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THE EFFECT OF POWER LAW BODY FORCES ON A THERMALLY-DRIVEN FLOW BETWEEN CONCENTRIC ROTATING SPHERES

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ABSTRACT

A numerical study is conducted to determine the effect of power-law body forces on a thermally-driven axisymmetric flow field confined between concentric co-rotating spheres. This study is motivated by Spacelab geophysical fluid-flow experiments, which use an electrostatic force on a dielectric fluid to simulate gravity; this force exhibits a $(1/r)^5$ distribution. Meridional velocity is found to increase when the electrostatic body force is imposed, relative to when the body force is uniform. Correlation among flow fields with uniform, inverse-square, and inverse-quintic force fields is obtained using a modified Grashof number.

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I. Background

Geophysical fluid flow experiments (Hart, 1976; Fowlis and Fichtl, 1977) designed to run aboard future shuttle flights will rely on a radial body force created by imposing an electric field across a dielectric fluid to simulate gravity. The fluid will be contained between concentric, rotating spheres with latitudinal "side" walls to confine experimental hardware (see figure 1). The dielectric force is proportional to $(1/r)^5$, (Hart, 1976). The effect of this power law on the flow field relative to a uniform radial body force or one following an inverse square law is investigated here. [Effects of varying rotation rates and boundary temperatures for a uniform gravitational field can be found in the literature (Douglass, 1975; Miller and Gall, 1983; Macaraeg, 1984).] This study utilizes a numerical model based on the incompressible Navier-Stokes equations in a vorticity/stream function formulation, discretized by a pseudospectral representation in the latitudinal direction and a finite difference representation of radial dependencies. For details of the numerical model see Macaraeg (1985).

II. Results of Numerical Study

The gravitional distribution is varied by changing the exponent for $1/r$ (where r is the radial coordinate) in the buoyancy term of the vorticity equation (Macaraeg, 1985), as given below:

$$\sin\theta \left(\frac{1}{r}\right)^n Gr E^2 \frac{\partial T}{\partial \theta}, \tag{1}$$

where Gr is the Grashof number, E is the Ekman number, T is temperature, and θ is the latitudinal coordinate. Values of 0, 2, and 5 are given to n to simulate a uniform gravitational field, and fields proportional to $(1/r)^2$ and $(1/r)^5$, respectively. Gr is a function of a reference gravitational acceleration (g_{ref}):

$$Gr = \frac{g_{ref} \ \alpha \ \Delta T \ r_0^3}{\nu^2} \ , \tag{2}$$

where α and ν are the coefficients of thermal expansion and kinematic viscosity, respectively, and ΔT is the maximum temperature difference on the boundaries. g_{ref} is calculated so that a mean value of gravity is maintained across the gap:

$$g_{ref} = g_{mean} \ (r_0 - r_i) \left[\int_{r_i}^{r_0} \left(\frac{r_0}{r} \right)^n dr \right]^{-1} \tag{3}$$

where r_0 and r_i are the outer and inner sphere radius.

In figure 2 a plot of maximum values of stream function versus rotation rate is given. Three families of curves are shown corresponding to maximum temperature differences on the solid boundaries of 1, 2, and 3°C. Within each family ψ_{max} increases with n. For the parameter range studied the effects of the gravitational distribution appear secondary relative to temperature and rotation.

However, figure 3 exhibits some characteristics of the flow which are influenced by the gravitational distribution. Figures 3a and 3b are stream function contours for a uniform field (n=0) and a field produced by the dielectric (n=5), respectively. Although overall features of the flow fields appear to be identical, the dielectric force produces a shift in the internal

meridional recirculation relative to a uniform gravitational field. Note that the inner cell is stronger in the case of the dielectric than the inner recirculating cell of the uniform field. Conversely, the outer recirculating cell is stronger for n=0. Since the inverse quintic power law of the dielectric produces stronger forces towards the inner sphere which drop off quickly towards the outer sphere, the above trend is expected.

