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CALCULATION OF ALLOWABLE ORBITAL SPACINGS

FOR THE FIXED-SATELLITE SERVICE

By

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July 1985
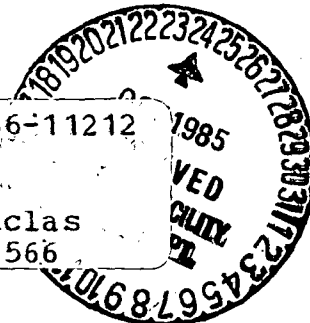**The Ohio State University**

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16. Abstract (Limit: 200 words)

Minimum satellite separations are calculated which satisfy a given carrier-to-interference protection ratio for the Fixed-Satellite Service (FSS) on a single-entry basis, assuming circular antenna beams. The results are presented in the form of universal contour curves, in which antenna-centered angles are the coordinates, and also in terms of the more conventional longitude and latitude separations. It is shown that orbit capacity increases with decreasing service-area size and that, for practical service areas, capacity is increased if the longitude of a satellite does not differ too greatly from that of the service area it serves.

17. Document Analysis   a. Descriptors

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b. Identifiers/Open-Ended Terms

Frequency Reuse

c. COSATI Field/Group

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CHAPTER I

INTRODUCTION

A. BACKGROUND

Satellite communication is suitable for a wide variety of applications. Its prime use is to provide long-distance communications, and this application is continuing to grow fast.

One of the reasons why satellite communication has become so popular is the unique synchronous property of the geostationary satellite orbit (GSO). The GSO can be described as an imaginary circle in the equatorial plane, approximately 22,300 miles (36,000 km) above the surface of the Earth. A satellite placed there orbits the Earth at the same angular velocity as the Earth rotates on its axis, so that it appears to be stationary to an observer on the Earth. Although there are some important satellite applications using non-geostationary orbits, without doubt the existence of the GSO has made satellite communication so significant.

Satellites cannot be placed arbitrarily close to one another along the GSO, not primarily because of danger of possible collision, but because of the resulting electromagnetic interference. Allowable interference levels dictate the minimum possible spacings between two satellites using the same frequency band. Therefore, the GSO can

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accommodate only a finite number of satellites using any given frequency band. It has been recognized that the GSO is a limited natural resource [1]. As use of the GSO becomes more intensive, the effective utilization of this natural resource has become an important issue.

Technical analyses on satellite communication services are performed by the International Radio Consultative Committee (CCIR) of the Internationl Telecommunication Union (ITU), using the work of CCIR study groups as input. This study utilizes the CCIR recommendations and reports as sources for technical parameters, such as antenna reference patterns.

The purpose of this study is to develop computational methods and to perform engineering calculations which will provide a basic understanding of satellite orbital locations. More specifically, minimum satellite separations are calculated which satisfy a given carrier-to-interference power ratio (C/I) for the Fixed-Satellite Service (FSS) on a single-entry basis. The results are presented in sets of contour curves as well as computer codes.

B. SOME PRELIMINARIES

A satellite communications link consists of an up link from an earth station to a satellite and a down link from the satellite to a destination earth station. Interference can occur on each of these half links. On the up link, interference may be due to the signals

from other up links, from other satellites, or from stations in the terrestrial service. In this study, it will be assumed that the up-link signals under consideration will be in different frequency bands from all down-link, inter-satellite, and terrestrial-service signals, or that separate coordination procedures will be used, so that in up-link calculations only the interference between up links needs to be considered. Similarly, it will be assumed that the down-link signals under consideration will be in frequency bands different from those assigned to up links, inter-satellites, and terrestrial services, or that these services will be coordinated, so that in down-link calculations only the interference between down links need be considered. Therefore, each half link will be treated separately. The situation is shown schematically in Figure 1.1 with the geometric parameters of interest.

As shown in Figure 1.1, only two satellite networks will be dealt with in this study, i.e., the analysis is on a single-entry basis. One is viewed as a wanted network, and the other as an interfering network. Earth station antennas are assumed to be pointed directly at their respective satellite antennas as required by good system design. Also, pointing and station keeping errors are ignored for simplicity, and only circular-beam antennas are considered here.

In actual system analysis, many interfering satellites or interfering earth stations are involved (aggregate interference) rather than just a single interfering source (single-entry interference). In order to obtain an estimate of multi-interference source situations,

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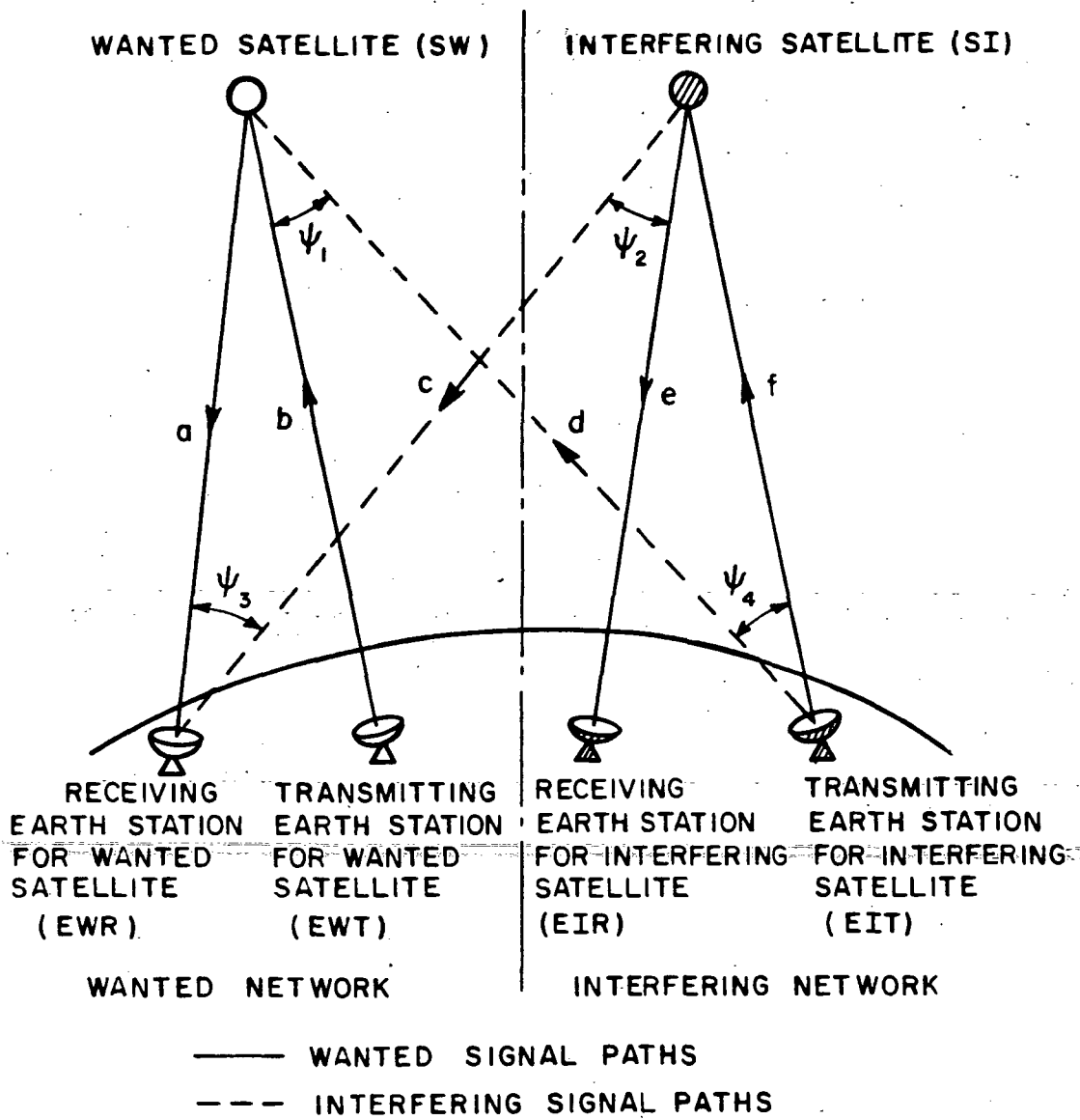
Figure 1.1 Interference geometry between satellite networks.

the carrier-to-interference power ratio (C/I), which will be used here as an interference criterion, should be raised by a suitable factor, such as 5 dB. For example, if the required C/I ratio on an aggregate basis is 30 dB, a single-entry C/I ratio of 35 dB due to one of the worst interference sources will often be satisfactory [2].

CHAPTER II

DOWN-LINK INTERFERENCE

A. INTERFERENCE GEOMETRY

The single-entry C/I ratio will be derived first for down links.
The interference geometry between the down-link parts of two FSS
networks is shown in Figures 2.1 and 2.2, where ψ_2, ψ_3, and ψ_5 denote
angles between the main-beam axis and the direction considered (off-axis
angles), and lowercase letters a, c, e, and g denote distances. The
wanted signal is received from satellite SW, while the interfering
signal arrives from satellite SI. The calculation of the C/I ratio is
performed at earth station EWR, and the resulting equation is used to
determine the allowable satellite spacing. The ground aim point of
satellite SI is designated by EIR; an earth station of the interfering
network may (or may not) be located there. Satellite SW is not
necessarily pointed directly at earth station EWR; however, earth
station EWR is located within a service area of satellite SW.

B. CARRIER AND INTERFERENCE POWERS

It is first necessary to determine the carrier signal power and the
interference signal power at earth station EWR. Signal power levels can
be obtained by application of the Friis transmission formula [3]
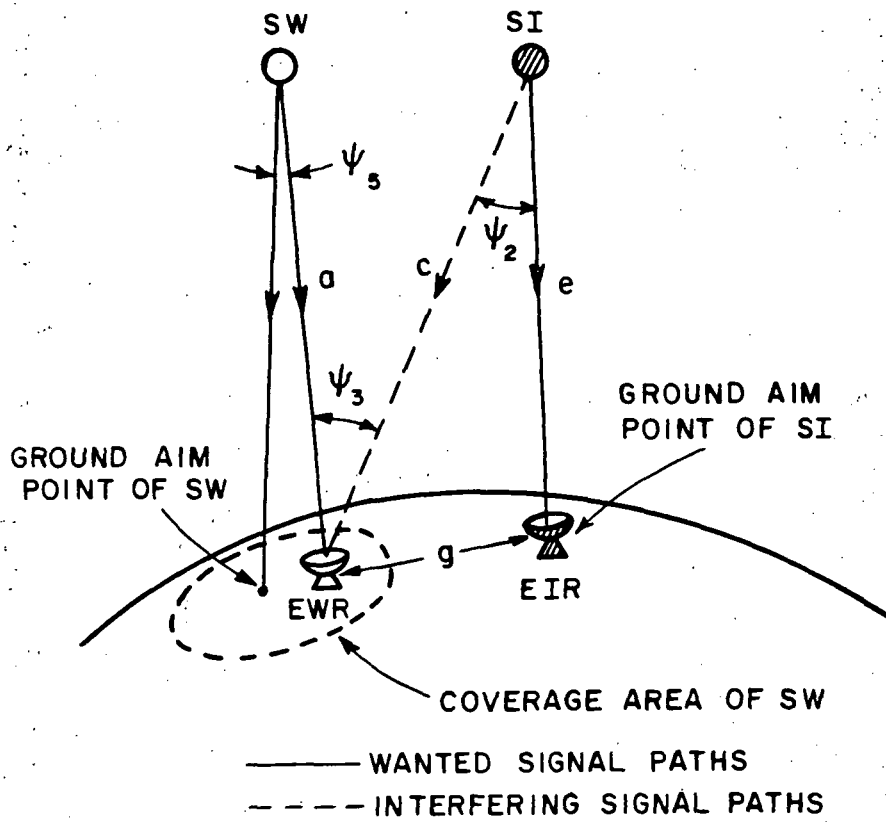
Figure 2.1 Interference geometry between down-link networks.
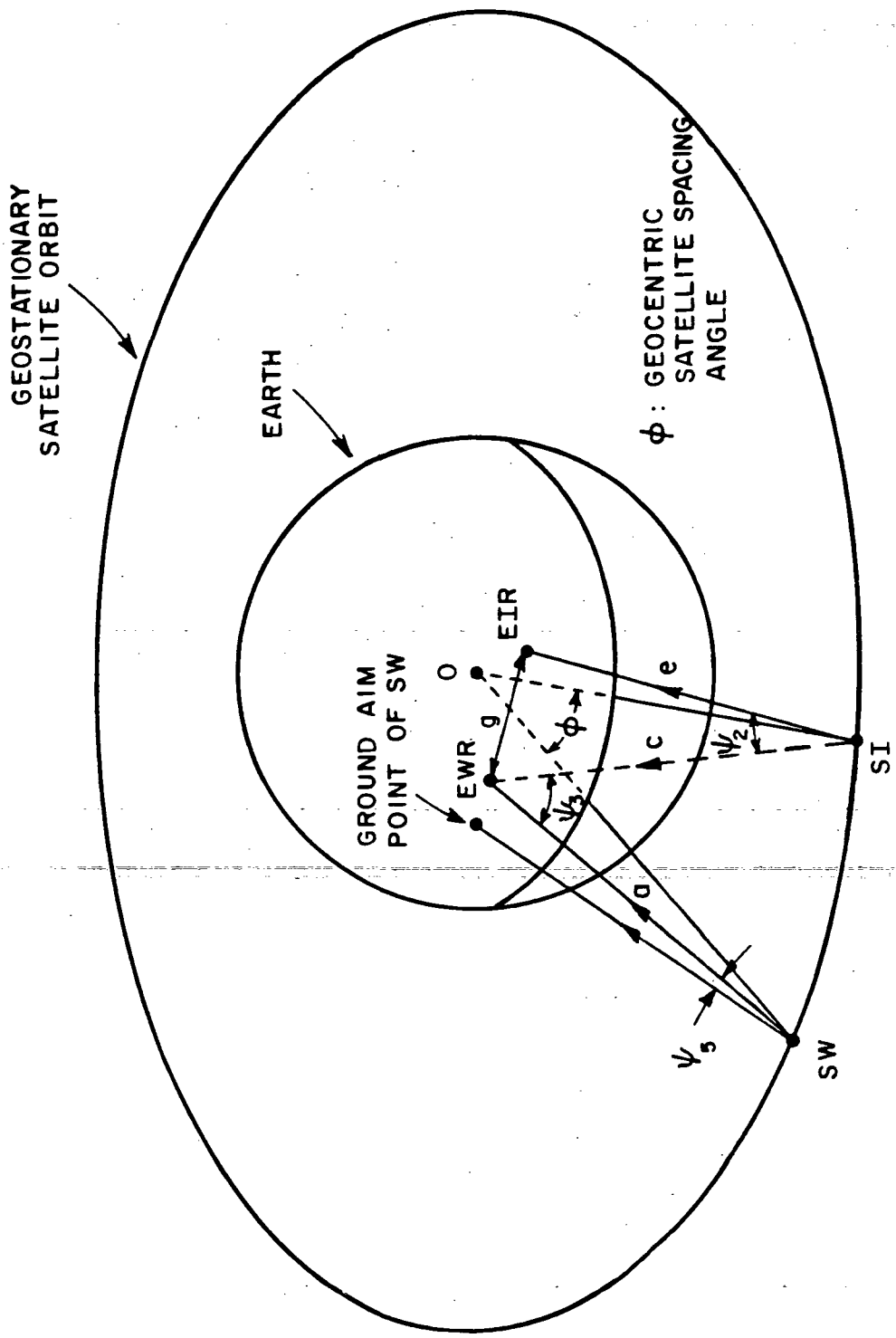
Figure 2.2   Alternative presentation of interference geometry between down-link networks (see Figure 2.1). Earth radius exaggerated for clarity.

$$P_R = \frac{P_T \, G_T \, G_R \, \lambda^2}{(4 \, \pi \, r)^2} \, , \qquad\qquad (2.1)$$

where P denotes power, G gain, λ wavelength, and r distance. The subscripts T and R indicate transmit and receive, respectively.

The carrier signal power C_{EWR} and the interference signal power I_{EWR} received by earth station EWR are given by

$$C_{EWR} = \frac{P_{SWT} \{G_{SWT} D_{SWT}(\psi_5, G_{SWT})\} G_{EWR} \lambda^2_W}{(4\pi a)^2} \qquad\qquad (2.2)$$

and

$$I_{EWR} = \frac{P_{SIT} \{G_{SIT} D_{SIT}(\psi_2, G_{SIT})\} \{G_{EWR} D_{EWR}(\psi_3, G_{EWR})\} \lambda^2_I}{(4\pi c)^2} \, , \qquad (2.3)$$

where the multiple subscript notation adheres to the following convention: the first letter identifies the location, S for satellite and E for earth station, of the parameters in question; the second letter indicates wanted (W) or interfering (I) network; and the third letter indicates transmit (T) or receive (R). Thus, the terms in Equations (2.2) and (2.3) are interpreted as follows:

P_{SWT} = power transmitted from the wanted satellite

P_{SIT} = power transmitted from the interfering satellite

G_{SWT} = transmitting gain of the wanted satellite antenna in the direction of the main beam axis

G_{SIT} = transmitting gain of the interfering satellite antenna in the direction of the main beam axis

G_{EWR} = receiving gain of the wanted satellite antenna in the direction of the main beam axis

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ψ_2 = angle between the axis of the interfering satellite antenna and the direction of the wanted earth station

ψ_3 = angle between the axis of the wanted earth station antenna and the direction of the interfering satellite

ψ_5 = angle between the axis of the wanted satellite antenna and the direction of the wanted earth station

$D_{SIT}(\psi_2, G_{SIT})$ = relative gain of the interfering satellite antenna in the direction of the wanted earth station

$D_{EWR}(\psi_3, G_{EWR})$ = relative gain of the wanted earth station antenna in the direction of the interfering satellite

$D_{SWT}(\psi_3, G_{SWT})$ = relative gain of the wanted satellite antenna in the direction of the wanted earth station

λ_W = wavelength of the wanted signal

λ_I = wavelength of the interfering signal

a = distance between the wanted satellite and the wanted earth station

c = distance between the interfering satellite and the wanted earth station

The units Watts, degrees, and meters are implied above.

C. REFERENCE ANTENNA PATTERNS

The relative gain $D_{SIT}(\psi_2, G_{SIT})$ is a function of the off-axis angle ψ_2 and the gain G_{SIT} indicated in the argument and is determined from the satellite antenna reference pattern shown in Figure 2.3, while the relative gain $D_{EWR}(\psi_3, G_{EWR})$ is a function of ψ_3 and G_{EWR} and is

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determined from the earth station antenna reference pattern in Figure
2.4. These reference patterns are adopted from CCIR Report 558-2 [4]
and from CCIR Report 391-4 [5], respectively. Note that the reference
patterns in Figures 2.3 and 2.4 are expressed in decibels.
Consequently, D_{SIT}, G_{SIT}, D_{EWR}, and G_{EWR} in these figures are all in dB,
not ratios. The notation dB is placed before the equation number to
indicte a decibel form throughout this article.

The half-power beamwidth ψ_0 in Figure 2.3 can be calculated from
the gain G_{SIT} by the relationship

$$\psi_0 = \sqrt{\frac{27000}{G_{SIT}}} \quad \text{degrees} \tag{2.4}$$

as suggested in CCIR Report 215-4 [6].

The antenna diameter-to-wavelength ratio d/λ in Figure 2.4 is
related to gain G_{EWR} as

$$G_{EWR} = \varepsilon\pi^2(d/\lambda)^2 \quad , \tag{2.5}$$

where the aperture efficiency ε is assumed to be 55% for circular
parabolic reflector antennas [7].

