

SOLAR NEUTRINO FLUX, COSMIC RAYS AND THE 11 YEAR SOLAR CYCLE

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ABSTRACT

It is shown that the results of maximum likelihood treatment of Monte Carlo simulation with constant production rate of 7.6 SNU and 1.8 SNU are consistent with the constant production rate when the tests of hypotheses (e.g. t-test, χ^2 -test, Wilcoxon-Mann-Whitney test, run test etc.) has been applied to the two groups of data formed from sunspot minimum range and sunspot maximum range whereas the real data pulsates with the solar activity cycle. It is shown that SN flux-change is in opposite phase to the solar activity cycle and lags behind the latter by about one year. A correlation between SN flux and the cosmic rays has also been suggested.

Raychaudhuri(1,2,3,4,5,6) and Gavrin et al(7) showed that the solar neutrino (SN) flux data is varying with the solar activity cycle. Raychaudhuri (3,4,5,6) showed that the fluxes will be higher in the sunspot minimum range than in the sunspot maximum range. This is supported by the data presented by Rowley et al(8) at the solar neutrino and neutrino astronomy conference held at Lead, South Dakota. Apart from the above variation Raychaudhuri(5,6) showed that there is also a quasibiennial variation from 1970-1975 and 1979-1984. This type of variation is already observed by Gnevyshev(9) and Filisetti(10) in the cosmic ray intensity (proton flare).

In this paper an estimate of the moving average (5 successive run number) data is considered to find the statistical significant variation of SN flux with the solar activity cycle of 11 years by t-test, χ^2 -test, Wilcoxon-Mann-Whitney test and run test. To search variation in a convincing manner the above tests of hypotheses have been applied to the data that had been generated by Monte Carlo simulation and background parameters are typical of those in the actual experiment.

SOLAR ACTIVITY CYCLE AND (a) MOVING AVERAGE DATA, (b) MONTE CARLO SIMULATED DATA WITH CONSTANT PRODUCTION RATE (i) 7.6 SNU AND (ii) 1.8 SNU.

Here we apply the same procedure as in Raychaudhuri(5,6).

I) Student t-test:- a(i) Moving average data

We form the two groups, the first group comprises sunspot minimum range (about 4.6 years) and the second group i.e., the rest comprises sunspot maximum range. Let us now collect the data from the run number 36 to 58 from August 1974 to February 1979 (about 4.6 years) for the first group. If we now set up the null hypothesis $H : m_1 = m_2$ against the alternatives $m_1 > m_2$. The statistics t is given by (here the difference of variance is not significant).

$$t = \sqrt{\frac{N_1 N_2}{N_1 + N_2}} \frac{\bar{x} - \bar{y}}{\sqrt{N_1 S_1^2 + N_2 S_2^2}}$$

where $\bar{x} = N_1 \bar{x}_1 + N_2 \bar{x}_2$. In the above case $N_1 = 23, N_2 = 27, \bar{x} = 48$ which gives $t = 4.54$. Thus we can conclude that 99.9% of the data pulsates with the solar activity cycle. The standard error of the difference of means is 4.54. Hence the data are inconsistent with the assumption that the two means are equal.

(ii) Yearly average of the moving average data:- Here one group comprises the yearly average data from 1975 to 1978 and the other group comprises rest of the moving average data. We get for 9 d.f. $t = 2.76$. The standard error of the difference of the means is 3.16 which suggests that the

