FORMULATION OF COSMIC-RAY SOLAR DAILY VARIATION AND ITS SEASONAL VARIATION, PRODUCED FROM GENERALIZED STATIONARY ANISOTROPY OF SOLAR ORIGIN

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1. Introduction. In the previous papers(1-3), we presented a formulation of cosmic-ray daily variations produced from solar anisotropies stationary through a year and also of their annual (or seasonal) modulation caused by the annual variation of the rotation axis of the Earth relative to that of the Sun. These anisotropies are symmetric for an arbitrary rotation around an axis. On the other hand, from observations of the tri-diurnal variation(5), it has been suggested that solar anisotropies also contain some axis-asymmetric term of the third order with respect to the IMF-axis. This suggestion has recently found support in a theoretical study by Munakata and Nagashima(4). According to their results, the terms of axis-asymmetry with respect to IMF-axis appear also in the 2nd order anisotropy, together with some different kinds of axis-symmetric terms.

The contribution of these anisotropies to the daily variation is different from that of those discussed previously. In the present paper, we extend the above mentioned formulation to a case of a generalized anisotropy.

2. Formulation. Following after the formulation by Munakata and Nagashima(4), we express the stationary anisotropy $\eta(\boldsymbol{r},p)$ through a year in the IMF-polar-coordinate system defined in fig. 1, as

$$\eta(\boldsymbol{r},p)=\sum_{n=1}^{\infty}\sum_{m=0}^{n}\{\eta_n^{mc}(\boldsymbol{r},p)\cos m\Phi'+\eta_n^{ms}(\boldsymbol{r},p)\sin m\Phi'\}P_n^m(\cos\Theta'), \tag{1}$$

where $\eta_n^{mc}(\boldsymbol{r},p)$ and $\eta_n^{ms}(\boldsymbol{r},p)$ are coefficients, and the angles Θ' and Φ' express the incident direction of cosmic rays with momentum p at a point $\boldsymbol{r}$. $P_n^m(\cos\Theta')$ in the equation is the semi-normalized associate Legendre function(6). It is noted that the terms with coefficients $\eta_n^{0c}(\boldsymbol{r},p)$'s have a symmetry with respect to the IMF-axis, and have been discussed in the previous papers(1-3). In eq.(1), each term with the coefficient η_n^{mc} or η_n^{ms} can be expressed in the equatorial coordinate system, as

$$\begin{cases}\eta_n^{mc}(\boldsymbol{r},p)P_n^m(\cos\Theta')\cos m\Phi'=\sum_{k=0}^{n}P_n^k(\cos\theta)S_n^k(\alpha|n,m,c)\\[1ex] \eta_n^{ms}(\boldsymbol{r},p)P_n^m(\cos\Theta')\sin m\Phi'=\sum_{k=0}^{n}P_n^k(\cos\theta)S_n^k(\alpha|n,m,s),\end{cases} \tag{2}$$

where

$$S_n^k(\alpha|n,m,\sigma)=\eta_n^{m\sigma}\{a_n^k(\alpha_S|n,m,\sigma)\cos k(\alpha-\alpha_S+\pi)+b_n^k(\alpha_S|n,m,\sigma)\sin k(\alpha-\alpha_S+\pi)\}, \tag{3}$$

in which the angles α and θ are, respectively, the right-ascension and the co-declination $(\theta=\pi-\delta)$ of the particle's incident direction. α_S in eq.(3) is the right-ascension of the Sun, and $\alpha-\alpha_S+\pi$ is related to solar local time t as $\alpha-\alpha_S+\pi=2\pi(t/24\text{hr})$. Using this relation, the incident direction can be expressed in terms of θ and t instead of θ and α. The coefficients a_n^k and b_n^k in eq.(3) vary with α_S, owing to the seasonal variation of the relative configuration between the equatorial- and IMF-coordinate systems arising from the Earth's revolution around the Sun. Applying the frequency modulation method(2), S_n^k of eq.(3) can be transformed as

$$S_n^k(\alpha|n,m,\sigma)=\sum_{l=-\infty}^{\infty} S_n^k(t_{l/k}|n,m,\sigma) \quad \text{for } \sigma=c \text{ or } s, \text{ and } k\neq 0, \tag{4}$$

