ANALYTICAL CALCULATION OF MUON INTENSITIES UNDER DEEP SEA-WATER

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1. Introduction. The study of the energy loss of high energy muons through different materials, such as rock and sea-water can cast light on characteristics of lepton interactions. There are less ambiguities for the values of atomic number (Z) and mass number (A) in sea-water than in rock. Muon intensities should be measured not only as fundamental data but also as back ground data for searching the fluxes of neutrino. The average rang-energy relation in sea-water is derived. The correction factors due to the range fluctuation is also computed. By applying these results, the intensities deep under sea are converted from a given muon energy spectra at sea-level. The spectra of conventional muons from π, K decays have secθ enhancement. On the other hand, the spectrum of prompt muons from charmed particles is almost isotropic. The effect of prompt muons is examined.

2. The energy loss of muon in sea-water. With respect to the Pacific Ocean near Hawaii, the salinity is taken as 34.5‰ and the density as 1.0275. Referring to (1, 2) we obtain the fundamental constants for this salinity as follows: $\langle Z\rangle = 7.471$, $\langle A\rangle = 14.873$, $\langle Z/A\rangle = 0.5525$ and $\langle Z^2/A\rangle = 3.779$ which differ a little bit from Varilov et al (3). The salinity and the inorganic composition of the sea-water over the apparatus is desirable to be directly measured.

The rate of muon energy loss is given by (4)

$$-dE/dt = k(E) + b(E)E \tag{1}$$

and $b(E) = b_P(E) + b_B(E) + B_N(E)$, (2)

which are the terms from pair production, bremsstrahlung and nuclear interaction, respectively. In a good approximation, we have

$$B_N(E) = 5.043 + 0.1165\ \ln E(\mathrm{TeV}) \quad 10^{-7}/\mathrm{gcm}^{-2}, \tag{3}$$

which corresponds to $\sigma(h\nu) = 125\mu b$ (5). The b_B term is corrected (6). The energy dependence of b is shown in Fig. 1, where those in standard rock (S.R.) and water are compared.

3. Average range-energy relation and correction factors. After integrating eq. (1) numerically, we get the average range D(E) corresponding to the muon energy E at sea-level. The E-D relation in sea-water is shown in Fig. 2, as well as in S.R. and water. One find 1$\sim$2% differences of D(E) in sea-water and in water near at E = 100 TeV. The corresponding zenith angles at DUMAND to ranges are added in Fig. 2. The vertical depth of DUMAND is taken as 4.8×10^5 g/cm² which muons with E = 3.1 TeV reach to.

The correction factor R is defined by $R = I_0(D)/I(D)$, where I and I_0 are intensities with and without the range fluctuations, respectively. The values of R depend on the exponent β of the integral energy spectrum at sea-level, i.e. $I(>E)\propto E^{-\beta}$. Following to the procedure in (4), the R values in sea-water are computed for $\beta = 2$ and 3. The R values with any β at a given depth D can be interpolated or extrapolated from the relation

$$\ln R(\beta) = -k_1\beta^{3/2} + k_2 \quad , \qquad (4)$$

where k_1 and k_2 are constants determined from R(2) and R(3). Fig. 3 shows the dependence of R with D for β = 3, 2.7, 2.5.

4. Intensities deep under sea. The muon energy spectrum at sea-level can be converted by using the results obtained above. So the spectra are assumed as follows. The conventional spectrum at vertical direction is referred to Komori and Mitsui (7). This spectrum has almost constant β, i.e. 2.67, above E = 10 TeV. Since the conventional spectrum is enhanced by secθ, the enhance factors are estimated from those given by Maeda (8). These spectra are shown in Fig. 4 for both vertical and horizontal directions. The latter fits well with MUTRON data up to E = 25 TeV (9). The dashed curves in Fig. 4 shows prompt muon spectra which have no dependence of θ up to E = 1000 TeV (10). The maximum contribution (MAX.PROMPT) is estimated under the assumptions that the diffractive characters of the produced charmed particles are extremely stressed and the intrinsic charm distribution is very hard. The minimum contribution (MIN.PROMPT) is taken out of charm production only in non-diffractive processes. The maximum and minimum prompt spectra are well described by

$$I^p(>E) = 2.11 \times 10^{-9}\ E(\mathrm{TeV})^{-1.44}\ (\mathrm{cm}^2\ \mathrm{s\ ster})^{\ 1}, \qquad (5)$$

$$I^p(>E) = 7.36 \times 10^{-11}\ E(\mathrm{TeV})^{-1.49}\ (\mathrm{cm}^2\ \mathrm{s\ ster})^{\ 1}, \qquad (6)$$

respectively.