To correlate the data, the Grashof number, (Eq. 2), which represents the ratio of buoyancy to viscous forces was plotted against ψ_{max} for the same cases given in figure 2. The data is badly scattered. This Grashof number is the parameter obtained from nondimensionalizing the equations and imposes the proper condition in the buoyancy term, i.e. that the mean value of gravity across the gap be constant for varying n. However, it is clearly not the proper similarity parameter for this flow.

A modified Grashof number (Gr') is defined which takes into account the gravitational distribution as given in equation (4),

$$Gr' = \frac{\alpha \, \Delta T \, r_o^3 \, g_{mean}}{(r_o - r_i) \, \nu^2} \int_{r_i}^{r_o} \left(\frac{r_o}{r} \right)^n \, dr \, . \tag{4}$$

The scatter reduces considerably. Figure 5 is a plot of ψ_{max} versus Gr'. The data collapses into three sets corresponding to a rotation rate of 1, 2, and 3 radians per second, respectively. For a given rotation rate the data distributes itself so that the highest temperature differences on the boundary correspond to the highest values of ψ_{max}. For a given ΔT the data shows that as n increases from 0 to 5, ψ_{max} also increases.

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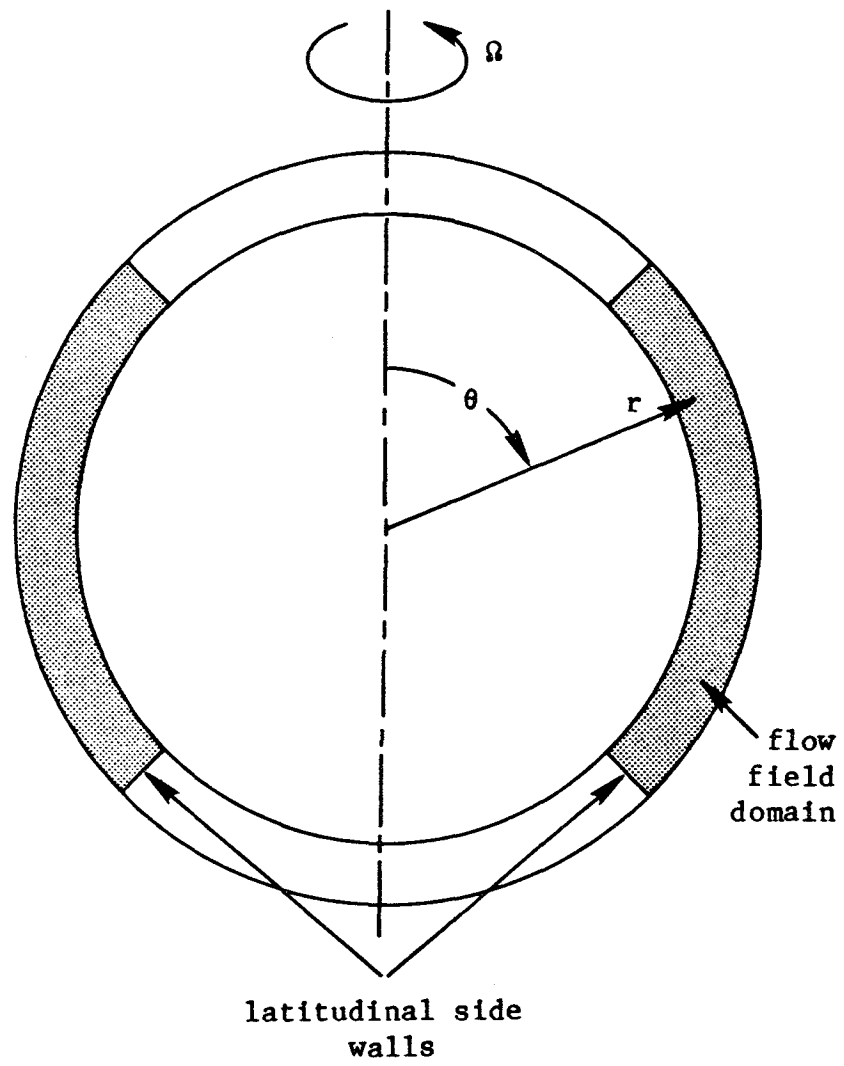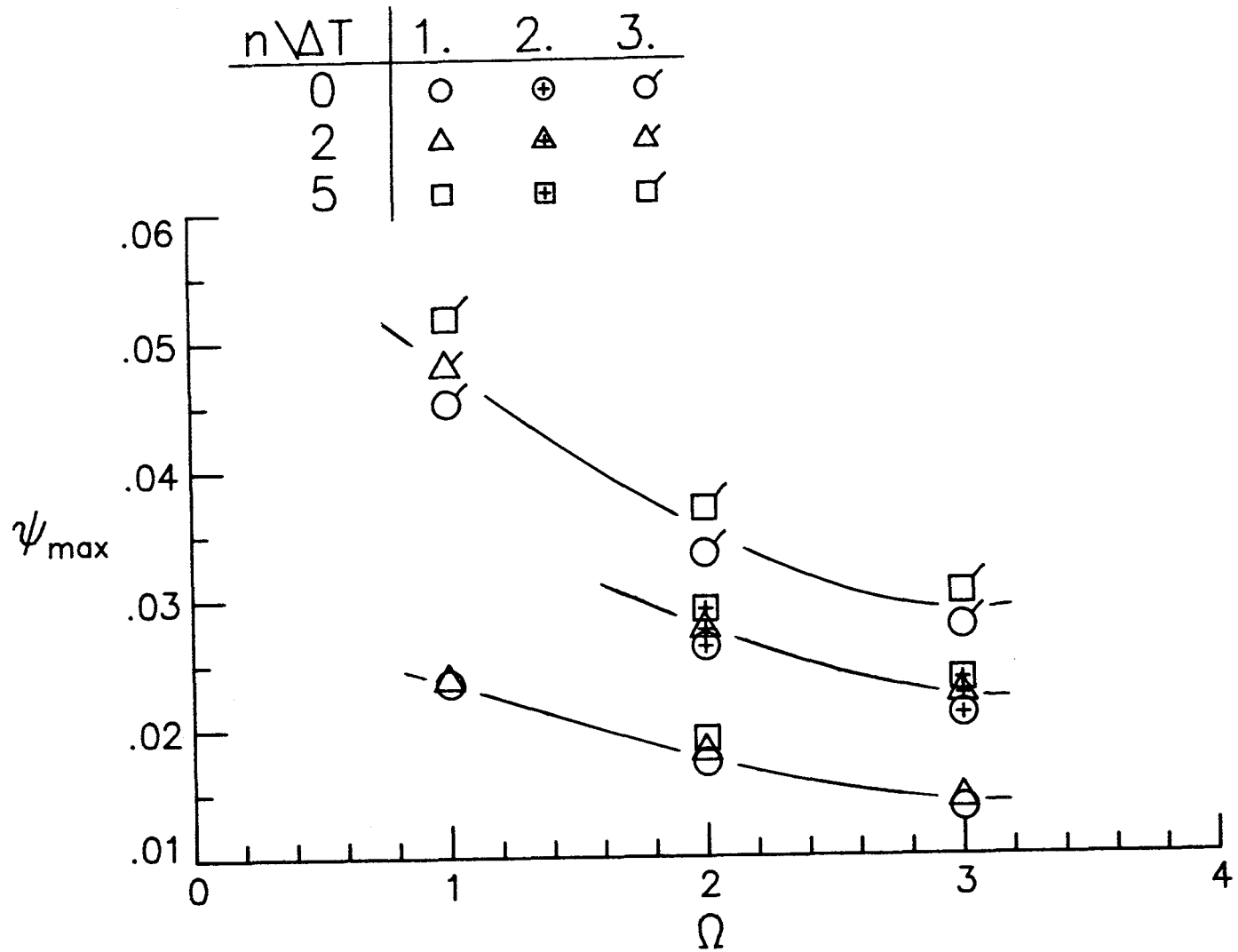
Figure 1. Numerical Model Schematic