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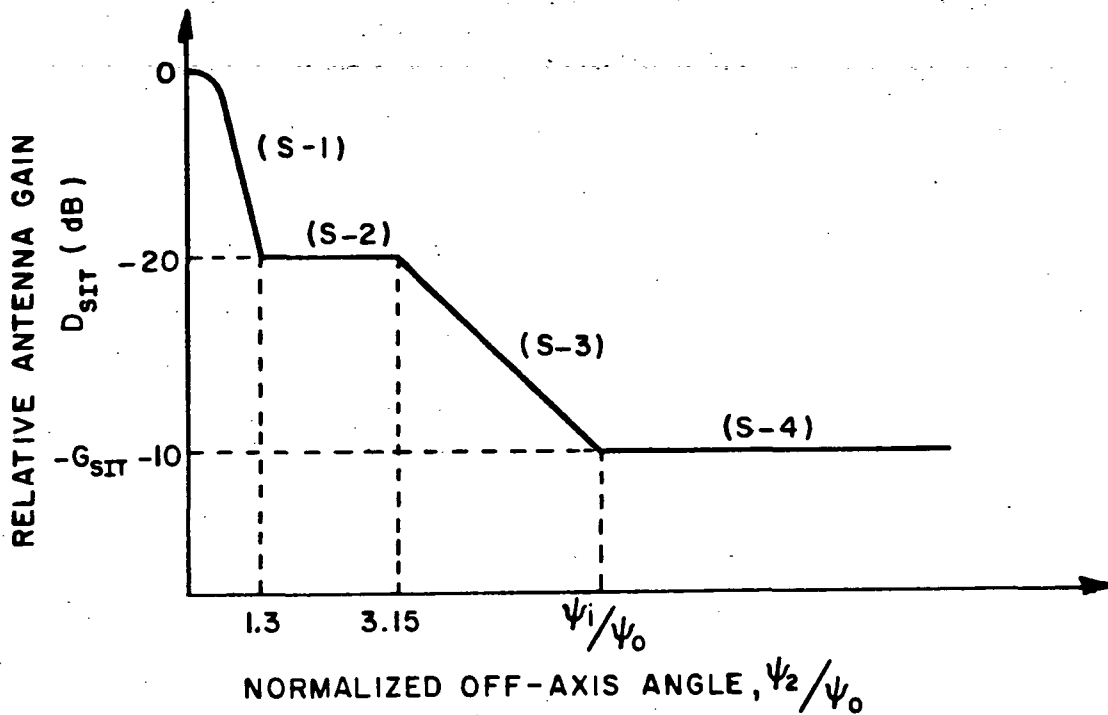
$$D_{SIT} = -12 \ (\psi_2/\psi_0)^2 \ dB \qquad \text{for} \quad 0° < \psi_2 < 1.3\psi_0 \qquad (S-1)$$

$$D_{SIT} = -20 \ dB \qquad \text{for} \quad 1.3\psi_0 < \psi_2 < 3.15\psi_0 \qquad (S-2)$$

$$D_{SIT} = -7.5 - 25 \ \log_{10}(\psi_2/\psi_0) \ dB$$

$$\text{for} \quad 3.15\psi_0 < \psi_2 < \psi_i \qquad (S-3)$$

$$D_{SIT} = -G_{SIT} - 10 \ dB \qquad \text{for} \quad \psi_i < \psi_2 \qquad (S-4)$$

where ψ_0 : half-power beamwidth

ψ_i : value of ψ_2 when D_{SIT} in region (S-3) is equal to $-G_{SIT} - 10$ dB

Figure 2.3 FSS satellite antenna reference pattern. This reference pattern is taken from CCIR Report 558-2 [4].
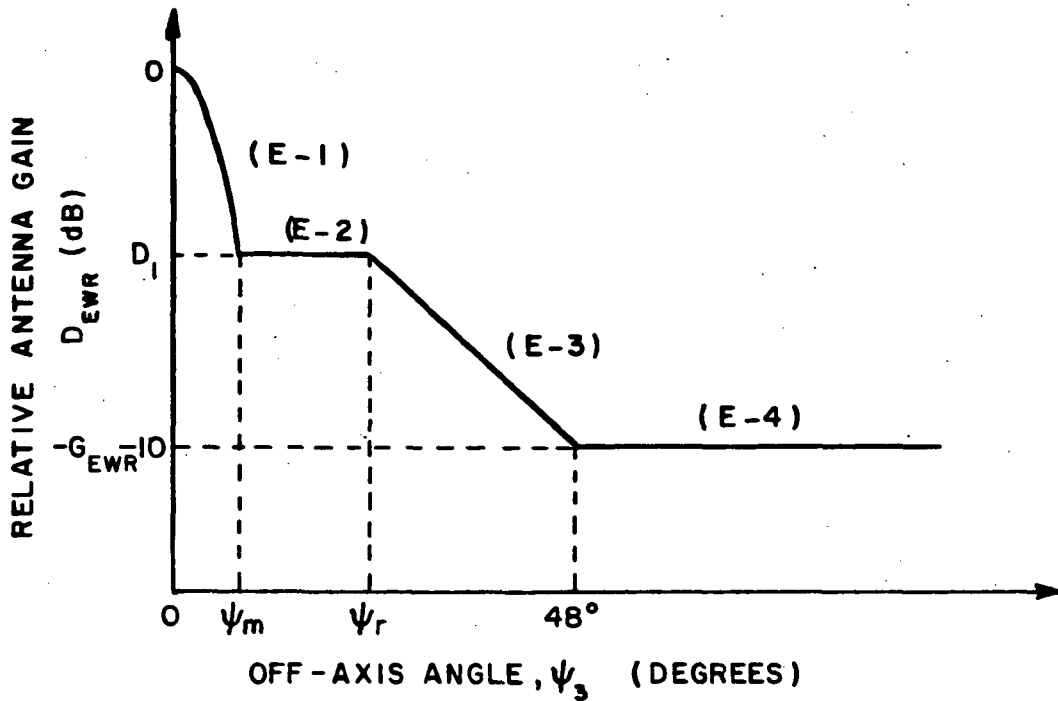
$D_{EWR} = -2.5 \times 10^{-3} (d/\lambda)^2 \psi_3^2$ dB   for $0° < \psi_3 < \psi_m$ (E-1)

$D_{EWR} = D_1$ dB   for $\psi_m < \psi_3 < \psi_r$ (E-2)

$D_{EWR} = 32 - 25 \log_{10}(\psi_3) - G_{EWR}$ dB   for $\psi_r < \psi_3 < 48°$ (E-3)

$D_{EWR} = -G_{EWR} - 10$ dB   for $48° < \psi_3$ (E-4)

where

    d : antenna diameter

    λ : wavelength

    D_1 : relative gain of the first sidelobe = $2 + 15 \log_{10}(d/\lambda) - G_{EWR}$

    $\psi_m = 20 \sqrt{-D_1} / (d/\lambda)$ degrees

    $\psi_r = 15.85 (d/\lambda)^{-0.6}$ degrees

Figure 2.4   FSS earth station antenna reference pattern. This reference
pattern is taken from CCIR Report 391-4 [5].

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D. CARRIER-TO-INTERFERENCE POWER RATIO

The single-entry C/I ratio at earth station EWR is obtained by dividing Equation (2.2) by Equation (2.3) as

$$(C/I)_{EWR} = \frac{P_{SWT}G_{SWT}}{P_{SIT}G_{SIT}} \frac{D_{SWT}(\psi_5, G_{SWT})}{D_{SIT}(\psi_2, G_{SIT})D_{EWR}(\psi_3, G_{EWR})} \frac{c^2}{a^2} , \qquad (2.6)$$

where it is assumed that the carrier signal and the interference signal use the same frequency, i.e., $\lambda_W = \lambda_I$.

A small notational simplification is available in Equation (2.6) by denoting

$$E_{SWT} = P_{SWT}G_{SWT} \qquad (2.7)$$

and

$$E_{SIT} = P_{SIT}G_{SIT} , \qquad (2.8)$$

which are the effective isotropically radiated powers (EIRP) of the satellite transmitters in the direction of their respective beam maxima. Equation (2.6) now becomes

$$(C/I)_{EWR} = \frac{E_{SWT}}{E_{SIT}} \frac{D_{SWT}(\psi_5, G_{SWT})}{D_{SIT}(\psi_2, G_{SIT})D_{EWR}(\psi_3, G_{EWR})} \frac{c^2}{a^2} . \qquad (2.9)$$

For reasonable antenna patterns, the minimum satellite spacing angles calculated by use of Equation (2.9) turn out to be relatively small, and consequently the distances a and c are nearly equal, i.e., c^2/a^2 is in the range of 1 ± 0.028 for 5° or smaller satellite separation. Note that Figures 1.1, 2.1, and 2.2 are not drawn to scale.

The actual ratio between the radius of the Earth and the radius of the geostationary satellite orbit is $1 : 6.6$. Thus, by assuming $c^2/a^2 = 1$, Equation (2.9) can be simplified to

$$(C/I)_{EWR} = \frac{E_{SWT}}{E_{SIT}} \cdot \frac{D_{SWT}(\psi_5, G_{SWT})}{D_{SIT}(\psi_2, G_{SIT}) D_{EWR}(\psi_3, G_{EWR})} \qquad . \qquad (2.10)$$

E. UNIVERSAL SYSTEM PARAMETER

Since the objective here is to find the minimum possible spacings between satellites and to present the results in a form which is helpful for system design, it is useful to reduce the number of parameters as much as possible in Equation (2.10) by combining some of them into a new system parameter. Defining such a parameter R_{DN} as

$$R_{DN} = (C/I)_{EWR} \frac{E_{SIT}}{E_{SWT}} \frac{1}{D_{SWT}(\psi_5, G_{SWT})} \qquad (2.11)$$

simplifies Equation (2.10) to

$$R_{DN} = \frac{1}{D_{SIT}(\psi_2, G_{SIT}) D_{EWR}(\psi_3, G_{EWR})} \qquad . \qquad (2.12)$$

Equations (2.11) and (2.12) can also be expressed in a decibel form as

$$R_{DN} = (C/I)_{EWR} - E_{SWT} + E_{SIT} - D_{SWT}(\psi_5, G_{SWT}) \text{ dB} \qquad (2.13)$$

and

$$R_{DN} = -D_{SIT}(\psi_2, G_{SIT}) - D_{EWR}(\psi_3, G_{EWR}) \text{ dB}, \qquad (2.14)$$

respectively. The decibel form equations will be used in the subsequent discussion.

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For each network system, the required $(C/I)_{EWR}$, E_{SWT}, and E_{SIT} in Equation (2.13) have given values, while D_{SWT} depends on ψ_5 and G_{SWT}. However, D_{SWT} varies only from 0 to -3 dB, i.e., D_{SWT} is ignored (0 dB) for EWR located at the SW aim point and is assumed to be -3 dB for EWR at the edge of the SW coverage area. Thus, D_{SWT} in Equation (2.13) can be estimated between 0 and -3 dB, depending on how closely EWR is located to the SW aim point.

Equation (2.14) can apply not only to the co-channel interference case, but also to the adjacent-channel interference case by obtaining the required C/I ratio for the adjacent-channel interference. An example is provided in Figure 2.5, which gives the adjacent channel protection ratio relative to the co-channel protection ratio as a function of the frequency offset. For example, suppose two channels are separated by 20 MHz, i.e., the center frequencies of the channels differ by 20 MHz, then the required C/I ratio is 8 dB below that for the co-channel interference case. Thus, R_{DN} calculated from Equation (2.13) is also 8 dB below that for the co-channel case.

The parameter R_{DN} will be used as a criterion reference level and called a universal system parameter in the subsequent discussion. It can also be interpreted physically as the total antenna discrimination from Equation (2.14). In application, the left side of Equation (2.14) is easily evaluated from Equation (2.13), while the right side is evaluated by computer and is presented in the form of a graph, i.e., the contour curve of ψ_3 against ψ_2 at each value of R_{DN} can be plotted for fixed values of G_{SWT} and G_{EWR} by using Equation (2.14). The computer code for this implementation is given in Appendix A.

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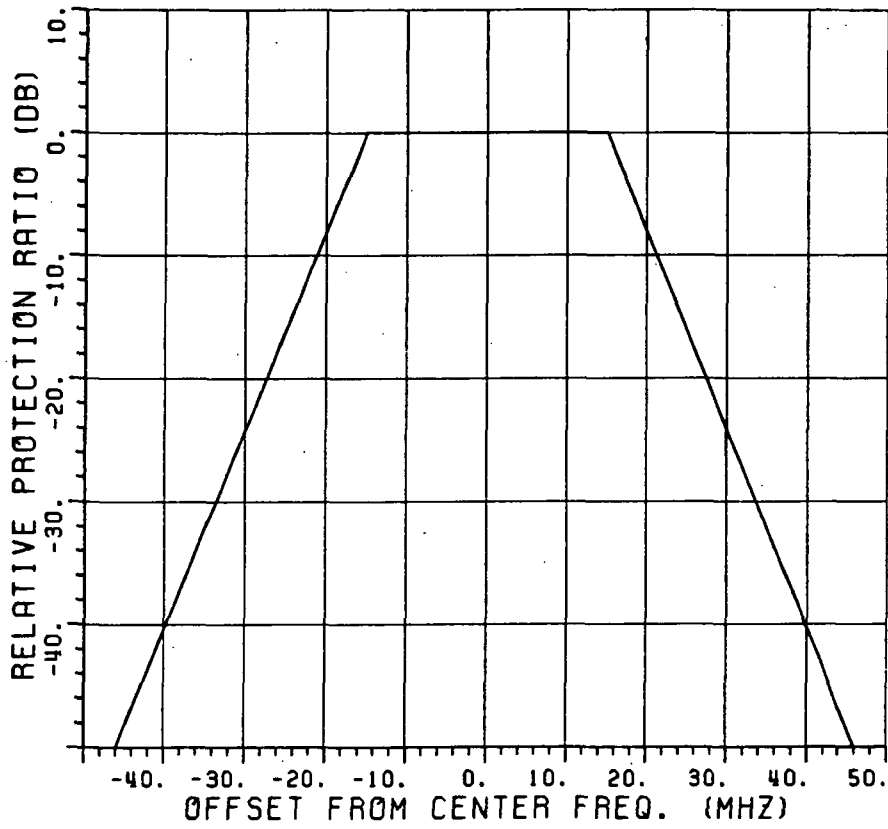
Figure 2.5   Protection ratio relative to co-channel values [8].

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F. GRAPHICAL PRESENTATION

The result from Equation (2.14) is presented graphically in Figure 2.6, where the minimum allowable satellite spacing angle ψ_3 is shown as a function of the off-axis angle ψ_2 as seen from satellite SI for R_{DN} = 30 dB, 35 dB, and 40 dB. The system parameter values G_{SIT} = 50 dB and G_{EWR} = 50 dB, corresponding to a half-power beamwidth of approximately 0.5°, are used for the calculation in Figure 2.6.

The off-axis angle ψ_2 is the angle between the main-beam axis of the SI antenna (toward EIR) and the direction of EWR, and can be interpreted as an earth station separation between EWR and EIR as seen from SI. It can be calculated from the two earth station locations and the orbital satellite location in terms of longitude and latitude by the Law of Cosines, i.e.,

$$\psi_2 = \cos^{-1} \frac{c^2 + e^2 - g^2}{2\,c\,e} , \tag{2.15}$$

where the distances c, e, and g, defined as in Figures 2.1 and 2.2, are obtained in the following way. Referring to Figure 2.7, the locations of EWR and EIR are represented in the rectangular coordinate form as

$$(r\cos\theta_{EWR}\cos\phi_{EWR},\ r\cos\theta_{EWR}\sin\phi_{EWR},\ r\sin\theta_{EWR})$$

and

$$(r\cos\theta_{EIR}\cos\phi_{EIR},\ r\cos\theta_{EIR}\sin\phi_{EIR},\ r\sin\theta_{EIR}),$$

respectively, and the location of SI is represented as
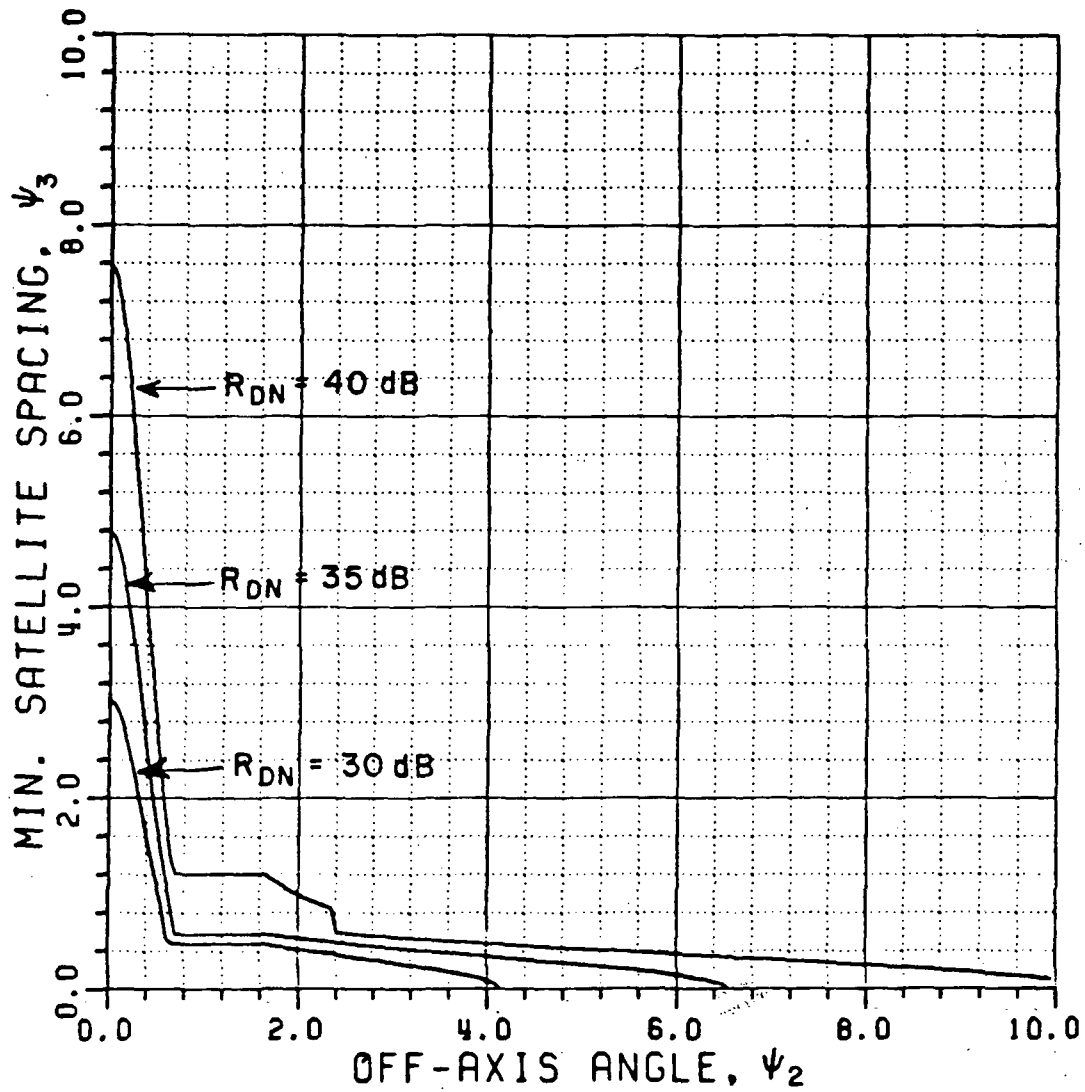
Figure 2.6 Minimum allowable satellite spacings as function of the earth station separation seen from the interfering satellite, based on down-link considerations. $G_{SIT} = G_{EWR} = 50$ dB.
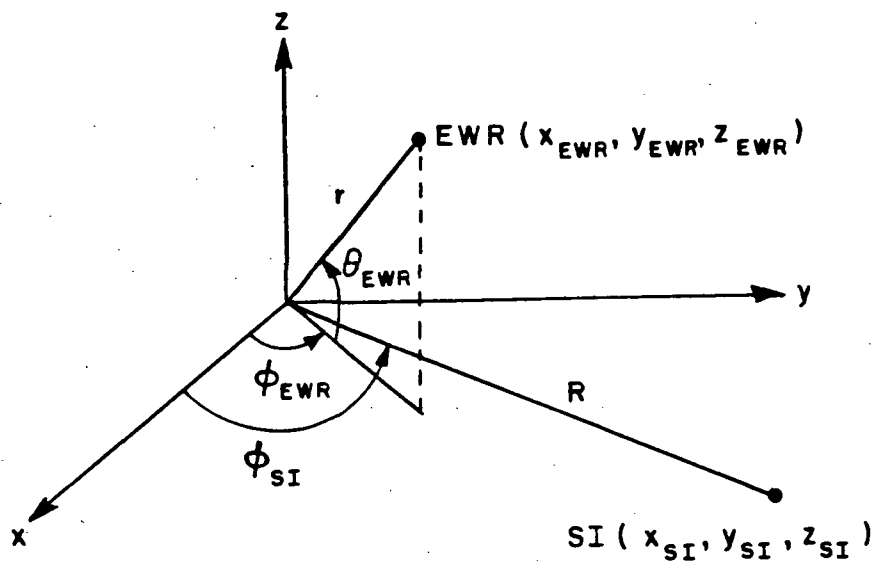
$$x_{EWR} = r\cos\theta_{EWR}\cos\phi_{EWR}$$

$$y_{EWR} = r\cos\theta_{EWR}\sin\phi_{EWR}$$

$$z_{EWR} = r\sin\theta_{EWR}$$

$$x_{SI} = R\cos\phi_{SI}$$

$$y_{SI} = R\sin\phi_{SI}$$

$$z_{SI} = 0$$

Figure 2.7   Relationship between earth station and satellite locations in terms of latitude and longitude and their rectangular coordinates.

$$(R\cos\phi_{SI}, R\sin\phi_{SI}, 0) \quad ,$$

where R indicates the radius of the geostatioary satellite orbit (4.2152×10^7 m), r indicates the earth radius (6.371×10^6 m), and θ and ϕ denote latitude and longitude, respectively. Note that latitude θ is negative in the southern hemisphere, and that longitude ϕ is negative in the western hemisphere. The subscript notation follows the same convention as before. Thus, the distances c, e, and g are calculated from

$$c^2 = R^2 + r^2 - 2Rr\cos(\phi_{SI}-\phi_{EWR})\cos\theta_{EWR} \quad , \tag{2.16}$$

$$e^2 = R^2 + r^2 - 2Rr\cos(\phi_{SI}-\phi_{EIR})\cos\theta_{EIR} \quad , \tag{2.17}$$

and

$$g^2 = 2r^2\{1 - \cos\theta_{EWR}\cos\theta_{EIR}\cos(\phi_{EWR}-\phi_{EIR})$$

$$- \sin\theta_{EWR}\sin\theta_{EIR}\} \quad . \tag{2.18}$$

It is evident that the calculations required to obtain the exact off-axis angle ψ_2 are numerous. A simple approximation is available for small values of ψ_2 by assuming that $c \approx e$. The distance c or e is expressed as 5.62r for a satellite operated at a high (90°) elevation angle and 6.54r for a satellite operated at a low (0°) elevation angle. Thus,

$$\sin(\psi_2/2) = \frac{g/2}{5.62r} \approx \frac{\psi_2}{2} \quad ,$$

i.e.,

$$\psi_2 \approx \frac{g}{5.62r} \quad \text{radians} \tag{2.19}$$

for high elevation angle. Similarly, one obtains

$$\psi_2 \approx \frac{g}{6.54r} \quad \text{radians} \tag{2.20}$$

for low elevation angle. Given the distance g, Equations (2.19) and (2.20) provide an upper bound of ψ_2 and a lower bound of ψ_2, respectively, and the bounds are quite narrow for small ψ_2.

