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HADRONIC COMPONENTS OF EAS BY RIGOROUS SADDLE POINT METHOD IN THE ENERGY RANGE BETWEEN 10^5 AND 10^8 GEV.

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Introduction : The study of hadronic components in the high energy range between 10^5 and 10^8 Gev exhibits by far the strongest mass sensitivity[1] since the primary energy spectrum as discussed by Linsley[2] and measured by many air shower experimental groups[3] indicates a change of slope from -1.7 to 2.0 in this energy range. This change of slope may be due to several reasons such as due to a genuine spectral feature of astrophysical origin, a confinement effect of galactic component or a rather rapid change of mass which problem has been attempted here to study in details.

Several attempts have already been done as discussed by Grider[3]. Here, we shall analyse the hadronic components of EAS at this energy range within the scheme of a scale breaking model of multiparticle production[4]. We have used a modified rigorous saddle point method[5] to calculate the inverse Mellin transform of diffusion equations. The hadron spectra has been calculated for primary energy range from 10^5 to 10^8 Gev at sea-level as well as at mountain height at 720 gm/m^2.

Method : The integro-differential cascade equation for nucleon N and pions N_π are respectively given by

$$\frac{\partial N}{\partial y}(E, y) = -\frac{N(E, y)}{\lambda_{in}} + \int_E^{E_0} \frac{N(E', y)}{\lambda_{in}} W_{NN}(E', E) \, dE' \tag{1}$$

$$\frac{\partial N_\pi}{\partial y}(E, y) = -\left(\frac{1}{\lambda_{in}} + \frac{B}{Ey}\right) N_\pi(E, y) + \int_E^{E_0} \frac{N(E', y)}{\lambda_{in}} W_{N\pi}(E, E') \, dE'$$

$$+ \int_E^{E_0} \frac{N_\pi(E', y)}{\lambda_{in}} W_{\pi\pi}(E', E) \, dE' \tag{2}$$

For simplification, we have considered (i) the nucleon interaction length $\lambda_N \approx$ pion interaction length λ_π at high energy[6].(ii) Limiting fragmentation is assumed to hold for F_{NN}. The unknown function $N_\pi(E, y)$ can be expressed in the form

$$N_\pi = n_\pi(E, y) \, e^{-y/\lambda_\pi} = n_\pi(E, y) \, e^{-y'} \tag{3}$$

Let $\sigma_{in} = \sigma_{in} (1 + a \ln E/100)$ (4)

i.e. $\dfrac{1}{\lambda_\pi} = (1 + a \ln E/100)$ where $a \simeq .037$.

Initially, this rigorous saddle point method was proposed for scaling model[5] where

$$n_\pi + N_n = \frac{C_0}{E} \sqrt{\frac{\pi}{y'}} \, e^{y'} \, (g(s) - 1) - \bar{s}_0 \ln w \qquad (5)$$

So is given by $\left.\dfrac{dg}{ds}\right|_{s = s_0} = \dfrac{\ln w}{y'}$ and $\bar{s}_0$ by $= s_0 - ay' + 1$

But according to our model the inclusive cross-section and the average multiplicity are given by

$$\frac{1}{\sigma_{in-el}} \left. E \frac{d\sigma}{d^3p}\right|_{pp \to \pi \, \bar{x}} \simeq \frac{9}{\langle n \rangle_{\pi^-}} \cdot \frac{25}{4} \cdot \frac{1}{4\pi} \frac{f_\rho \pi w}{16\pi^2} \exp\left[- \frac{26.88}{\langle n \rangle_{\pi^-}} - \frac{p_T^2}{(1-x)}\right]$$

$$\exp\left[- 2.38 \langle n \rangle_{\pi^-} x \right] \frac{mb}{Gev^2} \qquad (6)$$

where $\langle n \rangle_{\pi^+}^{pp} = \langle n \rangle_{\pi^-}^{pp} = \langle n \rangle_{\pi^0} = 0.35 \, s^{\frac{1}{3}}$ (Sin Gev^2)

Thus $F_{N\pi} = 1.06 \, e^{-2.38 \langle n \rangle_\pi} \cdot x$

$$F_{\pi\pi} = 1.062 \, e^{-2.38 \times 5/9 \langle n \rangle_\pi} \cdot x$$

Now at high energy we can write $F_{N\pi} = 1.06 \, e^{-2.38 \cdot 2^{\frac{1}{3}} x^{\frac{1}{3}} E_0^{\frac{1}{3}}}$ (7)

where $E/E' \sim E/E_0$.

