STANDARD VALUE FOR THE RADIATION LENGTH IN AIR

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ABSTRACT

Following the Paris ICRC I did some additional research on the radiation length in air, which was reported in *Proceedings of the Paris Workshop on Cascade Simulations*. However no standard value was recommended because a few calculations remained undone. These have now been finished. They give new values for t_o in atomic oxygen and nitrogen which are entirely free of dependence on the Thomas-Fermi approximate model. With the usual small corrections for atmospheric A and CO_2, these give $t_o|_{air} = 37.15$ g cm^{-2}, in close agreement with a value recommended by Dovzhenko and Pomanskii, but in contrast to $t_o|_{air} = 36.66$ g cm^{-2} obtained by Tsai using the Thomas-Fermi approximation.

1. Introduction. This concludes an inquiry concerning the radiation length in air, motivated by the importance of that unit in applying cascade theory to the interpretation of cosmic ray air shower data. The preceding installments provide background and more complete references (Linsley 1981a, 1981b).

2. Definitions. The radiation length t_o is customarily defined as follows:

$$(t_o)^{-1} = (N\sigma_o/A)[Z^2(L_{rad} - f) + ZL_{rad'}] \;, \qquad (1)$$

where N is Avagadro's number, $\sigma_o = 4\alpha r_o^2$ (α the fine-structure constant, r_o the classical radius of the electron), A is the atomic weight and Z is the atomic number of the target atom, f corrects for use of the Born approximation, L_{rad} is the usual radiation logarithm given by

$$L_{rad} = \int_0^1 (1-F)^2 \frac{dq}{q} + 1 \;, \qquad (2)$$

where q is the momentum transfer in units of $m_e c$ and F(q,Z) is the atomic form factor, and $L_{rad'}$, a quantity analogous to L_{rad}, takes into account collisions in which the scattering system is left in an excited state.

$$L_{rad'} = \int_0^1 S \frac{dq}{q} + 1 \;, \qquad (3)$$

where S(q,Z) is the so-called incoherent scattering function. There is general acceptance of the Bethe-Maximon formula for f:

$$f(z) = z \sum_1^\infty n(n^2+z) \sim 1.202z-1.0369z^2+1.008z^3/(1+z) \;, \qquad (4)$$

where $z = (Z/137)^2$. For N and O, f = 0.0031 and 0.0041, respectively.

3. Evaluation of Structure Dependent Terms. The effect of atomic structure on t_o is expressed mainly through L_{rad}. It was shown by Bethe (1934) that the Z-dependence of L_{rad} is given essentially by

$$L_{rad} = \ln(aZ^{-1/3}) \ , \qquad (5)$$

where a is called the elastic screening coefficient. Using the Thomas-Fermi model he obtained the widely quoted value 183 for a_{TF}.* Recognizing the hazard in relying on a statistical model for an atom as light as N, Wheeler and Lamb (1939) recalculated L_{rad} for that element (and for hydrogen) using self-consistent wave functions of the Hartree-Fock type, obtaining a result equivalent to $L_{rad}|_{SCF} = 4.56$.

The need for taking into account inelastic scattering was noted first by Landau and Rumer (1938). Their estimate $L_{rad'} \sim L_{rad}$ underestimates $L_{rad'}$ by about 20% in case of elements as light as N and O, leading to errors in t_o of about 3%. A correct quantum mechanical formula equivalent to Eq. 3 was given by Wheeler and Lamb (1939). They used it to calculate $L_{rad'}$ for N (and for hydrogen) using both the Thomas-Fermi model and self-consistent wave functions. Their result in the latter case was equivalent to $L_{rad'}|_{SCF} = 5.47$.

More recently, values of F and S based on accurate self-consistent wave functions have been tabulated for all the elements over a complete range of q-values (Hubbell *et al.* 1975). Using these data and Eq's (2) and (3) I have calculated $L_{rad}|_{SCF}$ and $L_{rad'}|_{SCF}$ for O as well as N, obtaining 4.490, 5.253 for O, and 4.554, 5.347 for N (in good agreement with the results of Wheeler and Lamb), respectively.