The off-axis angle ψ_3 is the topocentric (as seen from a point on the Earth) satellite spacing angle and not the geocentric (measured from the center of the Earth) satellite spacing angle. In practice, we are interested in small satellite spacings, and under this condition, these two angles are nearly equal. Also, the topocentric angle is always greater than the geocentric angle, and hence the minimum satellite spacing angle ψ_3 in Figure 2.6 is always conservative in this sense.

However, the difference between the two angles is not always negligible. Especially when satellites are near zenith, and an observation point on the Earth is near the equator, the error in substituting the topocentric angle for the geocentric angle may approach 15%. An approximate but accurate conversion formula from the topocentric angle ψ_t to the geocentric angle ϕ_g is given by

$$\phi_g = \sqrt{1.023 - 0.302 \cos(\phi_M - \phi_E)\cos\theta_E} \; \psi_t \quad \text{radians} \quad , \tag{2.21}$$

where ϕ_M is the longitude of the midpoint between two satellites, and θ_E

and ϕ_E are the latitude and longitude of an observation point on the Earth, respectively. The derivation of Equation (2.21) is shown in Appendix B.

Consider now Figure 2.6. For R_{DN} = 35 dB, the required satellite spacing is 4.8° when ψ_2 = 0°, i.e., when EWR and EIR are collocated. The required satellite spacing is reduced to 2.5° when ψ_2 = 0.4°, i.e., when EWR and EIR are separated. The condition ψ_2 = 0° implies that the satellite antenna beams are overlapped (overlapping service areas), and ψ_2 > 0° implies that the antenna beams are restricted (assigning limited service areas). Thus, in the case of ψ_2 = 0°, the EWR antenna is illuminated by the main beam of the SI antenna, and discrimination is achieved only by the EWR antenna in the near-sidelobe region, thus resulting in the worst satellite spacing, whereas, in the case of ψ_2 > 0°, the beam maximum of the SI antenna does not face the EWR antenna directly, and less discrimination by the EWR antenna is required, thus allowing closer satellite spacing. Thus, the concept of assigning limited service areas is crucial for the effective use of the orbital arcs. This concept seems to have been enunciated first by Kiebler [9].

G. UNIVERSAL CURVES

Figure 2.6 is shown for the particular values G_{SIT} = G_{EWR} = 50 dB. The assignment of specific values to these variables in Equation (2.14) allows the plotting of a family of curves with R_{DN} as a parameter. A similar family of curves can be constructed for each set of gain values,

but an unwieldy number of curves result. Instead, the curves will be
reduced to a more universal form.

Since the FSS satellite antenna reference pattern of Figure 2.3 is
represented in terms of ψ_2/ψ_0, where the half-power beamwidth ψ_0 is a
function of G_{SIT} as shown in Equation (2.4), the abscissa variable in
Figure 2.6 can be chosen as ψ_2/ψ_0 instead of ψ_2, thus no longer
requiring the specification of the value G_{SIT} for plotting the curves.

On the other hand, the FSS earth station antenna reference pattern
of Figure 2.4 is represented as a function of $(d/\lambda)\psi_3$ in the region
labeled (E-1), i.e.,

$$D_{EWR}(\psi_3, G_{EWR}) = -2.5 \times 10^{-3}\{(d/\lambda)\psi_3\}^2 \text{ dB} \quad , \tag{2.22}$$

and as a function of $\psi_3 G_{EWR}^{0.4}$ in the region labeled (E-3), i.e.,

$$D_{EWR}(\psi_3, G_{EWR}) = 32 - 25 \log_{10}(\psi_3) - 10 \log_{10}(G_{EWR})$$

$$= 32 - 25 \log_{10}(\psi_3 G_{EWR}^{0.4}) \text{ dB} \quad , \tag{2.23}$$

in which G_{EWR} is not in dB but is a ratio. Note that G_{EWR} is a function
of d/λ as shown in Equation (2.5). Thus, the ordinate variable in
Figure 2.6 can be transformed into $(d/\lambda)\psi_3$ for the portion of the curves
contributed by region (E-1), and into $\psi_3 G_{EWR}^{0.4}$ for the portion of the
curves contributed by region (E-3). At first sight, it seems that four
sets of curves (four different ordinates) are necessary in order to
present ψ_3 because there are four regions (E-1) through (E-4) in the
reference pattern of Figure 2.4. Fortunately, the (E-4) portion is not

necessary since the ψ_3 values of interest are the minimum satellite spacing angles, which never exceed 48° for any practical situation. Furthermore, the curves in Figure 2.6 merely have a discontinuity, as shown by the dotted lines in Figure 2.8, when the D_{EWR} value falls into the region (E-2). Therefore, two sets of universal curves, corresponding to the regions (E-1) and (E-3), suffice to present the minimum allowable satellite spacing angle ψ_3. The set of universal curves contributed by the region (E-3) is shown in Figure 2.9 with the ordinate as $\psi_3 G_{EWR}^{0.4}$, where the values of ψ_3 obtained from Figure 2.9 must satisfy

$$\psi_3 > \psi_r = 15.85 \ (d/\lambda)^{-0.6}$$

$$= 26.3 \ G_{EWR}^{-0.3} \quad \text{degrees} \quad , \qquad (2.24)$$

in which G_{EWR} is a ratio. The other set of universal curves, contributed by the region (E-1), is shown in Figure 2.10 with the ordinate as $(d/\lambda)\psi_3$, where the values of ψ_3 obtained from Figure 2.10 must satisfy

$$\psi_3 < \psi_m = 20 \ \sqrt{G_{EWR} - 2 - 15 \ \log_{10}(d/\lambda)} \ / \ (d/\lambda)$$

$$= 20 \ \sqrt{5.35 + 5 \ \log_{10}(d/\lambda)} \ / \ (d/\lambda) \quad \text{degrees.} \qquad (2.25)$$

The dashed lines in Figures 2.9 and 2.10 indicate where to switch from one set of curves to the other for G_{EWR} = 50 dB and 60 dB. The value of ψ_3 has to be obtained from the area above the dashed lines in Figure 2.9, or from the area below the dashed lines in Figure 2.10.
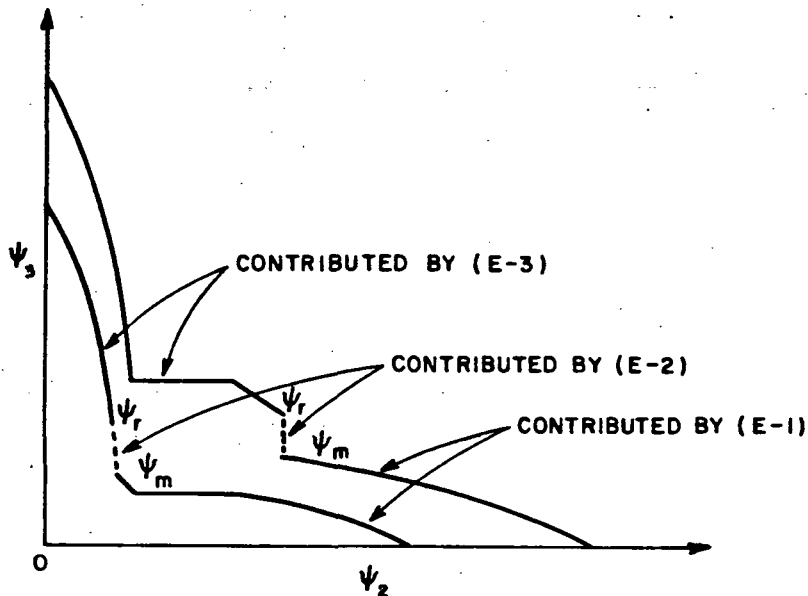
Figure 2.8 Discontinuity of contour curves in Figure 2.6.

Both sets of universal curves are based on Equation (2.14). In order to construct the first part of the universal curves, Equation (2.23) is substituted for D_{EWR} in Equation (2.14), and $(d/\lambda)\psi_3$ is calculated for each value of ψ_2/ψ_0 by means of an iterative search. Similarly, in order to construct the second part of the universal curves, Equation (2.22) is substituted for D_{EWR} in Equation (2.14), and $\psi_3 G_{EWR}$ is calculated for each value of ψ_2/ψ_0. The computer codes used for producing Figures 2.9 and 2.10 are given in Appendices C and D, respectively.

An example of how to use these curves will be given in the following. Given system parameters such as a required C/I ratio and an EIRP ratio, the universal system parameter R_{DN} can be computed from
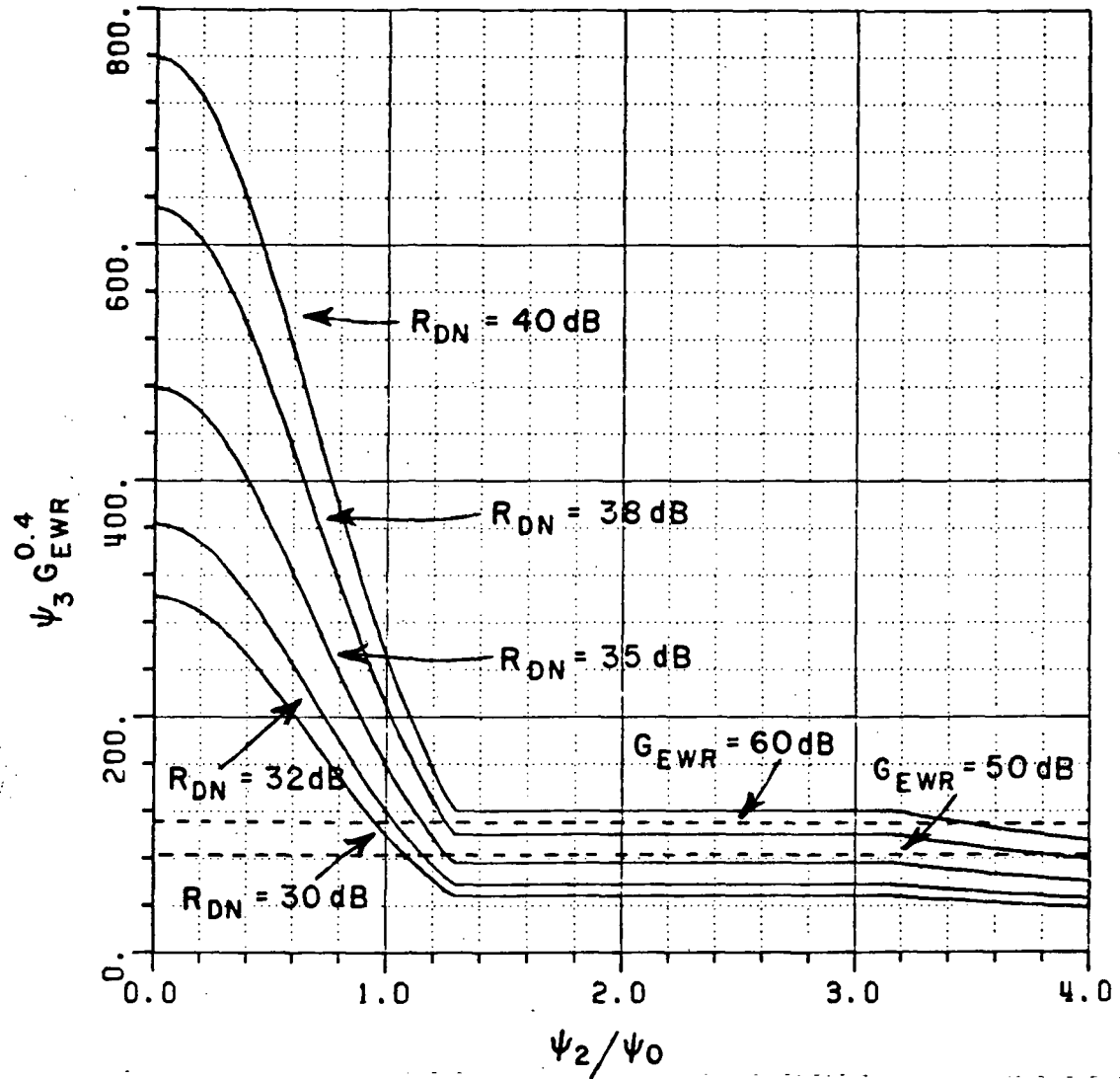
Figure 2.9   First part of universal curves for minimum allowable
satellite spacing angle ψ_3 as function of normalized
off-axis angle ψ_2/ψ_0, based on down-link consideration.
ψ_3 found from this figure must satisfy Equation (2.24).
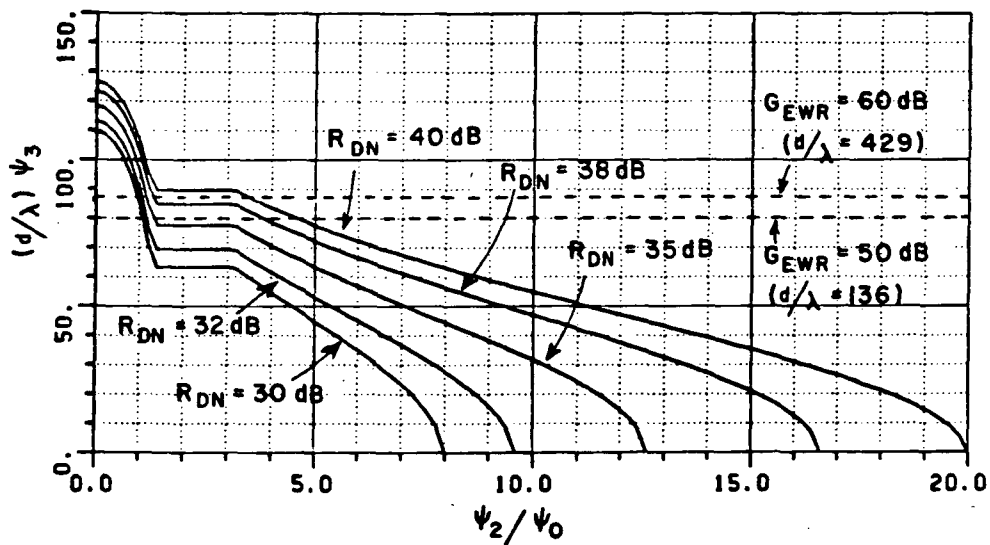
Figure 2.10   Second part of universal curves for minimum allowable
satellite spacing angle ψ_3 as function of normalized
off-axis angle ψ_2/ψ_0, based on down-link considerations.
ψ_3 found from this figure must satisfy Equation (2.25).

Equation (2.13). The off-axis angle ψ_2 is obtained as explained in the previous section F, and the half-power beamwidth ψ_0 is given in each system or is obtained from Equation (2.4). Suppose R_{DN} = 35 dB, G_{EWR} = 50 dB, and ψ_2 = 0.15°, then ψ_2/ψ_0 = 0.3. Figure 2.10 cannot be applied to obtain ψ_3 in this case because the value of $(d/\lambda)\psi_3$ at ψ_2/ψ_0 = 0.3 is not below the dashed line of G_{EWR} = 50 dB in Figure 2.10, since the resulting ψ_3 does not satisfy Equation (2.25). Thus, the $\psi_3 G_{EWR}$ value at ψ_2/ψ_0 = 0.3 is obtained from Figure 2.9 and is 320, above the dashed line of G_{EWR} = 50 dB. Since G_{EWR} = 10^5 (50 dB), the calculated value of ψ_3 is 3.2°, which also satisfies Equation (2.24). When satellites SW and SI are near zenith, and earth station EWR is near the equator, the conversion of the topocentric angle ψ_3 into the geocentric angle may be necessary by use of Equation (2.21).

H. PLOTS INVOLVING LONGITUDE AND LATITUDE

The minimum allowable satellite spacings in Figure 2.6 are shown as a function of the off-axis angle ψ_2. This angle is the natural coordinate for the calculation and allows the construction of universal curves, but it may not be optimum for system design. A presentation using earth station longitudes and latitudes directly can also be helpful. As an example, the results for a universal system parameter value R_{DN} = 35 dB are replotted in Figures 2.11, 2.12, 2.13, and 2.14, in which the longitude of SW relative to EWR is used as one coordinate and the required satellite separation is the other. Note that the

satellite separations are shown in terms of the geocentric angle. However, such a presentation is no longer universal. As shown in Figure 2.2, EWR is located inside the coverage area of SW and is not necessarily at the ground aim point of SW, but EIR designates the ground aim point of SI, whether or not an earth station is actually located there. The C/I ratio at EWR is used to determine the allowable satellite separations. In Figures 2.11 and 2.13, the longitude of EIR relative to EWR is shown as an explicit parameter, while the antenna gains, the earth station latitudes, and the universal system parameter R_{DN} are implicit parameters in the calculation. In Figures 2.12 and 2.14, the latitude of EIR is shown as an explicit parameter, while the antenna gains, the latitude of EWR, the longitude of EIR and the universal system parameter R_{DN} are implicit parameters in the calculation. The parameter values R_{DN} = 35 dB, G_{SIT} = 50 dB, and G_{EWR} = 50 dB are used in Figures 2.11 and 2.12, and R_{DN} = 35 dB, G_{SIT} = 40 dB, and G_{EWR} = 50 dB are used in Figures 2.13 and 2.14.

The calculation method regarding these four figures is based on Equation (2.14). First, the locations of EWR and EIR and the parameter values of R_{DN}, G_{SIT}, and G_{EWR} are fixed. Then, for each location of SW which is visible from EWR, the locations of SI (both east and west relative to SW) which satisfy Equation (2.14) are determined by means of an iterative search. Also, SI must be visible from both EWR and EIR in this calculation. The computer program used to produce Figures 2.11, 2.12, 2.13, and 2.14 is given in Appendix E.