The intensity at a given vertical depth D can be obtained by the following procedure. The corresponding energy E to D is given by the average range energy relation. After β around E is determined from I(>E) in vertical direction, R with D and β is got from eq. (5). I(>E)/R is the vertical intensity at D. The resultant intensity is shown in Fig. 5 as a solid curve. When the energy spectrum includes the miximum prompt part, the intensities changes into a dashed curve. Old measurements of OCU group (11) and Vavilov et al. (3) are also plotted. The recent simulation (12) gives the almost same results with curves of the figure.

The same procedure is applied to the intensity at a slant depth. Here we consider a measurement at the vertical depth 4.8×10^5 g/cm^2 (DUMAND). Since the conventional energy spectrum is enhanced in an inclined direction with θ as shown in Fig. 4, we have a different R as well as I(>E) from vertical one, where E corresponds to the slant depth 4.8×10^5 secθ g/cm^2. The relative contribution of prompt muons, if any, to total intensity becomes the less as θ becomes the larger. The intensities vs slant depths (or zenith angles at DUMAND) are presented in Fig. 6. If the maximum prompt muons contribute, the intensity is about twice of that without prompt muons at θ = 70°.

5. Conclusions and discussions. The average range energy relation and the correction factors due to the range fluctuation have been computed in sea-water. From a given energy spectra at sea-level which are conventional (π. k decays) and prompt (charmed particles decays), the intensities in the vertical direction deep under sea have been obtained (Fig. 5). The angular dependence of intensities at DUMAND has been made clear (Fig. 6). It is concluded that measurements at larger zenith angle than 70° can reveal the contribution of prompt muons. The present calculations are useful for the measurement of muon neutrino

flux. Because this flux can be estimated from the deviation of the observed flux from the intensity described here.

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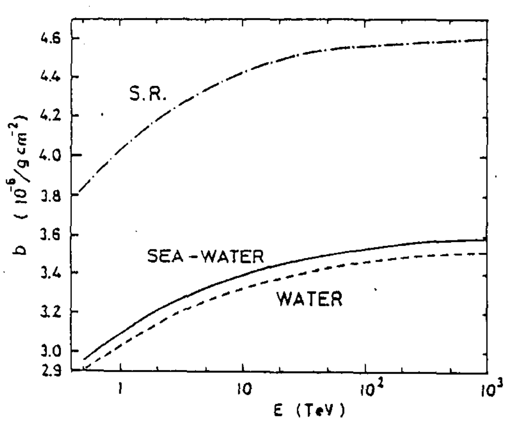

Fig.1 The energy dependence of b-term in sea-water, water and standard rock.

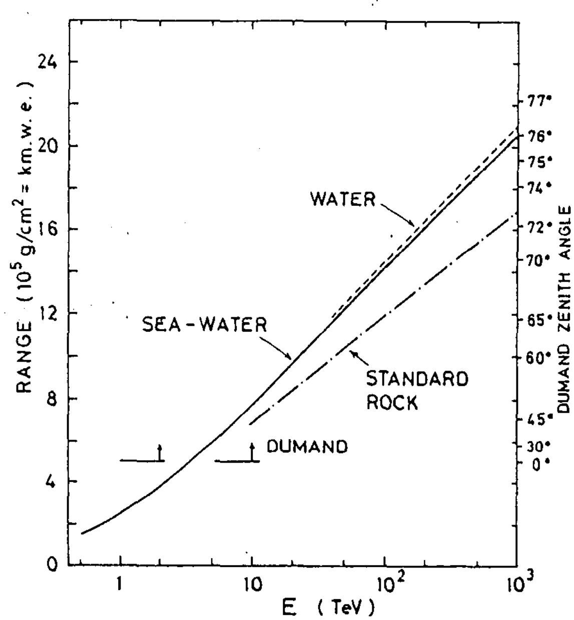

Fig.2 Average range energy relations in sea-water, water and standard rock. The zenith angles at DUMAND position (vertical depth 4.8×10^3 g/cm^2) are added.

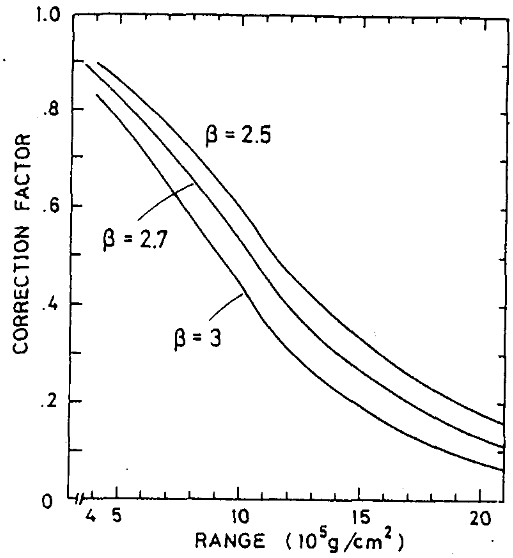

Fig.3 Correction factors in sea-water for β, the exponent of the integral energy spectrum of muons.

