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Three-dimensional supersonic flows of an ideal fluid in the neighborhood of bodies formed by being cut out along the streamlines of an axisymmetric flow are investigated. The flow consists of a region of isoentropic compression and a region of vortex flow. An exact solution with variable entropy is used to describe the flow in the vortex region. In the continuous flow region an approximate solution is constructed by expanding the solution in a series in a small parameter. The effect of the shape of the excission and the vorticity of the flow on compression of the jet and the total pressure loss coefficient is studied.

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DECELERATION OF A SUPERSONIC FLOW BEHIND A CURVED SHOCK WAVE WITH ISOENTROPIC PRECOMPRESSION

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This study investigates three-dimensional supersonic $\angle 72 *$ flows of an ideal fluid in the neighborhood of bodies formed by an excision along the streamlines of an axisymmetric flow. The uneven flow consists of two regions: regions of isoentropic compression and regions of vortex flow. An exact solution with variable entropy [l] is used to describe the flow in the vortex region. In the continuous flow region an approximate solution is constructed by expanding the solution in a series according to a small parameter. The effect of excision shape and flow vorticity on jet compression and the total pressure loss coefficient is studied.

1. Reference [l] investigates solving axisymmetric equations for an ideal gas with variable entropy. This flow is described with a common differential equation for characteristic function $z(h)$ :

$$
\begin{align*}
& \left.1-\alpha^{2}(r-1)-\frac{3}{4} \alpha^{2}\right] h,  \tag{1.1}\\
& \left.b=\frac{x-1}{x} h^{2} \right\rvert\, h\left(\% \alpha^{2}-4 x \alpha+3 \alpha^{2}\right)-4 \alpha \%(\alpha-1) 1, \quad
\end{align*}
$$

and total pressure distribution along the streamlines is written as $p_{0}=C\left(\psi_{0}-\psi\right)^{\alpha}\left(\alpha, c\right.$, and $\psi_{0}$ are randonm constants). Here and further all values are given dimensionless form: enthalpy $h$

[^0]relates to heat content $h_{m}$, values with velocity dimensionality relate to the maximum velocity of steady outflow in a vacum * $\sqrt{2 h_{m}}$, and pressure relates to total pressure po. Under these conditions, stream function $\Psi$ should be related to pol $/ \sqrt{2 h_{m}}$ where $l$ is the characteristic linear dimension.

Flow (1.1) is converted to plane (u, v) of a travel-time curve and physical plane ( $x, y$ ) with equations:

$$
\begin{gather*}
u=z-\alpha \frac{x-1}{x} h z^{\prime}, \quad v=\sqrt{1-h-\left(z-\alpha \frac{x-1}{x} h z^{\prime}\right)^{2}}  \tag{1.2}\\
x=\left.\sqrt{\frac{4-4}{\prime}}\right|_{h_{0}} ^{n} F i h, y=\sqrt{2 \frac{x-1}{x} \frac{\|_{11}-4}{p} h z^{\prime},}  \tag{1.3}\\
F=\frac{\left(\frac{x-1}{x} h z^{\prime}-\frac{z^{\prime \prime}}{x}\right) u-\frac{1}{\alpha} \cdot\left[\frac{1}{2}+u\left(z^{\prime}-\alpha \frac{x-1}{x} h z^{\prime \prime}-x^{x-1} \frac{x}{x} z^{\prime}\right)\right]}{\sqrt{2 \frac{x-1}{x} h z^{\prime} v}}
\end{gather*}
$$

In general, it is impossible to make flow (l.l) adjacent 173 to the even flow by means of a curved shock because the solution is not arbitrary. We will discuss the possibility of joining (1.1) with an even flow through an intermediate region of uneven flow $\Omega_{1}$ (Figure 1). If this is set up as an exact problem, it can only be solved numerically. To construct an analytic, approximate solution we assume that leading shock AC (See Figure 1) is weak and the flow in $\Omega_{1}$ is isoentropic. In $\Omega_{2}$, we have a vortex flow. This flow is described with equation (1.1) and is joined with $\Omega_{1}$ by means of curved shock wave $A B$, which diffuses in the uneven flow.