Consider first the case of the earth stations being collocated (satellite antenna beams being overlapped). For the assumed system parameters, the required geocentric satellite separation angles for EWR at 0°N, 20°N, 40°N, and 60°N are 4.1°, 4.2°, 4.4°, and 4.6°, respectively, at the highest elevation of SW as seen from its earth station (relative longitude of SW 0°E), and they all increase slightly to 4.8° at the lowest elevation of SW, i.e., these curves are fairly flat, and the elevation of SW has very little effect on the required satellite spacing when the antenna beam of SI is not restricted. Therefore, it can be said that, when satellites which use all allowed frequency bands and have no service area (beam) restrictions are placed over the orbital arcs with constant separation (about 4.8° in this example), the orbit would be "full". Also, note that since ψ_2 is always equal to 0° in this case, the corresponding topocentric satellite spacing angle obtained from Figure 2.6 or Figure 2.9 is 4.8°, regardless of the satellite elevation. This illustrates that the difference between the geocentric angle and the topocentric angle increases slightly with increasing satellite elevation.

Consider next the case of the earth stations being separated (satellite antenna beams being restricted). It is evident that much closer satellite spacing along the orbital arc is achievable with increasing the service area separation. Figures 2.11 and 2.13 show that the service area separation in the longitudinal direction greatly reduces the required satellite spacing at moderately high elevations of SW, whereas Figures 2.12 and 2.14 show that the service area separation

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Figure 2.11 Minimum geocentric satellite spacing angles when earth stations are separated in longitudinal direction. R_{DN} = 35 dB, G_{SIT} = G_{EWR} = 50 dB, with EWR and EIR at (a) 0°N of latitude, (b) 20°N, (c) 40°N, and (d) 60°N.

(c) EWR AT 40° N

(d) EWR AT 60° N
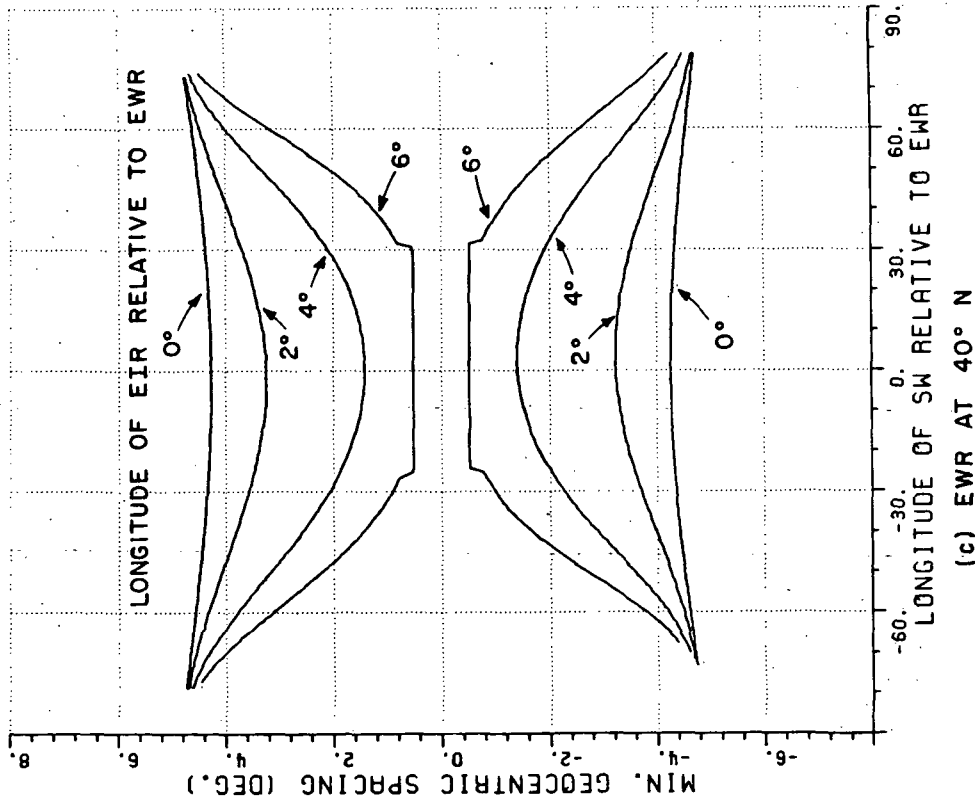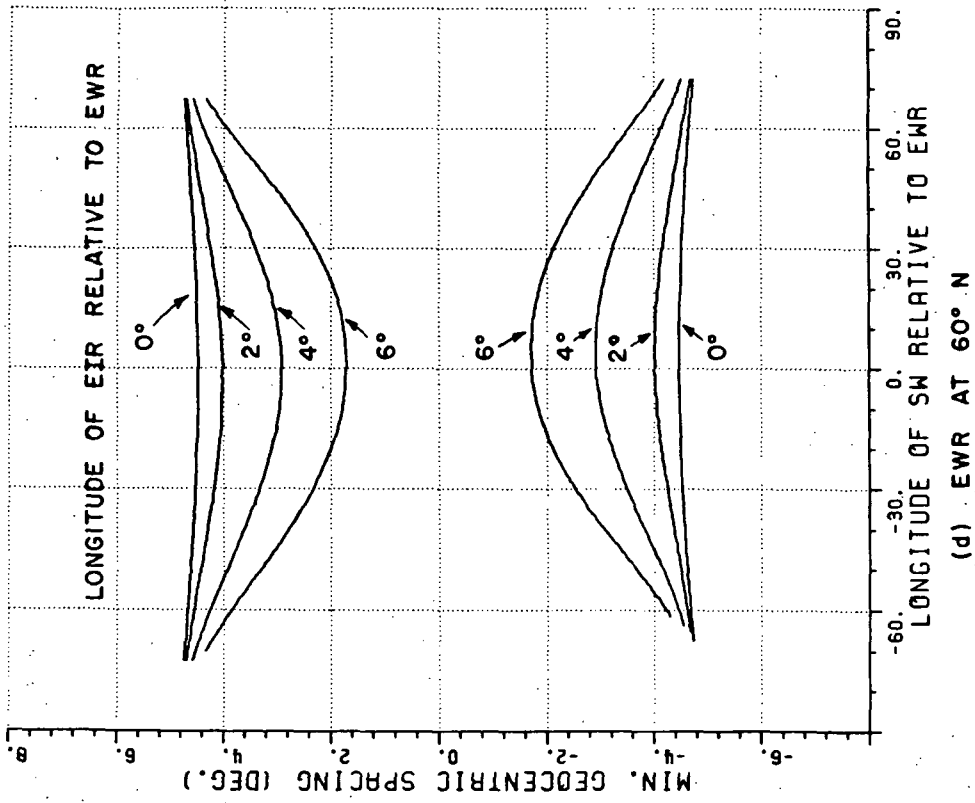
Figure 2.11 (Continued).

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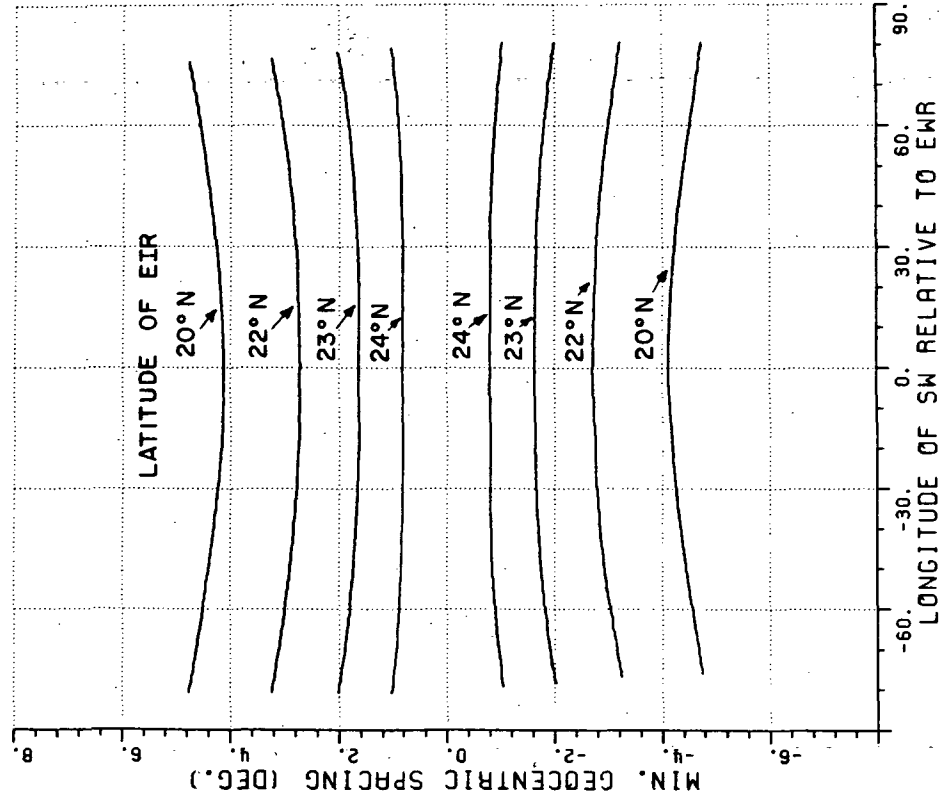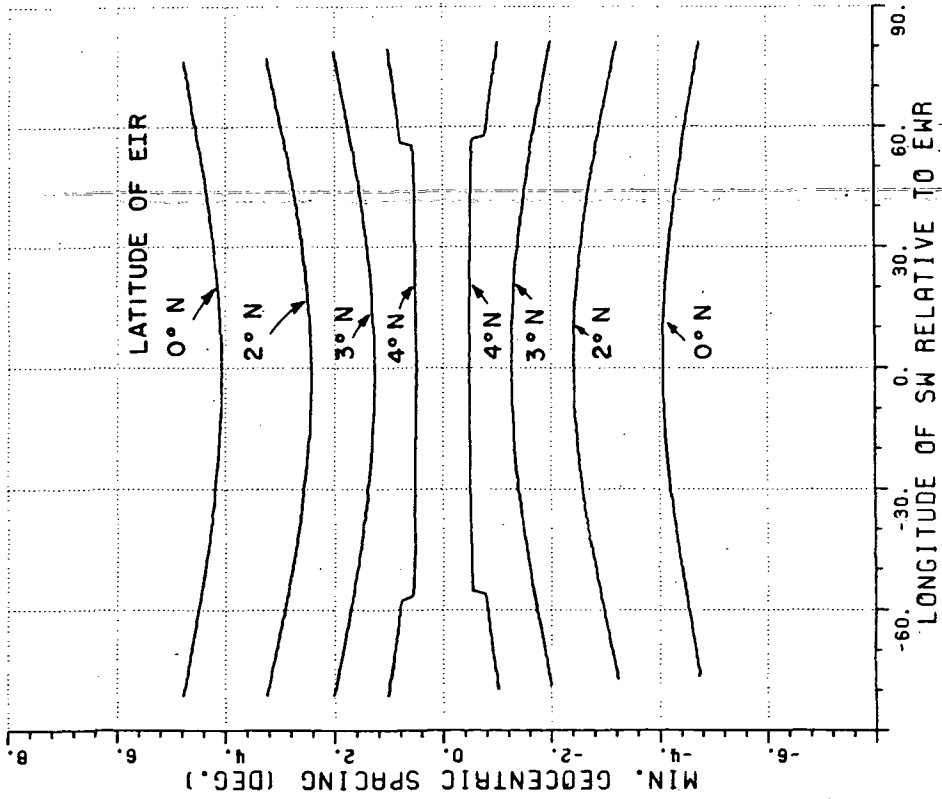
Figure 2.12 Minimum geocentric satellite spacing angles when earth stations are separated in latitudinal direction. R_{DN} = 35 dB, G_{SIT} = G_{EWR} = 50 dB, with EIR at the same longitude as EWR and EWR at (a) 0°N of latitude, (b) 20°N, (c), 40°N, and (d) 60°N.
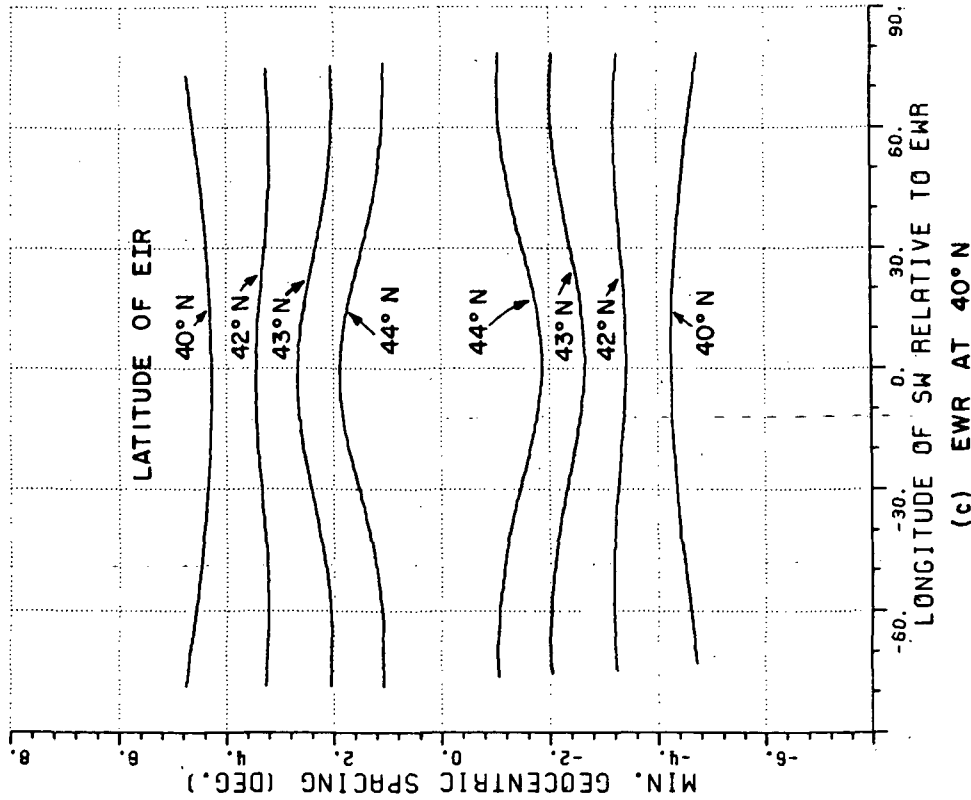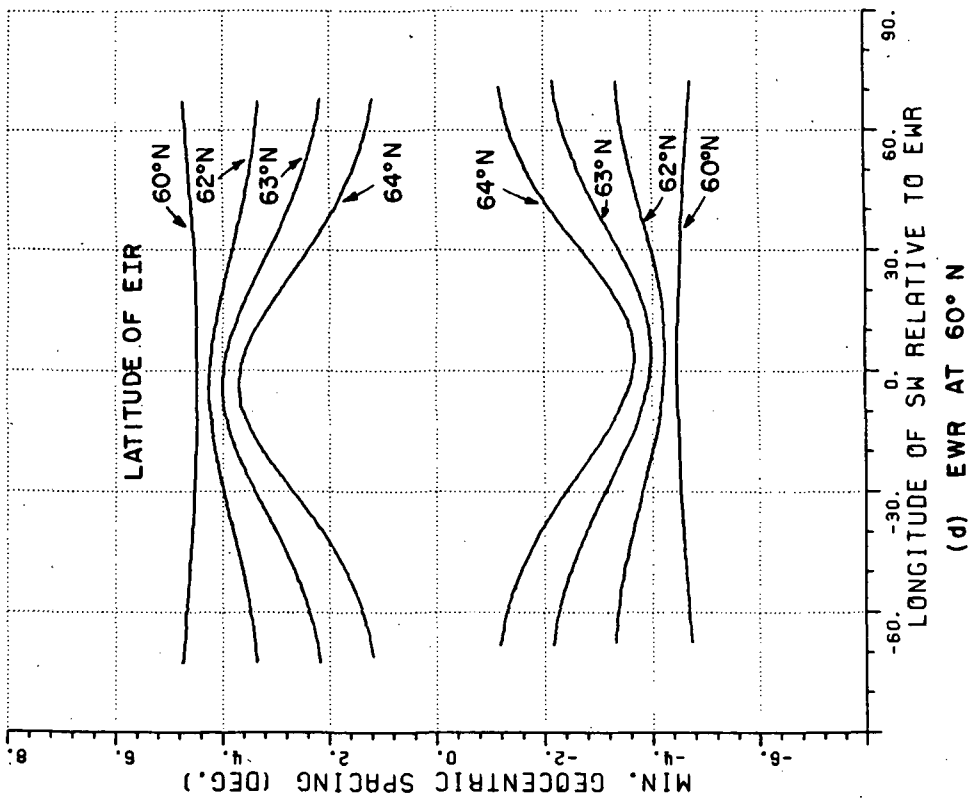
Figure 2.12 (Continued).