where

$$S_n^k(t_{l/k}|n,m,\sigma)=x_n^k(l|n,m,\sigma)\cos(2k\pi t_{l/k}/24\mathrm{hr}) + y_n^k(l|n,m,\sigma)\sin(2k\pi t_{l/k}/24\mathrm{hr}), \tag{5}$$

where

$$\begin{cases} x_n^k(l|n,m,\sigma)=\mathrm{T}_x(k,l|n,m,\sigma)\eta_n^{m\sigma} \\ y_n^k(l|n,m,\sigma)=\mathrm{T}_y(k,l|n,m,\sigma)\eta_n^{m\sigma}, \end{cases} \tag{6}$$

in which T_x and T_y are called the transformation coefficients. In eqs.(4) and (5), $t_{l/k}$ is the time in hours (0~24hr) in a day defined by 1year/(365+l/k) and therefore $P_n^k(\cos\theta)S_n^k(t_{l/k}|n,m,\sigma)$ expresses the k-th harmonic of the daily variation in the time frame of $t_{l/k}$ observed by a vertical telescope pointing toward the θ-direction in the outer space of the Earth.

The terms with l=0 are the yearly averaged solar k-th harmonic variations and are expressed by the symbol "SO", while those for $l\neq 0$ are called the extended sidereal ($l>0$) and anti-sidereal ($l<0$) daily variations(2) and are expressed by the symbol "$SI_{l/k}$" for $l>0$ and "$AS_{|l|/k}$" for $l<0$.

Assuming that the anisotropy is stationary through a year, all the transformation coefficients $\mathrm{T}_x(k,l|n,m,\sigma)$ and $\mathrm{T}_y(k,l|n,m,\sigma)$ in eq.(6) are obtained for $n\leq 3$ and $|l|\leq 3$ and are shown in Tables I, II and III. In the calculations, $\boldsymbol{B}$, assumed to have Parker's Archimedian spiral structure, is expressed in the heliographic polar co-ordinate system (r, θ_h, φ_h) as

$$\begin{aligned} B_r &= B_0(1\mathrm{a.u.}/r)^2, \\ B_{\theta_h} &= 0, \\ B_{\varphi_h} &= -B_0(1\mathrm{a.u.}/r)\sin\theta_h, \end{aligned} \tag{7}$$

where B_0 is positive(negative) for the away (toward) sector. The inclination of the solar equatorial plane with respect to the ecliptic plane is also taken into account in the calculations(3).

3. Discussion and Conclusion. The terms with a mark "c" in tables express the IMF-sense-*independent* terms, as the corresponding coefficients η_n^{mc}'s in eq.(2) are independent of IMF-sense(4). On the contrary, the terms with a mark "s" express the IMF-sense-*dependent* terms, as η_n^{ms}'s change their sign with the change of IMF-sense.

The No.1 and No.2 terms in Table I produce the solar diurnal variation of north-south symmetric type, whereas the No.3 term produces the IMF-sense-dependent sidereal diurnal variation of Swinson type(7).

The most important term in Table II is that of No.1. It is noteworthy that the No.4 and No.5 terms are IMF-sense-dependent and will be discussed in detail in a separate paper(8).

The importance of the No.1 term in Table III has been well acknowledged for the explanation of the solar tri-diurnal variation from the theoretical point of view, whereas the existence of the No.2 term has recently been pointed out by Munakata and Nagashima(4).

In conclusión, it is emphasized that the transformation coefficients(T_x and T_y) in these tables enable us to connect solar anisotropies produced from the diffusion-convection process with the observed cosmic-ray solar daily variation and its seasonal variation and, as the result, enable us to obtain the information of electromagnetic state in interplanetary space from the observation of solar daily variation of cosmic rays.