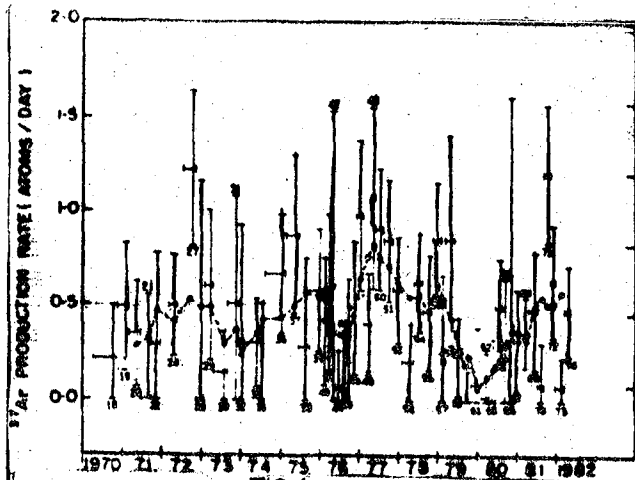


FIG. 1

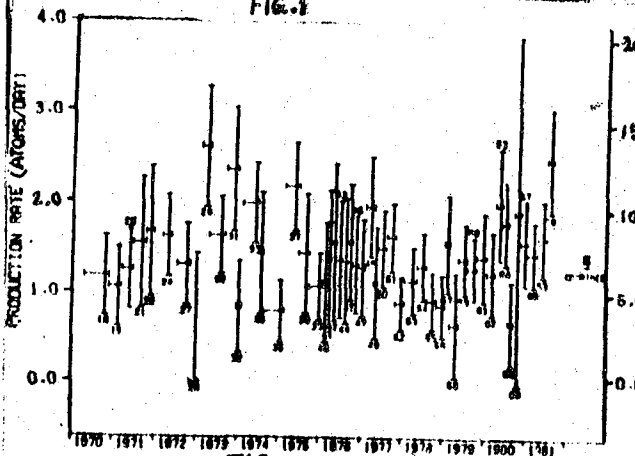


FIG. 2

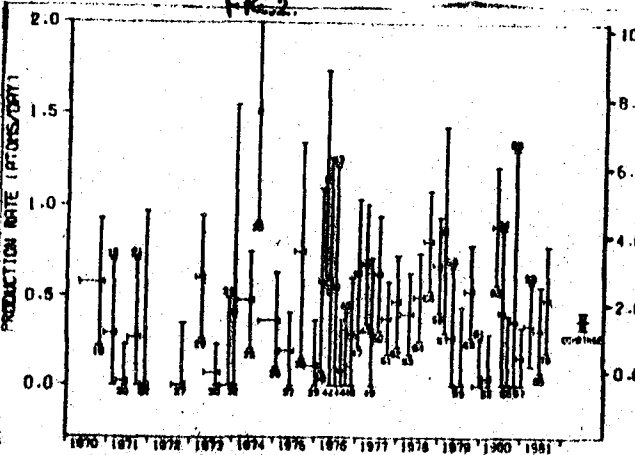


FIG. 3

Fig. 1. ^{37}Ar production rates observed in the Chlorine SN experiment 1970-1982 and the moving average data (dashed curve).

Fig. 2 and 3. Results of maximum likelihood treatment of Monte Carlo simulations with constant production rate of 7.6 SNU and 1.8 SNU respectively.

means of the two samples are not equal.

b) Monte Carlo simulated data: (i) 7.6 SNU and (ii) 1.8 SNU. We form the groups as in moving average data. In the case of 7.6 SNU the difference of variance is significant. We have to use the formula for the difference of two means.

$$t = \frac{\bar{x} - \bar{y}}{\left[\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2} \right]^{1/2}}$$

For d.f. 47, $t = 1.19$. The standard error of the difference of two means is 1.1 e . Hence the two means are equal i.e., the steady flux is acceptable. In the case of 1.8 SNU the difference of variance is not significant. Hence $t = 1.84$, d.f. 45. The standard error is found to be 1.8 e .

Hence the data are consistent with the assumption that the steady flux is acceptable. Next we take the yearly average of the Monte Carlo simulated data from fig 2 & 3 and form the two groups as in t-test. Here in both cases difference of variance is not significant. In case 7.6 SNU, $t = 0.43$ d.f. 9. In case 1.8 SNU, $t = 1.01$ d.f. 9. Here

the standard error is found to be (i) 0.53 e and (ii) 1 e respectively. Hence the data are consistent with the assumption that the steady flux is acceptable.

ii) χ^2 -test: (i) We form the group as in the t-test. For moving average data $\sigma^2 = 0.02$ and for d.f. 49, $\chi^2 = 73.19$. Thus the hypothesis of steady flux is not acceptable. For the case of 7.6 SNU, $\sigma^2 = 0.291$, $\chi^2 = 47.84$ d.f. 47. again for 1.8 SNU, $\sigma^2 = 0.10$, $\chi^2 = 46.83$ d.f. 45. Hence the difference of two samples are not significant for the case of 7.6 SNU and 1.8 SNU.

(ii) if we take the yearly average of the moving average data and Monte Carlo simulated data. We form the groups as in t-test. We get $\sigma^2 = 0.1$, $\chi^2 = 18.55$, d.f. 10 for moving average data. In this case steady SN flux is not acceptable. Again (i) $\sigma^2 = 0.09$, $\chi^2 = 9.39$, d.f. 9 for 7.6 SNU and (ii) $\sigma^2 = 0.5$, $\chi^2 = 10.48$, d.f. 9 for 1.8 SNU. Hence the steady SN flux is acceptable in Monte Carlo simulated data.

(III) Wilcoxon-Mann-Whitney test:- Here we take the yearly average data of moving average data and Monte Carlo simulated data 7.6 SNU and 1.8 SNU and form the two groups as in t-test and χ^2 -test. If we set up the null hypothesis H_0 : the distribution of x and y are identical. If a two sided test at 5% is desired for $m=4$, $n=7$ we use $3 < U < 25$ as the accepted region. For this test $\alpha = P_{H_0} (U \leq 3 \text{ or } U \geq 25) = 0.042$.

a) Moving average data:- we write the two sets of data as y: 3552, 4740, 3546, 4007, 3089, 2469, 4707 and x: 5305, 4437, 7482, 5421. Putting these numbers in order and keep track of which numbers came from x and which from y: yyyyyyxyxxx. There are 2 inversions of the y's and the rank of y's is $1+2+3+4+5+7+8 = 30 = 2 + \frac{7 \times 8}{2}$. Since the number of inversion of the y's is not within 3 and 25, we reject the null hypothesis, i.e. the constant SN flux is not acceptable.

b) (i) 7.6 SNU :- y: 1.408, 0.993, 1.430, 1.286, 1.277, 1.928 and x: 1600, 1.396, 1.320, 1.098. Putting these numbers in order and keep track of which numbers came from x and which came from y: yxyxyxyxy. There are 16 inversions of the y's and the rank of y's is $1+3+4+7+8+10+11 = 44$.