Figure 1

In variables $h$ and $\psi$, wave $A B$ (see figure 1 ) can be represented as $h=H(\Psi)$. From (1.3), we have the following in the physical plane:

$$
\begin{gather*}
x=\sqrt{\frac{\left(\psi_{n}-\psi\right)^{1-\alpha} H^{\frac{x}{x-1}}}{} /(H)=x(\psi), I}=\int F d h^{C},  \tag{1.4}\\
\therefore y=\sqrt{\frac{2(x-1)}{x} \cdot \frac{\left(\psi_{n}-\psi\right)^{1-\alpha}}{C} H^{-\frac{x}{x-1}}}=y(\psi) .
\end{gather*}
$$

We use $\xi$ to designate the square of the modulus of velocity in $\Omega_{1} ; \beta, v_{1}, v_{2}$ are the slope of shock wave $A B$ and velocity vectors $\Omega_{1}$ and $\Omega_{2}$ to the symmetry axis. Allowing for the fact that the flow in $\Omega_{1}$ is isoentropic, we can then write the laws of momentum and energy conservation on wave (1.4) as:

$$
\begin{gather*}
\xi-(1-H)\left[\frac{\cos \left(\beta-\theta_{2}\right)}{\cos \left(\beta-\theta_{1}\right)}\right]^{2} \\
\operatorname{tg}\left(\beta-0_{1}\right)=\frac{x-1}{\alpha+1} \frac{H M(1-H)-\sin ^{2}\left(\beta-\theta_{2}\right)}{\sin \left(\beta-\theta_{2}\right) \cos \left(\beta-\theta_{2}\right)^{2}}  \tag{1.5}\\
(1-\xi)^{\frac{x}{x-1}}=C\left(\psi_{0}-\psi\right)^{\alpha} \frac{x}{x-1}\left[1-\frac{2 \%}{x-1} \frac{1-H}{H} \operatorname{tg}\left(\beta-0_{1}\right) \sin \left(\theta_{2}-\theta_{1}\right)\right]
\end{gather*}
$$

The law of conservation of mass is satisfied, since the coordinate of $\Psi$ - stream function - is continuous during passage through the shock. The ratio $\operatorname{tg} \beta=y^{\prime}{ }_{\psi} / x^{\prime}{ }_{\psi}$ gives the equation for the wave

The set of equations, (1.1), (1.5), and (1.6), and ratio

$$
\begin{equation*}
\cos 0_{2}=\frac{2-\alpha \frac{x-1}{x} z^{2} H}{\sqrt{1-H}} \tag{1.7}
\end{equation*}
$$

obtained from (1.2) are a closed system of equations for $z, H, \beta$, $\mathcal{V}_{1}, \mathcal{V}_{2}$ and $\xi$. Solving this system defines flow in $\Omega_{2}$, the position of shock (1.4) and the distribution of gas dynamics parameters on the wave front.
2. To construct an approximate solution, we must make the transition to travel-time curve variables in the gas flow equations. For problems with axial symmetry, this transition is usually useless when studying exact problems and methods, since the equations become nonlinear. However, this transition can be quite convenient when constructing and analyzing approximate solutions.

In the statement for flow isoentropicity, i.e. $h=h(p)$, taken from the Eiler equation transformed to variables $\xi=u^{2}+$ $+v^{2}=1-h, n=v^{2}=(l-h) \sin ^{2} v_{1}$, it follows that there is a function $\phi(\xi, \eta)$ such that:

$$
\begin{equation*}
\varphi_{\xi}=\frac{y^{2}}{2 h_{p}}-\frac{\psi}{2 \sqrt{\xi-\eta}}, \quad \varphi_{\eta}=\frac{\psi}{2 \sqrt{\xi-\eta}} \tag{2.1}
\end{equation*}
$$

Any $\neq$ selected will satisfy the dynamic equations identically, and from the equation for continuity we obtain the ratios which determine coordinate $x$ :

$$
\begin{align*}
& x_{\xi}=\sqrt{h_{p}} \frac{2(\xi-\eta)\left[\varphi_{\xi}+i\left(\varphi_{\xi}+\varphi_{\eta}\right)\right]-\varphi_{\eta}}{\because \sqrt{8!(\xi-\eta)\left(\varphi_{\xi}+\varphi_{\eta}\right)}}, \quad x_{\eta}=\sqrt{h_{p}} \frac{\varphi_{\eta}+2(\xi-\eta) \varphi_{i n}}{\left.\sqrt{8 \eta(5-\eta)\left(\varphi_{\xi}+\varphi_{\eta}\right.}\right)} . \\
& j=-\frac{h_{p p}}{h_{p}^{2}} \tag{2.2}
\end{align*}
$$

