RANGE FLUCTUATIONS OF HIGH ENERGY MUONS PASSING THROUGH MATTER

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1. Introduction. The information about energy spectrum of sea level muons at high energies beyond magnetic spectrographs can be obtained from the underground intensity measurements if the fluctuation problems are solved. In the present paper we recalculate the correction factor R for the range fluctuations of high energy muons by analytical method of Zatsepin et. al[1], where most probable energy loss parameter are used. It is shown that by using the R at great depth together with the slope, Λ, of the vertical depth-intensity (D-I) curve in the form of $\exp(-t/\Lambda)$, the spectral index, γ, in the power law energy spectrum of muons at sea level can be easily obtained.

2. Formulation of the correction factor R. The R is defined as $R = I_0(t)/I(t)$, $I_0(t)$ is the intensity without fluctuations and $I(t)$ is the intensity, taking fluctuations into account. In order to find $I(t)$ the following diffusion equation has to be solved:

$$\partial I/\partial t - \beta(E_\mu)\,\partial I/\partial E_\mu = \int_0^1 W(E_\mu,v)\,[I(E_\mu/1-v,t) - I(E_\mu,t)]\,dv \qquad (1)$$

$$W(E_\mu,v) = (N/A)\Phi_B + N\Phi_N$$

(N: Avogadro's number, A: atomic mass number), where Φ_B and Φ_N are the differential cross section of bremsstrahlung and nuclear interactions, respectively. To get the solution of eq.(1), we make two assumptions and introduce three dimensionless variables as follows:

$$\text{(i)}\ \ \beta(E_\mu) = a + b_p E_\mu, \qquad \text{(ii)}\ \ f(v)/v = W/(b_B+b_N)$$

$$x = b_p t, \quad \varepsilon = b_p E_\mu/a, \quad b = (b_B+b_N)/b_p.$$

,where a is ionization loss and b_p, b_B and b_N are energy loss parameters due to pair creation, bremsstrahlung and nuclear interaction, respectively. Then eq.(1) becomes

$$\partial I/\partial x - (\varepsilon + 1)\partial I/\partial \varepsilon = b\int_0^1 f(v)/v[I(\varepsilon/1-v,x)-I(\varepsilon,x)]dv \quad (2)$$

In this notation the average energy loss may be written as

$$-d\varepsilon/dx = 1 + (1 + b)\varepsilon \quad (3)$$

$I_0(\varepsilon,x)$ is given by combining eq.(3) with the boundary condition $I(\varepsilon,0) = B\varepsilon^{-\gamma}$ as

$$I_0(\varepsilon,x)= B\cdot\exp[-\gamma(1+b)x][\varepsilon+(1-\exp(-(1+b)x))/(1+b)]^{-\gamma} \quad (4)$$

By analogy of eq.(4)we take the solution of eq.(2) in the form

$$I(\varepsilon,x)=B\cdot\exp[-\gamma(1+\chi b)x][\varepsilon+(1-\exp(-(1+\kappa b)x))/(1+\kappa b)]^{-\gamma} \times \exp[\phi(\varepsilon,x)] \quad (5)$$

From eqs.(4) and (5) we get R for $\varepsilon = 0$ in the following way

$$R(0,x)=[\exp(b(\chi-\kappa)x)\cdot(1+b)/(1+\kappa b)\cdot[\exp((1+\kappa b)x)-1]/[\exp((1+b)x)-1]]^{+\gamma}\exp[-\phi(0,x)] \quad (6)$$

At a great depth($t > 4000$ hg/cm^2) and under the assumption $\phi(0,t\to\infty)=0$, we get such a simple form as

$$R \approx [(b_p + b_f)/(b_p + \kappa b_f)]^{\gamma}\exp[-\gamma b_f(1-\chi)t] \quad (7)$$

,where $b_f = b_B + b_N$.

3. Derivation of spectral index γ. It is assumed that the D-I curves are expressed by a unique exponential law of the type $I(t) = C\cdot\exp(-t/\Lambda)$ (8). Then combining eqs.(4),(7) and (8) we can obtain a relationship between γ and Λ :

$$\gamma = [\Lambda(b_p + \chi b_f)]^{-1} \quad (9)$$

Here the constant χ is determined in such a way that $\phi(\varepsilon,x)$ approaches zero as $\varepsilon\to\infty$. By substituting eq.(5) into eq.(2) χ is given as

$$\chi = (1/\gamma)\int_0^1 f(v)/v[1-(1-v)^{\gamma}]dv \quad (10)$$

For Φ_B, Φ_N we take the following formulae

$$\Phi_B=0.95\alpha(2r_0m_e/m_\mu)^2Z(Z+\xi)(4/3+v^2-(4/3)v)(1/v)\ln(P/Q) \quad (11)$$

