an array-telescope, as well as on the detector separation and the timing resolution. It is also necessary to examine the meaning of 'particle density' in whatever type of detector is used. Results are given which can be used to predict the angular resolution of a given instrument for showers of various sizes, and to compare different instruments.

1. Introduction. Counter arrays for studying air showers are usually multi-purpose installations whose design represents a compromise between many requirements, including limits on construction and operation costs. Ordinarily the requirement for accuracy in measuring shower directions is not very stringent: angular resolving power of a few degrees is adequate. It is a fact of experience that resolving power of this order is achieved almost as a matter of course by applying the 'fast-timing' method of Bassi *et al.* (1953). Arrays which are physically small (diameter < 100 m) need to have 1-10 ns timing resolution, but this is not very difficult to achieve using scintillators. For giant arrays (diameter > 1 km) it might be difficult, but in this case the counter separations (the baselines for time of flight triangulation) are much greater, so the timing doesn't need to be as fast. The angular resolution characteristics of existing arrays have been studied with great care (see for example Clark *et al.* 1961, Linsley and Scarsi 1962, Lloyd-Evans 1982), but this seems always to have been done case by case, after the fact, so when I was asked recently, "What is the limiting precision of determining EAS directions? Is it possible, on a 300 m basis, to see the front within ± 1 ns, so as to achieve an angular resolution of 1 mrad?" (Cocconi 1984), I was unable to find a ready made answer. The question relates to a timely application: observing point sources of > 10^{15} eV γ-rays, in which the signal to noise ratio of an air shower array-telescope depends critically on the angular resolution $\Delta\theta$, being proportional to $(\Delta\theta)^2$.

2. Error in arrival time. For an isolated detector the error in the apparent arrival time will include the instrumental error and the effect of fluctuations due to the finite number of particles producing the signal. The contribution of fluctuations is given by $\delta t = \sigma_t/\sqrt{n}$, where n is the number of independent contributions to the signal and σ_t is the arrival time dispersion. This relation assumes that the fiducial time is the *average* arrival time. In practice it may be more convenient to use

instead $t_{\frac{1}{2}}$, the median time, or t_j, the time of the j-th particle, but this makes only a small difference (a factor of order 1) which I will neglect. For vertical showers on a thin scintillator of area A, $n = mSA$, where S is the 'particle density' as it usually is expressed, in units of the signal made by a relativistic muon. The factor m takes into account evidence that the average-size contribution to shower signals may be less than one muon (McDonald *et al.* 1977). For S I will use the NKG formula, $S = (N/r_o^2)C(r/r_o)^{s-2}(r/r_o + 1)^{s-4.5}$, where N is the shower size, s is the age, r_o is the Molière length, and $C = \Gamma(4.5-s)/[2\pi\Gamma(s)\Gamma(4.5-2s)]$. I find empirically that

$$\sigma_t = \sigma_{to}(r/r_t + 1)^b \tag{1}$$

with typical values $\sigma_{to} = 2.6$ ns, $r_t = 30$ m and $b = 1.5$.* Substituting in the formula for δt I obtain

$$\delta t = \frac{\sigma_{to} r_o}{\sqrt{CmNA}} (r/r_o)^{1-\frac{1}{2}s}(r/r_o+1)^{2\frac{1}{4}-\frac{1}{2}s}(r/r_t+1)^b \ . \tag{2}$$

This tells us that one method of improving the angular resolution is to increase the size of the detectors. For primary energies not too great it will also help to increase the altitude, because $\sqrt{N}$ will increase faster than $\sigma_{to}r_o$. To look at typical numbers I assume $s = 1$, which gives me $C \cong 0.4$. Then I take $r_o = 100$ m, $A = 1$ m^2, $N = 10^5$ and $m = 1$. I obtain

$$\delta t = (1.3 \text{ ns})(r/100)^{0.5}(r/100+1)^{1.75}(r/30+1)^{1.5}$$

and find these values:

r (m) =	10	20	50	100	200
δt (ns) =	0.8	1.7	8.1	39	267

Thus in order to locate a shower front within, say, 1 ns at greater and greater core distances one must use scintillators of rapidly increasing size. (In this example, whereas 3 m^2 detectors would be adequate up to 20 m, at 50 m the detector area would have to be 65 m^2.)