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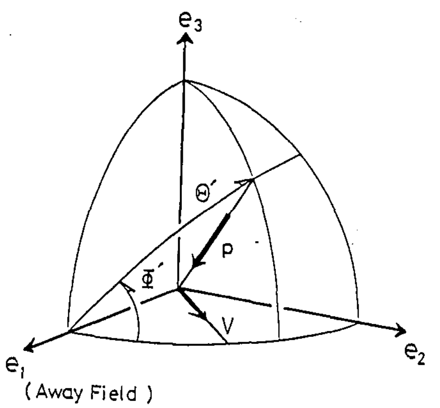

Fig. 1. The IMF-POLAR-COORDINATE SYSTEM.
p: Momentum of particle.
V: Solar wind velocity.
e_1 : Unit vector in the direction of "away" magnetic field.
e_3 : Unit vector in the direction of $e_1 \times V$.
$e_2 = e_3 \times e_1$.

Table I . The transformation coefficients $T_x(k,l|n,m,\sigma)$ and $T_y(k,l|n,m,\sigma)$ in eq.(6) for n=1, in units of 10^{-3}.

No.	$nm\sigma$	k	$l=3$ T_x T_y	$l=2$ T_x T_y	$l=1$ T_x T_y	$l=0$ T_x T_y	$l=-1$ T_x T_y	$l=-2$ T_x T_y	$l=-3$ T_x T_y
1	10c	1		-3, 47		696, -668		16, -15	
2	11c	1		32, -46		660, 671		13, 16	
3	11s	1	-4, 9		62, -410		-57, -25		-1, 0

Table II. The transformation coefficients $T_x(k,l|n,m,\sigma)$ and $T_y(k,l|n,m,\sigma)$ in eq.(6) for n=2, in units of 10^{-3}.

No.	$nm\sigma$	k	$l=3$ T_x T_y	$l=2$ T_x T_y	$l=1$ T_x T_y	$l=0$ T_x T_y	$l=-1$ T_x T_y	$l=-2$ T_x T_y	$l=-3$ T_x T_y
1	20c	1	-8, -12		-55, 285		-300, 59		-7, 1
		2		51, 60		34, -804		2, -36	
2	21c	1	-9, 16		57, 1		-45, -395		0, -9
		2		-41, -25		907, 25		40, 4	
3	22c	1	23, -8		-80, 579		266, [illegible]		6, 1
		2		134, 17		8, 421		-3, 17	
4	21s	1		22, 102		591, -553		28, 6	
		2	22, 13		-230, -330		-61, 13		-2, 1
5	22s	1		121, -100		531, 522		-12, 28	
		2	-25, -13		313, -227		-13, -59		-1, -2

Table III. The transformation coefficients $T_x(k,l|n,m,\sigma)$ and $T_y(k,l|n,m,\sigma)$ in eq.(6) for n=3, in units of 10^{-3}.

No.	$nm\sigma$	k	$l=3$ T_x T_y	$l=2$ T_x T_y	$l=1$ T_x T_y	$l=0$ T_x T_y	$l=-1$ T_x T_y	$l=-2$ T_x T_y	$l=-3$ T_x T_y
1	30c	1		-35, -110		-291, 274		82, 46	
		2	-30, -7		167, 247		-186, 276		-9, 12
		3		103, 10		-466, -532		-31, -36	
2	31c	1		-61, 40		-127, -101		-70, 122	
		2	10, 15		-60, 46		-343, -267		-14, -13
		3		-47, 51		627, -568		44, -36	
3	32c	1		-42, -132		-342, 293		-117, -75	
		2	-38, -15		268, 390		238, -228		12, -9
		3		104, -96		353, 350		19, 25	
4	33c	1		-223, 161		-342, -314		59, -74	
		2	52, 53		-410, 306		102, 139		3, 7
		3		84, 128		-113, 140		-9, 5	
5	31s	1	-19, -26		-83, 449		-373, 75		-9, -7
		2		137, 85		42, -563		28, -22	
		3	39, -15		-368, -77		-41, 46		-1, 3
6	32s	1	-25, 47		75, 11		-60, -537		12, -12
		2		-30, -57		655, 14		23, 38	
		3	-21, 28		84, -451		-57, -49		-4, -2
7	33s	1	63, 0		-83, 639		389, -8		8, 11
		2		256, 48		27, 345		-25, 9	
		3	2, -51		261, 38		27, -34		1, -2