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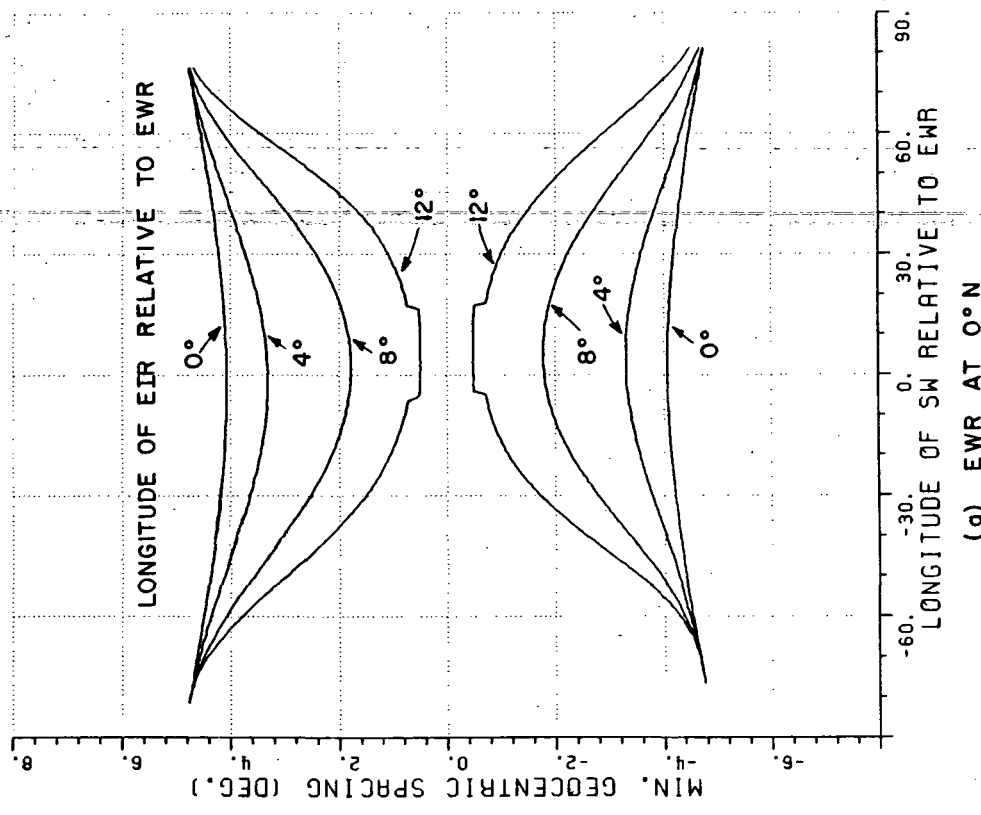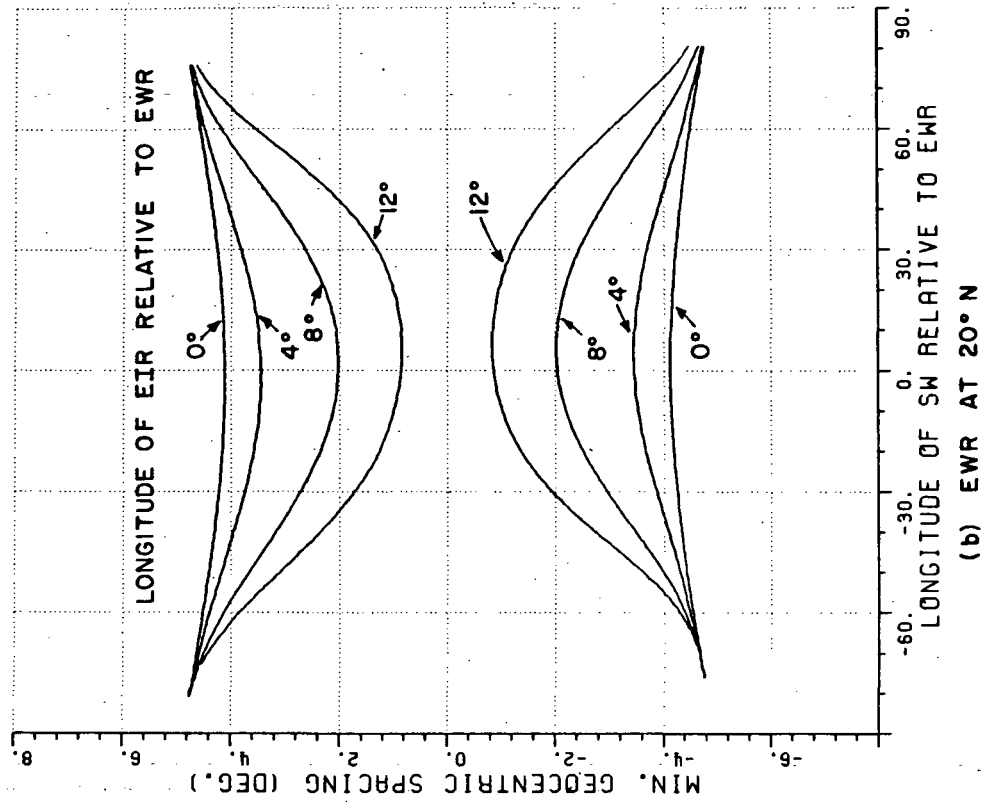
Figure 2.13 Minimum geocentric satellite spacing angles when earth stations are separated in longitudinal direction. R_{DN} = 35 dB, G_{SIT} = 40 dB, and G_{EWR} = 50 dB with R_{DN} = 35 dB, G_{SIT} = 40 dB, and G_{EWR} = 50 dB with EWR and EIR at (a) 0°N of latitude, (b) 20°N, (c) 40°N, and (d) 60°N.
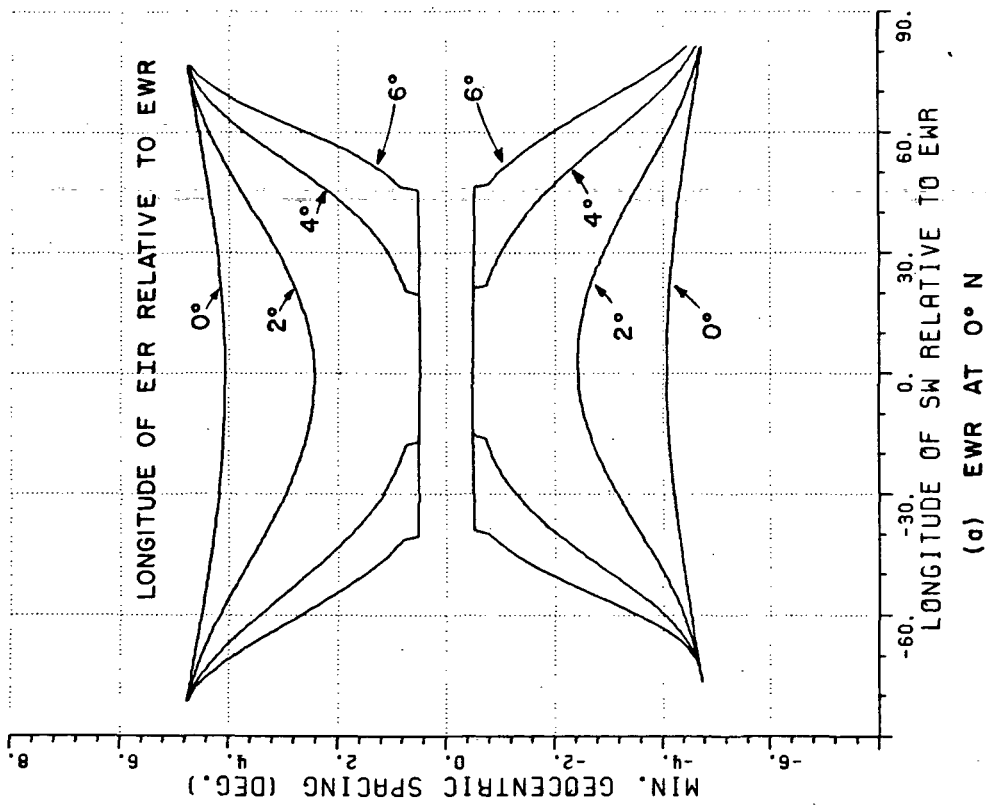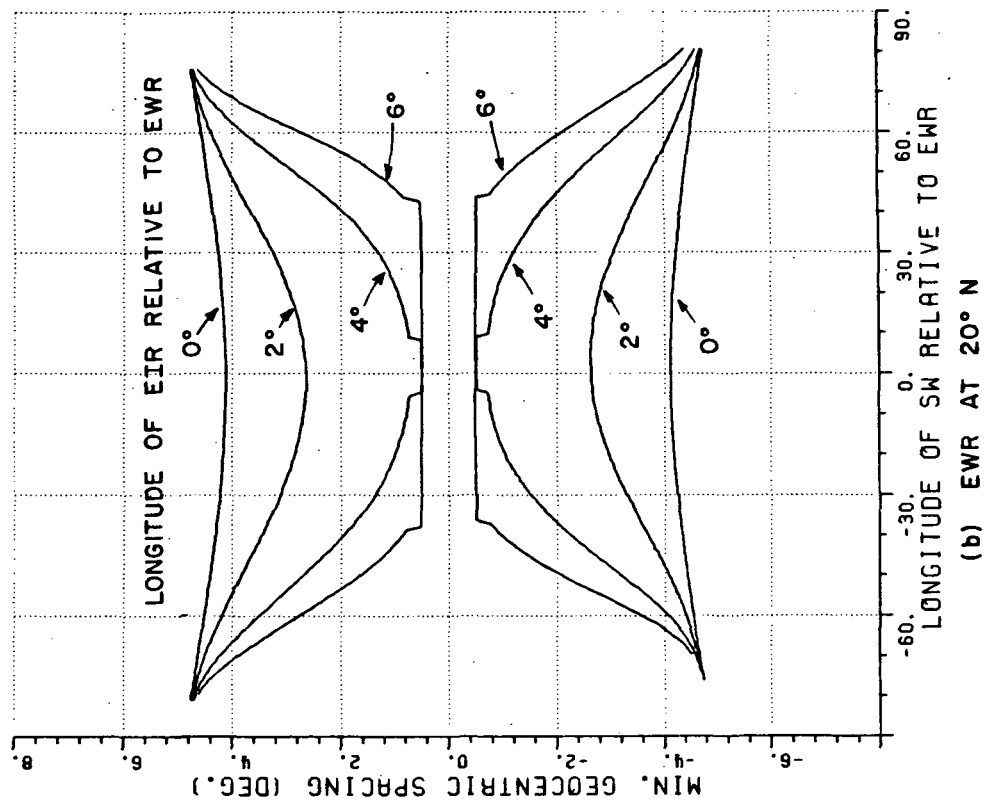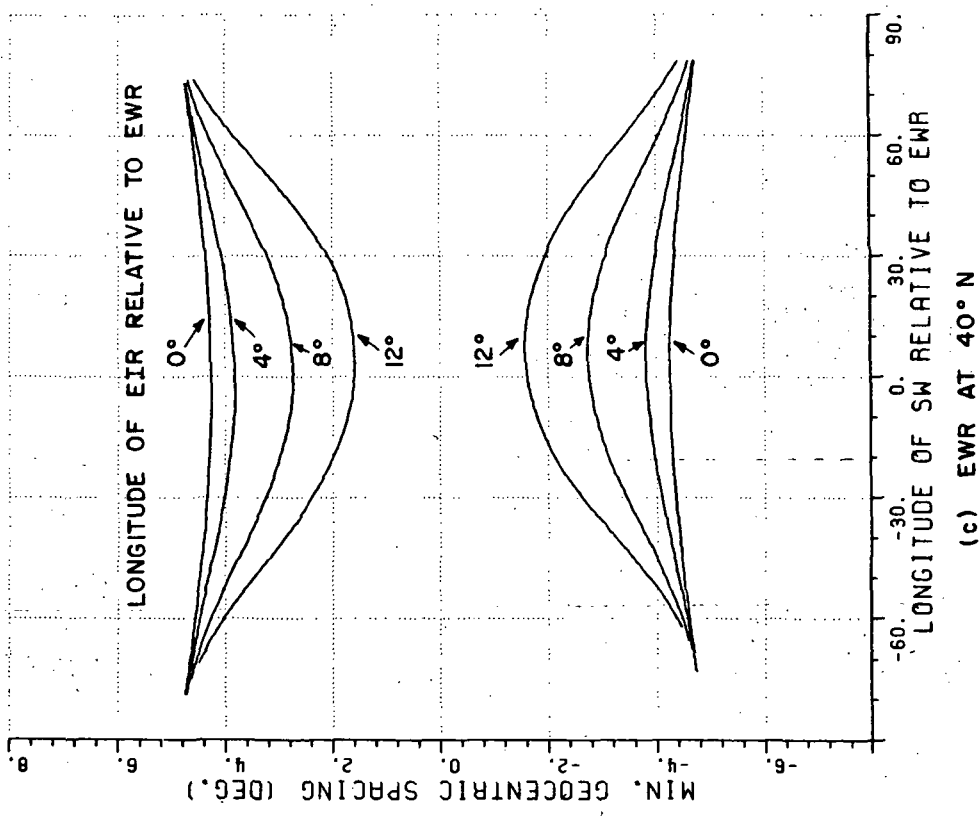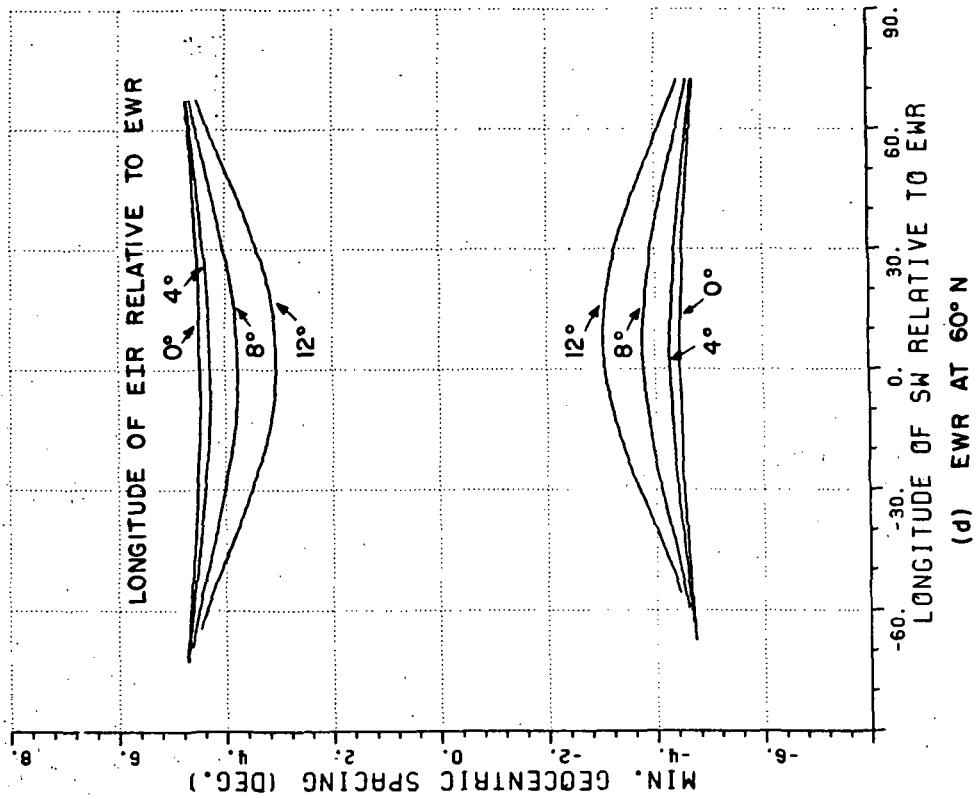
Figure 2.13 (Continued).

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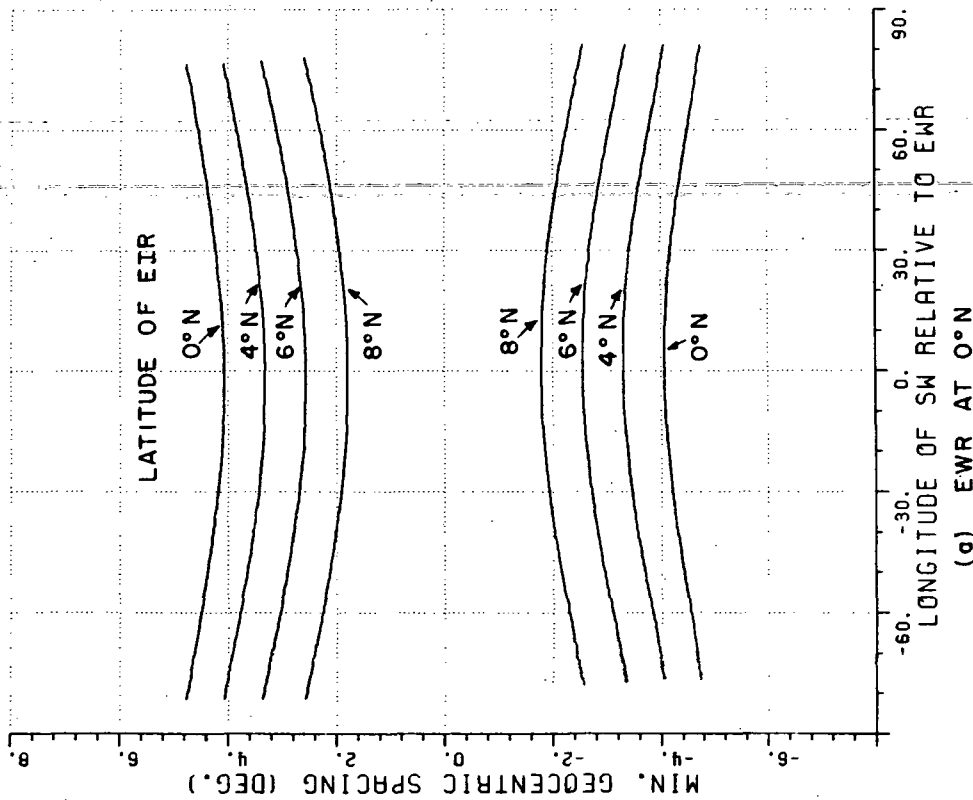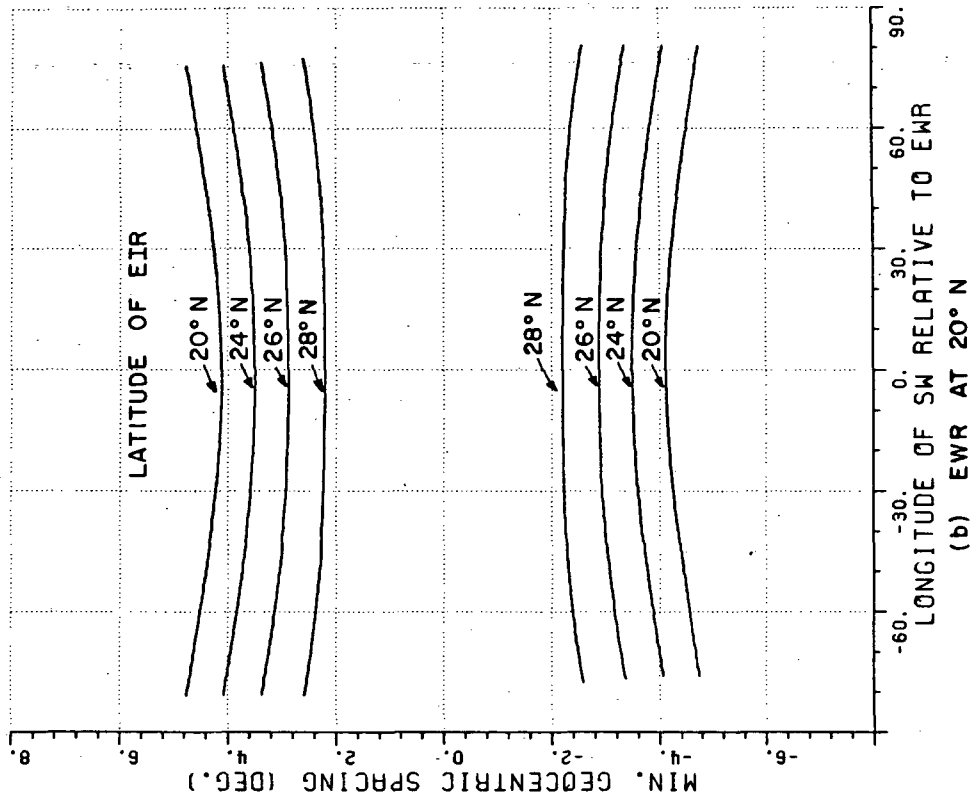
Figure 2.14 Minimum geocentric satellite spacing angles when earth stations are separated in latitudinal direction. RDN = 35 dB, GSIT = 40 dB, and GEWR = 50 dB, with EIR at the same longitude as EWR and EWR at (a) 0°N of latitude (b) 20°N, (c) 40°, and (d) 60°N.
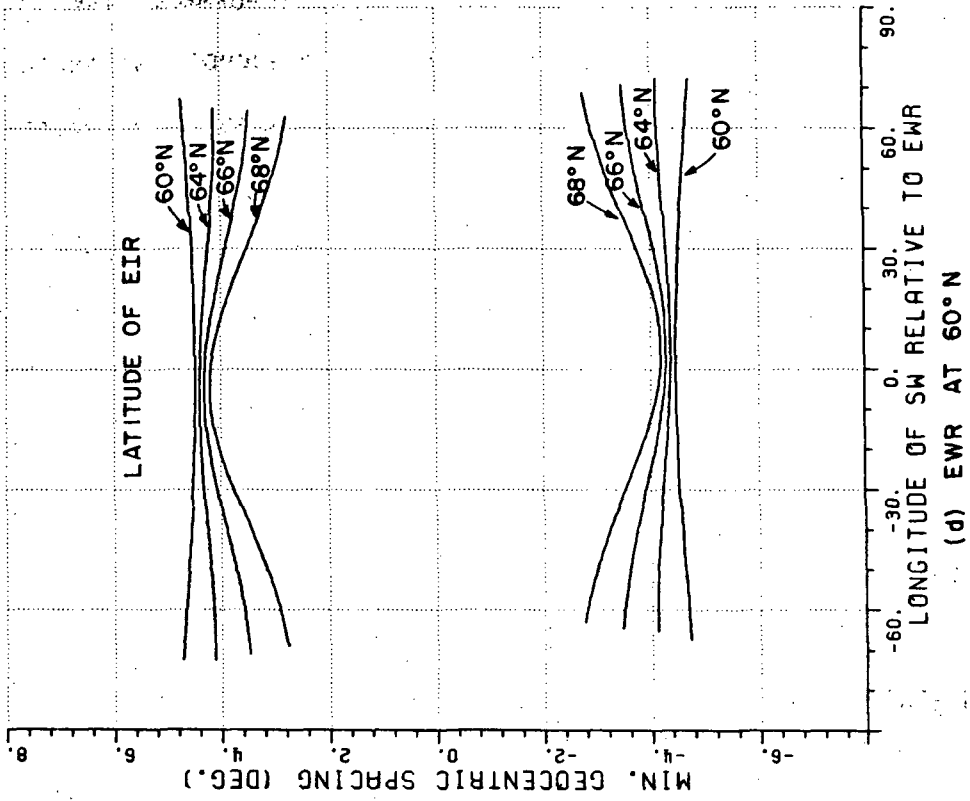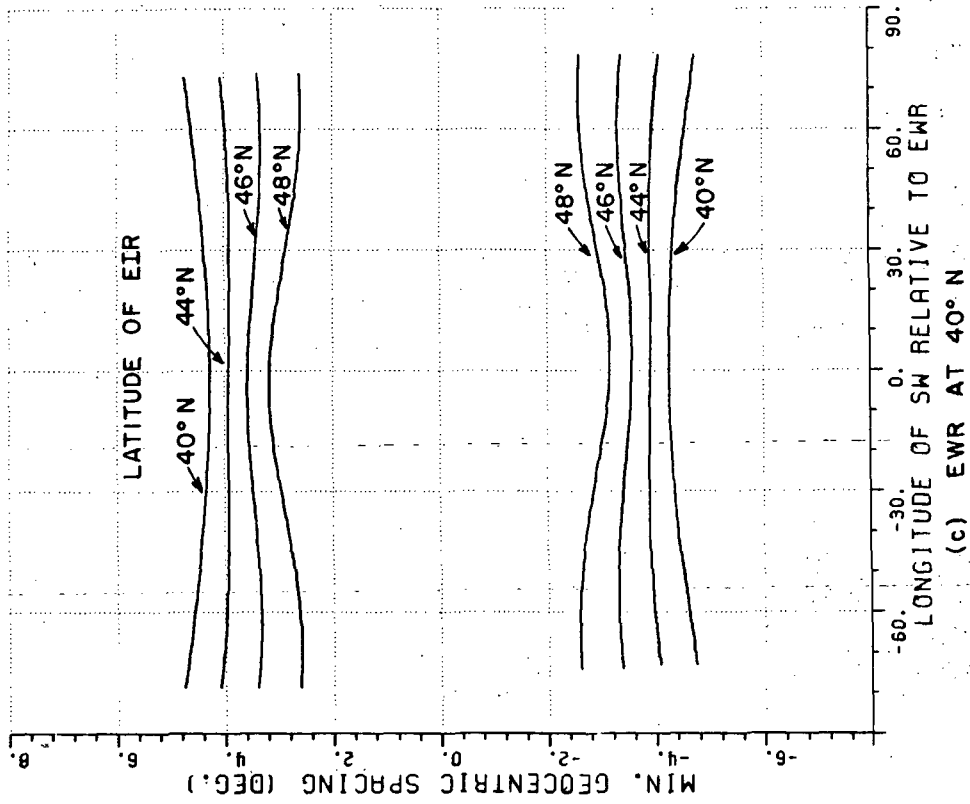
Figure 2.14 (Continued).

in the latitudinal direction also reduces the required satellite spacing but is not greatly affected by the elevation of SW except in the case of service areas located at high latitudes. Note that the service area separation implies the size of the off-axis angle ψ_2 rather than the actual distance between the earth stations, and that the required satellite spacing decreases with increasing ψ_2. Therefore, it is important to consider not only an adequate geographical separation between the service areas but also the satellite elevations in order to make the off-axis angle ψ_2 as large as possible and hence reduce the required satellite spacing.