By excluding $X$ from (2.2) with cross-differentiation, we obtain the equation which contains only one function for $\phi$ :

$$
\begin{align*}
& 2(\xi-\eta) \eta\left(\varphi_{\Sigma \eta}^{2}-\varphi_{s} \varphi_{\eta \eta}\right)-l(2 \xi-\eta) \varphi_{\eta}+2 \xi \varphi_{i} l \varphi_{\xi}-2 \eta\left(2 \varphi_{\xi}+2 \varphi_{\eta}\right) \varphi_{\xi \eta}- \\
& \left.-\eta\{2 \|-j(\xi-\eta)]\left(\varphi_{\xi}+\varphi_{\eta}\right)\right\} \varphi_{\eta \eta}-2 \xi j\left(\varphi_{\xi}+\varphi_{\eta}\right)^{2}+(1-\eta j) \varphi_{\eta}\left(\varphi_{\xi}+\varphi_{\eta}\right)=0 . \tag{2.3}
\end{align*}
$$

This is a Monzh-Ampere type equation with a quasi-linear part. All terms in the equation contain derivatives from the unknown function as products of two partial derivatives, and coefficients for the products are polynomials according to variable $\eta$ to no more than the second power. These properties of equation (2.3) make it possible to search for the solution as a series according to the powers of $\eta$. We enter the parameter $\varepsilon=\max _{\Omega_{1}} \eta$
and let

$$
\begin{equation*}
\varphi=\sum_{k=0}^{\infty} \varepsilon^{k} \varphi_{k}(\xi, \bar{\eta}), \eta=\varepsilon \eta . \tag{2.4}
\end{equation*}
$$

Inserting (2.4) in (2.3) gives the following equation for $\phi_{k}$ :

$$
\begin{equation*}
(2 \xi j-1) \varphi_{0, \eta}\left(\varphi_{h, \eta \eta}-\frac{1}{1} \varphi_{h, \eta}\right)=F_{h-1} . \tag{2.5}
\end{equation*}
$$

Function $F_{n}$ is expressed through $\phi_{k}$ and their derivatives are expressed with $k \leq n$.

$$
\begin{aligned}
& F_{n}=(2 \xi j-1)\left(\varphi_{s, \eta}-\eta \varphi_{\varepsilon, \eta \eta}\right) \varphi_{n-x, \eta}-2 \xi \eta \varphi_{8, \varepsilon, \eta} \varphi_{n-j, \xi n}+2 \eta 1(1-2 \xi j) \varphi_{c, k}+
\end{aligned}
$$

$$
\begin{aligned}
& 4
\end{aligned}
$$

Here, repetitive index $s$ indicates summation from 0 to $n$. If $2 \xi \mathrm{j}-\mathrm{l} \neq 0$ (i.e. Mach number $\mathrm{M} \neq 1$ ), formula (2.5) is a recurrent ratio for $\phi_{k}$. Obviously $F_{-1}=0$. Then, from (2.5), $\phi_{0}=A^{2} \times \eta^{2} / 2+B$, where $A$ and $B$ are random functions of $\xi$. According to (2.1), the flow corresponding to this solution is written with the following formulas:

These formulas determine flow in a channel with variable cross- 175 section area in a one-dimensional approximation (isobars for $\xi=$ $=$ const are the straight lines for $x=$ const). Further

$$
\begin{aligned}
& \text {. } \left.F_{0}=\frac{1}{2-\tilde{\xi}^{2} j}\left\{\frac{\xi\left(A^{2}-1\right)}{A}\left(A^{\prime \prime}+\frac{A^{2}}{A}\right) \div \frac{37^{-2 \xi}}{2} A A^{\prime}-5 j A^{2}\right] \eta-\frac{1+2 \xi j}{\eta} B^{\prime}\right\}, \\
& \left.\varphi_{1}=\frac{1}{1-2 \xi j}\left\{45\left(A A^{\prime \prime}-A^{\prime 2}\right) \div(5 \div 2 \xi j) A A^{\prime} \div 3 j A^{2}\right] \frac{\eta^{2}}{3} \div\left[(1+2 \xi j) B^{\prime}+4 B^{\prime \prime}\right] \eta\right\}
\end{aligned}
$$

The flow corresponding to approximate solution $\dot{\phi}=\phi_{0}+\varepsilon \phi_{1}$ is described with the following equations:

$$
\begin{align*}
& y^{2}=2 h_{p} A^{2} \eta+\frac{2 h_{p}}{2 \xi j-1}\left[\left[4 \xi\left(A A^{\prime \prime}-A^{\prime 2}\right)+4(1+\bar{s} j) A A^{\prime}+3 j A^{2}\right] \eta^{2}+\right. \tag{2.6}
\end{align*}
$$

Here $\varepsilon$ is set equal to one, which actually means a return to undeformed variable $\eta$. According to (2.6), the isobars are second order curves.

