The Nucleon Intensity in the Atmosphere and the p_t Distribution

A. Liland
N-9785 Veidnesklubben
Norway

ABSTRACT

The diffusion equation for cosmic ray nucleons in the atmosphere has been solved analytically, taking into account the transverse momentum distribution of nucleons produced in nucleon-air nucleus collisions. The effect of the transverse momentum distribution increases the nucleon intensity at large zenith angles and low energies.

1. Introduction

The intensity of cosmic ray nucleons in the atmosphere, due to cosmic ray primary protons, can be expressed by (1)

$$N(E,x) = C_p E^{-(\gamma+1)} e^{-x/\lambda} \tag{1}$$

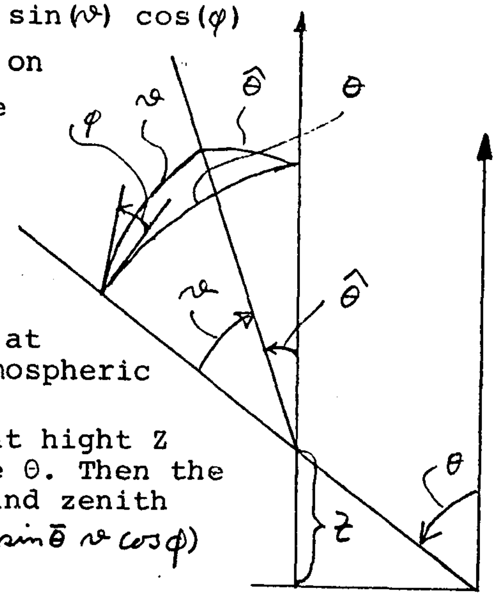
where $N(E,x=0) = C_p E^{-(\gamma+1)}$ is the primary proton spectrum, x the atmospheric depth and λ the diffusion length for nucleons in the atmosphere. This solution is obtained by the assumption that the secondary nucleons are moving in the same direction as their parent nucleons. It can be used for deriving the intensities of cosmic ray mesons in the atmosphere and muons at sea-level (1) because the energies of interest here are high such that the average transverse momentum of secondary nucleons $\langle p_t \rangle$ is very small compared to the energy E of the nucleon, $\langle p_t \rangle \ll E$. If $\langle p_t \rangle$ is no longer much smaller than E, as is the case, for instance, for low energy cosmic rays in the upper part of the atmosphere, equation (1) will give too low nucleon intensity at large zenith angles Θ, as we shall see below.

2. The Atmospheric Depth

From the U.S. Standard Atmosphere (1976) (2) we obtain the atmospheric depth at sea-level and zenith angle 0°, $x_o = 1035.6$ g/cm^2. We approximate the Standard Atmosphere by an exponential atmosphere and obtain for the atmospheric depth at hight Z (km) above sea-level $x = x_o \exp(-Z/H)$, $H = 6.95$ km. The error is less than 12% for $x < 1$ g/cm^2 compared to the Standard Atmosphere. For an inclined direction, $0^\circ < \Theta \leq 90^\circ$, we obtain for the atmospheric depth $x_{max}(\Theta)$, at sea-level,

$$\text{(2)} \qquad X_{max}(\theta) = \frac{\rho_o \sqrt{\frac{\pi}{2} H R}}{\frac{1+\alpha}{2} \sqrt{\frac{\pi}{2} \frac{R}{H}} \cos\theta + \beta} \qquad \begin{cases} \theta \geq 87^\circ & \begin{cases} \alpha = 0.398 \\ \beta = 0.96 \end{cases} \\ \theta < 87^\circ & \begin{cases} \alpha = 0.96 \\ \beta = 0.398 \end{cases} \end{cases}$$

ρ_o is the density of air at sea-level
R is the radius of the earth, 6366 km.
The error, due to approximations, is less than 5%.



