RE-EVALUATION OF COSMIC RAY CUTOFF TERMINOLOGY

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ABSTRACT

The study of cosmic ray access to locations inside the geomagnetic field has evolved in a manner that has led to some misunderstanding and misapplication of the terminology originally developed to describe particle access. This paper presents what is believed to be a useful set of definitions for cosmic ray cutoff terminology for use in theoretical and experimental cosmic ray studies.

1. Introduction. Early work on cosmic ray motion in axially symmetric representations of the field (Stormer, 1930; Lemaitre and Vallarta (1936a, b) was limited to analytic considerations. More recent studies, which rely on digital computer calculations of charged particle trajectories in high order mathematical representations of the field (Shea et al., 1965) emphasize the study of access to locations within the geomagnetic field as a function of particle rigidity, in contrast to the earlier studies which examined the broader directional access picture. The terms used in the early work to refer to the access regions were carried over into the numerical computations, although often in an imprecise way. Whereas the early theoretical workers viewed access conditions in what may be called a "direction picture", describing the directions from which particles of a specified rigidity could or could not arrive, the later computer calculations have usually used a "rigidity picture" in which accessibility is considered as a function of particle rigidity in a single arrival direction. The use of the old geometrical terms, appropriate to the direction picture, has caused considerable confusion in rigidity picture studies.

A re-examination of the characteristics of the access regions distinguished by Stormer and by Lemaitre and Vallarta, together with a comparison between the properties of these analytically distinguished regions and those detected by the digital computer method has resulted in definitions which, it is hoped, will meet present needs.

2. Definitions. The definitions required may conveniently be subdivided by viewpoint, as indicated in Table 1. The list is not exhaustive, but seeks to portray the most useful quantities in each situation.

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TABLE 1.

Summary of terms used in cutoff calculations. Quantities describing phenomena which are equivalent in both pictures are listed on the same line.

Direction Picture	Rigidity Picture
	Cutoff Rigidity
Allowed Cone	
Main Cone	Main Cutoff Rigidity
	First Discontinuity Rigidity
Shadow Cone	Shadow Cutoff Rigidity
Penumbra	Penumbra
Penumbral Band	Penumbral Band
	Primary Band
Stormer Cone	Stormer Cutoff Rigidity
Forbidden Cone	
	Upper Cutoff Rigidity
	Lower Cutoff Rigidity
	Horizon Limited Rigidity
	Effective Cutoff Rigidity
	Estimated Cutoff Rigidity

2.1. Directional Definitions. The following definitions appear to be appropriate for use with the directional picture. Each definition is for charged particles of a single specified rigidity value arriving at a particular point in the geomagnetic field.

Allowed Cone: The solid angle containing the directions of arrival of all trajectories which do not intersect the earth and which cannot possess sections asymptotic to bound periodic orbits (because the rigidity is too high to permit such sections to exist in the directions of arrival concerned).

Main Cone: The boundary of the allowed cone. The main cone is constituted in part by trajectories which are asymptotic to the simplest bound periodic orbits and in part by trajectories which graze the surface of the earth. (For this purpose the surface of the earth is generally taken to be at the top of the effective atmosphere.)

Forbidden Cone: The solid angle region within which all directions of arrival correspond to trajectories which, in the absence of the solid earth, would be permanently bound in the geomagnetic field. Access in these directions from outside the field is, therefore, impossible.

Stormer Cone: The boundary of the forbidden cone. In an axially symmetric field the surface forms a right circular cone.

Shadow Cone: The solid angle containing all directions of particle arrival which are excluded due to short range earth intersections of the approaching trajectories while loops within the local field line bundle are being traversed.

Penumbra: The solid angle region contained between the main cone and the Stormer cone. In general the penumbra contains a complex structure of allowed and forbidden bands of arrival directions.

Penumbral Band: A contiguous set of directions of arrival within the penumbra, the members of which are either all allowed or all forbidden.

2.2. Rigidity Picture Definitions. The following definitions are proposed as being applicable to the rigidity picture. Each definition refers to particles arriving at a particular site within the geomagnetic field from a specified direction.

Cutoff Rigidity: The location of a transition, in rigidity space, from allowed to forbidden trajectories, as rigidity is decreased. Unless otherwise defined, the value normally quoted in representing the results of computer calculations is, for practical reasons, the rigidity of the allowed member of the appropriate juxtaposed allowed/forbidden pair of trajectories computed as part of a spaced series of traces. Sometimes the term is employed to refer to the location of a notional transition from one region to another, for example at the Stormer cone, where an allowed trajectory may not perhaps exist at all.