For an rough estimate of approximation process accuracy, we compare it with certain exact solutions for gas flow equations. Since the basic approximation is an expansion according to the angle of flow divergence, a solution in which angle $v$ remains unchanged in the trajectories (i.e. the flow fronthe gas source. This flow, by the way, has the simplest structure) is the natural choice for comparison.


Figure 2

Figure 2 shows graphs of the approximate solutions' relative deviation $\delta(\%)$ from the exact solution as a function of the angle of cone $\theta$ for two Mach numbers at the inlet. Solid lines ( $x=$ const) are a one-dimensional approximation and dotted lines are approximation (2.6). As Figure 2 shows, the near-axial region is described most exactly, the second approximation nearly doubles accuracy, and when the Mach number at the inlet (and consequently
throughout the entire flow) increases, error decreases.
3. To construct the flow in $\Omega_{1}$ we use approximate solution (2.6). Wave $A C$ (Figure 1) is assumed to be weak, and the angles of turn on it are small. It is precisely in this case that expansion (2.4) is valid. In variables $\xi$ and $\eta$, weak shock $A C$ is described with the equation:

$$
\begin{equation*}
\eta_{4 c}=\frac{3}{M_{\infty}^{2}-1}\left(\sqrt{5_{\infty}}-1\right)^{n} \tag{3.1}
\end{equation*}
$$

From the laws of conservation of momentum and mass and the laws for joining the flow on $A B$, we obtain:

$$
\begin{align*}
& A^{\prime \prime}=\frac{2 \Xi j-1}{4 \tau_{r}}\left[\frac{y_{A n}^{2}}{2 h_{n}}-\frac{\lambda,}{2\left(i_{x}-\lambda\right)}-A^{2}\left(\eta_{11}-\eta_{A C}\right)\right]-\frac{A^{2}}{A_{A}} \\
& -\frac{1-\frac{\ddagger}{i} j}{\dot{z}} A^{\prime}+\frac{3 j}{4 j} A  \tag{3.2}\\
& B^{\prime \prime}=\frac{2 \xi-1}{4 \xi}\left[\frac{\eta_{A B}^{2}+\tau}{4 \tau} \frac{y_{A B}^{2}}{2 h_{p}}-\frac{2 \eta_{A B}^{2}+\eta_{A C}^{2}}{\eta_{A B}-\eta_{A C}} A-\frac{2}{2 \xi-1} B^{\prime}-\frac{\lambda \nu}{2 \tau\left(\lambda_{\infty}-\lambda\right)}\right] \\
& y_{A C}^{2}=\frac{v \sqrt{\xi}}{\lambda_{\infty}-\lambda}, \quad\left(\lambda=\frac{\sqrt{\xi}}{h_{\eta}}, \tau=\eta_{A B}^{2}-\eta_{A C}^{2}, v=2 B^{\prime} \frac{\left(\xi A^{2}\right)^{\prime}}{\xi_{i}} \eta_{A_{C} C}^{2}\right),
\end{align*}
$$

where $y_{A B}$ and $\eta_{A B}$ are functions of $\xi$, determinable with system (1.1) and (1.5) - (1.7); index $\infty$ relates to the corresponding leading undisturbed flow parameters.

The general system of equations (1.1), (1.5) - (1.7), (3.1) and (3.2) determines total flow, i.e. gives the solution to the problem of joining vortex flow (1.1) with even flow.

Shock wave $A B$ reaches the axis if $y=0$ in (1.3), i.e. if either $\psi=\Psi_{0}(a)$ or $z^{\prime}=0(b)$. In first case (a), we have po $=$ $=0$ for total pressure on the axis, which is impossible. We will
consider the second variation, $z^{\prime}=0$. For (l. l), this is a singular point through which the single-parameter family of equations $z=\sqrt{1-h_{0}}+k\left(h_{0}-h\right)^{2}$ passes (ho is the value for enthalpy on the axis and $k$ is the parameter). Here, according to (1.2), $v=0$ on the axis. It follows that the shock wave reaches the axis and approaches it at a straight angle.