n\ΔT	1.	2.	3.
0	○	⊕	◐
2	△	▲	◮
5	□	⊞	◰

ψ_{max}

.06

.05

.04

.03

.02

.01

0   1   2   3   4

Ω

MAX STREAM FNC vs. ROTATION RATE

Figure 2. ψ_{max} vs. Ω

STREAM FUNCTION CONTOURS (contour increment = 3×10^{-3})
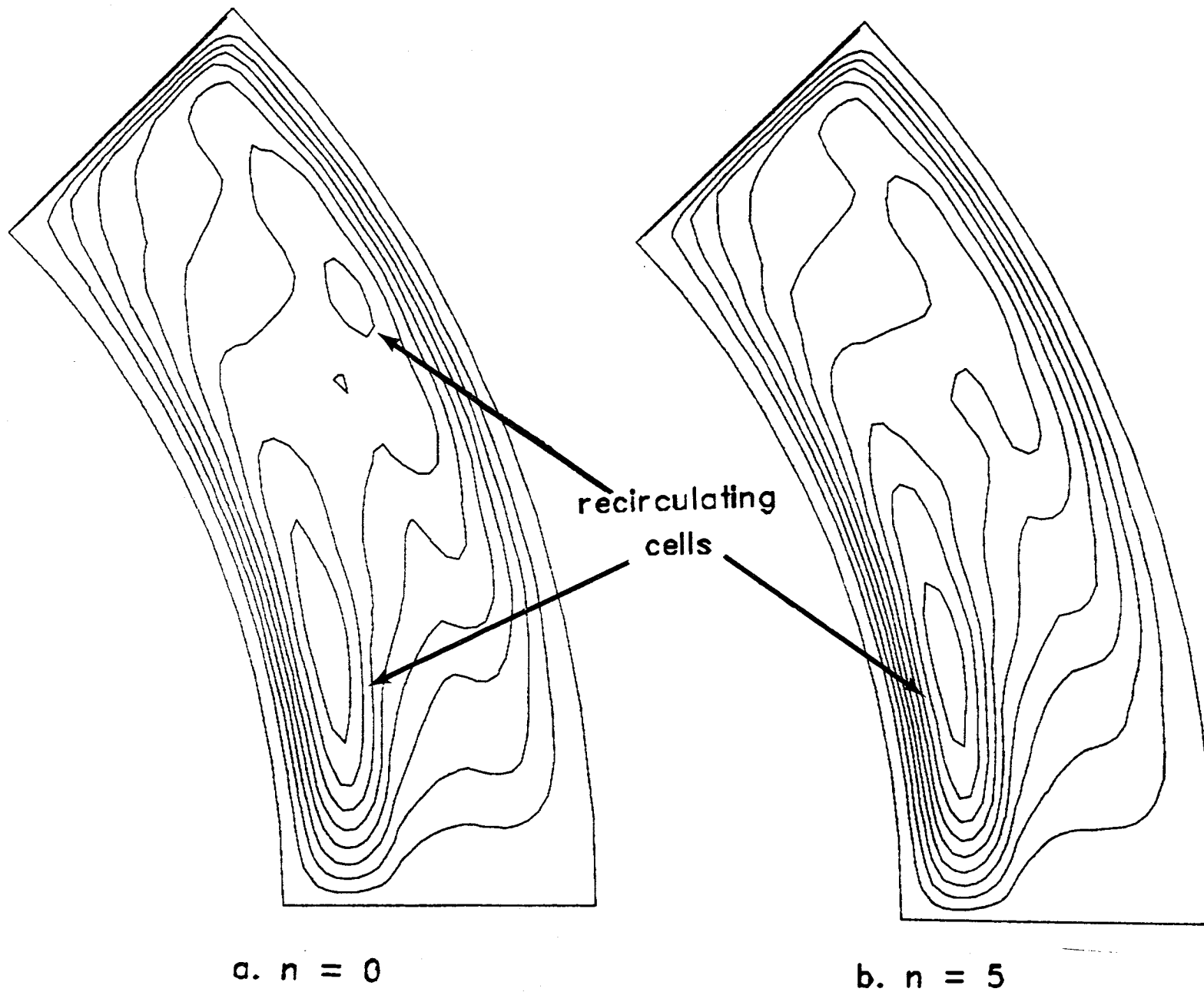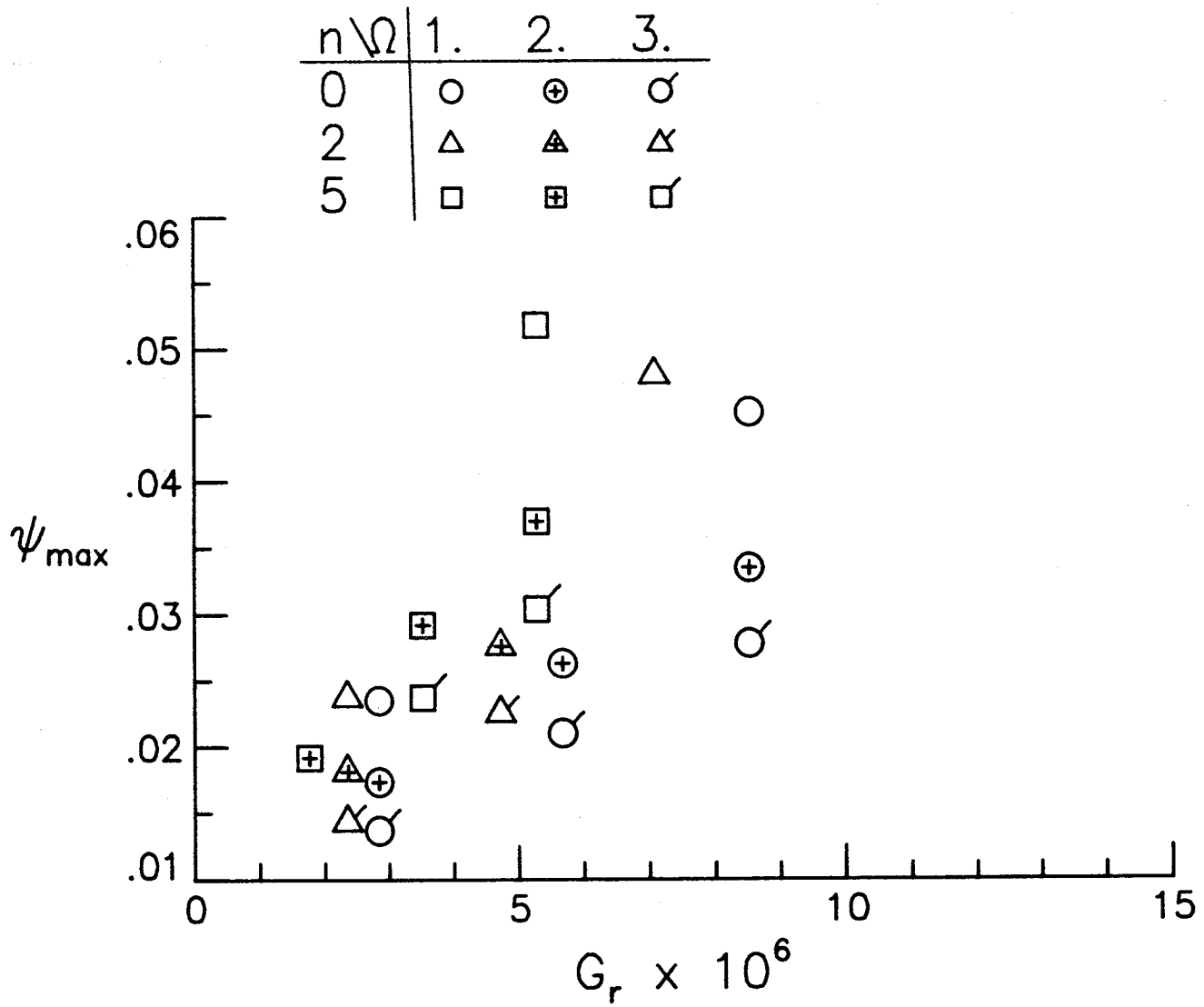