$$P=(2/3)k(m_\mu/m_e)Z^{-2/3},$$

$$Q=(k\sqrt{e}/2)(m_\mu^2/m_eE_\mu)(v/(1-v))Z^{-1/3}+1^{2)}$$

$$\Phi_N=\frac{\alpha}{\pi}\cdot\sigma_{\gamma N}\cdot\frac{1}{v}\left\{(v-1)+\left[1-v+\frac{v^2}{2}\left(1+\frac{2m_\mu^2}{\Lambda^2}\right)\right]\times\ln\frac{\frac{E^2(1-v)}{m_\mu^2}\left[1+\frac{m_\mu^2v^2}{\Lambda^2(1-v)}\right]}{\frac{Ev}{\Lambda}\left(\frac{Ev}{\Lambda}+\frac{\Lambda}{2M}\right)}\right\}^{3)} \quad (12)$$

$(\sigma_{\gamma N}=125\ \mu b$ and $\Lambda^2=0.4\ Gev^2)$

We calculated the values of χ as a function of γ for standard(Z=11,A=22) and K.G.F(Z=12.93,A=26.12) rocks at E_μ=1, 10,100 TeV. The results are shown in Fig.1. We find from this figure that the χ and γ has nearly a linear relation such that

$$\chi = m_1\gamma + m_2 \quad (13)$$

In the case of E_μ=10 TeV we have $m_1=-0.1085$, $m_2=0.9943$ for S rock and $m_1=-0.1092$, $m_2=0.9935$ for K.G.F rock. Substituting eq.(13) into eq.(9) we get the equation for γ.

$$m_1b_f\gamma^2 + (b_p + m_2b_f)\gamma - 100/\Lambda = 0 \quad (14)$$

(b_p,b_f are measured in units of 10^{-6} $cm^{-2}g$ and Λ in units of 10^4 gcm^{-2})

For b_p, we use the expression taken from Bugaev's book[4]:

$$b_p=6.01\cdot10^{-8}Z(Z+1)/A[0.97\ln(p_1/Q_1) + 2.15](g^{-1}cm^2) \quad (15)$$

$$P_1=183Z^{-1/3}, \quad Q_1=183Z^{-1/3}m_\mu^2/(2E_\mu m_e)+1$$

In the following we apply this equation together with the Λ of D-I curves for S rock and K.G.F rock to get γ. If we take $\Lambda=9.868\cdot10^4 gcm^{-2}$ ($8000 < t < 9000$ $hgcm^{-2}$) for S rock, which is derived from eq.(2) in ref.(5), and $\Lambda=9.00\cdot10^4$ gcm^{-2} ($t=10^4$ $hgcm^{-2}$) for K.G.F rock.[6]

The values of γ thus obtained is as follows:

γ = 2.70 (E $\approx$ 30 TeV) for S rock and

γ = 2.62 (E $\approx$ 50 TeV) for K.G.F rock.

The former is consistent with the value of 2.71 by Bergamasco et al.[5] and the latter is also consistent with that of 2.6 by Miyake et al.[7], where fluctuations are treated by Monte Carlo Method.

4. Conclusions. By using the approximate R at great depth, γ can be easily obtained if the D-I curve has the form $I(t) = C \cdot \exp(-t/\Lambda)$. It is found that there is a discrepancy of γ between S rock and K.G.F rock in the same energy region.

References.

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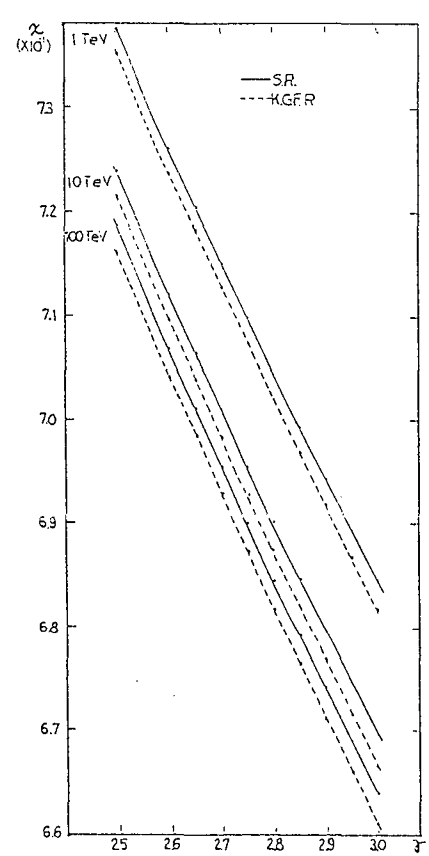

Fig.1 Dependence of χ on γ.