J. EARTH STATION ANTENNA-PATTERN EFFECTS ON SATELLITE SPACING

When the main beams of SI and SW overlap each other, or when SI is intended to serve a very wide area such as earth coverage, the allowable satellite spacing is determined only by the receiving antenna of EWR, i.e., Equation (2.14) is reduced to

$$R_{DN} = -D_{EWR}(\psi_3, G_{EWR}) \text{ dB} \quad . \tag{2.26}$$

For the R_{DN} values of interest, only the sidelobe region of the earth station antenna designated by (E-3) makes a contribution in Equation (2.26), i.e.,

$$R_{DN} = G_{EWR} - 32 + 25 \log_{10}(\psi_3) \text{ dB} \quad . \tag{2.27}$$

By assigning another universal system parameter as

$$Y_{DN} = -R_{DN} + G_{EWR} \quad dB$$

$$= -(C/I)_{EWR} + E_{SWT} - E_{SIT} + G_{EWR} \quad dB \quad , \tag{2.28}$$

Equation (2.27) is rewritten as

$$Y_{DN} = 32 - 25 \log_{10}(\psi_3) \quad dB \quad . \tag{2.29}$$

Suppose earth station antennas with a better sidelobe level are available, so that

$$D_{EWR}(\psi_3, G_{EWR}) = 29 - 25 \log_{10}(\psi_3) - G_{EWR} \quad dB \tag{2.30}$$

which is given in CCIR Rec. 580 [10]. Then Equation (2.29) is replaced by

$$Y_{DN} = 29 - 25 \log_{10}(\psi_3) \quad dB \quad . \tag{2.31}$$

Figure 2.15 shows the graphical results based on Equations (2.29) and (2.31). It is presented in the same format as in Figures 2.11 through 2.14 except that Y_{DN} is used as a parameter instead of R_{DN}, and there is therefore no need to specify the antenna gain values. The computer code in Appendix E is used to produce Figure 2.15. The program, as written, produces the solid curves corresponding to Equation (2.29); in order to produce the dashed curves corresponding to Equation (2.31), the value 32 in the subroutine REAL FUNCTION DISC_E in Appendix E is replaced with the value 29. It is obvious that better discrimination in the sidelobe region can provide closer satellite spacing.

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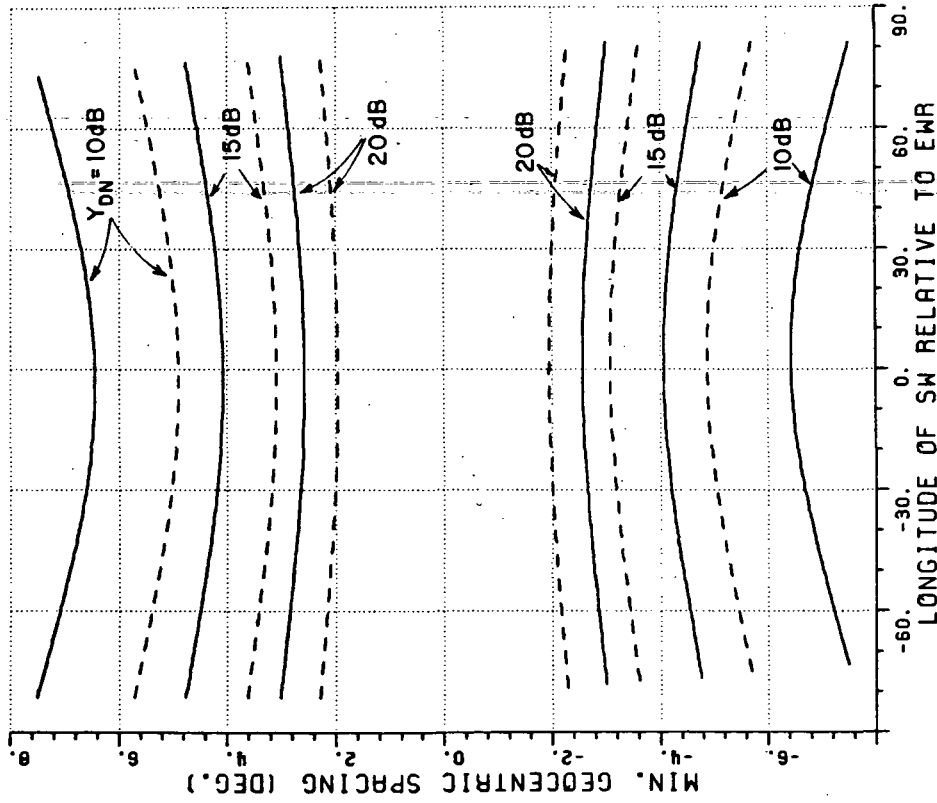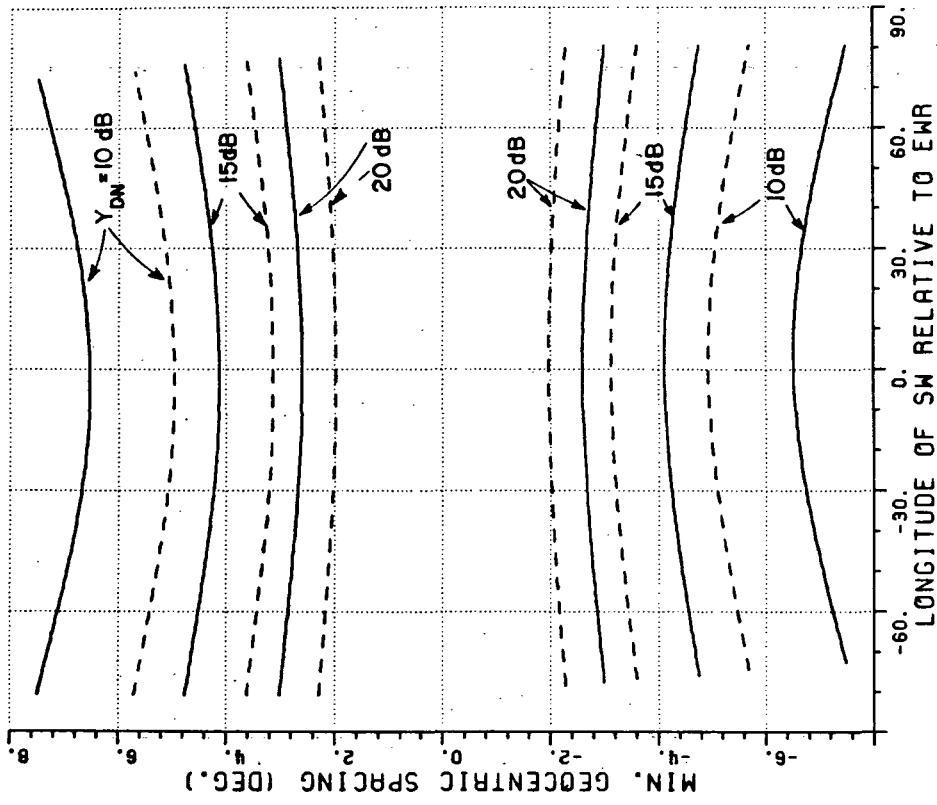
Figure 2.15 Minimum geocentric satellite spacing angles when earth stations are collocated (satellite antenna beams are overlapped). Solid lines represent satellite spacing angles corresponding to sidelobe gain $32-25\log_{10}(\psi_3)$, while dashed lines correspond to $29-25\log_{10}(\psi_3)$. Y_{DN} is defined in Equation (2.28).

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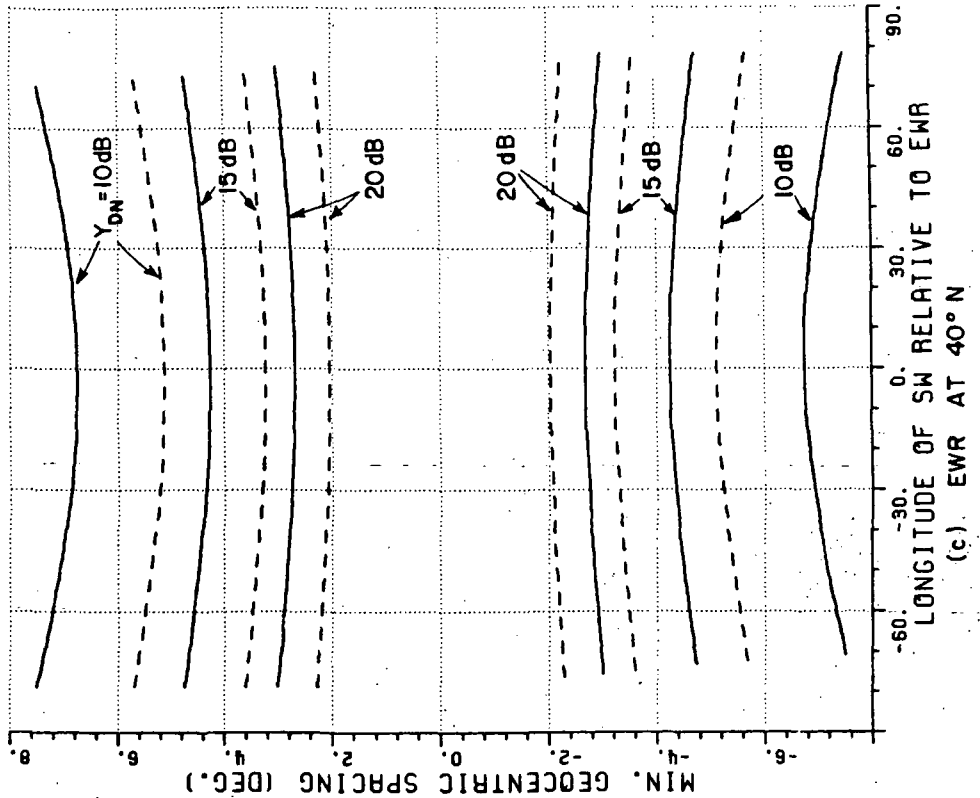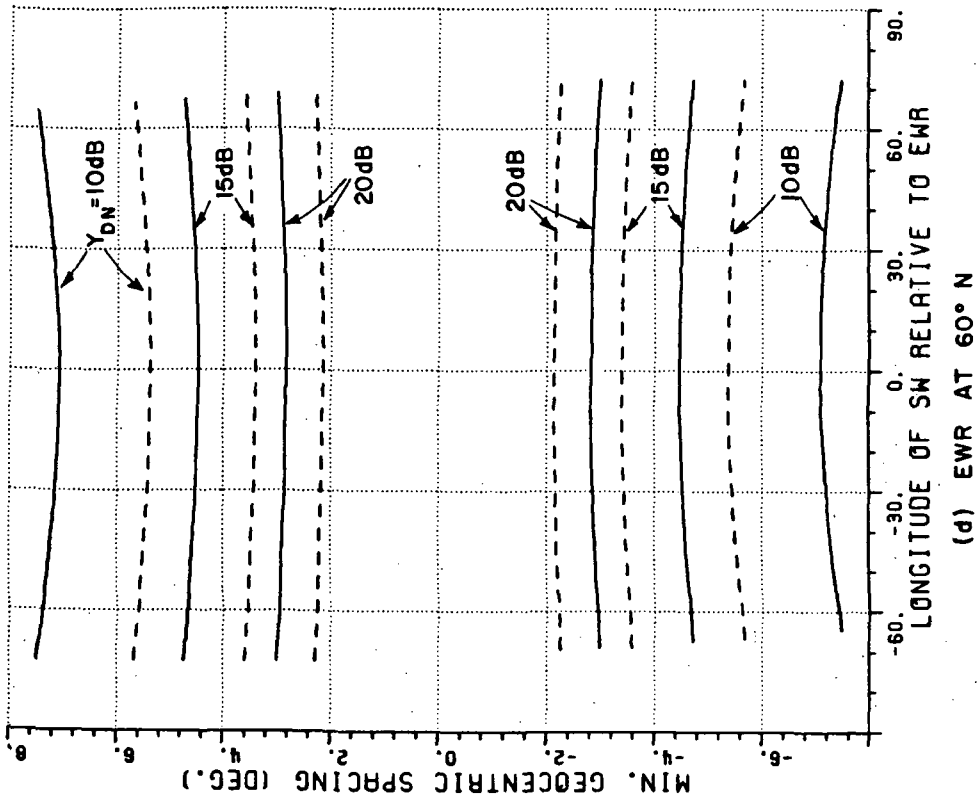
Figure 2.15 (Continued).

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CHAPTER III

UP-LINK INTERFERENCE

The single-entry C/I ratio on the up links will now be derived in a
manner similar to that used for the down links. The interference
geometry between up-link networks is shown in Figures 3.1 and 3.2 with
the geometric parameters of interest. The wanted signal is received
from earth station EWT, while the interference signal arrives from earth
station EIT. The calculation of the C/I ratio is performed at satellite
SW. Note that, since there are a multiple number of earth stations
transmitting to a satellite on the up link, it is not possible for its
satellite antenna to be pointed directly at all the earth station
antennas, i.e., the receiving antenna of SW is not necessarily pointed
directly at the transmitting antenna of EWT. However, the EWT and EIT
antennas are assumed to be pointed directly at their respective
satellites as required by good system design.

The wanted carrier power C_{SW} and the interference signal power I_{SW}
received at satellite SW are obtained by application of the Friis
transmission formula [3] as

$$C_{SW} = \frac{P_{EWT} G_{EWT} \{G_{SWR} D_{SWR}(\psi_6, G_{SWR})\} \lambda_W^2}{(4\pi b)^2} \tag{3.1}$$

and

$$I_{SW} = \frac{P_{EIT} \{G_{EIT} D_{EIT}(\psi_4, G_{EIT})\} \{G_{SWR} D_{SWR}(\psi_1, G_{SWR})\} \lambda_I^2}{(4\pi d)^2} , \tag{3.2}$$

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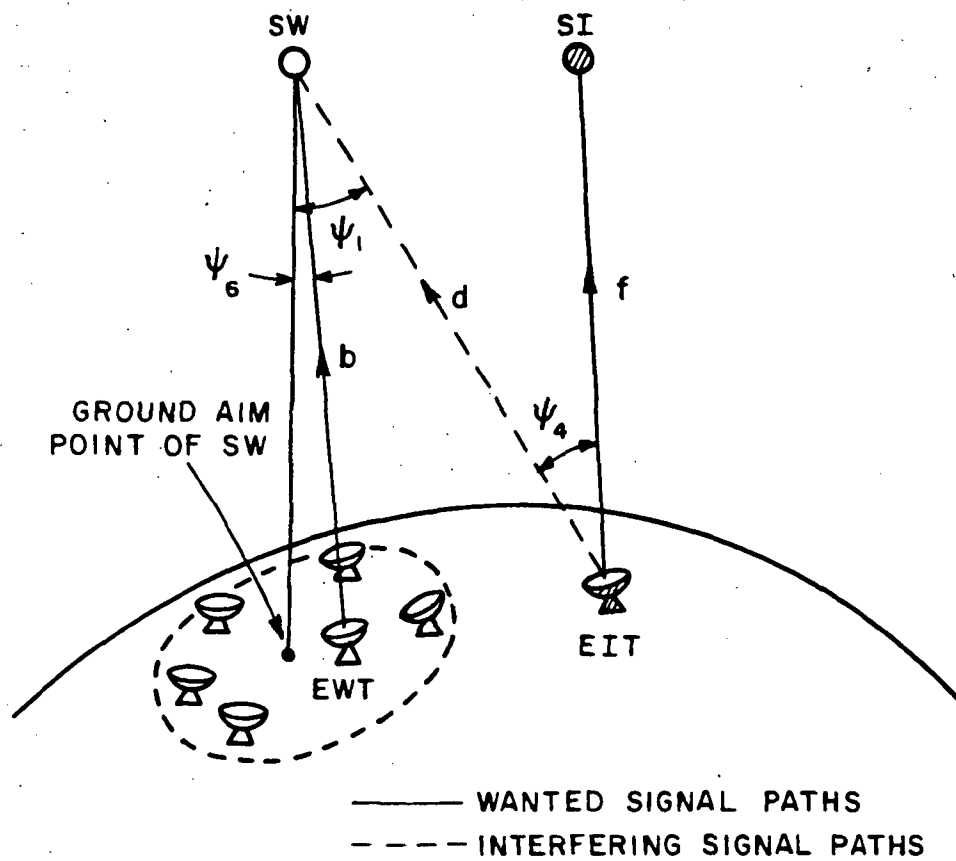
Figure 3.1 Interference geometry between up-link networks.
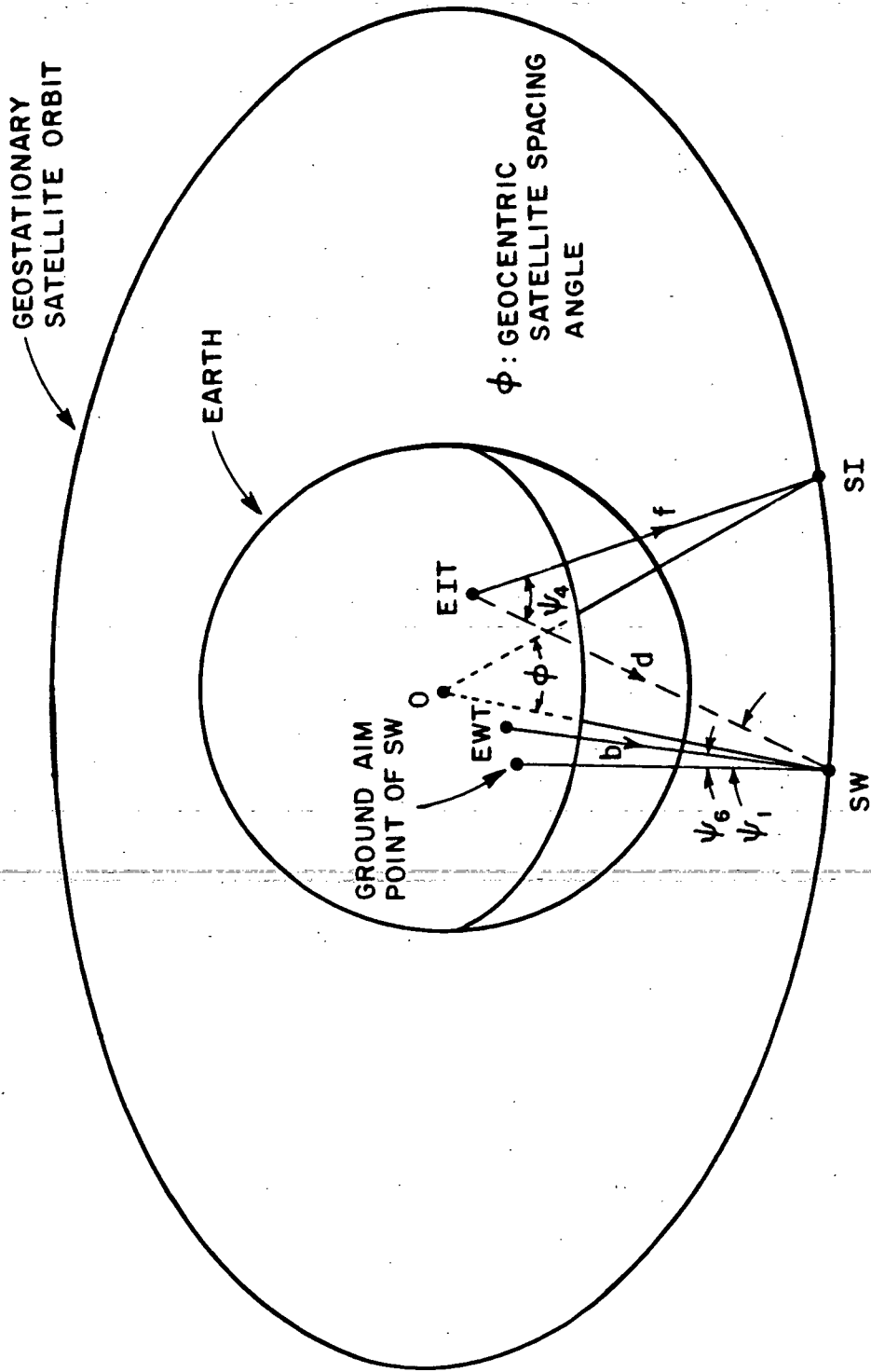
Figure 3.2 Alternative presentation of interference geometry between up-link networks (see Figure 3.1). Earth radius exaggerated for clarity.