3. Angular resolution. The exact angular resolution of an array-telescope will depend on details of the layout and the data processing, but if the system is well designed the result will be approximately

$$\Delta\theta = c\delta t(R)/R \ , \tag{3}$$

where the 'effective baseline' R is such that $\delta t(R)$ equals the instrumental timing error, which I will call Δt. The idea is that in a 'well designed' system the separation of the detectors will be small enough in relation to r_o, A, and shower sizes of interest, so that typical useful events will produce several signals with δt approximately equal to Δt. These signals will dominate the usual least-squares fitting procedure for computing the shower direction. Signals with $\delta t \ll \Delta t$ will have little weight because the corresponding baselines will be $\ll$ R; signals with δt

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* See conference paper HE4.7-14 for experimental results on $b(E,\theta)$ for $10^{17}<E<10^{20}$ eV, and more references. In that experiment, as in most others, the energy dependence was found to be too small to detect. Parameter b controls the large core distance behavior of σ_t, which is not the main issue here. Measurements near the core are more difficult to make and to interpret; more data are needed. The results of Woidneck and Böhm (1975) favor a smaller value for σ_{to}, around 1.6 ns. See also Clay and Dawson (1984) and McDonald *et al.* (1977).

>> Δt will have little weight in spite of having baselines > R, because δt is such a rapidly increasing function for large core distances.

Moving the radical $\sqrt{mNA}$ to the left hand side of (2), one sees that the right hand side depends only on certain constants and r. Solving for r (graphically or numerically), representing the result as $r = f(\sqrt{mNA}\cdot\delta t)$, and then letting r = R so that δt can be replaced by Δt, one obtains the desired result:

$$\Delta\theta = c\Delta t/f(\sqrt{mNA}\cdot\Delta t) \tag{4}$$

4. Discussion. Taking the Kiel array as an example (Bagge *et al.* 1979), Δt = 1 ns and A = 1 m^2, so for mN = 10^5 the effective baseline R is 13 m (by interpolation in the table above), and the angular resolution is predicted to be 23 mrad. This prediction is in good agreement with what is claimed by the Kiel group: $\Delta\theta$ better than 1° for N = 10^5-10^7. I have tested the predictions in other cases (MIT Agassiz, MIT Volcano Ranch, and Haverah Park arrays), each time finding satisfactory agreement.

Fig. 1 shows curves of constant angular resolution in coordinates log(Δt) vs log(mNA), calculated in the way I have explained, using (4). In region A the performance of an array-telescope is *particle statistics limited*. In this region, improving the instrumental time resolution does little to improve $\Delta\theta$. In region B the performance is limited instead by characteristics of the detectors and recording system, so there is room for improvement in $\Delta\theta$ by improving these characteristics.

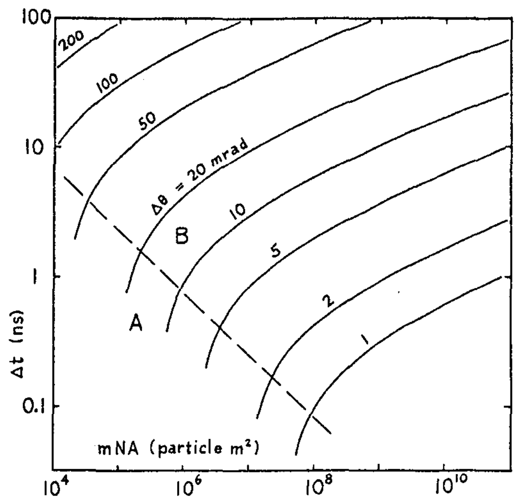

Fig. 1. Curves of constant angular resolution ($\Delta\theta$, mrad) for an array-telescope with given timing error Δt and area per detector A, for showers of size N. See text for explanation of m, and of regions A & B.

For E = 10^{15} eV (N = 10^5 at sea level, systems like the one at Kiel are already in region A. The most practical way to improve their performance is by greatly increasing the size (or density) of the detectors, or by moving to a higher altitude. A goal of 5 mrad angular resolution within the next decade seems to be realistic. This can be achieved with scintillation counters, without improving Δt. The improvement in signal to noise ratio for point sources of γ-rays, a factor of $(20/5)^2$, will be substantial.

Improving still further so as to attain $\Delta\theta$ = 1-2 mrad at this energy seems to require a radical change, from scintillators to some other kind of device with an intrinsically much faster response. Workers at SLAC have constructed planar spark counters (PSC's) with dimensions of 10 x 300 cm, and these have been tested at PEP giving on-line timing resolution better than 200 ps, as well as position resolution smaller than 4 mm

(Atwood *et al.* 1983, Ogawa *et al.* 1984). The timing resolution of similar 30 x 30 cm units constructed at Novosibirsk is reported to be as small as 24 ps (Fedotovich *et al.* 1982).

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