a. n = 0

b. n = 5

recirculating cells

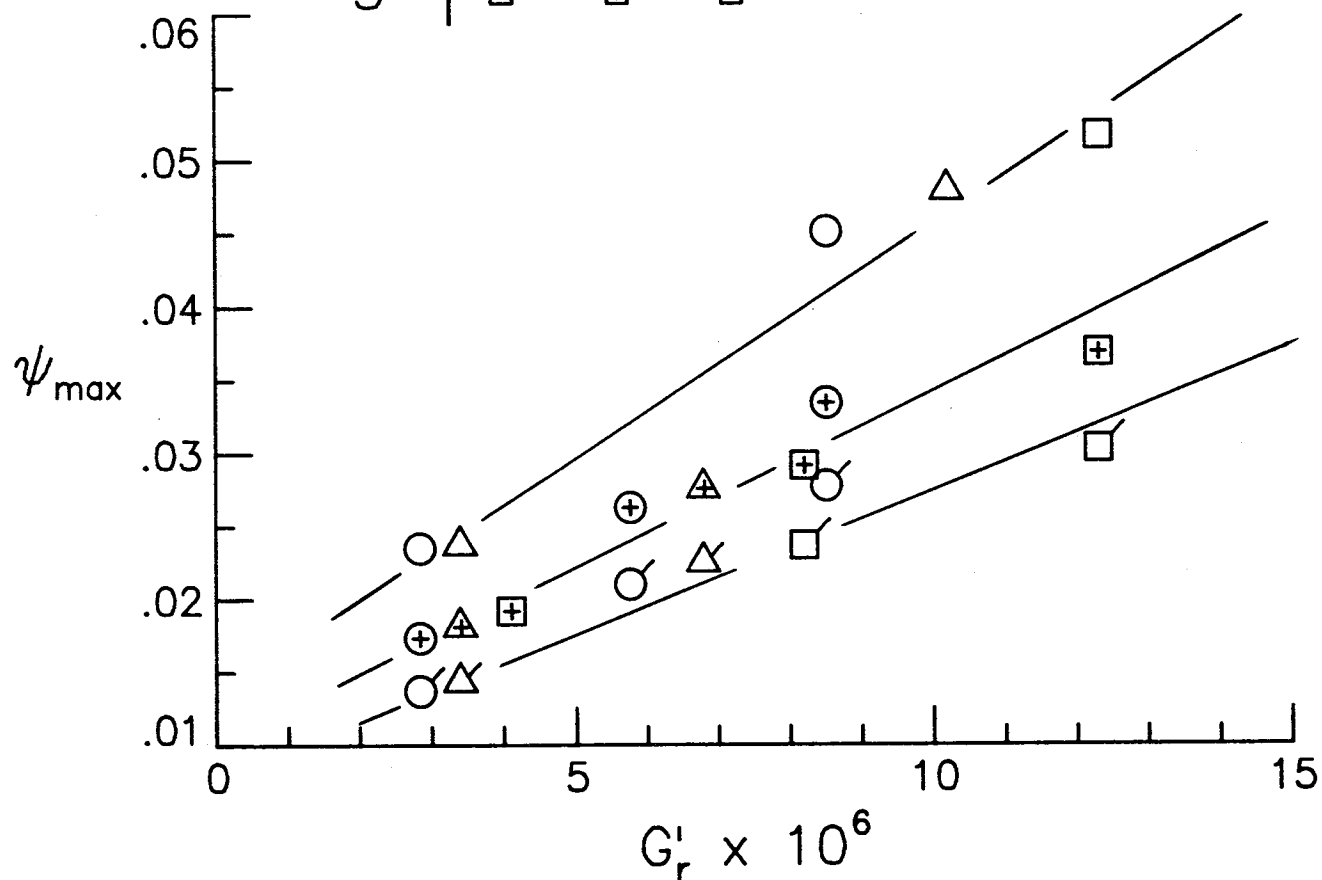Figure 3. Stream Function Contours. (contour increment = 3×10^{-3})

MAX STREAM FNC vs. GRASHOF NO.

Figure 4. ψ_{max} vs. Gr.

Figure 5. ψ_{max} vs. Gr'.

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