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where the symbols, notation, and units adhere to the same convention as in the down-link case.

The satellite antenna reference pattern of Figure 2.3 is used to determine the value of $D_{SWR}(\psi_1, G_{SWR})$ with appropriate parameter changes: ψ_1 and G_{SWR} on the up link correspond to ψ_2 and G_{SIT} on the down link, respectively. Similarly, the earth station antenna reference pattern of Figure 2.4 is used to determine the value of $D_{EIT}(\psi_4, G_{EIT})$, with ψ_4 and G_{EIT} on the up link corresponding to ψ_3 and G_{EWR} on the down link, respectively.

Dividing Equation (3.1) by Equation (3.2) gives the C/I ratio at satellite SW as

$$(C/I)_{SW} = \frac{P_{EWT} G_{EWT}}{P_{EIT} G_{EIT}} \frac{D_{SWR}(\psi_6, G_{SWR})}{D_{SWR}(\psi_1, G_{SWR}) D_{EIT}(\psi_4, G_{EIT})} \frac{d^2}{b^2} . \qquad (3.3)$$

Although the factor c^2/a^2 was eliminated from the down-link equation by assuming $c^2/a^2 = 1$, the corresponding factor d^2/b^2 cannot be dropped from Equation (3.3) because even for small values of ψ_1 the difference between the distances b and d is not necessarily negligible, and the assumption of $d^2/b^2 = 1$ is not appropriate. Instead, since the quantity d^2/f^2 is in the range of 1 ± 0.028 for satellite spacing angles of 5° or less, the factor d^2 can be replaced by f^2, and then Equation (3.3) is rewritten as

$$(C/I)_{SW} = \frac{P_{SW}}{P_{SI}} \frac{D_{SWR}(\psi_6, G_{SWR})}{D_{SWR}(\psi_1, G_{SWR}) D_{EIT}(\psi_4, G_{EIT})} , \qquad (3.4)$$

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where

$$P_{SW} = \frac{P_{EWT}G_{EWT}}{4\pi b^2} \qquad (3.5)$$

is the power density at SW due to EWT, and

$$P_{SI} = \frac{P_{EIT}G_{EIT}}{4\pi f^2} \qquad (3.6)$$

is the power density at SI due to EIT. Defining a universal system parameter on the up links as

$$R_{UP} = (C/I)_{SW} \frac{P_{SI}}{P_{SW}} \frac{1}{D_{SWR}(\psi_6, G_{SWR})} \qquad (3.7)$$

simplifies Equation (3.4) to

$$R_{UP} = \frac{1}{D_{SWR}(\psi_1, G_{SWR}) D_{EIT}(\psi_4, G_{EIT})} \qquad . \qquad (3.8)$$

As in the down-link case, $D_{SWR}(\psi_6, G_{SWR})$ in Equation (3.7) can be estimated between 0 and -3 dB, depending on how closely EWT is located to the SW ground aim point.

A few words may be in order with regard to the universal system parameter R_{UP}. On the up link, both the wanted and the interfering signals, which are transmitted by earth stations located at different points on the Earth, will fluctuate due to local precipitation but will not, in general, be attenuated to the same degree. Thus, attenuation factors which take the local precipitation effect into account should be introduced in calculating the C/I ratio for the up link. Such factors

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are to be included in the power densities P_{SW} and P_{SI} (although not included in Equations (3.5) and (3.6)). The universal system parameter R_{UP}, which is represented in terms of the power densities P_{SW} and P_{SI}, also includes the local precipitation effect. The relationship between rain rate and attenuation is discussed in Appendix F. On the down link, both the wanted and the interfering signals travel through approximately the same disturbed region. Therefore, their respective attenuations are likely to be correlated, and the local precipitation effect may usually be neglected.

Compare the up link-equation and the down-link equation shown below,

$$R_{UP} = -D_{SWR}(\psi_1, G_{SWR}) - D_{EIT}(\psi_4, G_{EIT}) \text{ dB} \tag{3.9}$$

and

$$R_{DN} = -D_{SIT}(\psi_2, G_{EIT}) - D_{EWR}(\psi_3, G_{EWR}) \text{ dB} . \tag{3.10}$$

It is evident that these two equations are of exactly the same form. This implies that the up-link and the down-link problems can be treated as dual problems, and that the contour curves produced for the down link can also be used for the up link by simply changing the down-link parameters R_{DN}, ψ_2, ψ_3, G_{SIT}, and G_{EWR} to the up-link parameters R_{UP}, ψ_1, ψ_4, G_{SWR}, and G_{EIT}, respectively. Thus, all the plots generated for the down link in Chapter II can also apply to the up link with the appropriate parameter changes.

The universal curves for the minimum allowable satellite spacing based on up-link considerations are shown in Figures 3.3 and 3.4. The values of ψ_4 obtained from Figures 3.3 and 3.4 must satisfy

$$\psi_4 > 26.3 \; G_{EIT}^{-0.3} \quad \text{degrees} \tag{3.11}$$

and

$$\psi_4 < 20 \sqrt{5.35 + 5 \log_{10}(d/\lambda)} \; / \; (d/\lambda) \quad \text{degrees,} \tag{3.12}$$

respectively. For convenience, the dashed lines in the figures indicate where to switch from one to the other in the particular case of $G_{EIT} = 50$ dB and 60 dB. Figure 3.3 is valid above the dashed lines for the given values of G_{EIT}, and Figure 3.4 is valid below the dashed lines.

The following observation can be made regarding the up link. When the transmitting earth station EIT is located at the ground aim point of the SW receiving antenna, i.e., $\psi_1 = 0°$, the worst satellite spacing results. Separating EIT from the ground aim point of SW, that is, increasing the off-axis angle ψ_1, reduces the required satellite spacing. The restriction on the transmitting earth station locations, i.e., the service area assignments on the up link, turns out to be significant in allowing closer satellite spacing.
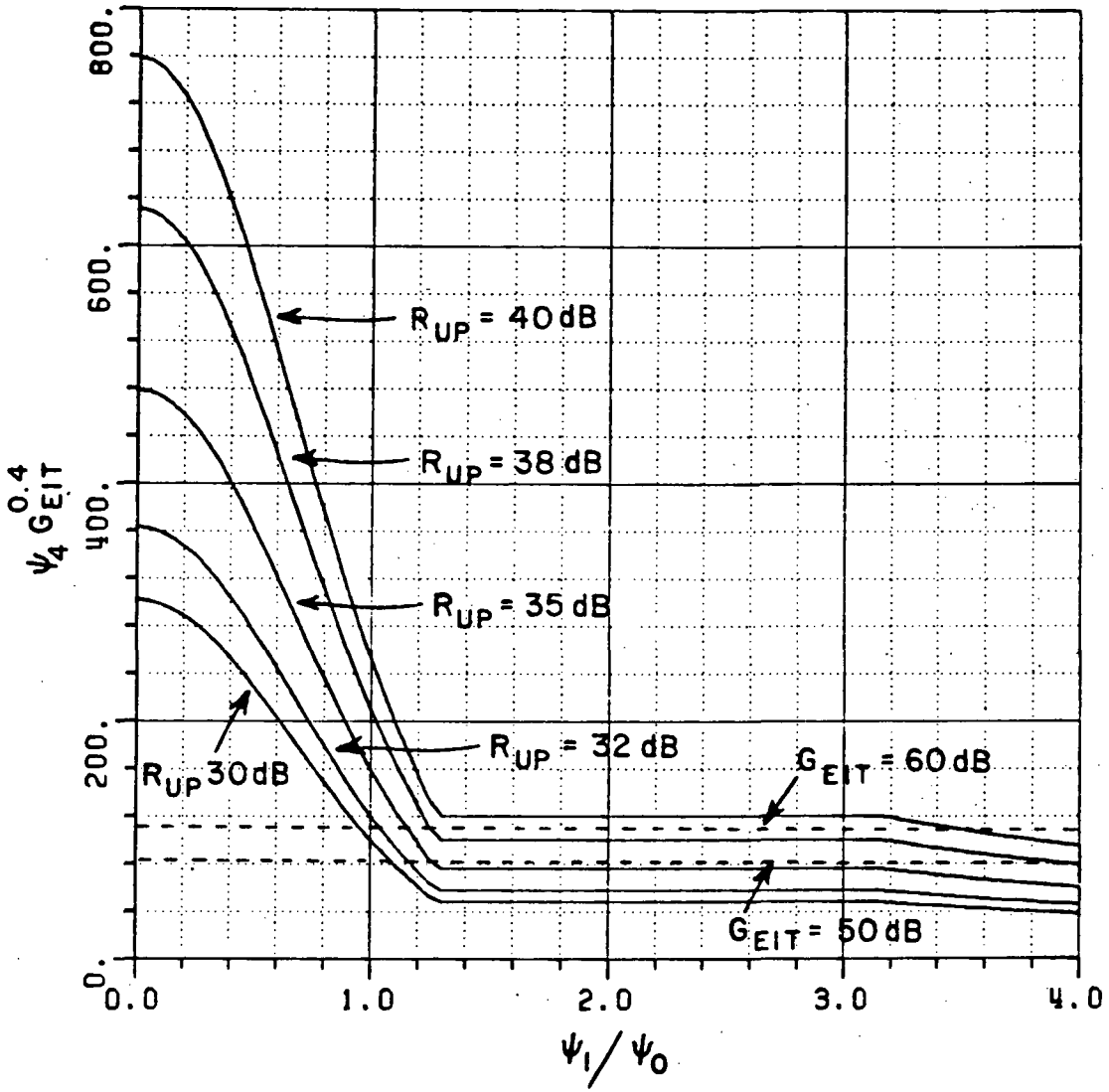
Figure 3.3   First part of universal curves for minimum allowable
satellite spacing angle ψ_4 as function of normalized
off-axis angle ψ_1/ψ_0, based on up-link considerations.
ψ_4 obtained from this figure must satisfy Equation (3.11).
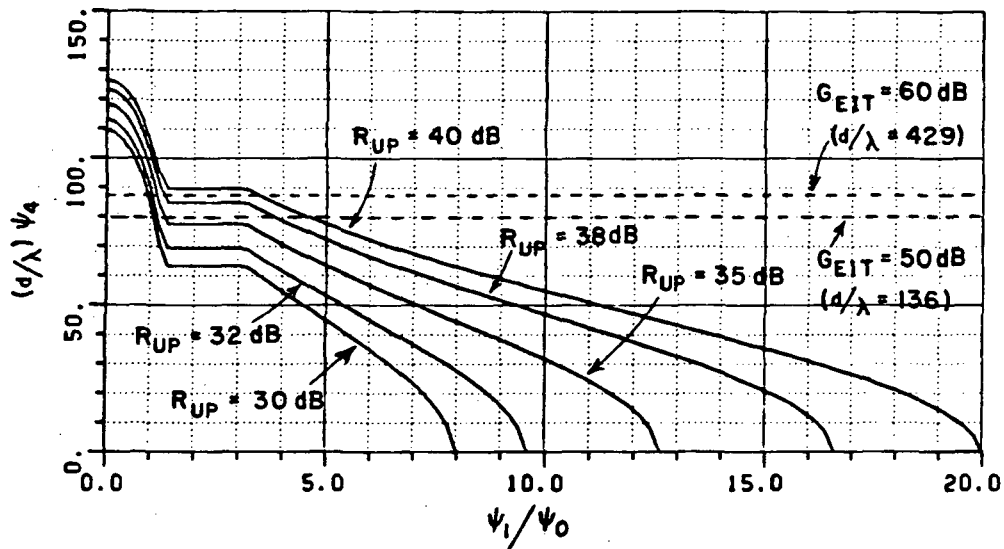
Figure 3.4   Second part of universal curves for minimum allowable
satellite spacing angle ψ_4 as function of normalized
off-axis angle ψ_1/ψ_0, based on up-link considerations.
ψ_4 obtained from this figure must satisfy Equation (3.12).

CHAPTER IV

CONCLUSION

This study deals with the minimum allowable satellite spacing for
FSS systems which attains an acceptable single-entry C/I ratio,
using co-polarized circular antenna beams conforming to Recommendations
and Reports of the CCIR.

Universal curves for the minimum allowable satellite spacings are
constructed, in which the natural coordinates are normalized off-axis
angles, defined as the angle between a main beam axis and an
interference direction, seen at the antenna. In the down-link case, the
independent (input) off-axis angle is that of the closest wanted earth
station, seen at the interfering satellite; in the up-link case, it is
that of the closest interfering earth station, seen at the wanted
satellite. The dependent (output) angle in each case is the topocentric
satellite spacing angle; however, the difference between the topocentric
angle and the geocentric angle is small, the topocentric angle is always
greater than the geocentric angle, and hence the topocentric angle used
as the minimum satellite spacing is conservative in this sense. The key
to the universal representation is the combination of the
multiplicative/divisor system parameters into a universal factor, which
can be set equal to the total antenna discrimination required. The
earth station separation then determines the part of this discrimination

supplied by a satellite antenna; the remainder must be supplied by an earth station antenna, and this leads to the required satellite separation.

Plots involving earth station longitudes and latitudes directly are also presented for some particular parameter values. When the satellite beams serve very wide areas such as earth coverage, the allowable satellite separation is achieved mainly by the earth station antenna discrimination patterns and is almost independent of the elevation angle. However, the allowable satellite spacing can be decreased substantially by defining service areas for the satellites; thus the assignment of limited service area turns out to be an important concept for the effective use of the geostationary satellite orbit. For longitudinal service area separations, the allowable satellite spacing is decreased most when the satellites are near zenith; for latitudinal service area separations, the improvement is relatively independent of the satellite location except in the case of service areas at high latitudes, which is of little practical importance.

For the case of no service area separation, universal curves are constructed to show the satellite spacing improvement available through the improvement of the earth-station sidelobe level.

In general, the considerations of both down-link and up-link interference are necessary to determine orbital satellite locations for the FSS systems. It is shown that the up-link and down-link calculations are dual, so that the same curves and computer codes can be used when suitable parameter substitutions are made.

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APPENDIX A

```fortran
C     PROGRAM  KAZU.FOR
C       This program computes minimum topocentric satellite spacing
C     angle (PSI3) as a function of off-axis angle (PSI2).
C
          REAL   GAINSI, GAINEW
          REAL   R_GIVEN, R_TEST
          REAL   PSI2, PSI3_COARSE, PSI3_FINE
C
          INTEGER  I, J, K
C
          WRITE(6,10)
  10      FORMAT(//' Enter R in dB.')
          READ(5,*) R_GIVEN
          WRITE(6,20)
  20      FORMAT(//' Enter satellite (SI) antenna gain and'
      $          /' earth station (EWR) antenna gain in dB.')
          READ(5,*) GAINSI, GAINEW
C
          OPEN(UNIT = 7, FILE = 'KAZU.DAT', TYPE = 'NEW')
          DO 100  I = 1, 200
            PSI2 = 0.05 * (I - 1)
C
          DO 200  J = 1, 100
            PSI3_COARSE = 0.1 * J
            R_TEST = -DISC_S(PSI2,GAINSI) - DISC_E(PSI3_COARSE,GAINEW)
            IF(R_TEST .GT. R_GIVEN)  GO TO 30
  200     CONTINUE
          GO TO 100
C
  30      DO 300  K = 1, 101
            PSI3_FINE = PSI3_COARSE - 0.1 + (K - 1) * 0.001
            R_TEST = -DISC_S(PSI2,GAINSI) - DISC_E(PSI3_FINE,GAINEW)
            IF(R_TEST .GE. R_GIVEN)  THEN
                WRITE(6,40) PSI2, PSI3_FINE
                WRITE(7,40) PSI2, PSI3_FINE
  40            FORMAT(2F10.2)
                GO TO 100
            END IF
  300     CONTINUE
  100     CONTINUE
          CLOSE(UNIT = 7)
          STOP
          END
```

```
C
C     Function DISC S calculates relative antenna gain of a satellite
C  in dB from off-axis angle (PSI) and antenna gain (GAIN).
C
      REAL FUNCTION DISC_S(PSI,GAIN)
      HPBW = SQRT(27000. / 10.**(GAIN/10.))
      PSI_HP = PSI / HPBW
      IF(PSI_HP .LE. 1.291)
     $    DISC_S = -12. * PSI_HP**2
      IF(PSI_HP .GT. 1.291 .AND. PSI_HP .LE. 3.1623)
     $    DISC_S = -20.
      IF(PSI_HP .GT. 3.1623)
     $    DISC_S = -7.5 - 25. * LOG10(PSI_HP)
      IF(DISC_S .LE. -GAIN - 10.)
     $    DISC_S = -GAIN - 10.
      RETURN
      END
C
C     Function DISC E calculates relative antenna gain of an earth
C  station in dB from off-axis angle (PSI) and antenna gain (GAIN).
C
      REAL FUNCTION DISC_E(PSI,GAIN)
      D_W = SQRT(10.**(GAIN/10.) / 5.428)
      PSI_M = 20. * SQRT(GAIN - 2. - 15. * LOG10(D_W)) / (D_W)
      PSI_R = 15.85 * D_W**(-0.6)
      IF(PSI .LT. PSI_M)
     $    DISC_E = -2.5 E-3 * (D_W * PSI)**2
      IF(PSI .GE. PSI_M .AND. PSI .LT. PSI_R)
     $    DISC_E = 2. + 15. * LOG10(D_W) - GAIN
      IF(PSI .GE.PSI_R .AND. PSI .LT. 48.)
     $    DISC_E = 32. - 25. * LOG10(PSI) - GAIN
      IF(PSI .GE. 48.)
     $    DISC_E = -GAIN - 10.
      RETURN
      END
```

APPENDIX B

A conversion formula from the topocentric angle ψ_t to the geocentric angle ϕ_g is derived as follows. Referring to Figure B.1,

$$\sin(\phi_g/2) = (\ell/2)/R \quad . \tag{B.1}$$

For small ϕ_g,

$$\phi_g \approx \frac{\ell}{R} = \frac{\sqrt{a^2 + c^2 - 2 a c \cos\psi_t}}{R} \quad . \tag{B.2}$$

Since $a \approx c$ for small ψ_t,

$$\phi_g \approx \frac{a}{R} \sqrt{2 (1-\cos\psi_t)} \approx \frac{c}{R} \sqrt{2 (1-\cos\psi_t)} \quad . \tag{B.3}$$
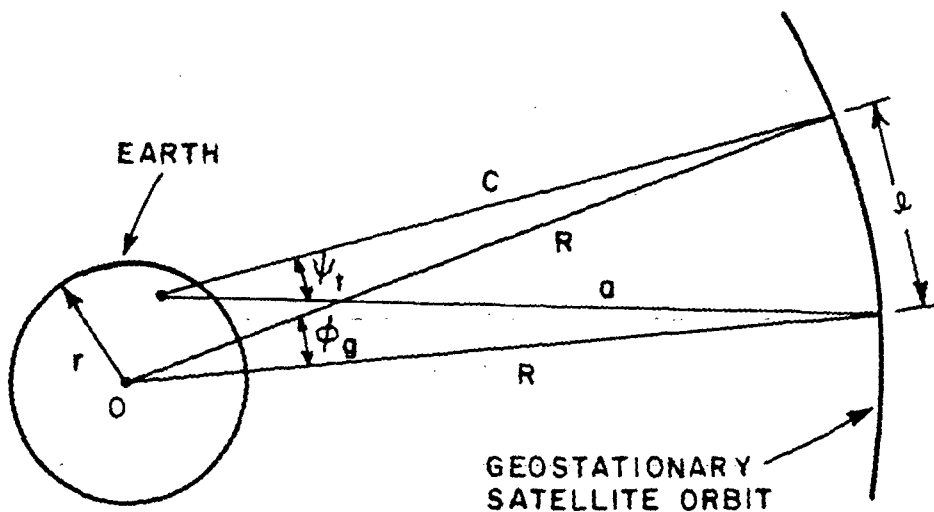
FIGURE B.1 Relationship between topocentric angle and geocentric angle.