From the figure we obtain:

$\cos(\hat{\Theta}) = \cos(\Theta)\cos(\vartheta) + \sin(\Theta)\sin(\vartheta)\cos(\varphi)$

ϑ will be given by $\frac{p_t}{E}$ later on and is small so that we can take

$\cos\hat{\theta} = \cos\theta + \sin\theta\,\vartheta\cos\varphi$

We introduce the reduced zenith angle $\bar{\theta}$,

$$\cos\bar{\theta} = \sqrt{1 - \left(\frac{\sin\theta}{1+\frac{2+3\cos\theta}{6366}}\right)^2}$$

which is the local zenith angle at approximately half the total atmospheric depth $x_{max}(\Theta)$.

Let x be the atmospheric depth at hight Z above sea-level and zenith angle Θ. Then the atmospheric depth $\hat{x}$ at hight Z and zenith angle $\hat{\Theta}$ is given by ($\cos\hat{\bar{\theta}} = \cos\bar{\theta} + \sin\bar{\theta}\,\vartheta\cos\varphi$)

$$\hat{x} = x\,\frac{X_{max}(\hat{\bar{\theta}})}{X_{max}(\bar{\theta})} = \frac{x}{1+\frac{\sin\bar{\theta}\,\vartheta\cos\varphi}{\cos\bar{\theta}+\eta}}$$

$$\eta = \frac{2\beta}{1+\alpha} = \begin{cases} 0.0362 & \theta \geq 87^\circ \\ 0.0107 & \theta < 87^\circ \end{cases}$$

3. The Diffusion Equation

Let the differential production cross section for nucleon production by nucleons have the following transverse momentum distribution

$$\frac{d^2\sigma}{dp_\parallel\, dp_\perp^2} \sim e^{-\alpha p_\perp^2} \qquad \alpha = \text{const.}$$

where $p_\parallel$ is the longitudinal momentum and p_t is the transverse momentum of the produced nucleons. With $\vartheta = \frac{p_\perp}{E}$ we then obtain for cosmic ray nucleons

$$g(E,E';\vartheta,\varphi) = g(E,E')\,\frac{1}{2\pi}\,\frac{\pi}{4}\,\frac{1}{\langle p_\perp\rangle^2}\,e^{-\frac{\pi}{4}\left(\frac{E}{\langle p_\perp\rangle}\right)^2\vartheta^2}$$

with $G_{NN} = \int_0^1 g(E,E/u)\,u^\gamma\,du \quad (1)$

We use the generation method and obtain the diffusion equation for the first generation nucleon

$$\frac{d\hat{N}_1}{dx} = -\frac{\hat{N}_1}{\lambda_N} + \int_E^\infty \frac{1}{\lambda_N} C_p E'^{-(\gamma+1)} \int_0^\pi\int_0^{2\pi} e^{-\frac{\hat{x}}{\lambda_N}}\, g(E';E,\vartheta,\varphi)\,d\varphi \cdot 2E^2\vartheta^2\,d\vartheta\,\frac{dE'}{E'}.$$

$\lambda_N = 84\ g/cm^2$ is the interaction length for nucleons, The analytic solution is

$$\hat{N}_1(E,X,\theta) = C_p E^{-(\gamma+1)} G_{NN} \frac{X}{\lambda_N} e^{-\frac{X}{\lambda_N}} \frac{1}{1-\left\{\varepsilon_1 \sqrt{\frac{\pi}{4}} \frac{X}{\lambda_N} \frac{\langle p_\perp \rangle}{E} T(\bar{\theta},E)\right\}^2}$$

$\varepsilon_1 = 0{,}2845$

$$T(\bar{\theta},E) = \frac{\sin\bar{\theta}/(\cos\bar{\theta}+\eta)}{1+(\sin\bar{\theta}\,\frac{\langle p_\perp \rangle}{E}\,\frac{\pi}{4})/(\cos\bar{\theta}+\eta)}$$

The solution for the m'th generation nucleon has the form

$$\hat{N}_m = C_p E^{-(\gamma+1)} \frac{1}{m!}\left(\frac{X}{\lambda_N} G_{NN}\right)^m e^{-\frac{X}{\lambda_N}} \prod_{n=1}^{m} g_{m,n}$$

$$g_{m,n} = \frac{t_n}{1-\left\{S_{m,n} \frac{X}{\lambda_N} \frac{\langle p_\perp \rangle}{E}\right\}^2}$$