Here x be the Feynman scaling variable as used in the previous paper on saddle point method[5]. So we can use the previous method to our case also. Now after some calculations it can be shown that s_0 admits of an approximate representation in terms of gamma function i.e.

$$\left.\frac{dg}{ds}\right|_{s = s_0} = \frac{\ln w}{y'}$$

Here the following approximation has been taken for gamma function

$$(s) = 1 + a_1 (s - 1) + a_2 (s - 1)^2 + a_3 (s - 1)^3 + a_4 (s - 1)^4$$
$$+ a_5 (s - 1)^5$$

where $a_1 = 0.5748$, $a_2 = 0.95123$, $a_3 = - 0.699$, $a_4 = 0.4245$
$a_5 = - 0.10106$.

Results : In Fig.1, the several data are shown together with the had-
ron spectrum obtainted by us from our model and also with that calcu-
lated by Grieder. These data cover a large size range centred around
    Ne   = 10^5 and this corresponds to primary energy 10^6 Gev. So dif-
ferent types of detecting system with large size window, explain this
large dispersion of data.

     But even with this dispersion we can emphasize from the compa-
rison of these data with our prediction for iron and proton initiated
showers of same total energy i.e. 10^6 Gev, that this energy range of
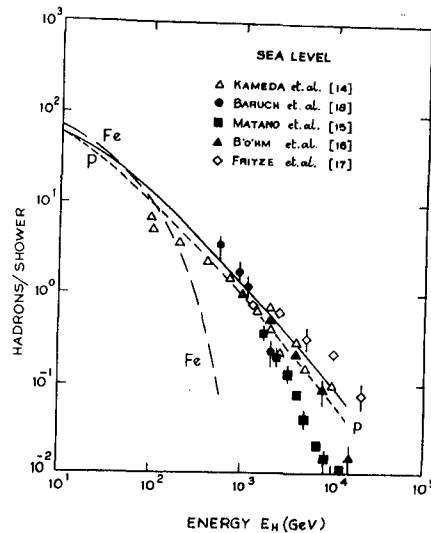primary spectrum can not be dominated by iron nuclei.

FIG.1

Hadron energy spectra at sea level in proton initiated showers of
10^6 Gev primary energy obtained from our present work (solid line).
Dashed and dot lines are from Grieder [1].

     In Fig.2, we have compared the data obtained by Dubovy et al[7]
at 720 gm/m^2 which shows a nice agreement with our predicted hadron
spectra with proton as primary particles whereas the spectra exp ect-
ed by taking heavy particle (Fe = 56) as primary is showing a sharp
cut off supporting our conclusion.

     Now it is evident from the above results that in the critical
energy region under consideration i.e. between 10^5 and 10^8 Gev, in
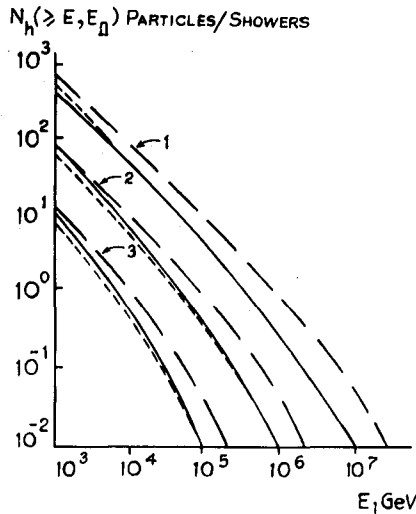particular around 10^6 Gev, is not dominated by iron primaries.

$N_h (\geqslant E, E_0)$ Particles/Showers

FIG.2

Integrated energy spectra of hadrons in EAS as calculated for $y = 720 \text{ gm/cm}^2$ and primary energy E_0 (Gev) $1-10^8$, $2-10^7$, $3-10^6$ and compared with those from Boyadzhyn et. al. [22] for $a_n = a_\pi = 0$ (dashed curves), $a_n = a_\pi = 0.03$ (dot curves) and prediction from present model (solid curves).

References :

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4] P. Bandopadhyay et. al. Nuovo Cimento, **49A**, 133 (1979) and references therein.

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