(ii) 1.8 SNU :- y: 1.545, 6001, 165, 805, 537, 307, 378 and x: 3755, 4900, 518, 581. There are 8 inversion of the y's and the rank of y's is 36. Since the number of inversion of the y is within 3 and 25, we accept the null hypothesis i.e., steady SN flux is acceptable in both the Monte Carlo cases.

(IV) Run test:- In the case of run up and down the total number of runs is approximately normally distributed under the assumption of randomness with mean and variance is given by $E(r) = \sqrt{3(2n-1)}$ and $V(r) = \sqrt{(90)(6n-29)}$.

For moving average data:- $n=50$, $E(r)=33$, $V(r)=8.57$. In this case there are 21 runs either positive or negative. If we take the critical region to be $\alpha = 0.20$ would yield a K of $33 - 0.84 \sqrt{8.57} = 30.543$. Thus we cannot accept the randomness since $21 < 30.543$. Thus we can suggest that the constant SN flux is not acceptable. For case 7.6 SNU, $n=49$, $E(r)=32.33$ and $V(r)=8.389$. In this case there are 31 runs either positive or negative. For the case 1.8 SNU, $n=47$, $E(r)=30.31$ and $V(r)=8.03$. In this case there are 29 runs is either positive or negative. If we take the critical region to be $r < K$, a choice of $\alpha = 0.20$ would yield a K of (i) $32.33 - 0.84 \sqrt{8.389} = 29.9$, (ii) $31 - 0.84 \sqrt{8.03} = 28.62$. Thus we accept the randomness in both the case since (i) $31 > 29.9$ and (ii) $29 > 28.62$. So we can conclude that the Monte Carlo simulated data do not pulsate with the solar activity cycle.

Sunspot cycle and SN flux:- We have taken the yearly average smooth sunspot from solar geophysical data, yearly average of the moving average data from fig. 1. and yearly average of the cosmic ray data I with $E = 0.1-5.8$ Gev(11). We have displayed the data in Table I.

The correlation coefficient between sunspot cycle and ^{37}Ar production data are $r(S, Q_M) = -0.61$, when $\Delta T = -1$, $r(S, Q_M) = -0.49$ when $\Delta T = 0$, $r(S, Q_M) = -0.35$ when $\Delta T = 2$ where $\Delta T = T_S - T_{P_M}$ (in years). Thus the ^{37}Ar production rate anticorrelates with the sunspot cycle. The correlation coefficient between the galactic cosmic ray intensity and ^{37}Ar production data $r(I_0, Q_M) = 0.58$ when $\Delta T = 0$, $r(I_0, Q_M) = 0.31$ when $\Delta T = -1$, $r(I_0, Q_M) = 0.39$ when /

Table 1. Yearly averages of smooth sunspot number (S), Ar production data (Q_M), yearly average of cosmic ray data I_0 with $E=0.1-5.8$ Gev.

	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981
Yearly average Sunspot number (S)	66.6	68.9	38.5	34.4	15.5	12.6	27.5	92.7	155.3	155	140
Moving Average of the yearly Ar production data (Q_M)	0.36	0.44	0.35	0.40	0.53	0.44	0.75	0.54	0.31	0.25	0.47
$I (10^3 \text{ m}^{-2} \text{ s}^{-1} \text{ st}^{-1})$ Galactic cosmic ray intensity	2.01	2.86	2.80	2.73	3.11	3.20	3.29	2.79	1.89	1.24	1.13

$\Delta T=1$. Where $\Delta T = T_{I_0} - T_{Q_M}$ (in years). Thus there is an important correlation between the SN flux in Davis' data and galactic cosmic rays. This shows that the galactic cosmic rays and SN flux undergo the essential changes in the solar activity cycle from 1970-1982.

From the SN flux data we see that SN flux is higher during the ascending phase (about 2-3 years) and descending phase (about 2-3 years) of the solar cycle avoiding the sunspot maximum time. We call the time 2-3 years before the sunspot maximum as the second minimum time. Thus SN flux is higher in both second minimum time and second maximum time and SN flux is lower in both first minimum and first sunspot maximum stage. In addition to two prominent maximum in the year 1957 and 1960 they (9, 10) have found third maximum of smaller amplitude in the cosmic ray intensity but very clear in 1963. From fig. 1 we see that run number 37 is connected with third maximum. This may have some connection with the quasi-biennial variation of SN flux found from 1970-1975 and 1979-1984. From the moving average data in fig. 1 it appears that SN flux varies with a period of about 2.4 to 2.5 years from 1970-1975.

In conclusion it may be remarked that the present SN data exhibits a high level of statistically significant variation of SN flux with the solar activity cycle. The above results strongly suggest that the solar activity cycle is due to the pulsating character of the nuclear energy generation inside the core of the sun. We suggest that the chlorine SN experiment should be continued to get the data like the sunspot data etc. and this data could be of great importance to our understanding of the sun and cosmic rays.

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