Main Cutoff Rigidity, R_M: The rigidity value at which the direction concerned is a generator of the main cone as defined in the "direction picture". The associated trajectory is either one which is asymptotic to the simplest bound periodic orbit, or (owing to the presence of the solid Earth) is one which is tangential to the earth's surface.

First discontinuity rigidity, R_1: The rigidity associated with the first discontinuity in asymptotic longitude as the trajectory calculations are performed for successively lower rigidities, starting with some value within the allowed cone. The value of R_1 is approximately equal to the main cutoff as defined above.

Shadow Cutoff Rigidity, R_{SH}: The rigidity value at which the edge of the shadow cone lies in the direction concerned.

Penumbra: The rigidity range lying between the main and the Stormer cutoff rigidities.

Penumbral band: A continuous set of rigidity values within the penumbra, all members of which have the same general access characteristics, either all allowed or all forbidden.

Primary Band: The stable forbidden penumbral band which is associated with the earth intersection of a low point in the loop which lies at the last equatorial crossing before the trajectory (or its virtual extension in the assumed absence of the earth) takes on the characteristic guiding centre motion down the local field line bundle.

Stormer Cutoff Rigidity, R_S: The rigidity value for which the Stormer cone lies in the given direction. In a dipole field (and perhaps also in the real geomagnetic field) access for particles of all rigidity values lower than the Stormer cutoff rigidity is forbidden from outside the field. In a dipole approximation to the geomagnetic field, one form of the Stormer Equation gives the Stormer cutoff rigidity, in GV, as:

$$R_S = M\cos^4\lambda \Big/ \left\{ r^2\left[1 + \left(1 - \cos^3\lambda \cos A \sin Z\right)^{1/2}\right]^2\right\}$$

where M is the dipole moment and has a normalized value of 59.6 when r is expressed in units of earth radii, λ is the magnetic latitude, r is the distance from the dipole in earth radii, A is the azimuthal angle measured clockwise from the geomagnetic east direction (for positive particles), and Z is the angle from the local magnetic zenith direction.

Upper Cutoff Rigidity, R_U: The rigidity value of the highest detected allowed/forbidden transition among a set of computed trajectories. The upper cutoff rigidity can correspond to the main cutoff if and only if no trajectories asymptotic to bound periodic orbits lie at rigidities higher than this value. This can be identified from the nature of the trajectory associated with the main cutoff.

Lower Cutoff Rigidity, R_L: The lowest detected cutoff value (i.e., the rigidity value of the lowest allowed/forbidden transition observed in a set of computer calculations). If no penumbra exists, R_L will equal R_U.

Horizon Limited Rigidity, R_H: The rigidity value of the most rigid allowed trajectory found in a set of computer calculations performed for a below horizon direction at a location above the surface of the earth.

Effective Cutoff Rigidity, R_C: The total effect of the penumbral structure in a given direction may be represented usefully, for many purposes, by the "effective cutoff rigidity" - a single numeric value which specifies the equivalent total accessible cosmic radiation within the penumbra in a specific direction. "Effective cutoffs" may be either linear averages of the allowed rigidity intervals in the penumbra or functions weighted for the cosmic ray spectrum and/or detector response.

Estimated Cutoff Rigidity, R_{est}: A value obtained by using an empirically normalized equation to approximate the cosmic ray cutoff variation in the location of a particular point within a magnetic field in order to estimate a cutoff value pertaining to the point. This value can be found by use of a variety of interpolation techniques, one of which is application of the Stormer equation given above. Because the Stormer equation characterises the spatial variation of the cutoff rigidity, with appropriate normalization it may be used to obtain useful estimates of the various cosmic ray cutoff rigidities over intervals of latitude, longitude, zenith and azimuth, for example. In practice, estimates of the value of any cutoff can be obtained from adjacent calculated values to a reasonable accuracy by employing this method.

3. Discussion and Conclusion.

Not all the cutoffs defined in the previous section exist for every location and direction. It should be borne in mind that, because the definitions have deliberately been kept usefully general, the application of the terms may require more detailed qualification in some individual circumstances. In addition, there is no doubt that other physically meaningful quantities exist. It is believed, however, that the cutoff concepts described in this paper presently have the greatest significance, and that the use of these definitions should alleviate most of the existing confusion and satisfy the current requirements of investigators involved in cosmic ray access studies.

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