A NEW WORLD SURVEY EXPRESSION FOR COSMIC RAY VERTICAL INTENSITY VS. DEPTH IN STANDARD ROCK

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ABSTRACT

The cosmic ray data on vertical intensity versus depth below 10^5 g cm^{-2} is fitted to a 5-parameter empirical formula to give an analytical expression for interpretation of muon fluxes in underground measurements. This expression updates earlier published results and complements the more precise curves obtained by numerical integration or Monte Carlo techniques in which the fit is made to an energy spectrum at the top of the atmosphere. The expression is valid in the transitional region where neutrino induced muons begin to be important, as well as at great depths where this component becomes dominant.

1. Introduction. A "World Survey" curve of cosmic ray intensity vs. depth underground was published by Cassiday, Keuffel and Thompson[1] in 1973. This was a largely empirical curve based on 14 data points. Miyake[2] has published a semi-empirical curve for the Kolar Gold Fields (KGF) data which is more physically meaningful. An improved world survey analysis carried out at Utah[3] was based on 22 data points. However the analysis was made by finding the best fit to the pion intensity and spectral index, with a numerical integration to give the underground muon intensity. As a result no analytic expression giving muon intensity vs. depth was determined. Similarly an extensive 1983 Monte Carlo analysis by Takahashi et al.[4] does not yield an analytical expression.

In the present work a direct fit of an empirical relation is made in order to give a convenient tool for analysis of underground measurements and planning of experiments. 31 data points are analyzed, including extensive 1978 data from South Africa[5] at great depths. Since data are relatively abundant, measurements made in horizontal tunnels or drifts under mountainous terrain are not included, because of inherent uncertainties in depth determination. An additional parameter is added to the fitting function to include the contribution of neutrino induced muons.

2. The World Survey Data. Table I lists the measurements on which the analysis is based. Depths are corrected to equivalent depth in standard rock. Measurements actually made at large zenith angle are corrected for the earth's curvature.

TABLE I.

h Depth in Standard Rock hg/cm^2	I_v Vertical Intensity (particles/cm^2 ster sec)	Error	Reference
1068	1.03 E-06	5.07 E-08	7
1500	3.90 E-07	1.46 E-08	8
1535	3.40 E-07	1.51 E-08	9
1574	3.31 E-07	1.17 E-08	10
1840	1.91 E-07	7.71 E-09	8
1853	2.00 E-07	1.36 E-08	11
1853	1.77 E-07	9.52 E-09	12
2235	9.70 E-08	6.11 E-09	7
3534	1.15 E-08	8.01 E-10	9
3562	1.42 E-08	1.53 E-09	12
4312	4.63 E-09	6.23 E-10	11
4508	3.24 E-09	4.04 E-10	12
6808	1.92 E-10	4.97 E-11	12
7486	1.10 E-10	2.15 E-11	13
8742	1.87 E-11	3.05 E-12	14
9141	1.13 E-11	6.30 E-12	5
9358	1.36 E-11	3.60 E-12	5
9660	4.59 E-12	1.30 E-12	5
10060	3.77 E-12	8.90 E-13	5
10580	2.56 E-12	5.90 E-13	5
11250	9.07 E-13	2.90 E-13	5
12100	6.84 E-13	2.00 E-13	5
13210	3.48 E-13	1.20 E-13	5
14660	2.57 E-13	8.90 E-14	5
16610	2.34 E-13	7.70 E-14	5
19320	1.67 E-13	6.00 E-14	5
23300	2.81 E-13	7.20 E-14	5
29620	2.15 E-13	5.90 E-14	5
41050	1.87 E-13	5.20 E-14	5
67440	1.88 E-13	4.90 E-14	5
182700	2.61 E-13	5.40 E-14	5

3. The Fitting Procedure. The fitting function used is

$$I_v(h) = \exp(A_1 + A_2 h) + \exp(A_3 + A_4 h) + A_5$$

This is the function used by the Utah group with the constant parameter A_5, the neutrino term, added.

The least squares fit was made using an algorithm due to Marquardt[6]. The program was developed independently, but good agreement was found with the Utah results when the same analysis was used with their input data. Table II gives the values obtained for the five parameters.

TABLE II

A_1	-11.24 ± 0.18
A_2	$-.00264 \pm .00014$
A_3	$-13.98 \pm .14$
A_4	$-.001227 \pm .000021$
A_5	$(2.18 \pm .21) \times 10^{-13}$

I_v is in $g^{-1}\ cm^2\ sr^{-1}$. h is in hg cm^{-2} (1 hg = 100 g. Therefore a 1 meter thick absorber represents 1 hg cm^{-2}, and, of course, 1 Meter Water Equivalent). The accompanying figure shows the 31 data points together with the fitted function described above.

For those not familiar with the subject, the intensity at a vertical depth h at an arbitrary zenith angle θ is given to a very good approximation by

$$I\ (h,\theta) = I_v\ (h/\cos\theta,\ \theta)/\cos\theta$$

That is, the intensity is that corresponding to a depth equal to the slant thickness of earth above the detector, with a "sec θ enhancement factor" due to the increased decay probability for pions traversing the atmosphere at large zenith angles. In the competition between decay and capture processes, obliquely traveling pions spend more time in regions of low atmospheric density.

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