4. The Radiation Length in Air. The radiation length in a complex substance is given by the following expression (Rossi 1952):

$$(t_o)^{-1} = \sum_i p_i/(t_o)_i \ , \qquad (6)$$

where p_i is the fraction by weight of the i-th atomic species. The data

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* The alternative value 191, frequently quoted although it is erroneous, apparently entered the literature through being recommended by Oppenheimer to Serber (1938, 1981). (In Serber's 1938 paper it is attributed to Bartlett, who visited Oppenheimer's group at Caltech about the time the paper was being written.) The value of a has been recalculated recently, but it is questionable whether the new result, a_{TF} = 184.15 (Tsai 1974), is any more accurate than Bethe's, since the new calculation used an approximate parametrization of the Thomas-Fermi potential.

adopted here for calculating $t_o|_{air}$ are given in Table 1. The present result is $t_o|_{air} = 37.15$ g cm^{-2}.

Table 1. Calculation of Radiation Length in Air

i	p_i	Z_i	A_i	f_i	$(L_{rad})_i$	$(L_{rad'})_i$	note
1	0.05	6	12.0111	0.0019	4.618	5.403	(a)
2	75.52	7	14.0067	0.0031	4.554	5.347	(b)
3	23.14	8	15.9994	0.0041	4.490	5.253	(b)
4	1.29	18	39.9480	0.0204	4.252	5.081	(a)

note (a): $L_{rad} = L_{rad}|_{TF} = \ln(183Z^{-1/3})$, $L_{rad'} = L_{rad'}|_{TF} = \ln(1194Z^{-2/3})$ (Tsai 1974)

note (b): $L_{rad} = L_{rad}|_{SCF}$, $L_{rad'} = L_{rad'}|_{SCF}$

5. Comparison with Previous Results. Values of t_o for C, N, O and A, as well as air, are compared in Table 2 with values recommended in widely quoted reviews: 1) by Dovzhenko and Pomanskii (1964), and 2) by Tsai (1974). (Tsai's article is the authority for values given in the *Particle Properties Data Booklet*.) The results by Tsai are based entirely on the Thomas-Fermi model. Dovzhenko and Pomanskii used their own SCF calculations of L_{rad}. For $L_{rad'}$ they used interpolated values based on Wheeler and Lamb's results for N.

Table 2. Radiation Length in C, N, O, A and air (units g cm^{-2})

material	Dovzhenko & Pomanskii	Tsai	present work
C	43.3	42.70	43.33
N	38.6	37.99	38.53
O	34.6	34.24	34.83
A	19.7	19.55	19.57
air	37.1	36.66	37.15

6. Remaining Questions. Out of 7 questions listed previously (Linsley 1981b) there are 2 which have not been given definitive answers:

1) How great is the molecular binding effect in N_2 and O_2?

2) What is the relation between t_o defined by Eq. 1 and the electromagnetic cascade unit X_o defined by $X_o = X_{max}/\ln(E/E_c)$, where E_c is the critical energy and X_{max} is the average depth of maximum development of simple cascades with energy E?

Regarding (1): Using the conventional definition of L_{rad} (Eq. 2), the value I reported previously for molecular nitrogen (Linsley 1981a) becomes 4.50, 1.2% *less than* the best SCF value for atomic nitrogen. On the other hand, Bernstein and Panofsky (1956) showed that in the complete screening limit the effect of molecular binding in hydrogen is to *increase* σ_{pair} by 2.8%.

Regarding (2): The point of the distinction is that the quantity appearing in the elongation-energy relation should logically be X_o rather than t_o. Clearly, X_o/t_o is independent of the stopping medium to a high degree of accuracy. Landau and Rumer (1938), for approximation A, and Snyder (1949), for approximation B of electromagnetic cascade theory, give $X_o/t_o = 1.01$. From tables given by Messel and Crawford (1970) I find $X_o/t_o = 1.04 \pm .02$. Can additional results of this kind be derived from existing work?

7. References.

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