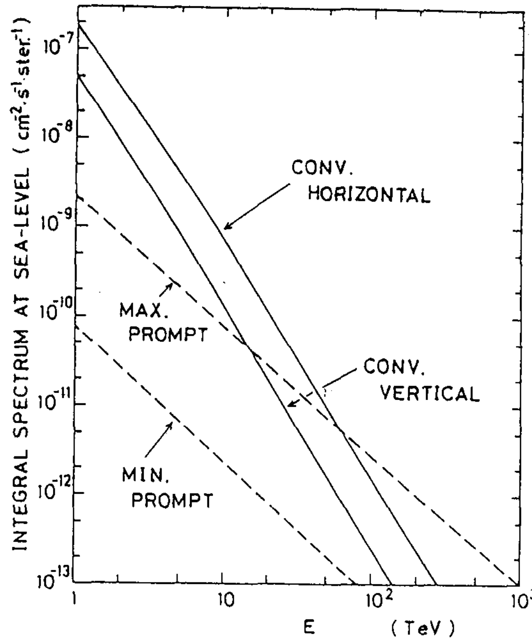

Fig.4 Adopted integral energy spectra of muons at sea-level: — conventional (π, k decays); - - - prompt (charmed particles decays).

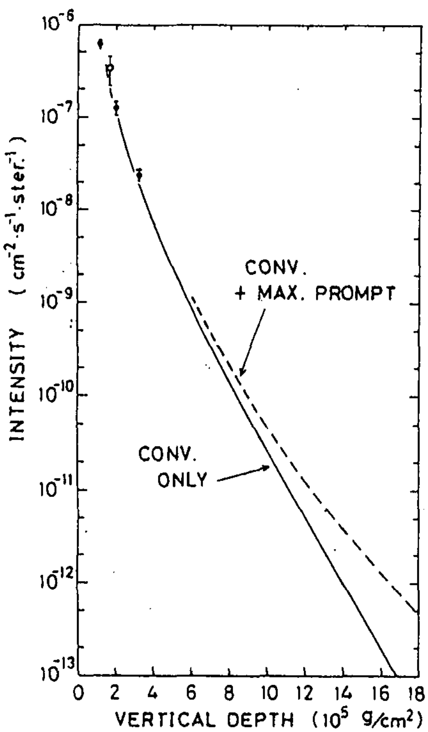

Fig.5 Intensity in the vertical direction at vertical depth under sea: — conventional only; - - - conventional plus maximum prompt; ○ OCU; ● Vavilov et al.

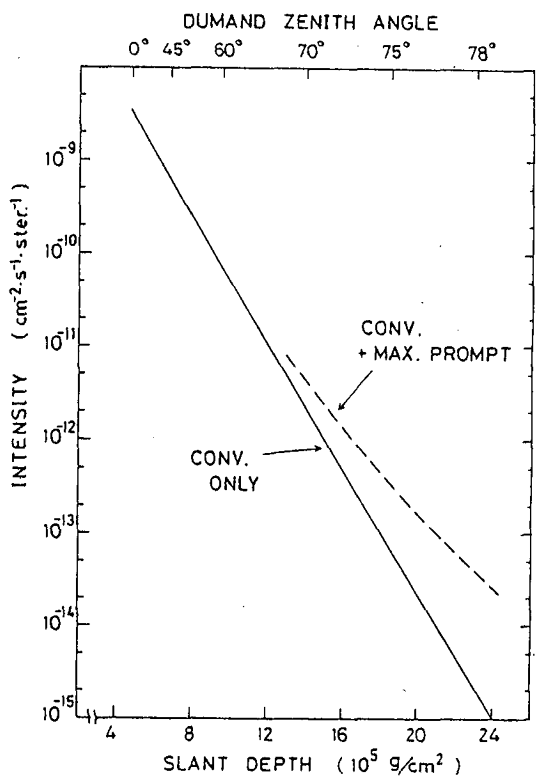

Fig.6 Intensity in the inclined directions at vertical depth 4.8×10^5 g/cm²: — conventional only; - - - conventional plus maximum prompt.