Calculations are made from the axis. Given $M_{\infty}$ and $\alpha$, we find parameter $k$, which fixes the integral curve in the family of solutions passing through the antinode point. Further, departing from the singular point according to the analytic solution, we solve the general system of common differential and transcendental equations (1.1), (1.5) - (1.7), and (3.1) - (3.2). At each integration step for the differential equations, the system of transcendental equations (1.5) is solveld by iteration [2]. Here a solution from the preceding step is selected as the initial approximation for the iteration process. Calculation continues $u p$ to $y=y_{\text {max }}$, whose value corresponds to flow lines $\Psi_{\max }=\psi_{0}-c^{-1 / \alpha}$.
4. The intensity of shock wave $A B$ decreases as $y_{A B}$ increases and falls to zero when $y=y_{\text {max }}$. Intensity changes due to solid wall distortion and uneven flow in $\Omega_{1}$. At values of $\alpha \leq 1$, the unevenness of flow in $\Omega_{2}$ is insignificant (when $\alpha=0.1$, the maximum value of $\Delta \mathfrak{V}_{1} / \Delta_{\psi}$ on the external side of the shock is 0.06 , and when $\alpha=0.01, \Delta v_{1} / \Delta_{\Psi}=10^{-4}$ ) and shock distortion is due basically to wall distortion. In this case leading wave $A C$ is almost a characteristic curve and the $\Omega_{2}$ zone has significant dimensions. As $\alpha$ increases, the flow in $\Omega_{1}$ becomes more uneven, $A C$ begins to differ from the characteristic curve, and zone $\Omega_{1}$ decreases. Here the vorticity in $\Omega_{2}$ can be explained not only by wall distortion, but also by uneven flow in front of the shock. Note that here the hypotheses on which the solution in $\Omega_{1}$ are based are met with decreasing accuracy.


Figure 3

We use the solution obtained to describe the flow in airscoops constructed by an excision along the styeamlines of an axisymmetric flow [3, 4]. We will discuss a sector excision with angle $2 \phi$ (Figure 3 a). The combined loss coefficient for total pressure is determined according to the formula

$$
\begin{equation*}
\mu_{z}=\iint p_{0} I S \tag{4.1}
\end{equation*}
$$

where $S$ is the area of the capturable gas jet.

The average recovery factor for total pressure for this diffusor formula is

$$
\begin{equation*}
\mu=\frac{\mu_{S}}{S}=(1-\alpha) C\left(\frac{S}{T}\right)^{\alpha \cdot(1-\alpha)} \tag{4.2}
\end{equation*}
$$

Figure 3 b shows $\mu$ as a function of the angle of sector $\phi$ at $\angle 77$ different values for $S$, and Figure 3 c shows $\mu$ as a function of index $\alpha$. The influence of $\alpha$ on the recovery factor increases with $\phi$. In Figure 4 the values of $\phi_{i}$ are determined with the equation $\phi_{i}=S_{i} / y_{\text {max }}$, which shows that $y \leq y_{\text {max }}$.

To evaluate the level of compression for the gas jet entering the airscoop, we determine pressure on $A D$ (see Figure 1). On $A D$ $h=$ const $=h_{i}$. Then

$$
\frac{1}{p_{\infty}}=C\left(\frac{h_{1}}{h_{\infty}}\right)_{1}^{\alpha_{1}^{\prime}(x-1)}\left(\psi_{0}-\psi\right)^{\infty}
$$

We obtain the following for total compression

$$
\begin{aligned}
& 1 \\
& \text { 耳 }_{2}=C_{1} x_{0} C_{1}=\left(\frac{h_{1}}{h_{\infty}}\right)^{x(x-1)}
\end{aligned}
$$

where $\mu_{x}$ is determined in (4.1).


The characteristics of intake diffusors with other excision shapes can be determined in a similar manner. Figure 4 shows a diffusor with a rectangular excision which can be characterized with two parameters: the area of captured jet $S$ and angle $\phi$, shown on Figure 4 a. Figure 4 b shows the curve for the ratio of diffusor recovery factor (Figure 4a) to the analogous value $/ 78$ (4.2) characterizing the influence of excision shape on aerodynamic characteristics.

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[^0]:    *Numbers in the margin indicate pagination in the foreign text.