If we define an average for $g_{m,n}$, for instance:

$$g(E,X,\theta) = \frac{\langle t_n \rangle}{1-\left\{\langle S_{m,n} \rangle \frac{X}{\lambda_N} \frac{\langle p_\perp \rangle}{E}\right\}^2}$$

we can sum over m and write an approximate solution for the nucleon intensity in the atmosphere

$$\hat{N}(E,X,\theta) = C_p E^{-(\gamma+1)} e^{-\frac{X}{\lambda_N}(1-G_{NN}\, g(E,X,\theta))}$$

$g(E,X,\theta) > 1$ and $\hat{N}(E,X,\theta)$ is increasing with increasing $\frac{X}{\lambda_N} \frac{\langle p_\perp \rangle}{E}$ and zenith angle θ.

If the differential production cross section is $\sim e^{-\alpha p_\perp}$ we obtain for the first generation nucleon

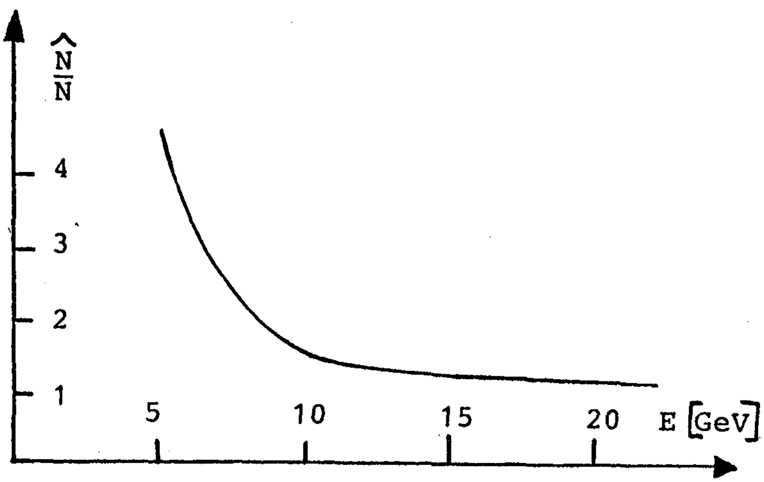
$$\hat{N}_1'(E,X,\theta) = C_p E^{-(\gamma+1)} G_{NN} \frac{X}{\lambda_N} e^{-\frac{X}{\lambda_N}} \frac{1}{1-\left\{\varepsilon_1' \frac{1}{2} \frac{X}{\lambda_N} \frac{\langle p_\perp \rangle}{E} T(\bar{\theta},E)\right\}^2}$$

$\varepsilon_1' \approx 1.$

and we see that this flatter trans verse momentum distribution gives a greater effect.

4. Discussion and conclusion

$\hat{N}(E,x,\theta)$ is the solution of the 3 - dimentional diffusion equation for nucleons in the atmosphere and $N(E,x)$ the 1-dimentional solution as given above. The figure shows, as an example, the ratio $\hat{N}(E,x,\theta)/N(E,x)$ at 30 km above sea-level ($x(\theta = 0^\circ) = 12.4$ g/cm^2) for $\theta = 90^\circ$, $x = 350$ g/cm^2 as a function of energy E,



We see that below 10 GeV the ratio $\hat{N}/N$ is increasing rapidely with decreasing energy. This is because $\hat{N}/N$ is a function of E^{-2} as we see from the solution for $\hat{N}$ above.

In the case of $\frac{d^2\sigma}{dp_{\parallel}\, dp_{\perp}^2} \sim e^{-\alpha p_{\perp}}$, we obtain approximately the same curve, but with the energy a factor 2 higher and the rapide increase is between 20 GeV and 10 GeV. We see that the effect of the transverse momentum distribution is not only a function of the average transverse momentum $\langle p_t \rangle$, but depends also on the shape of the transverse momentum distribution.

References

1. A. Liland, Fortschritte der Physik 23, 571 (1975)

2. U. S. Standard Atmosphere (1976), Handbook of Chemistry and Physics. The Chemical Rubber Co., 1984.