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Since $\cos \psi_t \approx 1 - \dfrac{\psi_t^2}{2}$ for small ψ_t,

$$\sqrt{2(1-\cos\psi_t)} \approx \psi_t \, . \qquad\qquad (B.4)$$

Therefore,

$$\phi_g \approx \frac{a}{R}\,\psi_t \approx \frac{c}{R}\,\psi_t \quad , \qquad\qquad (B.5)$$

and

$$\frac{a}{R} \approx \frac{c}{R} \approx \sqrt{1 + \left(\frac{r}{R}\right)^2 - \frac{2r}{R}\cos(\phi_M - \phi_E)\cos\theta_E}$$

$$= \sqrt{1.023 - 0.302\cos(\phi_M - \phi_E)\cos\theta_E} \quad , \qquad\qquad (B.6)$$

where ϕ_M is a midpoint (longitude) between two satellites, and θ_E and ϕ_E are latitude and longitude of an observation point on the Earth, respectively.

APPENDIX C

```
C     PROGRAM  UNIV1.FOR
C       This program computes PSI3*(GAIN**0.4) as a function of PSI2/HPBW.
C
        REAL   R_GIVEN, R_TEST
        REAL   PSI2_HP, ARG_COARSE, ARG_FINE
C
        INTEGER  I, J, K
C
        WRITE(6,10)
 10     FORMAT(//' Enter R in dB.')
        READ(5,*) R_GIVEN
C
        OPEN(UNIT = 7, FILE = 'UNIV1.DAT', TYPE = 'NEW')
C
        DO 100  I = 1, 101
          PSI2_HP = 0.05 * (I - 1)
C
        DO 200  J = 1, 800
          ARG_COARSE = 1. * J
          R_TEST = -DISC_E(ARG_COARSE) - DISC_S(PSI2_HP)
          IF(R_TEST .GT. R_GIVEN)  GO TO 30
 200    CONTINUE
        GO TO 100
C
 30     DO 300  K = 1, 101
          ARG_FINE = ARG_COARSE - 1. + (K - 1) * 0.01
          R_TEST = -DISC_E(ARG_FINE) - DISC_S(PSI2_HP)
          IF(R_TEST .GE. R_GIVEN)  THEN
            WRITE(6,40) PSI2_HP, ARG_FINE
            WRITE(7,40) PSI2_HP, ARG_FINE
 40         FORMAT(2F10.2)
            GO TO 100
          END IF
 300    CONTINUE
 100    CONTINUE
        CLOSE(UNIT = 7)
        STOP
        END
```

```
C
C     Function DISC_S computes relative gain of a satellite antenna
C     in dB from normalized off-axis angle (PSI/HPBW).
C
      REAL FUNCTION DISC_S(PSI_HP)
      IF(PSI_HP .LE. 1.291)
     $    DISC_S = -12. * PSI_HP**2
      IF(PSI_HP .GT. 1.291 .AND. PSI_HP .LE. 3.1623)
     $    DISC_S = -20.
      IF(PSI_HP .GT. 3.1623)
     $    DISC_S = -7.5 - 25. * LOG10(PSI_HP)
      RETURN
      END
C
C     Function DISC_E computes relative gain of second sidelobe of
C     an earth station antenna.
C
      REAL FUNCTION DISC_E(ARG)
C     ARG = PSI3 * (GAIN**0.4)
      DISC_E = 32. - 25. * LOG10(ARG)
      RETURN
      END
```

APPENDIX D

```fortran
C    PROGRAM  UNIV2.FOR
C      This program computes PSI3*(d/lambda) as a function of PS12/HPBW.
C
       REAL   R_GIVEN, R_TEST
       REAL   PSI2_HP, ARG_COARSE, ARG_FINE
C
       INTEGER  I, J, K
C
       WRITE(6,10)
 10    FORMAT(//' Enter R in dB.')
       READ(5,*) R_GIVEN
C
       OPEN(UNIT = 7, FILE = 'UNIV2.DAT', TYPE = 'NEW')
C
       DO 100  I = 1, 101
         PSI2_HP = 0.2 * (I - 1)
C
       DO 200  J = 1, 150
         ARG_COARSE = 1. * J
         R_TEST = -DISC_E(ARG_COARSE) - DISC_S(PSI2_HP)
         IF(R_TEST .GT. R_GIVEN)  GO TO 30
 200   CONTINUE
       GO TO 100
C
 30    DO 300  K = 1, 101
         ARG_FINE = ARG_COARSE - 1. + (K - 1) * 0.01
         R_TEST = -DISC_E(ARG_FINE) - DISC_S(PSI2_HP)
         IF(R_TEST .GE. R_GIVEN)  THEN
           WRITE(6,40) PSI2_HP, ARG_FINE
           WRITE(7,40) PSI2_HP, ARG_FINE
 40        FORMAT(2F10.2)
           GO TO 100
         END IF
 300   CONTINUE
 100   CONTINUE
       CLOSE(UNIT = 7)
       STOP
       END
```

```
C
C     Function DISC_S computes relative gain of a satellite antenna
C     in dB from normalized off-axis angle (PSI/HPBW).
C
      REAL FUNCTION DISC_S(PSI_HP)
      IF(PSI_HP .LE. 1.291)
     $    DISC_S = -12. * PSI_HP**2
      IF(PSI_HP .GT. 1.291 .AND. PSI_HP .LE. 3.1623)
     $    DISC_S = -20.
      IF(PSI_HP .GT. 3.1623)
     $    DISC_S = -7.5 - 25. * LOG10(PSI_HP)
      RETURN
      END
C
C     Function DISC_E computes relative gain of main lobe of an earth
C     station antenna.
C
      REAL FUNCTION DISC_E(ARG)
C     ARG = PSI3 * (d/lambda)
      DISC_E = -2.5 E-3 * (ARG)**2
      RETURN
      END
```

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```
C    PROGRAM  MURA.FOR
C      This program computes minimum geocentric satellite angles
C    when locations of two earth stations are fixed.
C
         REAL  RGSO, RE
         REAL  LONGSW, LONGSI_COARSE, LONGSI_FINE
         REAL  LONGEW, LATIEW, LONGEI, LATIEI
         REAL  GAINSI, GAINEW
         REAL  R_GIVEN, R_TEST
         REAL  L_EWEI, L_SWEW, L_SIEW, L_SIEI, L_SWSI
         REAL  PSI2, PSI3
         REAL  EL_EWSI, EL_EISI
         REAL  RANGE1, RANGE2, SEPA, DELTA
C
         INTEGER  I, J, K, L, M, N
C
         RGSO = 4.2152 E7
         RE = 6.371 E6
C
         WRITE(6,10)
  10     FORMAT(//' Enter earth station (EWR) location in degrees.',
     $          /' (longitude, latitude)')
         WRITE(6,20)
  20     FORMAT('0', ' The longitude value is positive for the eastern',
     $           ' hemisphere', /' and negative for the western',
     $           ' hemisphere.  Likewise,', /' the latitude value is',
     $           ' positive for the northern hemisphere' /' and negative',
     $           ' for the southern hemisphere.')
         READ(5,*) LONGEW, LATIEW
         WRITE(6,30)
  30     FORMAT(//' Enter earth station (EIR) location in degrees.',
     $          /' (longitude, latitude)')
         READ(5,*) LONGEI, LATIEI
         WRITE(6,40)
  40     FORMAT(//' Enter R in dB.')
         READ(5,*) R_GIVEN
         WRITE(6,50)
  50     FORMAT(//' Enter satellite (SI) antenna gain and',
     $          /' earth station (EWR) antenna gain in dB.')
         READ(5,*) GAINSI, GAINEW
```

```
C
        OPEN(UNIT = 7, FILE = 'MURAE.DAT', TYPE = 'NEW')
        OPEN(UNIT = 8, FILE = 'MURAW.DAT', TYPE = 'NEW')
C
C    Satellite (SW) can be seen from earth station (EWR) between
C   (RANGE1) deg. and (RANGE2) deg. of longitude.
C
        RANGE1 = LONGEW - ACOSD(RE/(RGSO*COSD(LATIEW)))
        RANGE2 = LONGEW + ACOSD(RE/(RGSO*COSD(LATIEW)))
        DELTA = (RANGE2 - RANGE1) / 200.
C
        L_EWEI = DIST(RE, LATIEW, LONGEW, RE, LATIEI, LONGEI)
C
C   The case of satellite (SI) being located in the east of
C   satellite (SW).
C
        DO 100  I = 1, 201
          LONGSW = RANGE1 + DELTA * (I - 1)
C
          L_SWEW = DIST(RGSO, 0., LONGSW, RE, LATIEW, LONGEW)
C
        DO 200  J = 1, 20
          LONGSI_COARSE = LONGSW + J * 0.5
C
          L_SIEW = DIST(RGSO, 0., LONGSI_COARSE, RE, LATIEW, LONGEW)
          L_SIEI = DIST(RGSO, 0., LONGSI_COARSE, RE, LATIEI, LONGEI)
          L_SWSI = DIST(RGSO, 0., LONGSW, RGSO, 0., LONGSI_COARSE)
C
          PSI2 = OFF_AXIS(L_SIEW, L_SIEI, L_EWEI)
          PSI3 = OFF_AXIS(L_SWEW, L_SIEW, L_SWSI)
C
          R_TEST = -DISC_S(PSI2,GAINSI) - DISC_E(PSI3,GAINEW)
C
          IF(R_TEST .GT. R_GIVEN)  GO TO 60
200       CONTINUE
          GO TO 100
C
 60     DO 300  K = 1, 101
          LONGSI_FINE = LONGSI_COARSE - 0.5 + (K - 1) * 0.005
C
          L_SIEW = DIST(RGSO, 0., LONGSI_FINE, RE, LATIEW, LONGEW)
          L_SIEI = DIST(RGSO, 0., LONGSI_FINE, RE, LATIEI, LONGEI)
          L_SWSI = DIST(RGSO, 0., LONGSW, RGSO, 0., LONGSI_FINE)
C
          PSI2 = OFF_AXIS(L_SIEW, L_SIEI, L_EWEI)
          PSI3 = OFF_AXIS(L_SWEW, L_SIEW, L_SWSI)
C
          R_TEST = -DISC_S(PSI2,GAINSI) - DISC_E(PSI3,GAINEW)
C
          IF(R_TEST .GE. R_GIVEN)  GO TO 70
300       CONTINUE
```

```
C
 70       EL_EWSI = ELEV(LATIEW, LONGEW, LONGSI_FINE)
          EL_EISI = ELEV(LATIEI, LONGEI, LONGSI_FINE)
C
          IF(EL_EWSI .GE. 0. .AND. EL_EISI .GE. 0.)  THEN
            SEPA = LONGSI_FINE - LONGSW
            WRITE(6,80) LONGSW, SEPA
            WRITE(7,80) LONGSW, SEPA
 80         FORMAT(2F10.2)
          END IF
100       CONTINUE
C
C    The case of satellite (SI) being located in the west of
C    satellite (SW).

          DO 400  L = 1, 201
            LONGSW = RANGE1 + DELTA * (L - 1)
C
            L_SWEW = DIST(RGSO, 0., LONGSW, RE, LATIEW, LONGEW)
C
          DO 500  M = 1, 20
            LONGSI_COARSE = LONGSW - M * 0.5
C
            L_SIEW = DIST(RGSO, 0., LONGSI_COARSE, RE, LATIEW, LONGEW)
            L_SIEI = DIST(RGSO, 0., LONGSI_COARSE, RE, LATIEI, LONGEI)
            L_SWSI = DIST(RGSO, 0., LONGSW, RGSO, 0., LONGSI_COARSE)
C
            PSI2 = OFF_AXIS(L_SIEW, L_SIEI, L_EWEI)
            PSI3 = OFF_AXIS(L_SWEW, L_SIEW, L_SWSI)
C
            R_TEST = -DISC_S(PSI2,GAINSI) - DISC_E(PSI3,GAINEW)
C
            IF(R_TEST .GT. R_GIVEN)  GO TO 90
500       CONTINUE
          GO TO 400
C
 90       DO 600  N = 1, 101
            LONGSI_FINE = LONGSI_COARSE + 0.5 - (N - 1) * 0.005
C
            L_SIEW = DIST(RGSO, 0., LONGSI_FINE, RE, LATIEW, LONGEW)
            L_SIEI = DIST(RGSO, 0., LONGSI_FINE, RE, LATIEI, LONGEI)
            L_SWSI = DIST(RGSO, 0., LONGSW, RGSO, 0., LONGSI_FINE)
C
            PSI2 = OFF_AXIS(L_SIEW, L_SIEI, L_EWEI)
            PSI3 = OFF_AXIS(L_SWEW, L_SIEW, L_SWSI)
C
            R_TEST = -DISC_S(PSI2,GAINSI) - DISC_E(PSI3,GAINEW)
C
            IF(R_TEST .GE. R_GIVEN)  GO TO 95
600       CONTINUE
```

```
C
 95      EL_EWSI = ELEV(LATIEW, LONGEW, LONGSI_FINE)
         EL_EISI = ELEV(LATIEI, LONGEI, LONGSI_FINE)
C
         IF(EL_EWSI .GE. 0. .AND. EL_EISI .GE. 0.)  THEN
           SEPA = LONGSI_FINE - LONGSW
           WRITE(6,80) LONGSW, SEPA
           WRITE(8,80) LONGSW, SEPA
         END IF
400      CONTINUE
         CLOSE(UNIT = 7)
         CLOSE(UNIT = 8)
         STOP
         END
C
C    Function DISC_S computes relative gain of a satellite antenna
C    in dB from off-axis angle (PSI) and antenna gain (GAIN).
C
         REAL FUNCTION DISC_S(PSI,GAIN)
         HPBW = SQRT(27000. / 10.**(GAIN/10.))
         PSI_HP = PSI / HPBW
         IF(PSI_HP .LE. 1.291)
     $      DISC_S = -12. * PSI_HP**2
         IF(PSI_HP .GT. 1.291 .AND. PSI_HP .LE. 3.1623)
     $      DISC_S = -20.
         IF(PSI_HP .GT. 3.1623)
     $      DISC_S = -7.5 - 25. * LOG10(PSI_HP)
         IF(DISC_S .LE. -GAIN - 10.)
     $      DISC_S = -GAIN - 10.
         RETURN
         END
C
C    Function DISC_E computes relative gain of an earth station
C    antenna in dB from off-axis angle (PSI) and antenna gain (GAIN).
C
         REAL FUNCTION DISC_E(PSI,GAIN)
         D_W = SQRT(10.**(GAIN/10.) / 5.428)
         PSI_M = 20. * SQRT(GAIN - 2. - 15. * LOG10(D_W)) / D_W
         PSI_R = 15.85 * D_W**(-0.6)
         IF(PSI .LT. PSI_M)
     $     DISC_E = -2.5 E-3 * (D_W * PSI)**2
         IF(PSI .GE. PSI_M .AND. PSI .LT. PSI_R)
     $     DISC_E = 2. + 15. * LOG10(D_W) - GAIN
         IF(PSI .GE. PSI_R .AND. PSI .LT. 48.)
     $     DISC_E = 32. - 25. * LOG10(PSI) - GAIN
         IF(PSI .GE. 48.)
     $     DISC_E = -GAIN - 10.
         RETURN
         END
```

```fortran
C
C     Function DIST computes distance between two points
C     represented in the cylindrical coordinate system.
C     (radius, latitude, longitude)
C
      REAL FUNCTION DIST(R1, THETA1, PHI1, R2, THETA2, PHI2)
      X1 = R1 * COSD(THETA1) * COSD(PHI1)
      Y1 = R1 * COSD(THETA1) * SIND(PHI1)
      Z1 = R1 * SIND(THETA1)
      X2 = R2 * COSD(THETA2) * COSD(PHI2)
      Y2 = R2 * COSD(THETA2) * SIND(PHI2)
      Z2 = R2 * SIND(THETA2)
      DIST = SQRT((X1-X2)**2 + (Y1-Y2)**2 + (Z1-Z2)**2)
      RETURN
      END

C
C     Function ELEV computes elevation angle of a satellite (PHI_S)
C     seen from an earth station (THETA_E, PHI_E).
C
      REAL FUNCTION ELEV(THETA_E, PHI_E, PHI_S)
      RGSO = 4.2152 E7
      RE = 6.371 E6
      IF(SQRT(1. - (COSD(THETA_E))**2 * (COSD(PHI_E-PHI_S))**2)
     $    .EQ. 0.)  THEN
        ELEV = 90.
      ELSE
        ELEV = ATAND((COSD(THETA_E) * COSD(PHI_E-PHI_S) - (RE/RGSO))
     $         / (SQRT(1. - (COSD(THETA_E))**2 * (COSD(PHI_E-PHI_S))
     $         **2)))
      END IF
      RETURN
      END

C
C     Function OFF_AXIS computes off-axis angle (PSI).
C
      REAL FUNCTION OFF_AXIS(X, Y, Z)
      ARG_OFF_AXIS = (X**2 + Y**2 - Z**2) / (2. * X * Y)
      IF(ARG_OFF_AXIS .LT. 1. )  THEN
        OFF_AXIS = ACOSD(ARG_OFF_AXIS)
      ELSE
        OFF_AXIS = 0.
      END IF
      RETURN
      END
```

APPENDIX F

The relationship between rain rate R (mm/hr.), as measured at the surface of the Earth, and specific attenuation α (dB/km) can be approximated by the equation [11]

$$\alpha = aR^b \quad dB/km \quad , \tag{F.1}$$

where a and b are frequency- and slightly temperature-dependent constants. Table F.1 presents a listing of the a and b coefficients for several frequencies of interest for satellite communications. Also, values of specific attenuation at each frequency are listed for rain rates of 10, 50, and 100 mm/hr. For example, assuming an effective path length of 4 km through rain, the magnitude of the rain attenuation is 1.2 dB at 6 GHz for a 50 mm/hr. rain and is 2.63 dB at 6 GHz for a 100 mm/hr. rain. Therefore, for regions of bad rain climate, it may be necessary to assume a certain value (dB) of rain attenuation in the wanted signal and none in the interfering signal and to adjust the C/I value used in R_{up} accordingly when calculating the allowable satellite separation for the up links.

TABLE F.1

COEFFICIENTS a AND b OF EQUATION (F.1) FOR CALCULATION

OF RAIN ATTENUATION AT RAIN TEMPERATURE OF 0°C [11]

Frequency, GHz	Coefficient		α, dB/km, for R specified in mm/hr		
	a	b	10	50	100
2	0.000345	0.891	0.003	0.011	0.021
4	.00147	1.016	.015	.078	.158
6	.00371	1.124	.049	.30	.657
12	.0215	1.136	.29	1.83	4.02
15	.0368	1.118	.48	2.92	6.34
20	.0719	1.097	.90	5.25	11.24
30	.186	1.043	2.05	11.0	22.7
40	.362	.972	3.39	16.2	31.8
94	1.402	.744	7.78	25.8	43.1